Report of the Backtesting of American Express Stock Exchange Rate

(a)Data description: I have chosen American Express daily stock data from 1 November 2015 to 30 December 2019 for the analysis. The time points of the first observation is first November 2015 and the last observation is 30 December 2019.

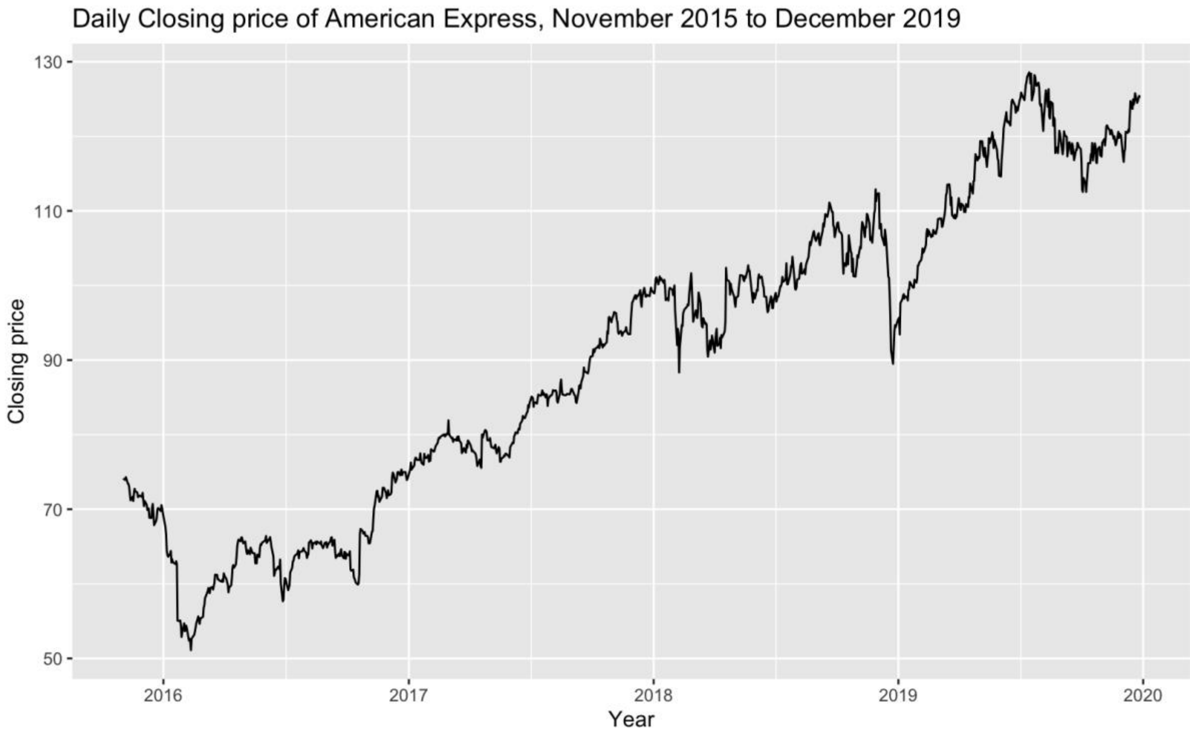

Fig: 1 Daily closing price of American Express

The closing price slightly declined from late 2015 to early 2016, dropping from around $70 to below $60. Again After the decline, the stock price begins to recover steadily throughout 2016. The price continues to rise, reaching approximately $90 by early 2018 and reaching the top in the middle of 2019 at most $130.

(b) Log returns of the selected closing price series:

The log returns of the daily stock price of American Express show that the series is stationary and exhibits volatility clustering.

Stationarity: The log-returns graph shows stationary, as there is no significant change in the mean over time. The log returns fluctuate around a constant mean, indicating stationarity.

Volatility: These graphs exhibit volatility clustering. Notable clusters of high volatility can be seen around 2016, late 2017, and early 2018. These periods show larger spikes and greater fluctuation in the graph, indicating times when the stock price was more volatile.

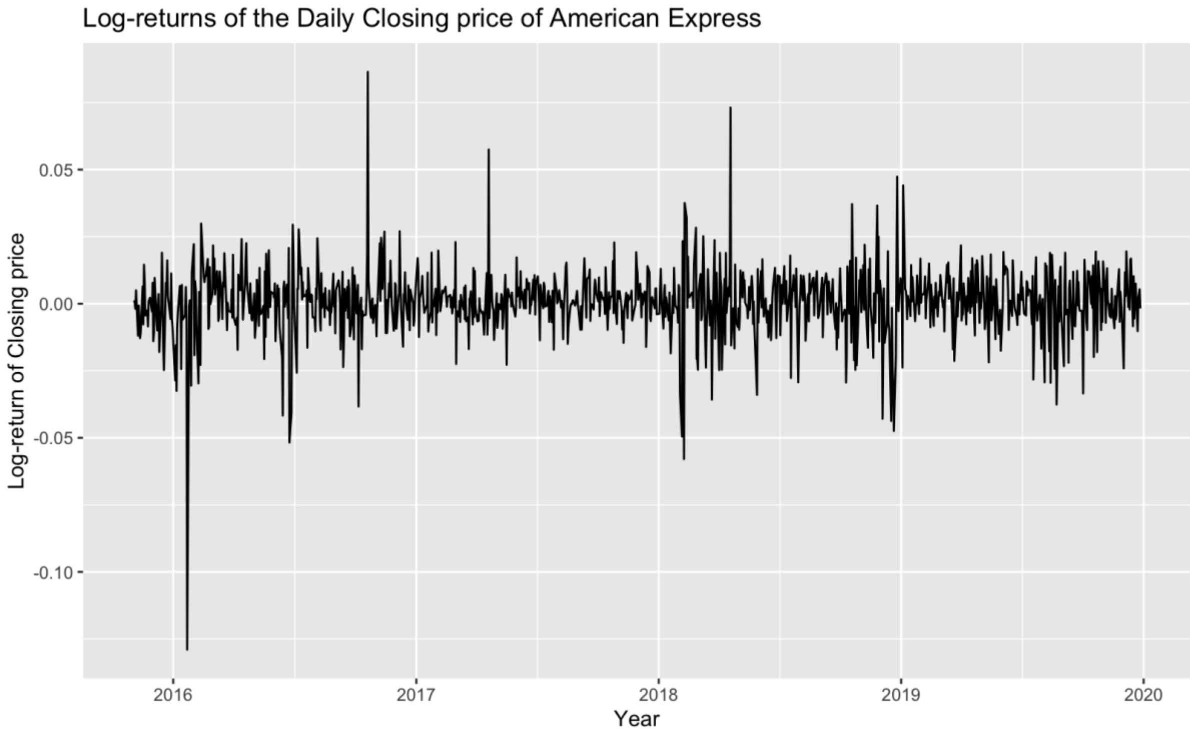

Fig : 2 Log returns of the selected closing price series

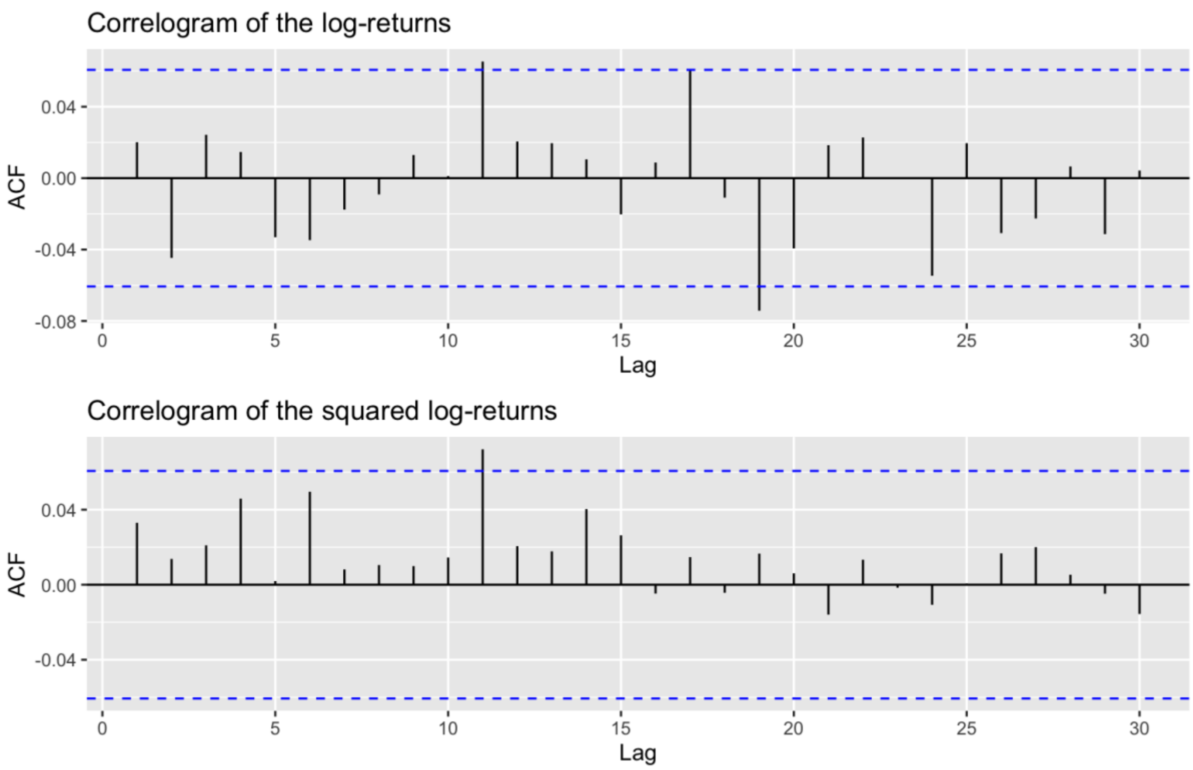

Fig: 3 Correlogram of the Log-Returns and the Squared Log-Returns

Correlogram of the Log-Returns

Most of the autocorrelations are within the 95% confidence bands (blue dashed lines), indicating that the log returns are largely uncorrelated. There are a few significant spikes, notably at lag 10, suggesting some periodic dependencies at these specific lags.

Correlogram of the Squared Log-Returns

Several significant autocorrelations are observed, with notable spikes within the first 10 lags. This indicates that there is significant autocorrelation in the squared log returns, suggesting high volatility and the presence of volatility clustering.

Conclusion: The log-returns series appears stationary as the autocorrelations of the log-returns themselves are largely within the confidence intervals, showing no significant autocorrelation except for a few specific lags. The significant autocorrelation in the squared log returns confirms the presence of volatility clustering. This implies that the volatility of the returns is not random but follows a pattern where high and low volatility periods are clustered together.

(c) The backtesting method result through the traffic light test:

Traffic light test result for GARCH (1,1) model: Red
Traffic light test result for APARCH (1,1) model: Yellow

The results indicate that the APARCH(1,1) model is more reliable than the GARCH(1,1) model for estimating risk in this context. However, neither model achieves a "Green" rating, meaning both have limitations. The traffic light test highlights the importance of selecting an appropriate risk model. In this case, while the APARCH(1,1) model is preferable, further refinement is necessary to achieve a more accurate and reliable risk assessment tool.

(d) The corresponding 99%-VaR and 97.5%-ES values for the test data:

The observed number of violations we see in the graph. This indicates that the 99%-VaR model is underestimating the risk, as the actual losses exceeded the VaR threshold more frequently than expected. This aligns with the "Red" traffic light test result for the GARCH(1,1) model, suggesting a need for improvement or adjustment in the model to more accurately capture the risk.

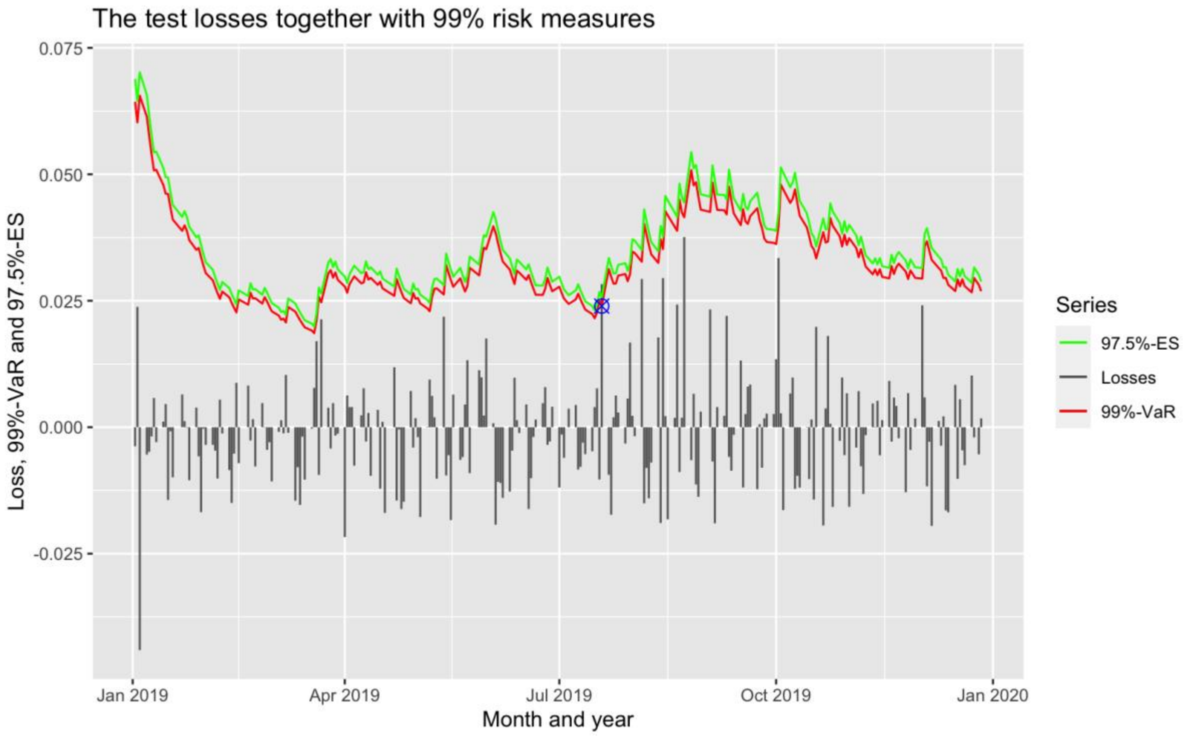

Fig: 4 The corresponding 99%-VaR and 97.5%-ES values

(e)

The fitted GARCHt(1,1) fitted model for the smallest BIC is:

$$Y_t = 1.28 \times 10^{-3} + \epsilon_{t,} \quad \epsilon_{t,} = \sqrt{h_t}\,\eta_t\,,$$

$$h_t = 4.162 \times 10^{-6} + 0.1123\epsilon_{t-1}^2 + 0.8865h_{t-1,}$$

The fitted APARCH(1,1) fitted model for the smallest BIC is:

$$Y_t = 6.779 \times 10^{-4} + \epsilon_{t,} \quad \epsilon_{t,} = \sigma_t \eta_t \quad \eta_t \sim t(3.6571)$$

$$\boldsymbol{\sigma}_t^{0.8234} = 6.9457 \times 10^{-4} + 0.06078(|\epsilon_{t-1}| - 1.\epsilon_{t-1})^{0.8234} + 0.9369\boldsymbol{\sigma}_{t-1}^{0.8234}$$