ECSE 403 - Control

Laboratory 5 - Report

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1) Considering the position as the output and the coefficients from the previous laboratory, we obtain the transfer function: $G(s)=\frac{1200}{s*(s+8.569)}$.

   We obtain the bode plot shown below with a gain margin of infinity and a phase margin of 19.8 deg.

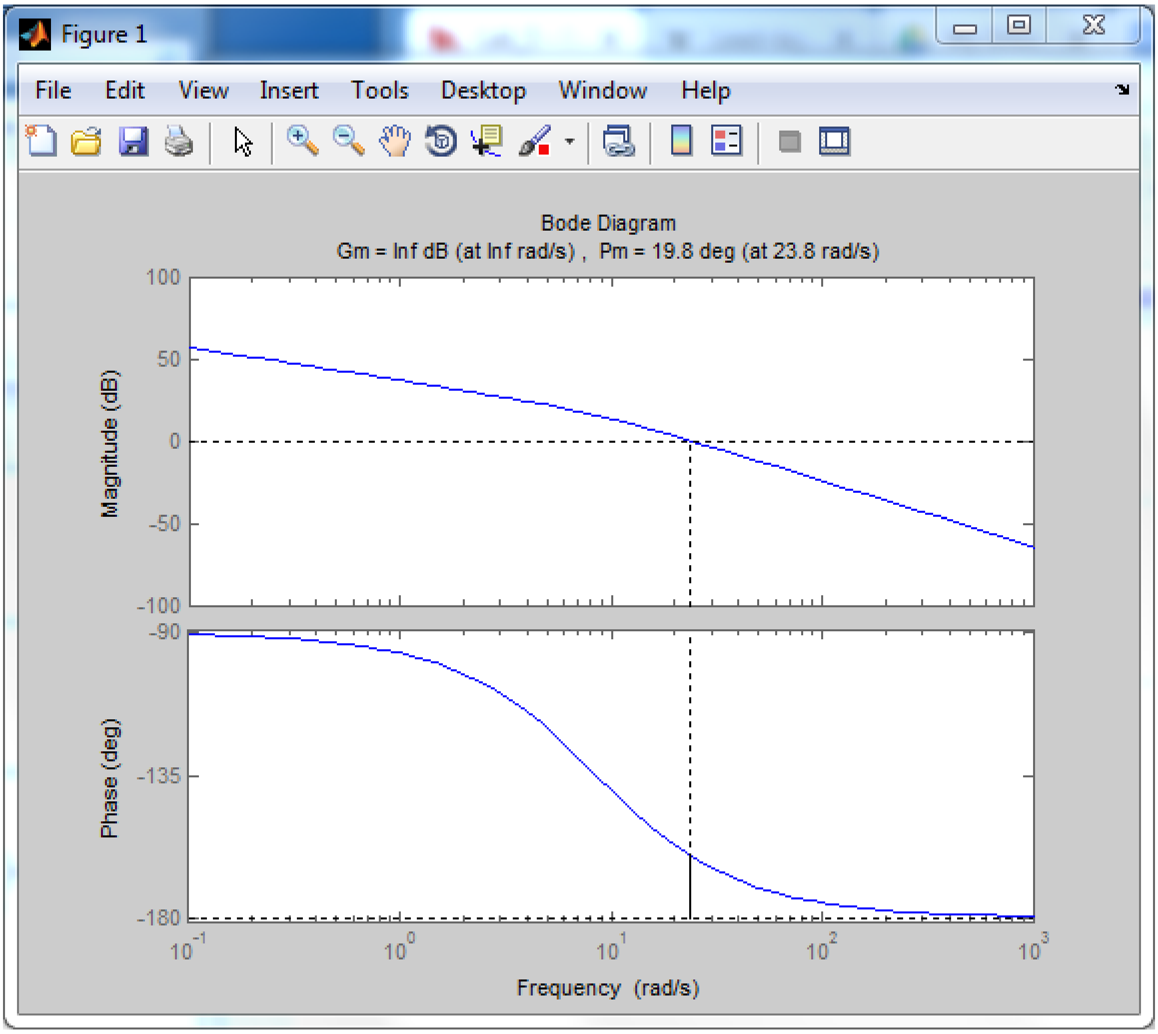

Figure: Bode Diagram of the system

2) We use Kp = 0.45 and we obtain a phase margin of 20.9 deg and a gain margin of infinity.

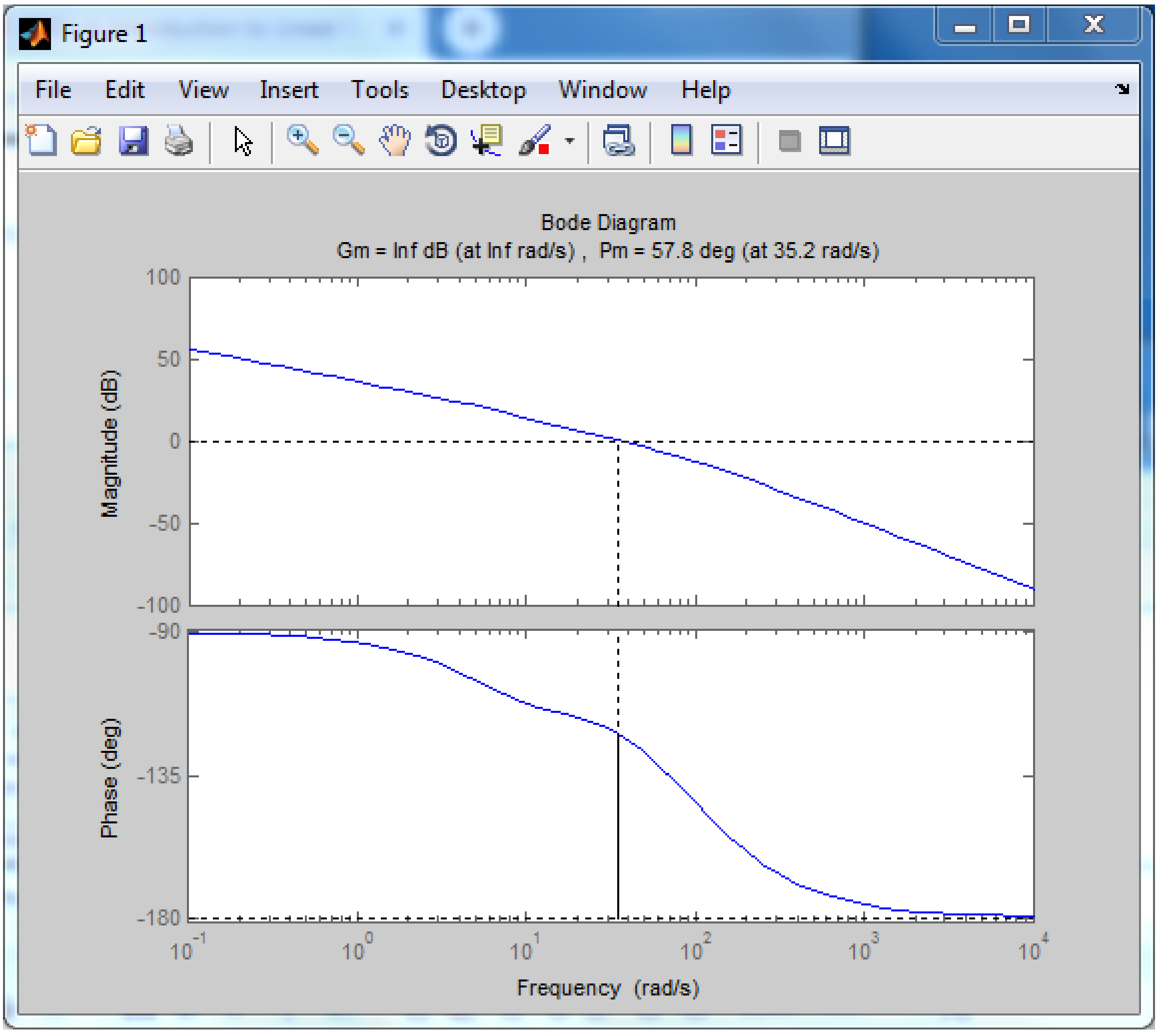

Figure: Open Loope Bode Diagram

a) Done
b) Done
c) The current phase margin of the open-loop system is 20.9 deg
d) Taking a safety margin of 5 degrees, the required phase is 44.1 deg.
e) We have a required phase of 44.1 degrees. Solving the equation:

   $\phi=\sin^{-1}\left(\frac{\alpha-1}{\alpha+1}\right)$. We get $\alpha=5.579$

f) By looking at the Bode plot we find a desired frequency wm of 35.2 rad/sec
g) We get, $T=\frac{1}{wm*\sqrt{\alpha}}$= 0.012.

   Hence, our lead controller is $C_{lead}=\frac{0.067\,s+1}{0.012\,s+1}$

   With this lead compensator, we reach a phase margin of 57.8 deg.

3)

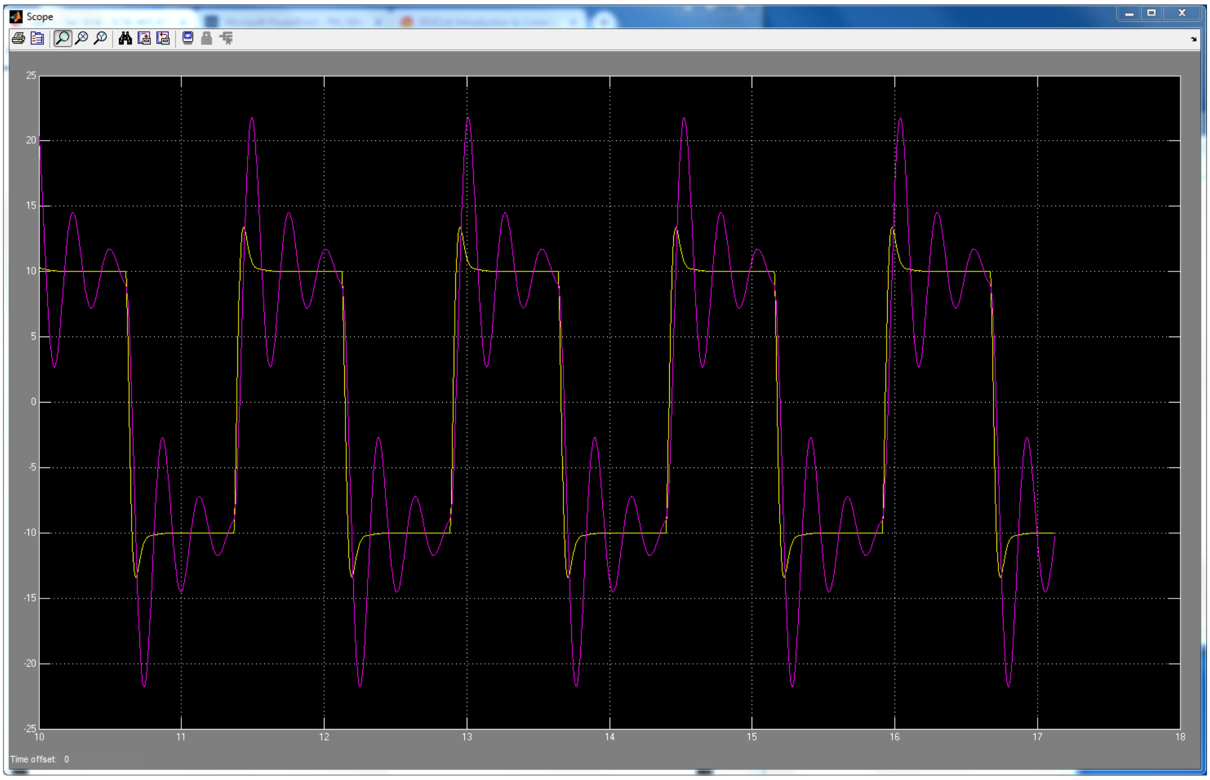

Figure: Step response of proportional feedback controller, with and without the compensator - Model

4)

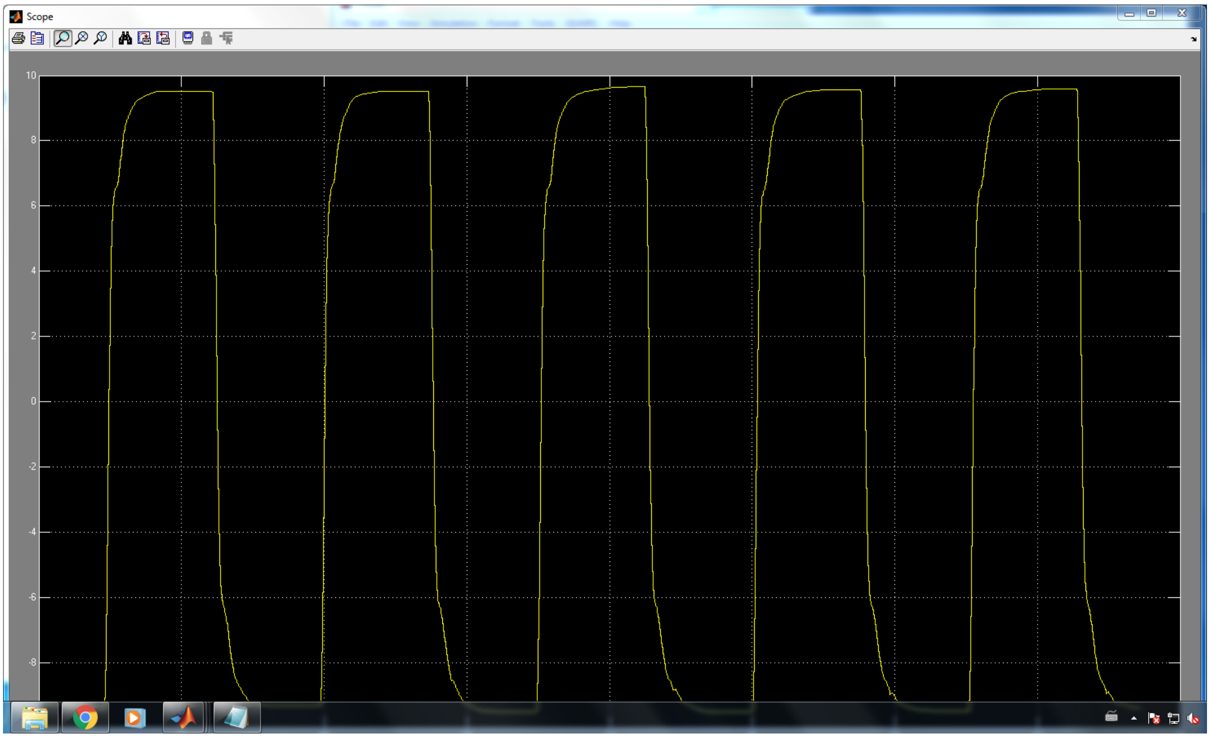

Figure: Step response of the system to proportional feedback controller with Lead compensator - Physical

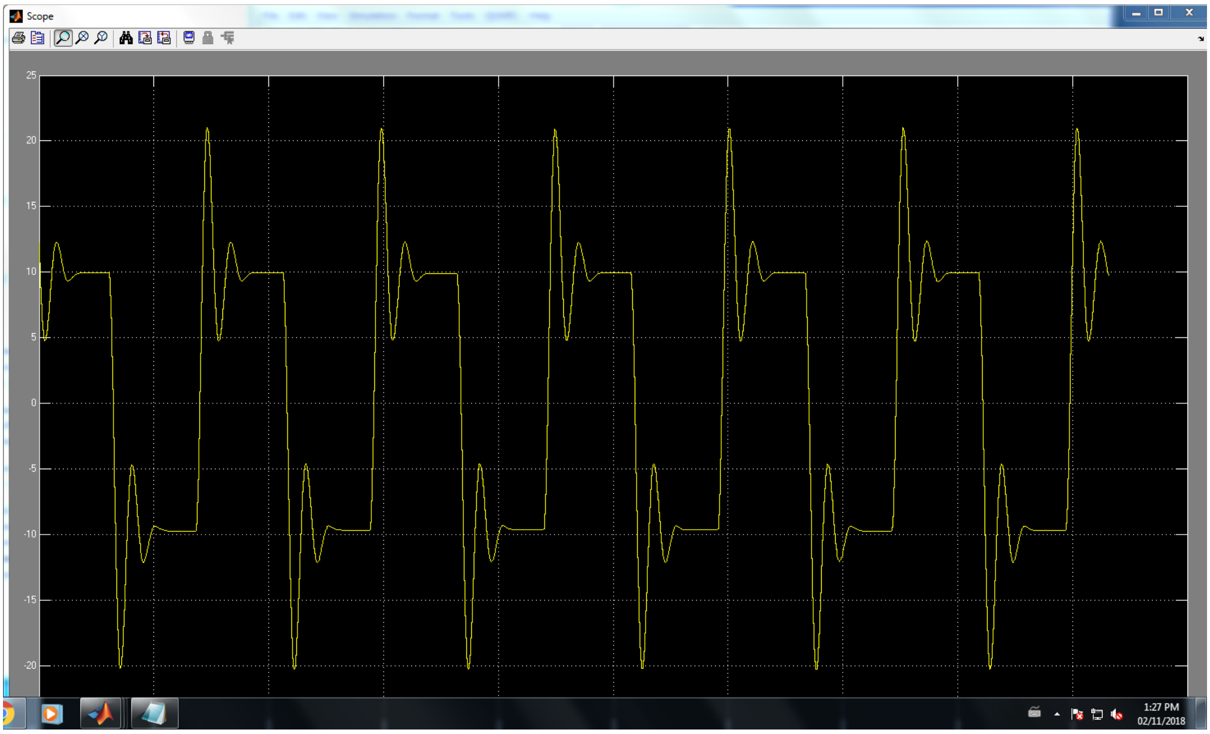

Figure: Step response of the system to proportional feedback controller without Lead compensator - Physical

5) We can see that adding the lead compensator allows the system to get rid of the overshoot and to have a slower rise time by having a bigger Kp. We also have a bigger phase margin which makes the system more stable. We have a steady state error of 2.3%.
The bandwidth of the closed loop system is at 51 rad/s (frequency at which the magnitude goes to -3dB).

6)

a) Done

b) Taking the limit of $s*H(s)*K_{lag}$ as s goes to 0, we find:
$K_v = K_{lag} * 63.02$

c) We had a steady state error of 2.3% using the lead compensator. Hence, we now want a steady state error of 0.23% with the lead-lag compensator as asked. We thus get: $e_{ss} = 0.23\,\%$
We thus obtain $K_{lag} = 6.899$

d) By looking at the bode plot, we find that at w = 6.92 rad/sec, we have the required phase margin of -110 deg.

e) We find using w = 6.92 rad/sec a dB-drop 17.8 dB and we get $\alpha = 7.7625$

f) We finally get T = 1.4451

Hence, our lead lag compensator is given by:

$$C\square_{lag} = K\square_{lag}\frac{Ts+1}{\alpha Ts+1} = 6.898 \times \frac{1.4451\,s+1}{7.7625 \times 1.4451 \times s+1}$$

Plotting the Bode Plot, we obtain a phase margin of 57.9.

7)

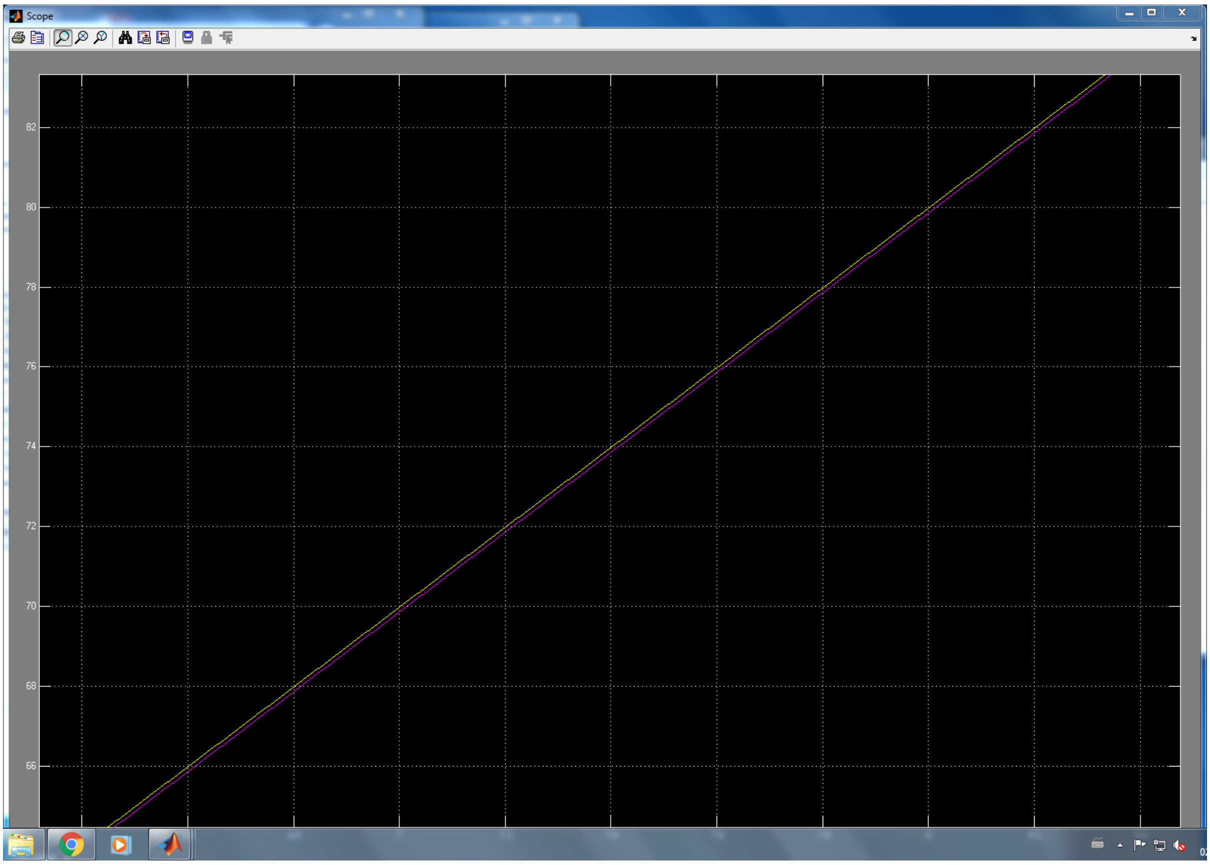

Figure: step response of proportional feedback controller with LEAD and LEAD-LAG compensators

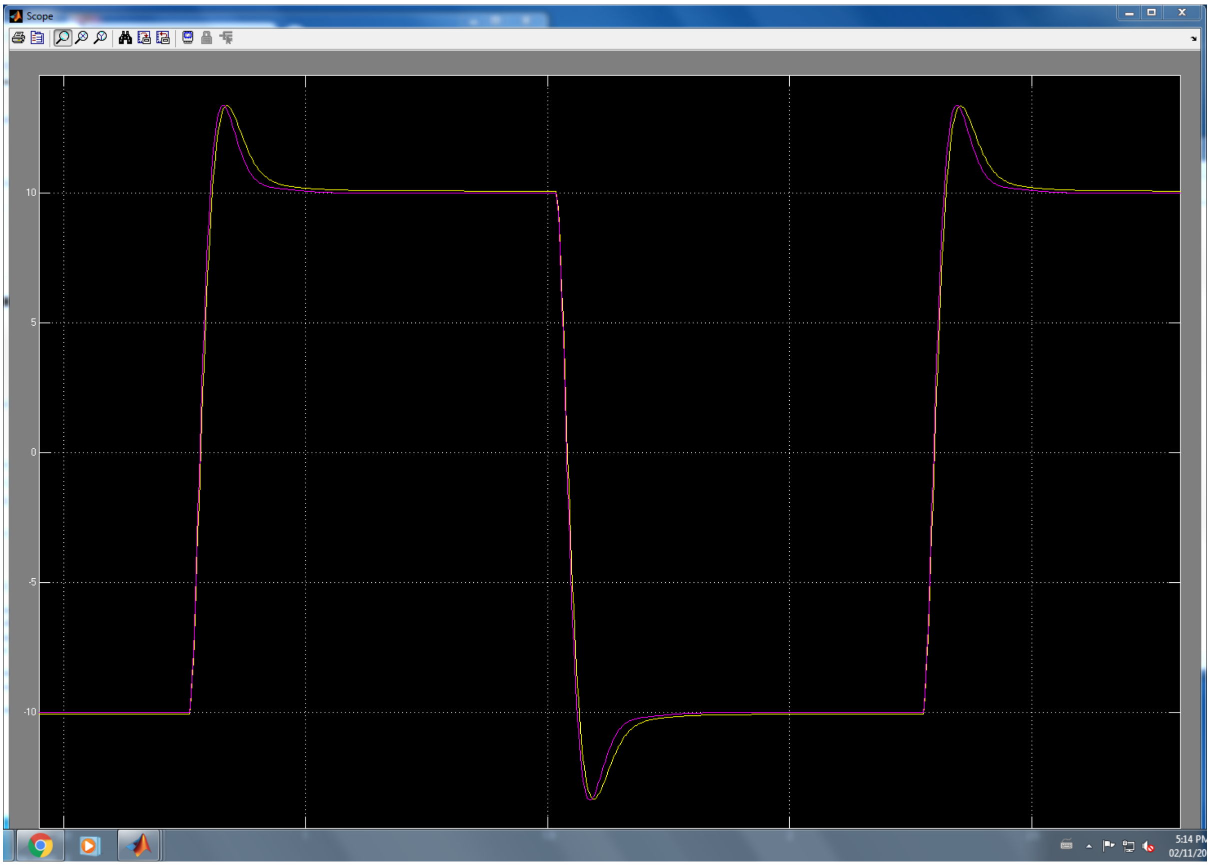

Figure: Ramp Response of proportional feedback controller with LEAD and LEAD-LAG compensators

8)

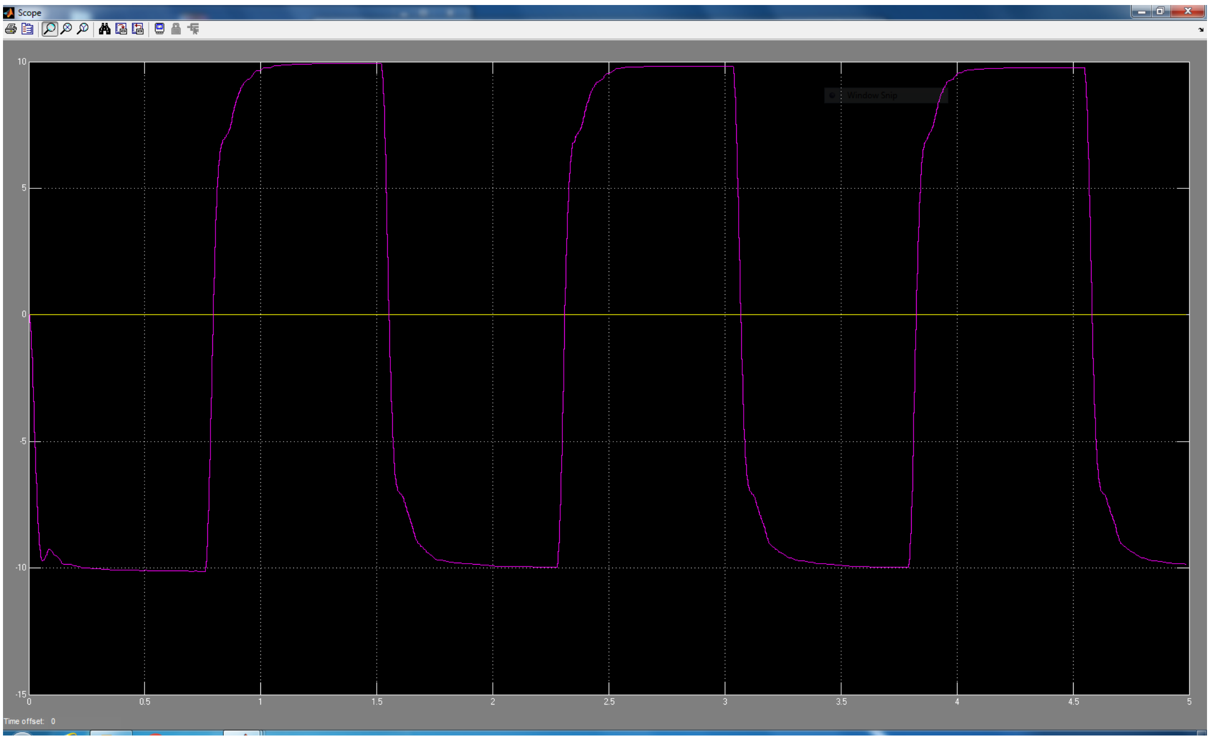

Figure: Step response with Lead-Lag compensators

9) The Lead-Lag has smaller rise but higher settling time than PID. Lead-Lag also has lower steady state error and no overshoot.

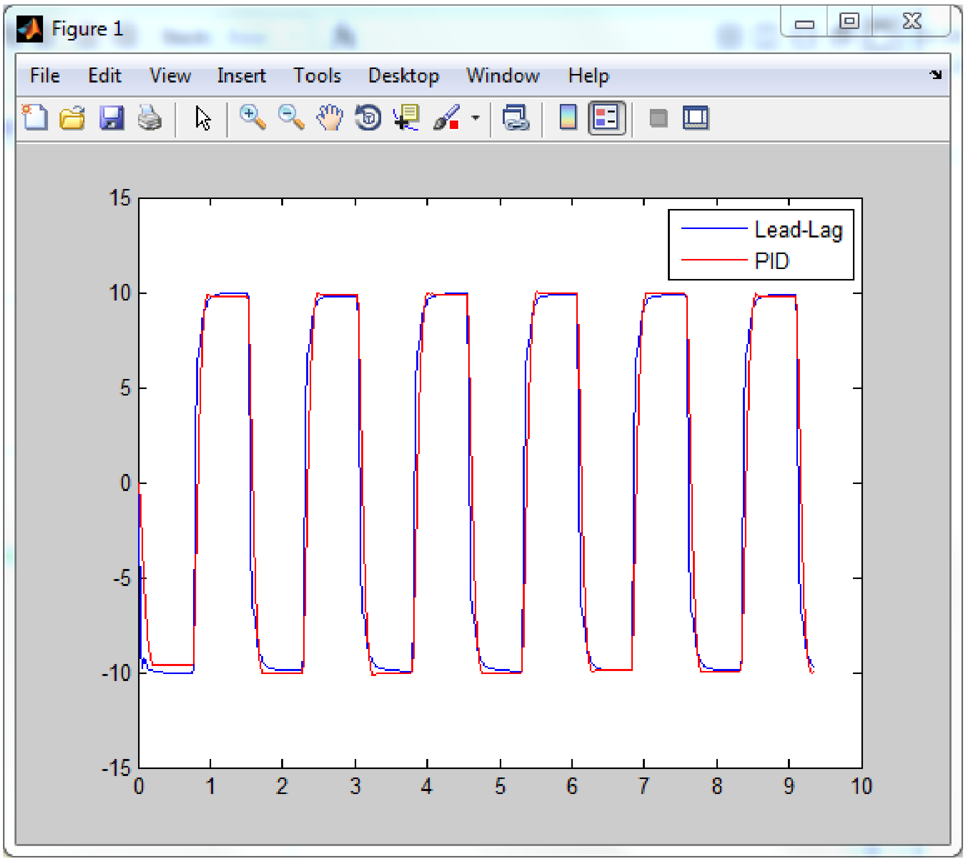

Figure: Lead-Lag compensators vs PID

Appendix

```
%% Q1
s = tf('s');
G = 1200/(s*(s+8.569));


%% Q2
H = 1200*0.45/(s*(s+8.569));
bode(H)
margin(H)

%% Q5
C = (0.067*s+1)/(0.012*s+1);

Z=C*H;
margin(Z);
bode(Z);

%% Q6
X = 6.899*(1.4451*s+1)/(1.4451*7.7625*s+1);
margin(Z*X)
bode(Z*X);

%% Q9
% time = (simout.time);
% H1 = (simout.signals.values);
% H2 = (simout.signals.values); %kp = 0.178, ki = 0.092, kd = 0.0075

% used to find length
% length(H1); length 4722
% length(H2); length 4682
% length(time); length 4682
% H_t = length(H2); %4682
%
% H3 = H1(1:H_t); %extract data of set size H_t
% H4 = H2(1:H_t); %extract data of set size H_t
% t = time(1:H_t);

plot(t,H3,t,H2,'r');
legend('Lead-Lag', 'PID');
```