Lab 7 Report 403

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1)

```
%% Q1
Km=0.0077;
Rm=2.6;
Lm=180;
Vmax=5.0;
Kg=3.7;
rg=0.0064;
mc=0.526;
Kx=2.28*(10^-5);
Kth=0.00153;
mp=0.106*2;
lp=0.168*2;
g=9.81;

b = (Kg*Km)/(rg);

A = [0,1,0,0;
    0,(-b^2)/(mc*Rm),-(mp*g)/(mc),0;
    0,0,0,1;
    0, (b^2)/(mc*lp*Rm),((mc+mp)*g)/(mc*lp),0]

B = [0;
    b/(mc*Rm);
    0;
    -b/(mc*lp*Rm)]

C = eye(4)

D = [0,0,0,0]'
```

Figure1: Code to define the state space system of the cart

```
A =

         0    1.0000         0         0
         0  -14.4899   -3.9538         0
         0         0         0    1.0000
         0   43.1247   40.9638         0


B =

         0
    3.2550
         0
   -9.6876


C =

     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0     1


D =

     0
     0
     0
     0
```

Figure 2: Matrices A, B, C and D

```
%% Q2
sys  = ss(A,B,C,D);
tf_sys = ss2tf(A,B,C,D);

s = tf('s');
G = tf(sys);
```

```
tf_sys =

         0         0    3.2550   -0.0000  -95.0349
         0    3.2550   -0.0000  -95.0349         0
         0         0   -9.6876    0.0000         0
         0   -9.6876    0.0000         0         0


G =

  From input to output...
          3.255 s^2 - 2.891e-15 s - 95.03
   1:  -------------------------------------
       s^4 + 14.49 s^3 - 40.96 s^2 - 423.1 s

              3.255 s^2 - 95.03
   2:  ---------------------------------
       s^3 + 14.49 s^2 - 40.96 s - 423.1

                  -9.688 s
   3:  ---------------------------------
       s^3 + 14.49 s^2 - 40.96 s - 423.1

                 -9.688 s^2
   4:  ---------------------------------
       s^3 + 14.49 s^2 - 40.96 s - 423.1

Continuous-time transfer function.
```

Figure 3: Definition of the transfer function on Matlab

```
%% Q3
pzmap(G);
```

Figure 4: pzmap function for pole placement

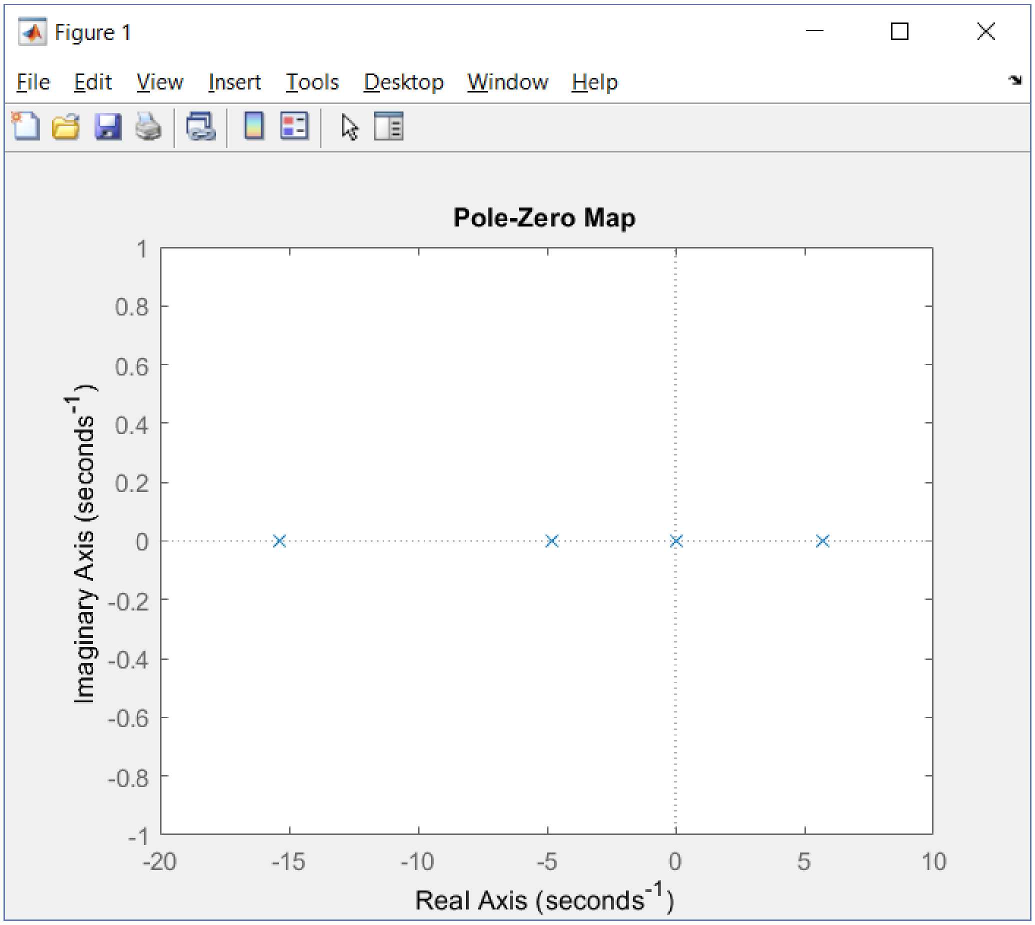

Figure 5: Pole placement

```
%% Q4
eig(A)
```

```
ans =

         0
  -15.3639
   -4.8286
    5.7026
```

Figure 6: Eigenvalues of A

The eigenvalues of the system are equal to the poles of the system.

```
%% Q5
Co = ctrb(A,B);
k = rank(Co);
```

```
Co =

   1.0e+04 *

         0    0.0003   -0.0047    0.0722
    0.0003   -0.0047    0.0722   -1.1013
         0   -0.0010    0.0140   -0.2431
   -0.0010    0.0140   -0.2431    3.6874


k =

     4
```

Figure 7: Controllability matrix

The system is controllable. The controllability has full rank.

We do not need to check for Observability because matrix C is full rank

```
%% Q7
Mp = 8.5;  % ~6 deg
Z = ((log(Mp)^2)/(pi^2+(log(Mp)^2)))^0.5;
Ts = 2;
Wn = 3.9/(Z*Ts);
Pd1 = -(Z*Wn) + 1i* Wn*(1-Z^2)^0.5;
Pd2 = -(Z*Wn) - 1i* Wn*(1-Z^2)^0.5;
```

```
Mp =

   11.5000


Z =

    0.6138


Ts =

    2.5000
```

```
Wn =

    2.5417


Pd1 =

  -1.5600 + 2.0066i


Pd2 =

  -1.5600 - 2.0066i
```

```
%% Q8
Nd1 = 10*Pd1;
Nd2 = 10*Pd2;
```

```
Nd1 =

  -15.6000 +20.0663i


Nd2 =

  -15.6000 -20.0663i
```

```
%% Q9 PP for Con
p = [Pd1, Pd2, Nd1, Nd2];
K = place(A,B,p);
```

```
p =

  -1.5600 + 2.0066i  -1.5600 - 2.0066i -15.6000 +20.0663i -15.6000 -20.0663i


K =

  -43.9140  -27.7812  -96.3840  -11.3814
```

The scope of position, velocity, angle and angle velocity were identical to that of the small stick, shown in *Lab 6 Report*.

2-6)

```
Q=15*[20,0,0,0;
    0,0.666,0,0;
    0,0,15,0;
    0,0,0,0];
R= 0.01;
N= 0;

[K,S,e] = lqr(A,B,Q,R,N);
```

By gradually changing the values of R and Q, and with the help of the Lab Instructor we were able to narrow down on the values. We conducted tests with 20 different values.

```
Q =

  300.0000         0         0         0
         0    9.9900         0         0
         0         0  225.0000         0
         0         0         0         0


R =

    0.0100


N =

     0


K =

 -173.2051  -99.7644 -232.0346  -23.0626


S =

  165.0867   40.3286   39.9456    6.8646
   40.3286   18.1040   15.1170    3.0930
   39.9456   15.1170   22.3361    2.6594
    6.8646    3.0930    2.6594    0.5315
 e =

  -99.6244 + 0.0000i
  -31.6622 + 0.0000i
   -2.6544 + 1.8414i
   -2.6544 - 1.8414i
```

Increasing the value of K will decrease rise time and steady state error of the four quantities - position, velocity, angle, angle velocity - but increase the overshoot.

7)

```
Q =

  150.0000         0         0         0
         0    9.9900         0         0
         0         0  150.0000         0
         0         0         0         0


R =

    0.1000


N =

     0


K =

  -38.7298  -32.8067  -90.8924  -14.9673


S =

  109.8190   34.2149   57.9682   11.8960
   34.2149   18.9486   30.5441    6.7054
   57.9682   30.5441   63.9329   11.2011
   11.8960    6.7054   11.2011    2.4075
```

```
e =

 -33.9073 + 0.0000i
 -13.8529 + 0.0000i
  -2.4701 + 1.3171i
  -2.4701 - 1.3171i
```

8) Matrix Q contains the costs and matrix K has the feedback gains.

System behavior after adding LQR gains, with the longer pendulum.

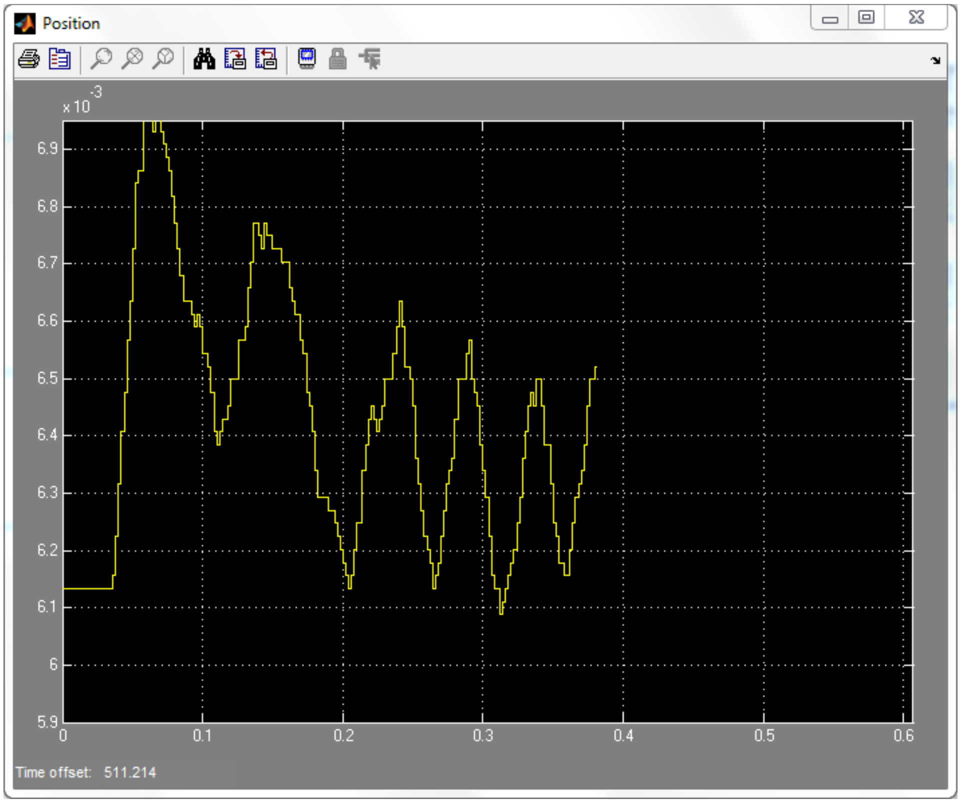

Figure: Position

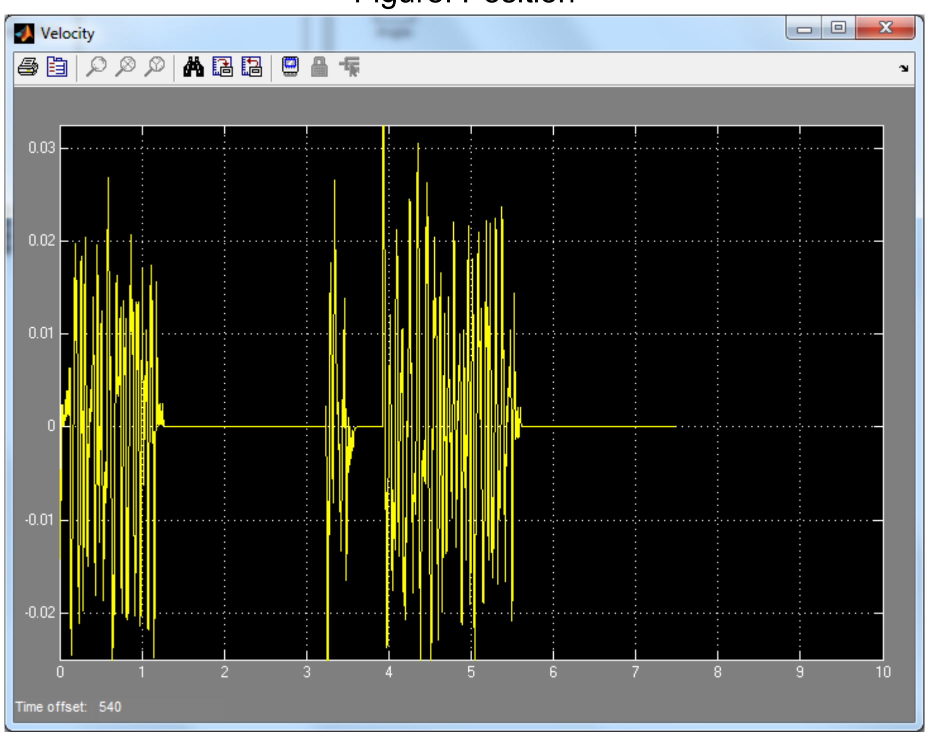

Figure: Velocity

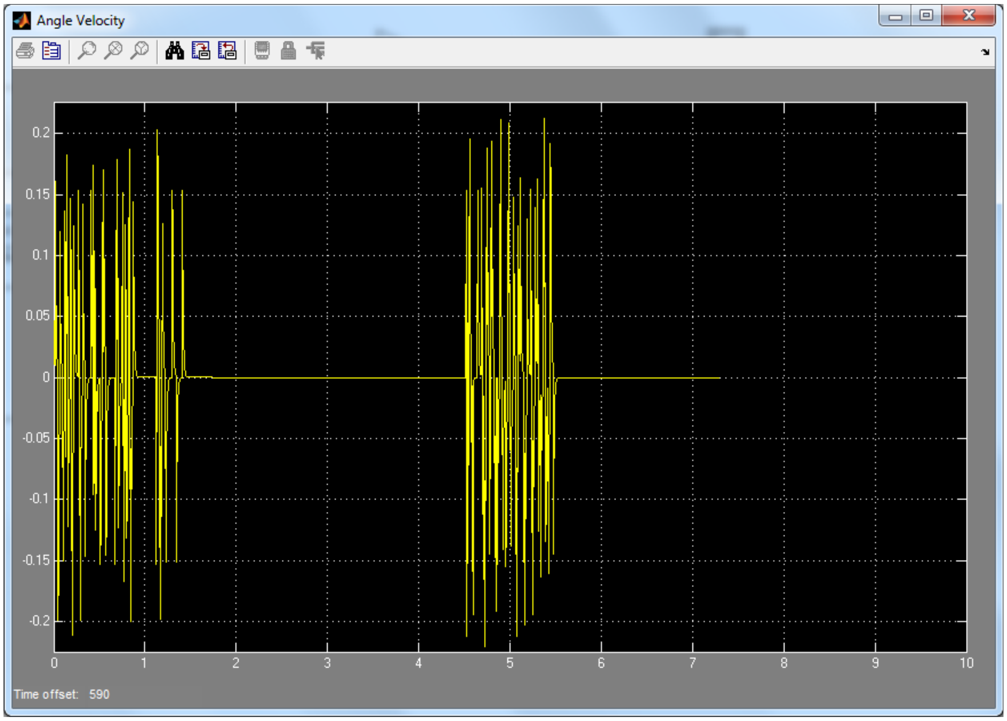

Figure: Angle

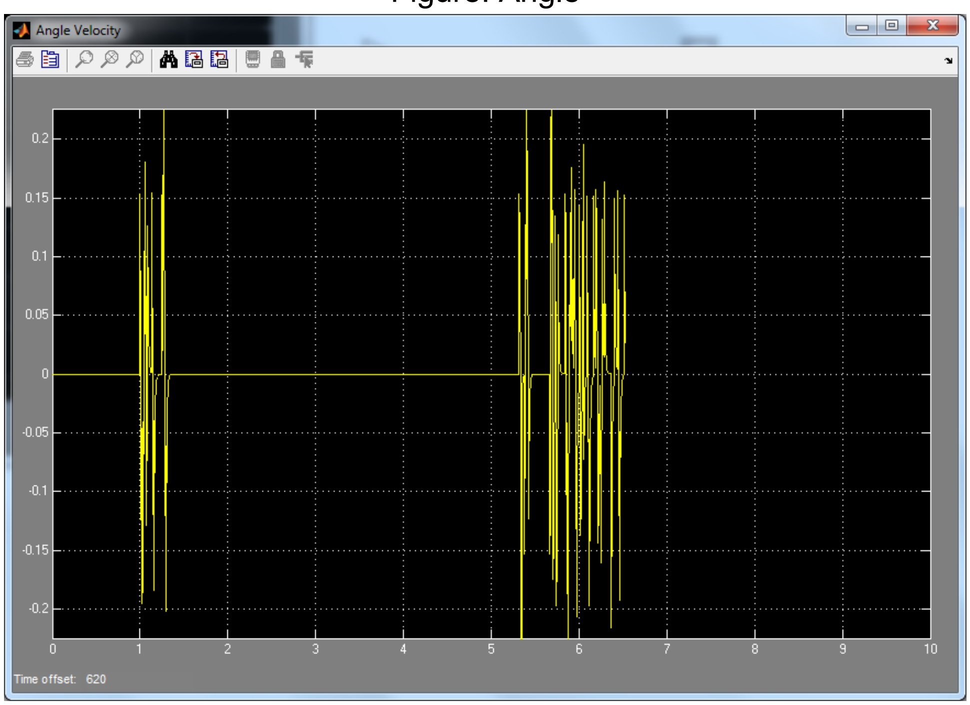

Figure: Angle Velocity

Appendix

```
%% Q1
Km=0.0077;
Rm=2.6;
Lm=180;
Vmax=5.0;
Kg=3.7;
rg=0.0064;
mc=0.526;
Kx=2.28*(10^-5);
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mp=0.106*2;
lp=0.168*2;
g=9.81;

b = (Kg*Km)/(rg);

A = [0,1,0,0;
    0,(-b^2)/(mc*Rm),-(mp*g)/(mc),0;
    0,0,0,1;
    0, (b^2)/(mc*lp*Rm),((mc+mp)*g)/(mc*lp),0];

B = [0;
    b/(mc*Rm);
    0;
    -b/(mc*lp*Rm)];

C = eye(4);

D = [0,0,0,0]';

%%
sys  = ss(A,B,C,D);
tf_sys = ss2tf(A,B,C,D);

s = tf('s');
G = tf(sys);

%%
pzmap(G);

%%
eig(A);

%% Q
Co = ctrb(A,B);
k = rank(Co);
```

```
%%
Mp = 11.5;  % 11.5
Z = ((log(Mp)^2)/(pi^2+(log(Mp)^2)))^0.5;
Ts = 2.5;
Wn = 3.9/(Z*Ts);
Pd1 = -(Z*Wn) + 1i* Wn*(1-Z^2)^0.5;
Pd2 = -(Z*Wn) - 1i* Wn*(1-Z^2)^0.5;

%%
Nd1 = 10*Pd1;
Nd2 = 10*Pd2;

%% PP for Con
p = [Pd1, Pd2, Nd1, Nd2];
K = place(A,B,p);

Q=15*[10,0,0,0;
    0,0.666,0,0;
    0,0,10,0;
    0,0,0,0];
R= 0.1;
N= 0;

[K,S,e] = lqr(A,B,Q,R,N);
```