

MAT104 Problem Saati

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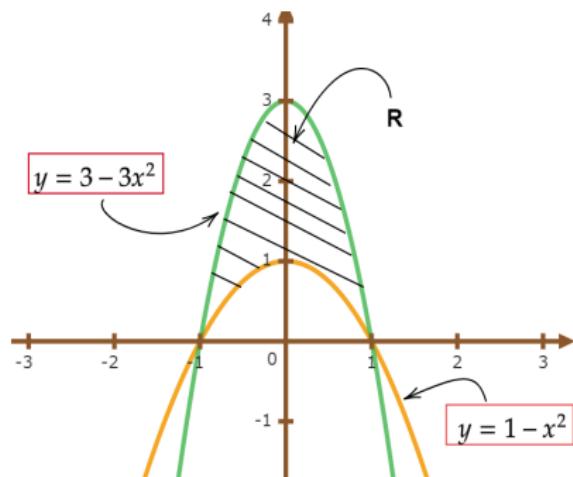
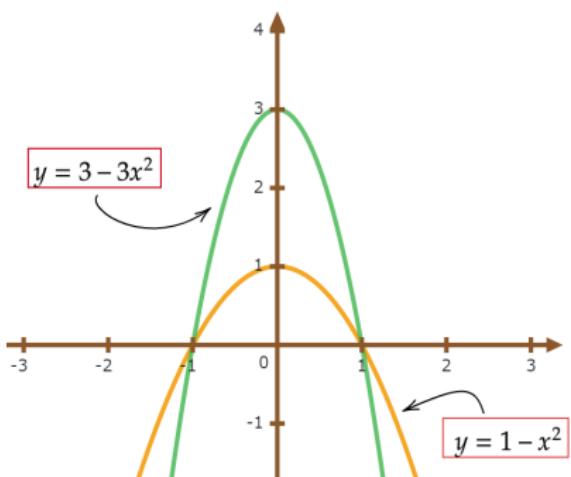
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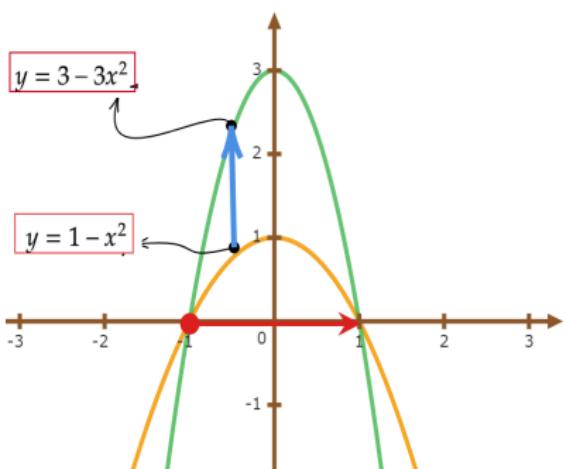
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Soru 1.

R , $y = 1 - x^2$, $y = 3 - 3x^2$ eğrileri ile sınırlandırılmış bölge olmak üzere
 $\iint_R (x + 2y) dA$ integralini hesaplayınız.

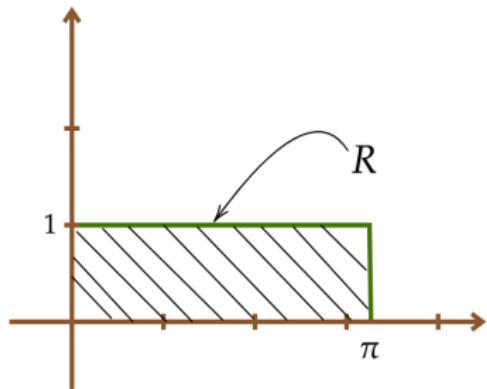




$$\begin{aligned}
 \iint_R (x+2y) dA &= \int_{-1}^1 \int_{1-x^2}^{3-3x^2} (x+2y) dy dx \\
 &= \int_{-1}^1 \left[xy + y^2 \right]_{1-x^2}^{3-3x^2} dx \\
 &= \int_{-1}^1 \left[x(3-3x^2) + (3-3x^2)^2 - x(1-x^2) + (1-x^2)^2 \right] dx \\
 &= \int_{-1}^1 (8x^4 - 2x^3 - 16x^2 + 2x + 8) dx \\
 &= \left. \frac{8x^5}{5} - \frac{2x^4}{4} - \frac{16x^3}{3} + x^2 + 8x \right|_{x=-1}^{x=1} \\
 &= \frac{128}{5} //
 \end{aligned}$$

Soru 2.

Eğer $R = [0, \pi] \times [0, 1]$, $\iint_R y \cos xy \, dA = ?$

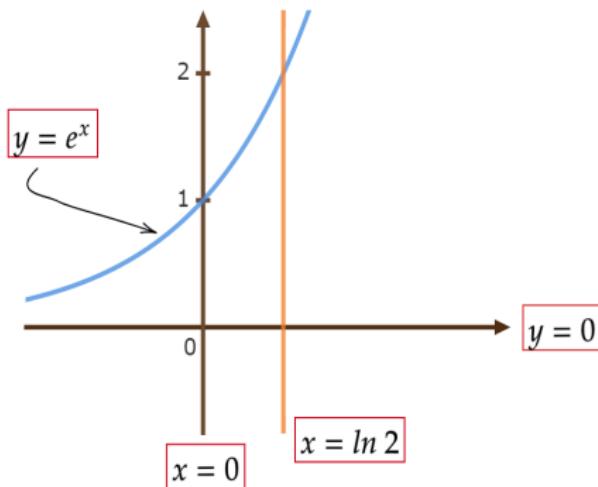


Fubini Teoreminde,

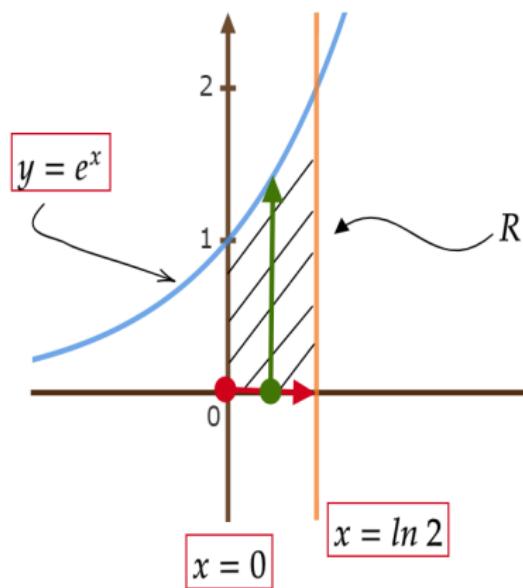
$$\iint_R y \cos(xy) \, dy \, dx = \int_0^{\pi} \int_0^1 y \cos(xy) \, dy \, dx$$
$$= \int_0^{\pi} \left[\sin(xy) \right]_{x=0}^{x=\pi} \, dy$$
$$= \int_0^{\pi} \sin(\pi y) \, dy$$
$$= - \frac{\cos(\pi y)}{\pi} \Big|_{y=0}^{y=1} = 2/\pi$$

Soru 3.

İki katlı integral kullanarak $y = e^x$, $y = 0$, $x = 0$, $x = \ln 2$ eğrileri arasında kalan bölgenin alanını hesaplayınız.



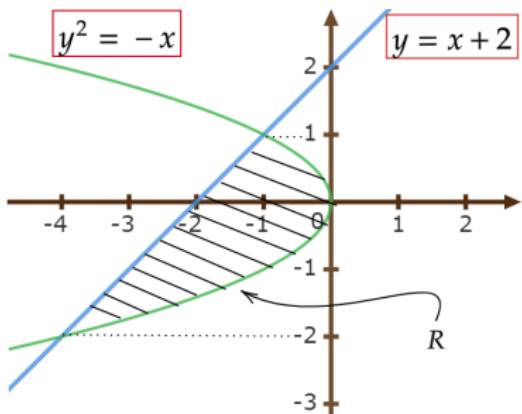
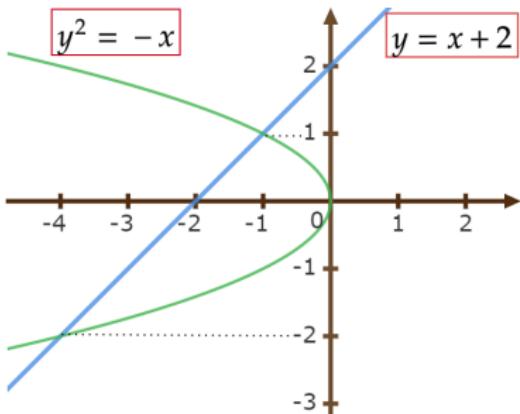
$$\text{Alan} = \iint_R dA$$



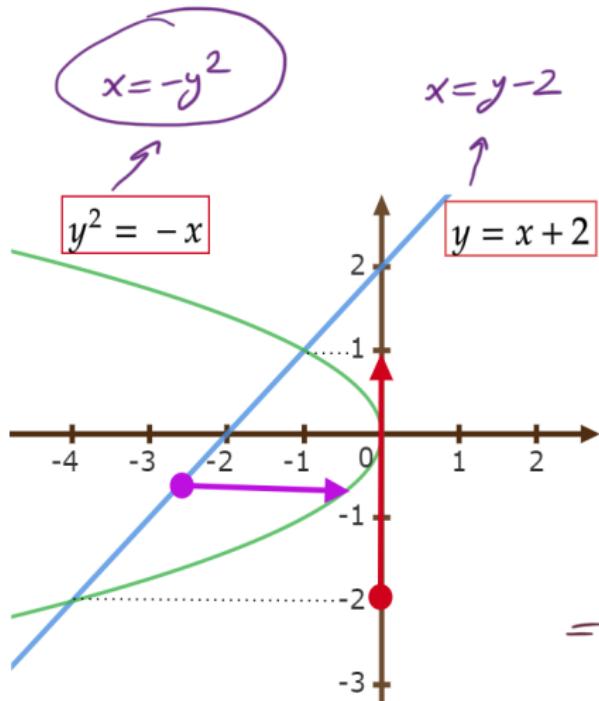
$$\begin{aligned}
 A_{\text{lon}} &= \iint_R dA \\
 &= \int_0^{\ln 2} \int_0^{e^x} dy dx \\
 &= \int_0^{\ln 2} e^x dx \\
 &= \left[e^x \right]_{x=0}^{x=\ln 2} \\
 &= e^{\ln 2} - e^0 = 1 //
 \end{aligned}$$

Soru 4.

İki katlı integral kullanarak $y^2 = -x$ ve $y = x + 2$ eğrileri arasında kalan bölgenin alanını hesaplayınız.



$$\left. \begin{array}{l} y = x + 2 \\ y^2 = -x \end{array} \right\} \Rightarrow y^2 = 2 - y \Rightarrow y^2 + y - 2 = 0 \Rightarrow \underline{\underline{y = -2}} \quad \text{ve} \quad \underline{\underline{y = 1}}$$

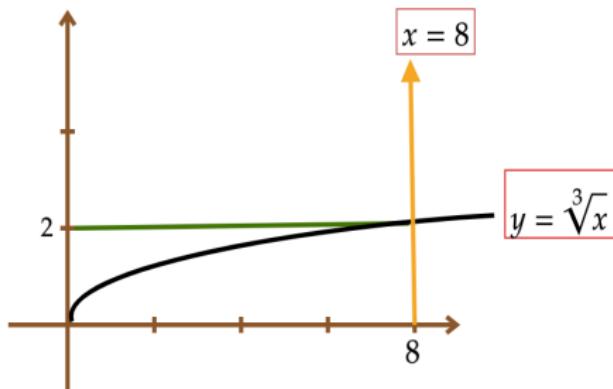


$$\begin{aligned}
 \text{Alan} &= \iint_R dA \\
 &= \iint_{-2}^1 dx dy \\
 &= \int_{-2}^1 \left(-y^2 - y + 2 \right) dy \\
 &= \left[-\frac{y^3}{3} - \frac{y^2}{2} + 2y \right]_{y=-2}^{y=1} \\
 &= \left(-\frac{1}{3} - \frac{1}{2} + 2 \right) - \left(\frac{8}{3} - 2 - 4 \right) \\
 &= \frac{9}{2}
 \end{aligned}$$

Soru 5.

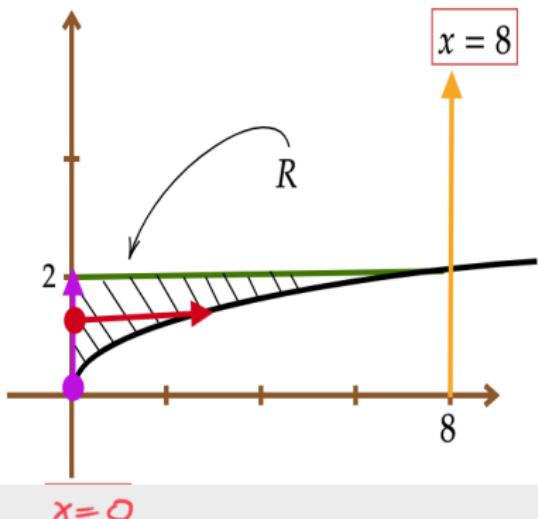
$$\int_0^8 \int_{\sqrt[3]{x}}^2 \frac{1}{1+y^4} dy dx$$
 integralini hesaplayınız.

R : $x = 0$ ve $x = 8$ doğruları arasında $y = \sqrt[3]{x}$ eğrisi ve $y = 2$ doğrularının sınırladığı bölge



Fubini teoreminden,

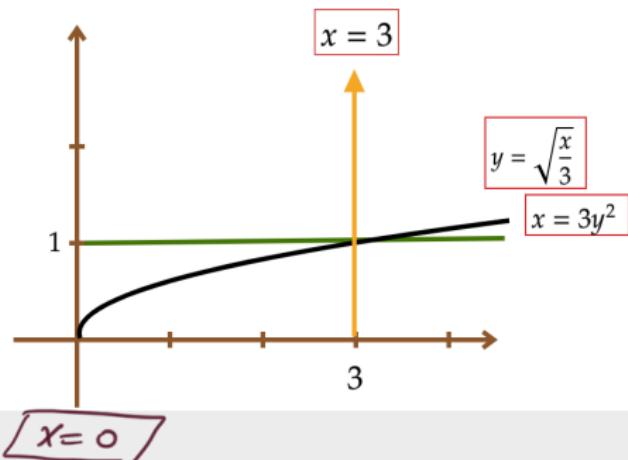
$$\int_0^8 \int_{\sqrt[3]{x}}^2 \frac{1}{1+y^4} dy dx = \int_0^2 \int_0^{y^3} \frac{1}{1+y^4} dx dy$$
$$= \int_0^2 \left(\frac{x}{1+y^4} \right) \Big|_{x=0}^{x=y^3} dy$$
$$= \int_0^2 \frac{y^3}{1+y^4} dy$$
$$= \frac{\ln(1+y^4)}{4} \Big|_{y=0}^{y=2}$$
$$= \frac{\ln 17}{4} \cdot \frac{1}{11}$$



Soru 6.

$$\int_0^3 \int_{\sqrt{\frac{x}{3}}}^1 e^{y^3} dy dx$$
 integralini hesaplayınız.

$R : x = 0$ ve $x = 3$ doğruları arasında $y = \sqrt{\frac{x}{3}}$ eğrisi ve $y = 1$ doğrularının sınırladığı bölge



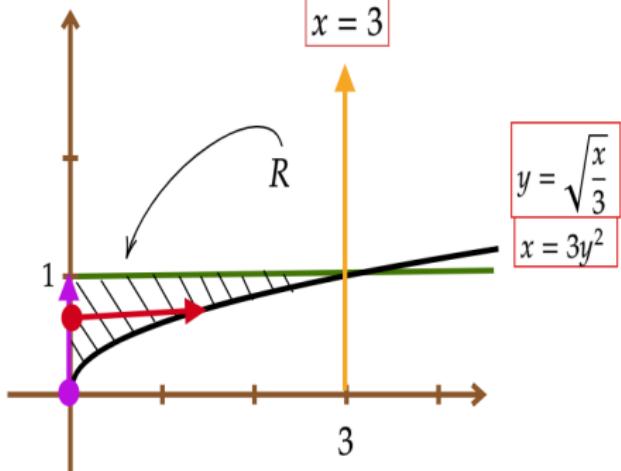
Fubini teoreminden,

$$\int_0^3 \int_{\sqrt{\frac{x}{3}}}^1 e^{y^3} dy dx = \int_0^1 \int_0^{3y^2} e^{y^3} dx dy$$
$$= \int_0^1 \left[e^{y^3} x \right]_{x=0}^{x=3y^2} dy$$

$$= \int_0^1 3y^2 e^{y^3} dy$$

$$= \left. e^{y^3} \right|_{y=0}^{y=1}$$

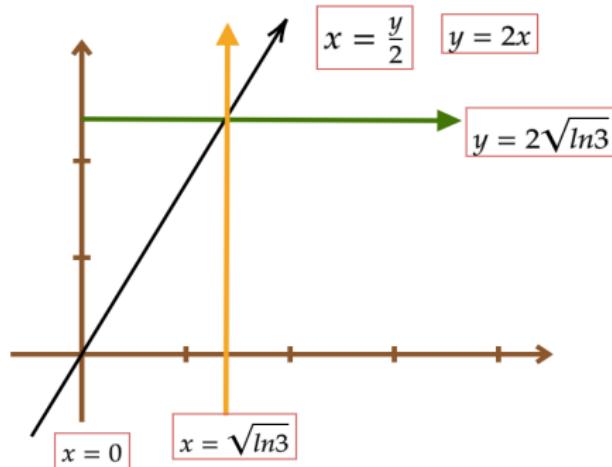
$$= e - 1 //$$



Soru 7.

$$\int_0^{2\sqrt{\ln 3}} \int_{y/2}^{\sqrt{\ln 3}} e^{x^2} dx dy$$
 integralini hesaplayınız.

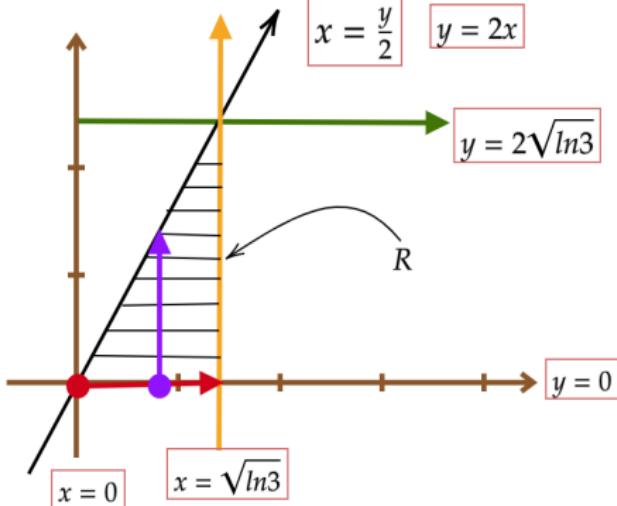
$R : y = 0$ ve $y = 2\sqrt{\ln 3}$ doğruları arasında $x = \frac{y}{2}$ eğrisi ve $x = \sqrt{\ln 3}$ doğrusunun sınırladığı bölge



Fubini teoreminden,

$$\int_0^{2\sqrt{\ln 3}} \int_{y/2}^{\sqrt{\ln 3}} e^{x^2} dx dy = \int_0^{\sqrt{\ln 3}} \int_{\frac{y}{2}}^{2x} e^{x^2} dy dx$$

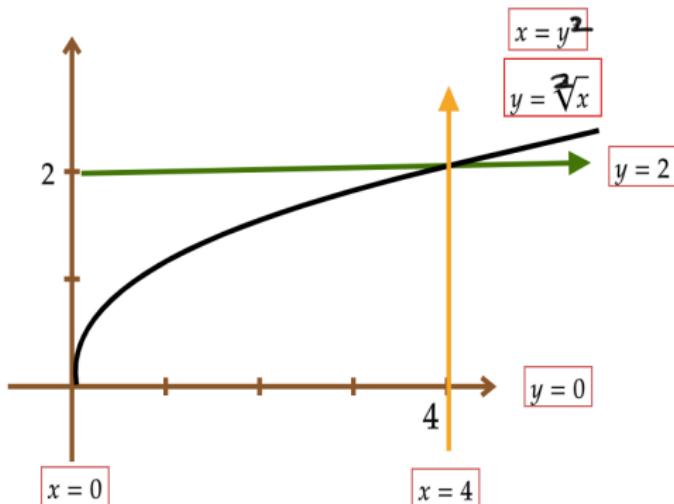
$$\begin{aligned}&= \int_0^{\sqrt{\ln 3}} \left[y e^{x^2} \right]_{y=0}^{y=2x} dx \\&= \int_0^{\sqrt{\ln 3}} 2x e^{x^2} dx \\&= \left. e^{x^2} \right|_{x=0}^{x=\sqrt{\ln 3}} \\&= 3 - 1 = 2 //\end{aligned}$$



Soru 8.

$$\int_0^2 \int_{y^2}^4 \frac{3}{2} e^{y/\sqrt{x}} dx dy$$
 integralini hesaplayınız ?

$R : y = 0$ ve $y = 2$ doğruları arasında $x = y^2$ eğrisi ve $x = 4$ doğrusunun sınırladığı bölge



Fubini teoreminden,

$$\int_0^2 \int_{y^2}^4 \frac{3}{2} e^{y/\sqrt{x}} dx dy = \int_0^4 \int_0^{\sqrt{x}} \frac{3}{2} e^{y/\sqrt{x}} dy dx$$
$$= \int_0^4 \frac{3}{2} \cdot \frac{1}{\sqrt{x}} \cdot e^{\sqrt{x}} \Big|_{y=0} dx$$
$$= \int_0^4 \left(\frac{3}{2} e^{\sqrt{x}} - \frac{3}{2} \sqrt{x} \right) dx$$
$$= \left. x (e - 1) \right|_{x=0}^{x=4}$$
$$= 8(e - 1), //$$

