Hierarchical Control

Laboratory

Exercise 3

Direct method of coordination

The aim of this exercise is to apply the direct method of coordination to a complex static system composed of three cross-coupled subsystems. During the exercise students will learn about some practical aspects of applying the method.

1 Direct method of coordination: a simple example

Let's suppose that the minimal value for the objective function in the form:

$$p(v, m_1, m_2) = p_1(v, m_1) + p_2(v, m_2)$$

has to be found. The p_1 and p_2 functions are defined as follows:

$$p_1(v, m_1) = -v^2 + vm_1 - m_1^2$$
$$p_2(v, m_2) = -v^2 + 2vm_2 - 3m_2^2$$

There are also constraints for all the variables:

$$2 \leq v \leq 10$$
$$1 \leq m_1 \leq 5$$

$$1 \leq m_2 \leq 5$$

In order to solve such a problem with Matlab, all the functions have to be declared:

```
function x = p(vector)
  x = p1(vector) + p2(vector);

function x = p1(vector)
  v = vector(1);
  m1 = vector(2);
  x = -v.*v + v.*m1 - m1.*m1;

function x = p2(vector)
  v = vector(1);
  m2 = vector(3);
  x = -v.*v + 2.*v.*m2 - 3.*m2.*m2;
```

If the minimal value of the p function was to be found without using the direct method of coordination, only one Matlab command had to be used:

```
[x_opt,f_val] = ...
 ... fmincon( @p, x0, [], [], [], [], [2 1 1]', [10 5 5]' )
```

`x0` is the starting vector (the initial values of variables). In this example, the optimal values of the variables are: $v = 10$, $m_1 = 1$, $m_2 = 5$. The minimal value of the performance index is -166.

Using the direct method of coordination is a little more complicated. For each subfunction an optimization have to be performed, and one more optimization have to be performed for the objective function. The purpose of the first optimizations (local ones) is to find optimal values for decision variables m_1 and m_2. The value of v should be constant. It can be achieved by making lower and upper bounds for v equal:

```
lb_sub = lb;
lb_sub(1) = x0(1);
ub_sub = ub;
ub_sub(1) = x0(1);
```

Let's start with the variable m_1:

```
x1 = fmincon( @p1, x0, [], [], [], [], lb_sub, ub_sub );
m1_opt = x1(2);
```

The value of the variable m_2 can be achieved in the same way:

```
x2 = fmincon( @p2, x0, [], [], [], [], lb_sub, ub_sub );
m2_opt = x2(3);
```

When the values of decision variables are known, the values of coordination variables have to be found. In this example, this is only one coordination variable v. In the last optimization, the values of decision variables should be constant, which can be achieved by using proper lower and upper bounds:

```
lb_coord = lb;
lb_coord(2) = m1_opt;
lb_coord(3) = m2_opt;
ub_coord = ub;
ub_coord(2) = m1_opt;
ub_coord(3) = m2_opt;
```

Finally, the value of the coordination variable v can be found:

```
[x_opt,f_val] = ...
 ... fmincon( @p, x0, [], [], [], [], lb_coord, ub_coord );
```

The initial values vector `x0` might be set to values from the middle of the ranges for all the variables:

```
x0 = [5 2.5 2.5]';
```

In such a case, the result of optimization using the direct method of coordination will be as follows: $v = 10$, $m_1 = 2.5$, $m_2 = 5$, and the objective function value will be -156.25. This result isn't optimal. The problem is the nature of numerical computations; they are not precise enough. If the direct method of coordination were repeated with starting point set to `x0 = [10 2.5 5]';`, the result would be optimal.

2 The complex system problem

The local performance indexes are given below:

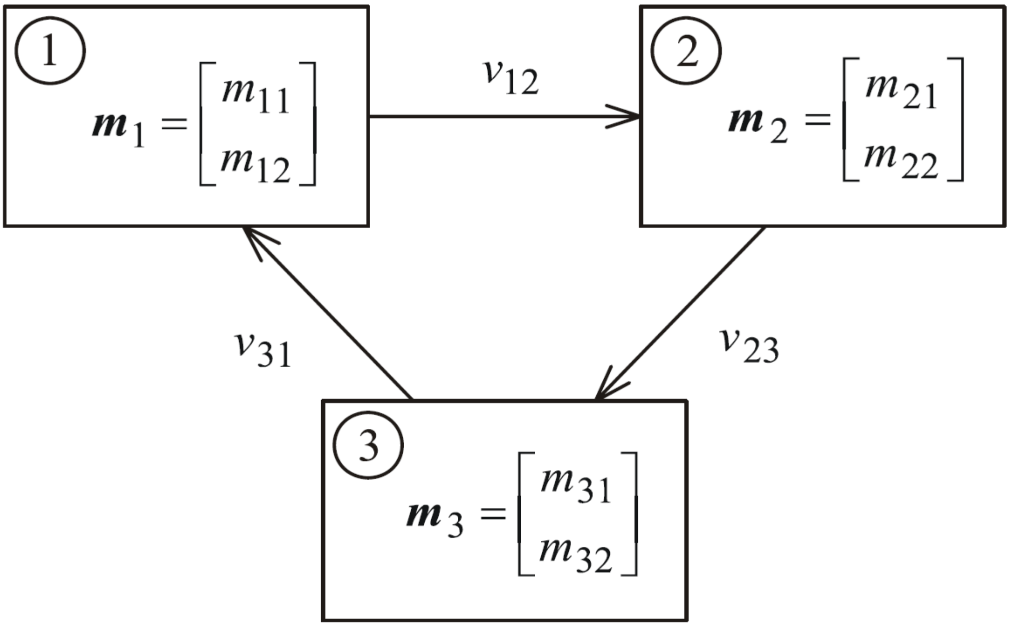

Figure 1: The complex system scheme.

$$\begin{aligned}
p_1(\underline{v}, \underline{m_1}) &= p_1(v_{31}, v_{12}, m_{11}, m_{12}) = \\
&= -(v_{31}-2)^2 - 2(v_{12}-3)^2 - v_{31}m_{12} - (m_{11}-v_{12})^2 + m_{11} + 40 \\
p_2(\underline{v}, \underline{m_2}) &= p_2(v_{12}, v_{23}, m_{21}, m_{22}) = \\
&= -3(v_{12}-4)^2 - (v_{23}-1)^2 + v_{23}m_{21} - (m_{22}-v_{12})^2 - m_{22} + 20 \\
p_3(\underline{v}, \underline{m_3}) &= p_3(v_{23}, v_{31}, m_{31}, m_{32}) = \\
&= -2(v_{23}-5)^2 - 4(v_{31}-1)^2 - v_{23}m_{31} - (m_{32}-v_{31})^2 + m_{32} + 30
\end{aligned}$$

During the exercise, each section will be given a unique set of variables constraints. These values will be used also in the next exercise.

3 The exercise program

1. Write a Matlab script which solves the complex system problem without any coordination method. The script should provide the following information:

   - the values of the coordination variables v_{12}, v_{23} and v_{31}

- the values of the decision variables m_{11}, m_{12}, m_{21}, m_{22}, m_{31} and m_{32}
- the value of the performance index

2. Write a Matlab script which solves the complex system problem using the direct method of coordination. The optimization should be repeated until the difference between two subsequent values of performance index is small enough. The script should provide the following information:

   - the values of the coordination variables v_{12}, v_{23} and v_{31}
   - the values of the decision variables m_{11}, m_{12}, m_{21}, m_{22}, m_{31} and m_{32}
   - the value of the performance index
   - the number of performed iterations

3. Check the influence of the initial variables values on the results of both scripts and the number of performed iterations in the second script.