Hierarchical Control

Laboratory

Exercise: Sensitivity analysis of complex static systems

During the laboratory students perform sensitivity analysis for complex static non-linear models. Sensitivity coefficients (partial derivatives) are generated using both: sensitivity model and the adjoint system. The obtained gradient is used during minimum searching for analyzed non-linear function.

Students are supposed to know how to construct:

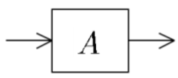
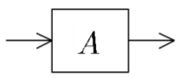
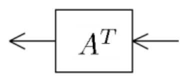
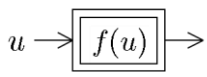
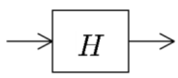
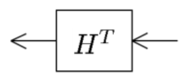
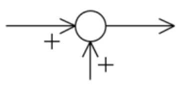
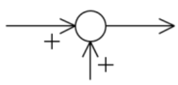
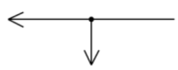
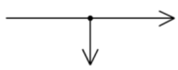
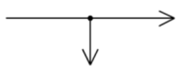
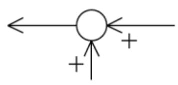
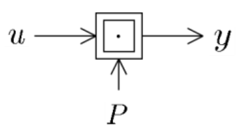
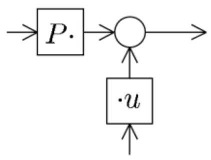
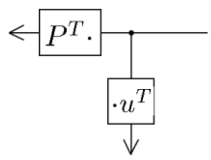
- sensitivity model,
- adjoint system

for any non-linear static system given in a structural form (as a block diagram). Furthermore, basic knowledge about gradient optimization methods will be required.

Rules for construction of the sensitivity model and the adjoint system

When the analyzed non-linear static system is given in a block diagram form, then the sensitivity model and the adjoint system may be constructed using set of rules collected in Table 1.

Table 1. Rules for construction of the sensitivity model and the adjoint system

Element of the original model		Element of the sensitivity model	Element of the adjoint system
Name	Symbol		
Linear element			
Non-linear element		 $H = \left.\frac{\partial f}{\partial u}\right\|_{nom}$	
Summing junction			
Branching node			
Product of two signals			

Hints for construction of the sensitivity model and the adjoint system in Matlab-Simulink.

Special attention should be paid during creation parts of the sensitivity model and the adjoint system that correspond to non-linear elements. Let us consider as an example a non-linear element realizing a cube function, presented in Fig. 1a. Its sensitivity model, for signal variations $\bar{u}$ and $\bar{y}$, is presented in Fig. 1b. Obtained sensitivity model is the *linear* element and its gain equals $3u^2$. Note that the gain depends on the value of the signal u appearing in the original system. Taking the above into account, the simultaneous realization of both systems (the original and the sensitivity model) in Matlab-Simulink environment looks as presented in Fig 1c. Similar remark applies to the adjoint system.

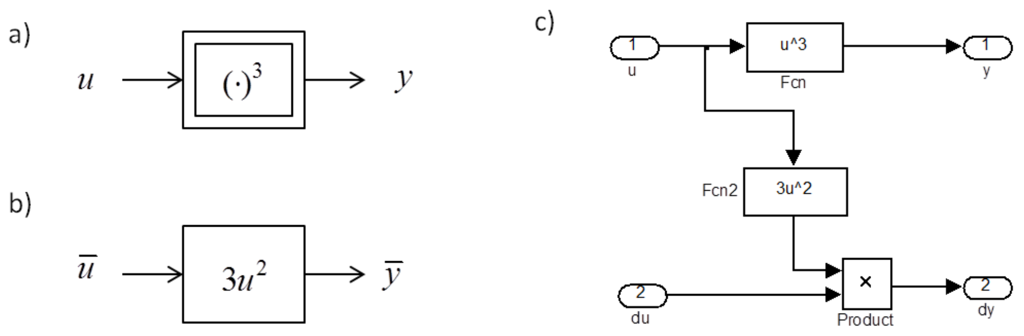


Figure 1. The non-linear element (a), the corresponding sensitivity model (b), and their Matlab-Simulink realization(c)

Plan of the laboratory exercise

1. Show the plot (contour plot or 3D plot) of given non-linear scalar function of two arguments.
2. Present given function in the structural form.
3. Construct the sensitivity models (one model for each argument).
4. Construct the adjoint model.
5. Use models from points 3 and 4 to generate the gradient of the analyzed function.
6. Compare the values of gradients obtained by both approaches.
7. Implement the simple gradient descent minimizing algorithm that uses gradients obtained in point 5.
8. Compare both minimizing algorithms for different starting points.

Test exercise

Perform points 2-4 for the Rosenbrock function (banana function)

$$f(x, y) = (x - a)^2 + b(y - x^2)^2$$

where a i b are given positive constants.