INSTRUCTIONS

- The assignment consists of one question for a total of 100 marks.
- Submit your assignment **electronically** via `https://odtuclass.metu.edu.tr/`.
- The report should be written via LaTeX.
- Please, do not forget send .m files, .tex file of your report with the .pdf form.
- Please, give a name for your pdf document as **your name, surname, and student ID**. For instance, **mkutuk_2389369**.
- The report for a programming assignment should consist of
  - Brief description of the problem;
  - Results such as data, graphs etc. Try to avoid attaching large set of data unless it is really necessary;
  - Discussion, comments, explanation and conclusion on your numerical observations. This part is equally important as your computer codes and the data you collect, and it helps with understanding concepts, algorithms, or other relevant issues not discussed during lectures.
- Every .m file should be documented, such as:

```
% CA2_numberExercises.m
%
% Author: (Your Name)
%
% Description:
% (Give a brief description of what your program does.)
%
% Input:
% (State what the program inputs are.)
%
% Output:
% (State what the program outputs are.)
%
% Usage:
% (Give an example of how to use your program.)
```

- For one day late submission, you will get 75% of your grade. For two days late submission, you will get 50% of your grade. After 2 days late, your homework will not be graded.

Consider the following linear programming

$$
\begin{aligned}
\min_{x} \quad & -x_1 + x_2 \\
\text{subject to} \quad & x_1 + x_2 \leq 40, \\
& 2x_1 + x_2 \leq 60, \\
& x \geq 0.
\end{aligned}
$$

(a) Implement the general primal–dual interior point method given in Algorithm 1.

(b) Implement Mehrotra predictor–corrector algorithm given in Algorithm 2.

(c) Use Matlab built–in function `linprog` to solve the problem.

Display your results for all cases with different initial values (choose also infeasible initial guesses) as Table 1 and choose the tolerance value as `tol=1e-6`. Compare your results and explain your observations.

Table 1: Numerical results for Algorithm X with different initial values (tol=1e-6)

x_0	λ_0	s_0	x	No. of iteration
$[\cdot,\cdot]$	$\cdot$	$\cdot$	$\cdot$	$\cdot$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Algorithm 1 Primal–Dual Interior Point Method

Initial guess $(\mathbf{x}_0, \lambda_0, \mathbf{s}_0) \in \mathcal{F}^0$
for $k = 1, 2, \ldots$ **do**
Set $\sigma_k \in [0,1]$ and $\mu_k = \frac{x_k^T s_k}{n}$
Solve

$$
\begin{bmatrix} 0 & \mathbf{A}^T & \mathbf{I} \\ \mathbf{A} & 0 & 0 \\ \mathbf{S}_k & 0 & \mathbf{X}_k \end{bmatrix}
\begin{bmatrix} \Delta \mathbf{x}_k \\ \Delta \lambda_k \\ \Delta \mathbf{s}_k \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ -\mathbf{X}^k \mathbf{S}^k \mathbf{e} + \sigma_k \mu_k \mathbf{e} \end{bmatrix},
$$

where $\mathbf{X}^k = diag(\mathbf{x}_k)$, $\mathbf{S}^k = diag(\mathbf{s}_k)$, and $\mathbf{e} = (1, ..., 1)^T$.
Set

$$
(\mathbf{x}_{k+1}, \lambda_{k+1}, \mathbf{s}_{k+1}) \leftarrow (\mathbf{x}_k, \lambda_k, \mathbf{s}_k) + \alpha_k (\Delta \mathbf{x}_k, \Delta \lambda_k, \Delta \mathbf{s}_k),
$$

choosing α_k so that $(x_{k+1}, s_{k+1}) > 0$.
end for

Algorithm 2 Mehrotra Predictor–Corrector Algorithm

Initial guess $(\mathbf{x}_0, \lambda_0, \mathbf{s}_0) \in \mathcal{F}^0$ with $(x_0, s_0) > 0$

for $k = 1, 2, \dots$ **do**

Compute the duality measure $\mu_k = \frac{x_k^T s_k}{n}$

Solve

$$\begin{bmatrix} 0 & \mathbf{A}^T & \mathbf{I} \\ \mathbf{A} & 0 & 0 \\ \mathbf{S}_k & 0 & \mathbf{X}_k \end{bmatrix} \begin{bmatrix} \Delta \mathbf{x}_k \\ \Delta \lambda_k \\ \Delta \mathbf{s}_k \end{bmatrix} = \begin{bmatrix} -\mathbf{A}^T \lambda_k - \mathbf{s}_k + \mathbf{c} \\ -\mathbf{A}\mathbf{x} + \mathbf{b} \\ -\mathbf{X}^k \mathbf{S}^k \mathbf{e} \end{bmatrix},$$

for $\Delta \mathbf{x}^{aff}, \Delta \lambda^{aff}, \Delta \mathbf{s}^{aff}$.

Calculate $\alpha_{aff}^{pri}, \alpha_{aff}^{dual}$, and μ_{aff} as follows

$$\begin{aligned} \alpha_{aff}^{pri} &= \min \left(1, \min_{i: \Delta x_i^{aff} < 0} -\frac{x_i}{\Delta x_i^{aff}} \right) \\ \alpha_{aff}^{dual} &= \min \left(1, \min_{i: \Delta s_i^{aff} < 0} -\frac{s_i}{\Delta s_i^{aff}} \right) \\ \mu_{aff} &= \left(\mathbf{x}_k + \alpha_{aff}^{pri} \Delta x^{aff} \right)^T \left(\mathbf{s}_k + \alpha_{aff}^{pri} \Delta s^{aff} \right) / n \end{aligned}$$

Set

$$\sigma = \left(\frac{\mu_{aff}}{\mu_k} \right)^3$$

Solve

$$\begin{bmatrix} 0 & \mathbf{A}^T & \mathbf{I} \\ \mathbf{A} & 0 & 0 \\ \mathbf{S}_k & 0 & \mathbf{X}_k \end{bmatrix} \begin{bmatrix} \Delta \mathbf{x}_k \\ \Delta \lambda_k \\ \Delta \mathbf{s}_k \end{bmatrix} = \begin{bmatrix} -\mathbf{A}^T \lambda_k - \mathbf{s}_k + \mathbf{c} \\ -\mathbf{A}\mathbf{x} + \mathbf{b} \\ -\mathbf{X}^k \mathbf{S}^k \mathbf{e} - \Delta \mathbf{X}^{aff} \Delta \mathbf{S}^{aff} \mathbf{e} + \sigma \mu_k \mathbf{e} \end{bmatrix},$$

Calculate α_k^{pri} and α_k^{dual} as follows

$$\begin{aligned} \alpha_k^{pri} &= \min \left(1, \eta \alpha_{max}^{pri} \right) \\ \alpha_k^{dual} &= \min \left(1, \eta \alpha_{max}^{dual} \right) \end{aligned}$$

where $\eta \in [0.99, 1.0)$ and α_{max}^{pri}, $\eta \alpha_{max}^{dual}$ are given by

$$\begin{aligned} \alpha_{max}^{pri} &= \min \left(1, \min_{i: \Delta x_i < 0} -\frac{x_i}{\Delta x_i} \right) \\ \alpha_{max}^{dual} &= \min \left(1, \min_{i: \Delta s_i < 0} -\frac{s_i}{\Delta s_i} \right) \end{aligned}$$

Set

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{x}_k + \alpha_k^{pri} \Delta \mathbf{x} \\ (\lambda_{k+1}, \mathbf{s}_{k+1}) &= (\lambda_k, \mathbf{s}_k) + \alpha_k^{dual} (\Delta \lambda, \Delta \mathbf{s}) \end{aligned}$$

end for