İsmail Öksüz
150119516

BASIC QUESTIONS

1) Vectors and Matrices

$$X=\begin{pmatrix}9 & 8\\ 7 & 6\end{pmatrix} \quad y=\begin{pmatrix}9\\ 8\end{pmatrix} \quad z=\begin{pmatrix}7\\ 6\end{pmatrix}$$

1- $y^T z = 9.7 + 8.6 = 63 + 48 = \boxed{111}$

2- $Xy = \begin{matrix} 9.9 + 8.8 = 81 + 64 = 145 \\ 7.9 + 6.8 = 63 + 48 = 111 \end{matrix} \quad \boxed{\begin{matrix}145\\111\end{matrix}}$

3- $\underbrace{\left(\begin{array}{cc|cc} 9 & 8 & 1 & 0 \\ 7 & 6 & 0 & 1 \end{array}\right)}_{\text{Augmented matrix}} \xrightarrow[\text{1st column pivot}]{R1=\frac{R1}{9}} \left(\begin{array}{cc|cc} \frac{9}{9} & \frac{8}{9} & \frac{1}{9} & \frac{0}{9} \\ 7 & 6 & 0 & 1 \end{array}\right)$

$R2 = R2 - 7R1$

$\left(\begin{array}{cc|cc} 1 & \frac{8}{9} & \frac{1}{9} & 0 \\ 0 & 1 & \frac{7}{2} & -\frac{9}{2} \end{array}\right) \xleftarrow[R_2 = -\frac{9R_2}{2}]{\text{make 2nd column pivot}} \left(\begin{array}{cc|cc} 1 & \frac{8}{9} & \frac{1}{9} & 0 \\ 0 & -\frac{2}{9} & -\frac{7}{9} & 1 \end{array}\right)$

Eliminate 2nd column by $R1 = R1 - \frac{8R_2}{9}$

$\left(\begin{array}{cc|cc} 1 & 0 & -3 & 4 \\ 0 & 1 & \frac{7}{2} & -\frac{9}{2} \end{array}\right)$

identity matrix | inverse matrix

$$\boxed{\begin{pmatrix}-3 & 4\\ \frac{7}{2} & -\frac{9}{2}\end{pmatrix}}$$

Yes, X is invertible.

4- X is invertible, as I did above.

So Rank of X = # of rows. = $\boxed{2}$

EĞİTİM KURUMLARI

İsmail Öksüz
1521195616

② Calculus

1- $y = 4x^3 - x^2 + 7$

$$\frac{dy}{dx} = 3.4x^2 - 2.x = \boxed{12x^2 - 2x}$$

2- $y = \tan(z)x^{6z} - \ln\left(\frac{7x+z}{x^4}\right)$

$$\frac{\partial y}{\partial x} = \frac{\partial}{\partial x}\left(\tan(z)x^{6z}\right) - \frac{\partial}{\partial x}\left(\ln\left(\frac{7x+z}{x^4}\right)\right)$$

$$\frac{\partial}{\partial x}\left(\tan(z).x^{6z}\right) = \tan z.\frac{\partial}{\partial x}\left(x^{6z}\right) = \tan z.6z.x^{6z-1}$$

$$\frac{\partial}{\partial x}\left(\ln\left(\frac{7x+z}{x^4}\right)\right) = \frac{x^4}{7x+z}\cdot\frac{\partial}{\partial x}\left(\frac{7x+z}{x^4}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{7x+z}{x^4}\right) = \frac{7.x^4 - (7x+z).4x^3}{x^4.x^4} = \frac{7x^4 - 28x^4 - 4zx^3}{x^8}$$

$$= \frac{7x - 28x - 4z}{x^5} = \frac{-21x - 4z}{x^5}$$

$$\frac{\partial}{\partial x}\left(\ln\left(\frac{7x+z}{x^4}\right)\right) = \frac{\cancel{x^4}}{7x+z}\cdot\frac{-21x-4z}{\cancel{x^5}\,x} = \frac{-21x-4z}{7x^2+xz}$$

$$\boxed{\left(\tan(z).6z.x^{6z-1}\right) - \left(\frac{-21x-4z}{7x^2+xz}\right)}$$

İsmail ÖKSÜZ
150119516

YEK EĞİTİM KURUMLARI

(3) Probability and Statistic

$S = \{0, 1, 1, 0, 0, 1, 1\}$

1- # of numbers in data = 7
Sum = 0+1+1+0+0+1+1 = 4

Sample mean = $\boxed{\frac{4}{7}}$

2- $s^2 = \frac{1}{7-1} \sum_{i=1}^{7} (x_i - \frac{4}{7})^2 = \frac{1}{6} \left[(-\frac{4}{7})^2 + (\frac{3}{7})^2 + \dots \right]$

$= \frac{1}{6} \cdot \left[3 \cdot (-\frac{4}{7})^2 + 4 \cdot (\frac{3}{7})^2 \right] = \frac{1}{6} \left[\frac{48}{49} + \frac{36}{49} \right] = \frac{1}{\cancel{6}} \cdot \frac{\overset{14}{\cancel{84}}}{49} = \boxed{\frac{2}{7}}$ //

3- $\frac{3}{10} \times \frac{7}{10} \times \frac{7}{10} \times \frac{3}{10} \times \frac{3}{10} \times \frac{7}{10} \times \frac{7}{10} = (\frac{3}{10})^3 \cdot (\frac{7}{10})^4 = \frac{27 \times 2401}{10^7} = \boxed{0.0065}$

4- The mean value maximizes the probability of the sample.

$\boxed{\frac{4}{7}}$

5-

A	B	P(A, B)
0	0	0.1
0	1	0.4
1	0	0.2
1	1	0.3

a- $P(A=0, B=0) = \boxed{0.1}$

b- $P(A=1) = 0.2 + 0.3 = \boxed{0.5}$

c- $P(A=0 | B=1) = P(A=0, B=1) / P(B=1) = \frac{0.4}{0.4+0.3} = \frac{0.4}{0.7} = \boxed{\frac{4}{7}}$

d- $P(A=0 \vee B=0) = P(A=0) + P(B=0) - P(A, B) = (0.4+0.1) + (0.1+0.2) - (0.1)$

$= 0.5 + 0.3 - 0.1 = \boxed{0.7}$

İsmail ÖKSÜZ
150119516

4) Big-O Notation

False

1- $O(f(n)) = O(\frac{n}{2}) = O(n)$

$O(g(n)) = O(\log_2(n)) = O(\log n)$

$O(n) \neq O(\log n)$

$\boxed{g(n) = O(f(n))}$

True

2- $O(f(n)) = O(\ln(n)) = O(\log n)$

$O(g(n)) = O(\log_2(n)) = O(\log n)$

$O(\log n) = O(\log n)$

Both are correct (boxed)

False

3- $O(f(n)) = O(n^{100})$

$O(g(n)) = O(100^n)$

$O(n^{100}) \neq O(100^n)$

$\boxed{f(n) = O(g(n))}$

İsmail ÖKSÜZ
150119516

MEDIUM-LEVEL QUESTIONS

5) Algorithm

```
findZero (arr, lower, upper)
    while (lower < upper)
        mid = (lower + upper)/2
        if (arr[mid] == 0)
            return mid
        else if (arr[mid] < 0)
            return findZero (arr, mid+1, upper)
        else
            return findZero (arr, lower, mid-1)
    return 0
```

- Created function, lower is initially 0 and the upper = len (arr) - 1

- While lower less then the upper, the function continues.

- Declared a new variable mid, mid is the average of lower and upper.

- If midth elemnt of arr is 0, function returns mid.

- Else if midth elemnt is less then 0, I called findZero again. But the lower value is mid+1.

- Else, I called findZero, upper value is mid-1.

- If there is no 0, returns 0.

İsmail Öksüz
150119516

6 Probability and Random Variables

6.1) 1- $P(A|B)P(B) = \frac{P(A,B)}{P(B)} P(B) = P(A,B)$

$P(B|A)P(A) = \frac{P(B,A)}{P(A)} P(A) = P(B,A)$

$P(A,B) = P(B,A)$ $\boxed{\text{True}}$

2- $P(A \cup B) = P(A) + P(B) - P(A,B)$

$P(A|B) = \frac{P(A,B)}{P(B)} \Rightarrow Eq. = P(A) + P(B) - \frac{P(A,B)}{P(B)}$

"$P(B) \neq 1$, 1. So, $\boxed{\text{False}}$

3- Let $B \cup C = X$, Equation becomes;

$\frac{P(A \cup X)}{P(X)} \geq P(A|X)P(X)$

$\frac{P(A \cup X)}{P(X)} \geq \frac{P(X|A)P(A)}{\cancel{P(X)}} \cancel{P(X)}$

$P(A \cup X) \geq \underbrace{P(X|A)P(A)P(X)}$

$\frac{P(X,A)}{P(A)} P(A)P(X) = P(X,A)P(X)$

$P(A) + P(X) - P(A,X) \geq P(X,A)P(X)$ $\boxed{\text{True}}$

$P(X)[1 - P(X,A)] \geq P(A,X) - P(A)$

4- $P(A^c|B) + P(A|B) = 1$, but it is not. $\boxed{\text{False}}$

5- $\boxed{\text{True}}$

İsmail Öksüz
150119516

EĞİTİM KURUMLARI

6.2- Discrete and Continuous Distributions

Laplace → h

Multinomial → i

Poisson → l

Dirichlet → k

Gamma → j

6.3 - Mean and Variance

1- $X \sim Binomial(n,p)$

a) $\mu = np$ $\boxed{np}$

b) $\sigma^2 = np(1-p)$ $\boxed{np(1-p)}$

2- $E[X]=1$, $Var(X)=1$

a) $E[3X] = 3E[X] = 3.1 = \boxed{3}$

b) $Var(3X) = 3^2 Var(X) = 9.1 = \boxed{9}$

c) $Var(X+3) = Var(X) = \boxed{1}$

EĞİTİM KURUMLARI

İsmail Ökgöz
150119516

6.4- Mutual and Conditional Independence

1- $E[XY] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \underbrace{f(x,y)}_{f(x).f(y)}.x.y.\,dx.dy = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x)f(y)\,x\,y\,dx\,dy$

We know that X, Y are indepented. So equation becomes;

$$\underbrace{\int_{-\infty}^{\infty} f(x)\,x\,dx}_{\Downarrow\ E[X]}\ \underbrace{\int_{-\infty}^{\infty} f(y)\,y\,dy}_{\Downarrow\ E[Y]}$$

$$\boxed{E[XY] = E[X]\,E[Y]}$$

2- Cov(X,Y) becomes "0" if X and Y are independent variables.

$Var(X+Y) = Var(X) + 2\,Cov(X,Y) + Var(Y)$ was given.

We know that $Cov(X,Y) = 0$;

$\boxed{\text{So } Var(X+Y) = Var(X) + Var(Y)}$

3- As those 2 dices are behave independently of each other, the result of the second die will be totally independent from the first die. Because they behave independently, they are not related to each other.

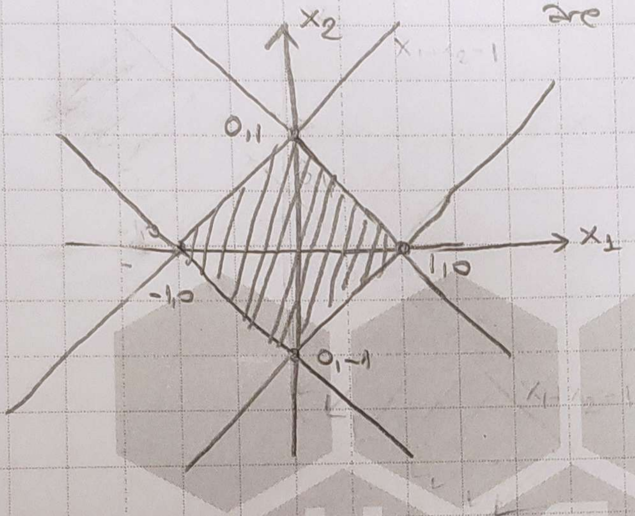
İsmail Öksüz
150119516

7 Linear Algebra

7.1 - Norms

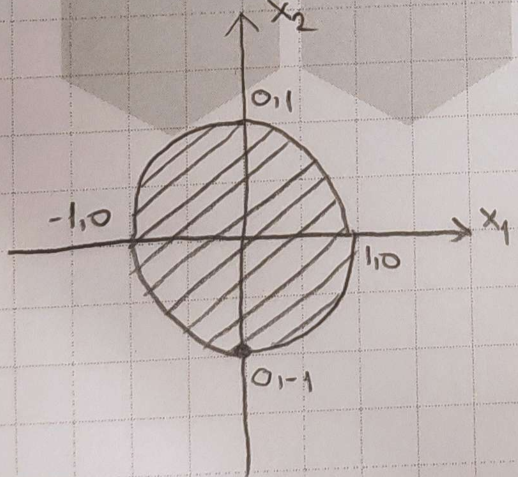
1- $\|x\|_1 \leq 1$, $\|x\|_1 = \sum_i |x_i| \Rightarrow |x_1| + |x_2| \leq 1$

intersection points on the top of region are $(1,0)$, $(0,1)$, $(0,-1)$ and $(-1,0)$



2- $\|x\|_2 \leq 1$, $\|x\|_2 = \sqrt{\sum_i x_i^2} \Rightarrow x_1^2 + x_2^2 \leq 1$

$(0,1)$, $(1,0)$, $(-1,0)$, $(0,-1)$ are the points intersected with axis on top of the region.



EĞİTİM KURUMLARI

İsmail Öksüz
150119516 ....../....../......

3- $\|x\|_\infty \leq 1$, $\|x\|_\infty = \max_i |x_i|$ $\Rightarrow$ $\underbrace{\max\{|x_1|,|x_2|\} \leq 1}$

if $x_1 > x_2$, we take x_1. $x_1 \leq 1$

if $x_2 > x_1$, we take x_2. $x_2 \leq 1$

else, let $\underbrace{m = x_1 = x_2 \quad m \leq 1}_{x_1 \leq 1 \ \& \ x_2 \leq 1}$