System of Linear Equations with 2 Variable

Systems of equations

System 1

$a + b = 13$

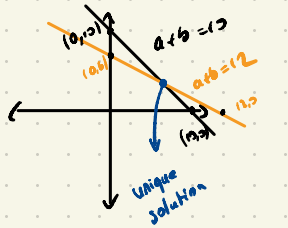
$a + b = 12$

Unique solution:

$a = 8$

$b = 2$

complete and non-singular



every line brings something new

System 2

$a+b = 10$

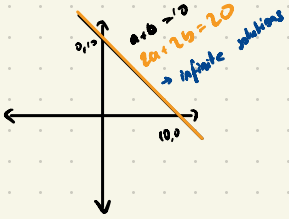
$2a+2b = 20$

Infinite Solution

$a = 8, 2, 10, \dots$

$b = 2, 3, 1, \dots$

redundant and singular



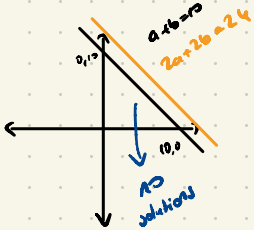
System 3

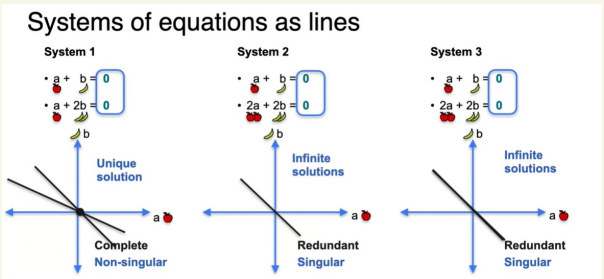
$a+b = 13$

$2a+2b = 24$

No solution

contradictory and singular





* if two lines intersect at a unique point, system will be non-singular

Linear

$6.4b = 10$

$2a + 3b = 15$

$3.4a - 46.99b + 2c = 122.5$

Non-Linear

$a^2 + b^2 = 13$

$\sin(a) + b^3 = 1$

$2^a - 3^b = 0$

Matrix

system 1

$a+b=0$
$a+2b=0$

$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$

Rows are linearly independent

determinant: $1\times2-1\times1=1$

system 2

$a+b=0$
$2a+2b=0$

$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$ → second row is a multiple of the first row

Rows are linearly dependent

determinant: $1\times2-2\times1=0$

*** if determinant 0, matrix is singular ; if determinant is non-0, matrix is non-singular

System 1

$a+b+c=10$
$a+2b+c=15$
$a+b+2c=12$

$a=3$
$b=5$
$c=2$

Unique solution

$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$

no relations
Independent (non-singular)

System 2

$a+b+c=10$
$a+b+2c=15$
$a+b+3c=20$

$c=5$
$a+b=5$
(0,0,5), (1,4,5), (2,3,5)...

Infinitely many solution

$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 3 & 2 & 3 \end{bmatrix}$

3Row1+2Row2-Row3
Dependent (singular)

System 3

$a+b+c=10$
$a+b+2c=15$
$a+b+3c=18$

$c=5$
$c=3$

No solution

$\begin{bmatrix} 1 & 2 & 5 \\ 0 & 3 & -2 \\ 2 & 4 & 10 \end{bmatrix}$

2Row1=Row3
Dependent (singular)

System 4

$a+b+c=10$
$2a+2b+2c=20$
$3a+3b+3c=30$

Infinitely many solutions

*** if determinant 0, matrix is singular ; if determinant is non-0, matrix is non-singular

3x3 matrix det.

$\begin{bmatrix} -2 & -1 & 4 \\ 6 & -3 & -2 \\ 4 & 1 & 2 \end{bmatrix}$ $\begin{matrix} -2 & -1 \\ 0 & -3 \\ 4 & 1 \end{matrix}$

-48 +4 -12
12 8 24

$(12+8+24)-(-48+4+(-12))$
$=100$

$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 3 & 3 \end{bmatrix}$ → Triangular matrices

main diagonal product = determinant

$1\times2\times3=6=\det.$