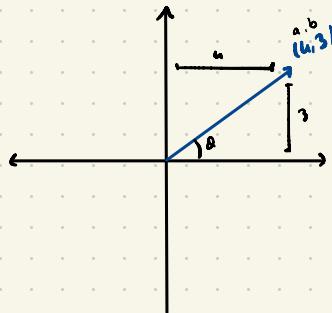


# Vectors

A vector is simply group numbers. Vectors have magnitude (norm) and direction



magnitude: length of vector

$$L-1 \text{ norm: } \|u, v\| = |a| + |b|$$

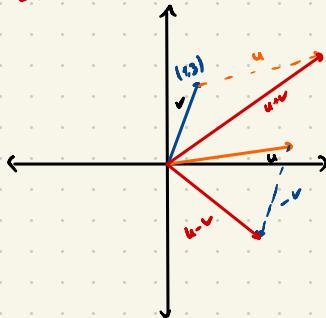
$$L-2 \text{ norm: } \|u, v\|_2 = \sqrt{a^2 + b^2}$$

most of time we use this

direction:  $\tan(\theta) = \frac{b}{a}$

$$\theta = \arctan(\frac{b}{a}) = 0.64 = 36.87^\circ$$

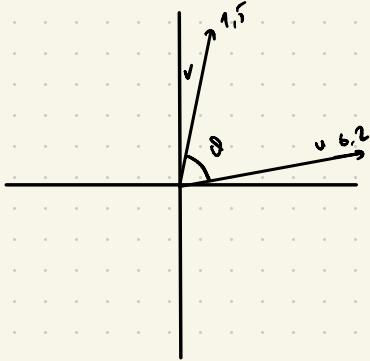
## Sum and Difference



$$\text{sum: } u+v = (4+1, 1+2) = (5, 3)$$

$$\text{difference: } u-v = (4-1, 1-2) = (3, -1)$$

## Distance Between Vectors



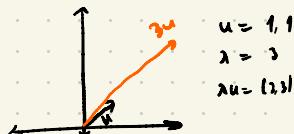
$$L1\text{-Distance: } \|u-v\|_1 = |5| + |3| = 8$$

$$L2\text{-Distance: } \|u-v\|_2 = \sqrt{5^2 + 3^2} = 5.83$$

Cosine Distance:  $\cos(\theta)$

If  $\cos \text{ dist} = 1$  that means two vector have 100% similarity  
if two vector is perpendicular to each other, similarity is 0

## Multiplying with scalar



$$u = 1, 1$$

$$\lambda = 3$$

$$\lambda u = (3, 3)$$

## Dot Product

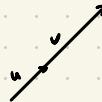
$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 5 \end{bmatrix} = 67 \quad u \cdot v = u_1 \cdot v_1 + u_2 \cdot v_2 + \dots + u_n \cdot v_n$$

$$L^2 - \text{Norm} = \sqrt{\text{dot product}(u, u)}$$

$$\|u\|_2 = \sqrt{\langle u, u \rangle}$$



$$\langle u, u \rangle = \|u\|^2 = \|u\| \cdot \|u\|$$



$$\langle u, v \rangle = \|u\| \cdot \|v\|$$



$$\langle u, v \rangle = 0$$



$$\begin{aligned} \langle u, v \rangle &= \|u\| \cdot \|v\| \\ &= \|u\| \cdot \|v\| \cdot \cos \theta \end{aligned}$$

dot product indicates that two vector has same direction or not

if dot product is positive then two vector have same direction

if dot product is negative, two vector have opposite direction

## Multiplying a matrix by a vector

$$a+b+c=10$$

$$a+2b+c=15$$

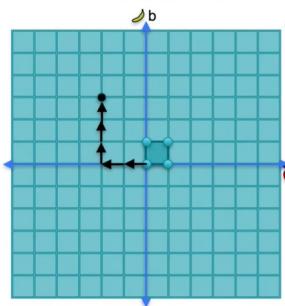
$$a+b+2c=12$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 10 \\ 15 \\ 12 \end{bmatrix}$$

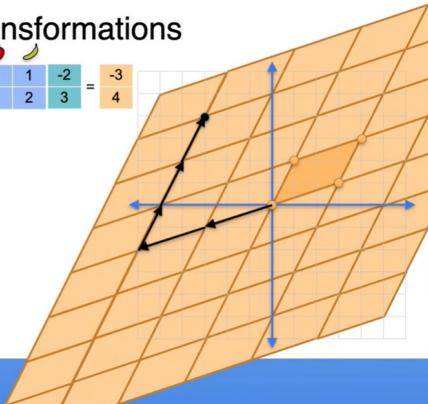
## Linear Transformation

(Linear transformations are works similar like functions  
matrix  $\times$  vector  $\rightarrow$  result ; equations  $\times$  f(x)  $\rightarrow$  result)

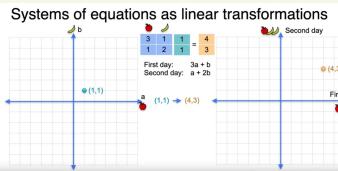
### Matrices as linear transformations



$$\begin{bmatrix} 3 & 1 & -2 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$



- $(0,0) \rightarrow (0,0)$
- $(1,0) \rightarrow (3,1)$
- $(0,1) \rightarrow (1,2)$
- $(1,1) \rightarrow (4,3)$



in linear transformation, we alter the entire space

## Matrix Multiplication

Matrix multiplication corresponds to combining two linear transformations into a third one.

The diagram shows the multiplication of two 2x2 matrices. On the left, a blue matrix  $\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}$  and an orange matrix  $\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$  are multiplied. The result is shown on the right as a 2x2 grid of four smaller boxes. The top-left box contains the product of the first row of the first matrix and the first column of the second matrix, which is  $\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 0 & 4 \end{bmatrix}$ . The other three boxes are zero matrices, indicating that the second transformation (orange) does not affect the result of the first transformation (blue).

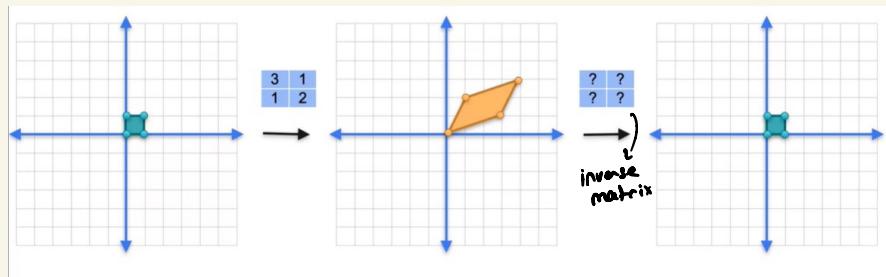
## Identity Matrix

The identity matrix is the matrix that when multiplied by any other matrix, it gives the same matrix.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

## Matrix Inverse

In linear transformation, the inverse matrix is the one that undoes the job of the original matrix, namely the one that returns the plane to where it was at the beginning.



$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

↑  
inverse  
matrix  
 $\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$

| if matrix is non-singular and  $\det \neq 0$   
| this matrix have inverse

| if matrix is singular and  $\det = 0$  this matrix doesn't have inverse