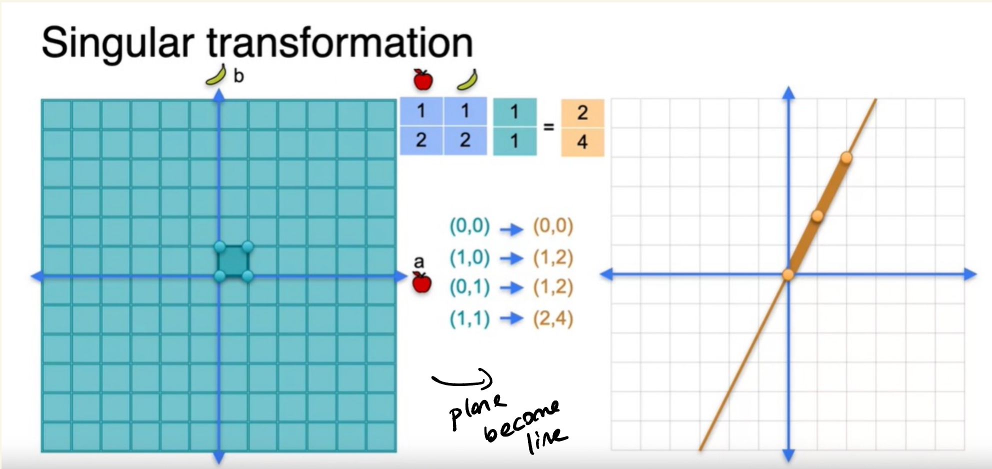
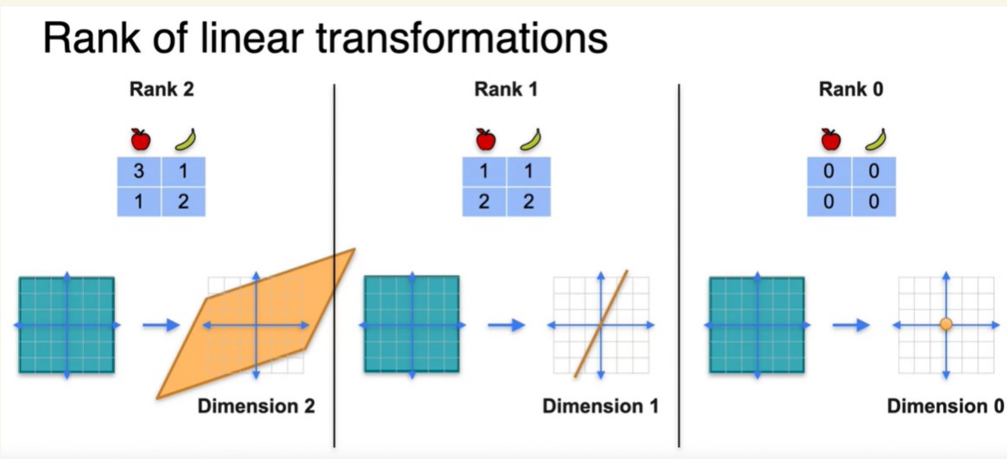
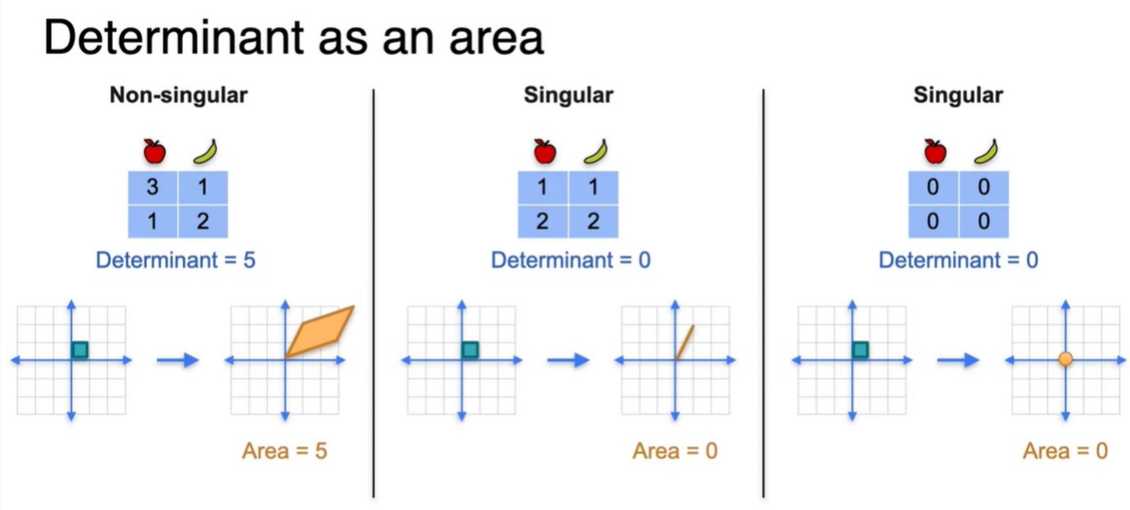
Determinants



if matrix is singular, if we implement transformation new matrix does not cover entire plane (space)

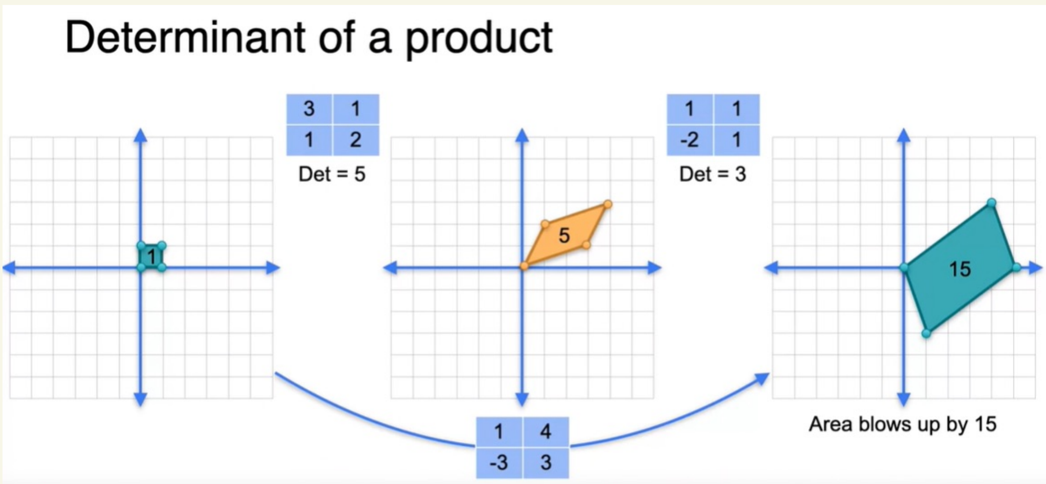


singular non- singular non - singular



determinant indicates change of volume of space. if $det=2$, that means space grow 2 times

if determinant is non-zero than our matrix is non-singular

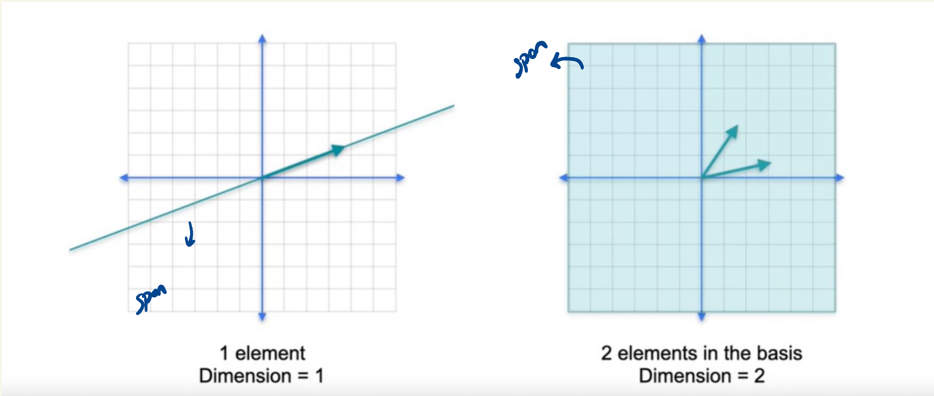


* $det(AB) = det(A) . det(B)$

Eigenvalues and Eigenvectors → extremely important for PCA

Every point in the space can be expressed as a linear combination of elements in the basis

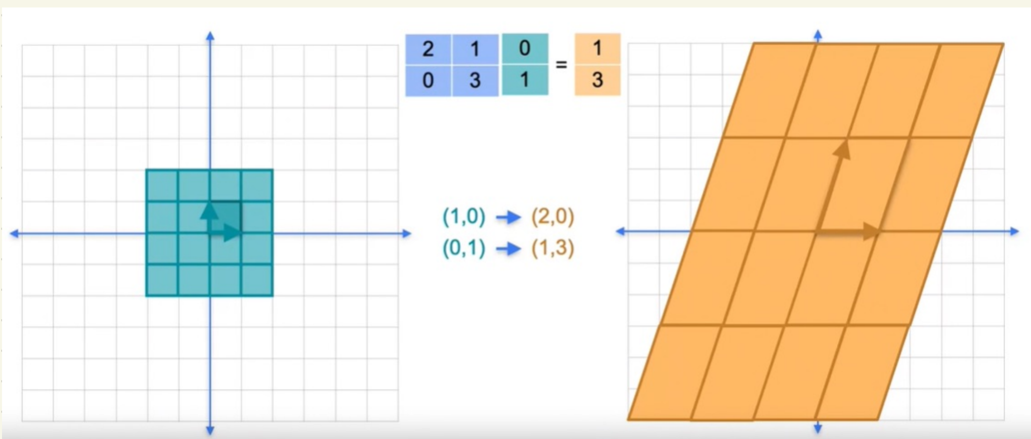
Span is basically set of points that can be reach with direction of basis vectors in any combination.

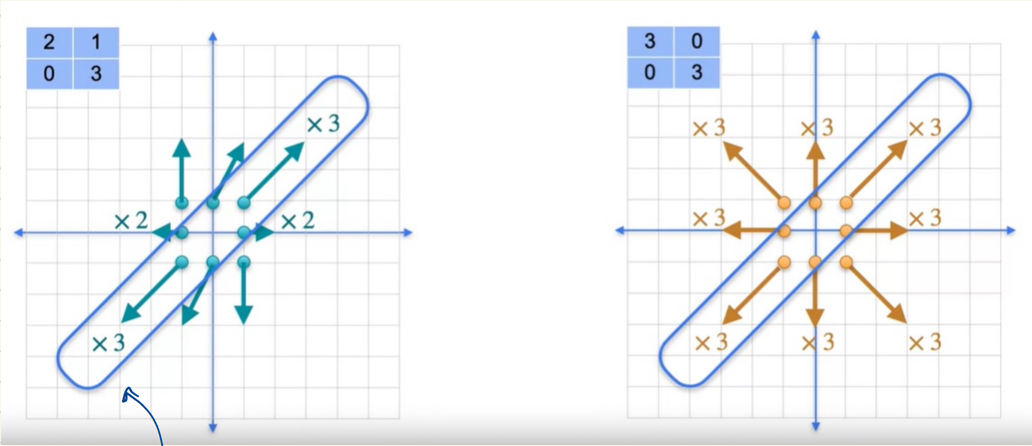


Eigen basis

some basis are more useful than others. but eigenbasis is the most useful among them

eigenbasis is basis of eigenvector





these two transformations are not the same but they act the exact same way for a specific line. And we call this line as eigen vector, and eigenvalue as value of change of this vector in transformation

exmp: find eigen values and eigen vectors of this matrix : $\begin{bmatrix} 9 & 4 \\ 4 & 3 \end{bmatrix}$

first step

$$\begin{bmatrix} 9 & 4 \\ 4 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 9-\lambda & 4 \\ 4 & 3-\lambda \end{bmatrix}$$

second step

$$\det \begin{bmatrix} 9-\lambda & 4 \\ 4 & 3-\lambda \end{bmatrix} = (9-\lambda).(3-\lambda) - 4.4 = 0$$

characteristic polynomial

eigenvalues $\begin{cases} \lambda = 11 \\ \lambda = 1 \end{cases}$

third step

$$\begin{bmatrix} -2 & 4 \\ 4 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} 8 & 4 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad \dots$$