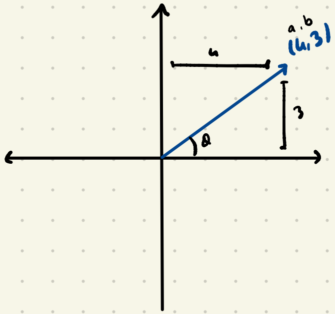
Vectors

A vector is simply group numbers. Vectors have magnitude (norm) and direction



magnitude: length of vector

L-1 norm: $\|(a,b)\| = |a| + |b|$

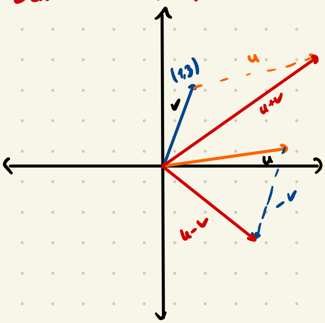
L-2 norm: $\|(a,b)\|_2 = \sqrt{a^2+b^2}$

↳ most of time we use this

direction: $\tan(\theta) = 3/4$

$\theta = \arctan(3/4) = 0.64 = 36.87°$

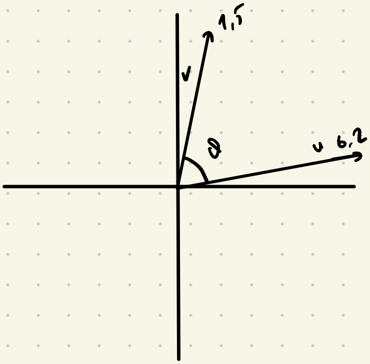
Sum and Difference



sum: $u+v = (4+1,\ 1+3) = (5,4)$

difference: $u-v = (4-1,\ 1-3) = (3,-2)$

Distance Between Vectors



L1 - Distance: $|u-v|_1 = |5| + |-3| = 5$

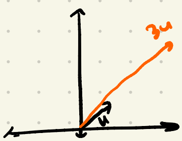
L2 - Distance: $|u-v|_2 = \sqrt{5^2+3^2} = 5.83$

Cosine Distance: $\cos(\theta)$

↳ if cos dist = 1 that means two vector have 100% similarity

if two vector is perpendecular to each other, similarity is 0

Multiplying with scalar



$u = 1, 1$

$\lambda = 3$

$\lambda u = (3,3)$

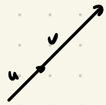
Dot Product

$\begin{bmatrix}1\\2\\3\end{bmatrix} \cdot [4 \ 5 \ 6]$ $\quad u \cdot v = u_1 \cdot v_1 + u_2 \cdot v_2 + \dots + u_n \cdot v_n$

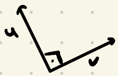
$L2\text{-norm} = \sqrt{\text{dot product}(u,u)}$

$|u|_2 = \sqrt{\langle u,u \rangle}$

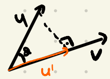
$\langle u,u \rangle = |u|^2 = |u| \cdot |u|$



$\langle u,v \rangle = |u| \cdot |v|$



$\langle u,v \rangle = 0$



$\langle u,v \rangle = |u'| \cdot |v|$

$= |u| \cdot |v| \cdot \cos\theta$

dot product indicates that two vector has same direction or not

if dot product is positive than two vector have same direction

if dot product is negative, two vector have opposite direction

Multiplying a matrix by a vector

$a + b + c = 10$

$a + 2b + c = 15$

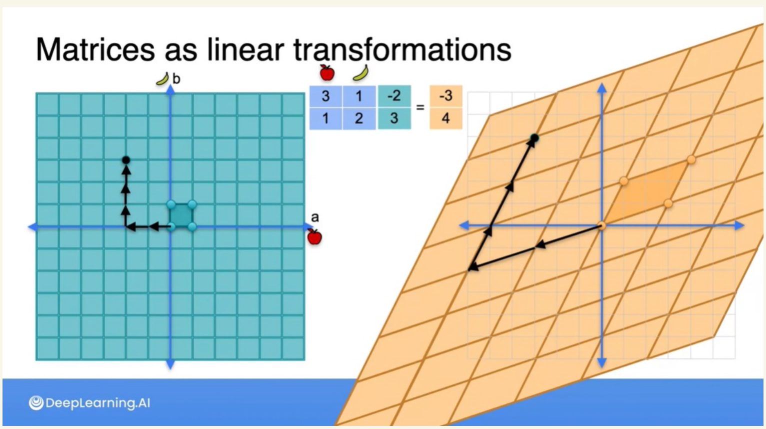
$a + b + 2c = 12$

$\Rightarrow \begin{bmatrix}1&1&1\\1&2&1\\1&1&2\end{bmatrix} \cdot \begin{bmatrix}a\\b\\c\end{bmatrix} = \begin{matrix}10\\15\\12\end{matrix}$

Linear Transformation

(linear transformations are works similar like functions

matrix x vector → result , equations , f(x) → result)

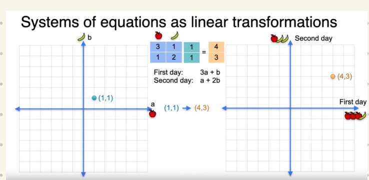


$(0,0) \to (0,0)$

$(1,0) \to (3,1)$

$(0,1) \to (1,2)$

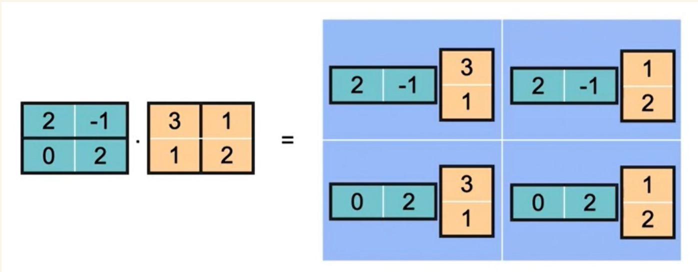
$(1,1) \to (4,3)$



in linear transformation, we alter the entire space

Matrix Multiplication

Matrix multiplication corresponds to combining two linear transformations into a third one



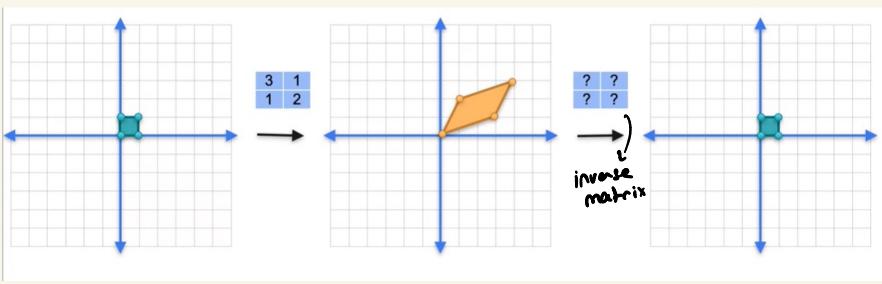
Identity Matrix

The identity matrix is the matrix that when multiplied by any other matrix, it gives the same matrix.

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix} \quad \begin{bmatrix}1&0\\0&1\end{bmatrix} \quad \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

Matrix Inverse

In linear transformation, the inverse matrix is the one that undoes the job of the original matrix, namely the one that returns the plane to where it was at the beginning



$$\begin{bmatrix}3&1\\1&2\end{bmatrix} \cdot \begin{bmatrix}a&b\\c&d\end{bmatrix} = \begin{matrix}1&0\\0&1\end{matrix}$$

inverse matrix $\begin{bmatrix}3&1\\1&2\end{bmatrix}^{-1}$

if matrix is non-singular and det $\neq 0$ this matrix have inverse

if matrix is singular and det $= 0$ this matrix doesn't have inverse