

Solving System of Linear Equations

Original system

$$\begin{aligned} 5a + b &= 12 \\ 4a - 3b &= 6 \end{aligned}$$

$$\begin{bmatrix} 5 & 1 \\ 4 & -3 \end{bmatrix}$$

Original matrix

Intermediate System

$$\begin{aligned} a + 0.2b &= 3.4 \\ b &= 2 \end{aligned}$$

$$\begin{bmatrix} 1 & 0.2 \\ 0 & 1 \end{bmatrix}$$

upper diagonal matrix
row echelon form

Solved system

$$\begin{aligned} a &= 3 \\ b &= 2 \end{aligned}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

diagonal matrix
reduced row echelon form

$$\begin{aligned} a + b &= 10 \\ 2a + 7b &= 20 \end{aligned}$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 7 \end{bmatrix}$$

$$\begin{aligned} a + b &= 10 \\ 0a + 0b &= 0 \end{aligned}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

row echelon form

$$\begin{aligned} 5a + b &= 11 \\ 0a + 2b &= 22 \end{aligned}$$

$$\begin{bmatrix} 5 & 1 \\ 0 & 2 \end{bmatrix}$$

$$\begin{aligned} a + 0.2b &= 2.2 \\ 0a + 0b &= 0 \end{aligned}$$

$$\begin{bmatrix} 1 & 0.2 \\ 0 & 0 \end{bmatrix}$$

row echelon form

Row Echelon Form

$$\begin{bmatrix} 1 & * & * & * & * \\ 0 & 1 & * & * & * \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Main diagonal can be all 1, all 0 or bunch of 1 and zero

below diagonal should be 0

right of the diagonal: can be anything if diagonal 1
or be zero if diagonal 0

Row operations

Switching Rows

$$\begin{vmatrix} 5 & 1 \\ 4 & 3 \end{vmatrix} \rightarrow \begin{vmatrix} 4 & 3 \\ 5 & 1 \end{vmatrix}$$

$\det = 5 \cdot 3 - 1 \cdot 4 = 11$ $\det = 4 \cdot 1 - 3 \cdot 5 = -11$

* new det. always negative of the original det.

Multiplying a row by (non-zero) scalar

$$\begin{vmatrix} 5 & 1 \\ 4 & 3 \end{vmatrix} \times 10 \rightarrow \begin{vmatrix} 50 & 10 \\ 4 & 3 \end{vmatrix}$$

$\det = 5 \cdot 3 - 1 \cdot 4 = 11$ $\det = 5 \cdot (10 \cdot 3) - 1 \cdot (10 \cdot 4) = 10 \cdot 11$

multiplying matrix new det., multiplying number \times old det.

Adding a row to another row

$$\begin{vmatrix} 5 & 1 \\ 4 & 3 \end{vmatrix} \rightarrow \begin{vmatrix} 9 & 4 \\ 4 & 3 \end{vmatrix}$$

$\det = 5 \cdot 3 - 4 \cdot 1 = 11$ $\det = 9 \cdot 3 - 4 \cdot 4 = 11$

* adding one row to another row don't change the det

*** these operations preserve singularity and non-singularity

Rank of a matrix

- The maximum number of its linearly independent columns (or rows) of a matrix

System 1

$$\begin{aligned} a + b &= 0 \\ a + 2b &= 0 \end{aligned}$$

Two equations

Two pieces of information
Rank = 2

System 2

$$\begin{aligned} a + b &= 0 \\ 2a + 2b &= 0 \end{aligned}$$

Two equations

One piece of information
Rank = 1

System 3

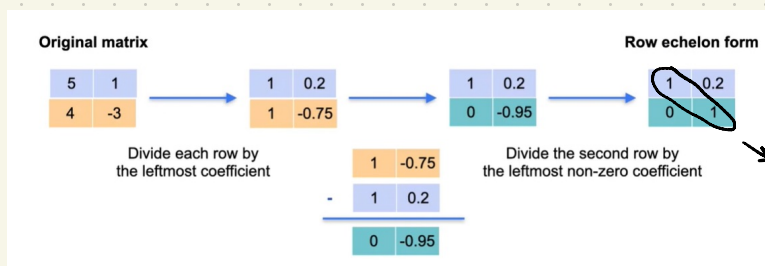
$$\begin{aligned} 0a + 0b &= 0 \\ 0a + 0b &= 0 \end{aligned}$$

Two equations

Zero piece of information
Rank = 0

*** a matrix is non-singular if and only if it has full rank

Row echelon form for finding rank



Rank of a row echelon form matrix is number of 1 in main diagonal

★ - A matrix is non-singular if main diagonal only contain 1

Rank of a matrix examples:

Matrix 1

1	1	1
1	2	1
1	1	2

Rank = 3

Matrix 2

1	1	1
1	1	2
1	1	3

Rank = 2

Matrix 3

1	1	1
2	2	2
3	3	3

Rank = 1

Matrix 4

0	0	0
0	0	0
0	0	0

Rank = 0

Row echelon forms

1	1	1
0	1	0
0	0	1

Number of pivots = 3

1	1	1
0	0	1
0	0	0

Number of pivots = 2

1	1	1
0	0	0
0	0	0

Number of pivots = 1

0	0	0
0	0	0
0	0	0

Number of pivots = 0

Reduced row echelon form

