Quantitative Financial Economics

Matlab tutorials

Problem set on present value models

Apply the VAR approach to test the Expectations Hypothesis on the US bond market. The data contains annual observations from 1954 to 2019 on the yield spread between 10-year Treasury notes and 1-year Treasury bills, as well as the first difference of the latter. All variables are measured in logs. This problem set is a natural extension of the analysis performed on the US stock market in the Matlab program already available on Blackboard. You can solve the problem set by making prober adjustments of the already provided program. Hint: Make use of the symbolic toolbox to derive the restrictions and their derivatives.

Variable	Description	Variable	Description
spr	Yield spread $\left(y_t^{(10)} - y_t^{(1)} \right)$	dy1	First difference of the short yield $\left(y_t^{(1)} - y_{t-1}^{(1)} \right)$

1. Estimate the VAR(1) model

$$\begin{bmatrix} s_{t+1}^{(10)} \\ \Delta y_{t+1}^{(1)} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} s_t^{(10)} \\ \Delta y_t^{(1)} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t+1} \\ \varepsilon_{2,t+1} \end{bmatrix}$$

   where $s_t^{(10)} = y_t^{(10)} - y_t^{(1)}$ and $\Delta y_t^{(1)} = y_t^{(1)} - y_{t-1}^{(1)}$.

2. Test the restrictions imposed on the present value model by the Expectations Hypothesis

$$e1' = e2'A \left[I - \frac{1}{10}(I - A^{10})(I - A)^{-1} \right] (I - A)^{-1}$$

   where $e1' = \begin{bmatrix} 1 & 0 \end{bmatrix}$, $e2' = \begin{bmatrix} 0 & 1 \end{bmatrix}$, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, and $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$.

3. Compute the theoretical yield spread as

$$s_t'^{(10)} = e2'A \left[I - \frac{1}{10}(I - A^{10})(I - A)^{-1} \right] (I - A)^{-1} Z_t$$

   where $Z_t = \begin{bmatrix} s_t^{(10)} \\ \Delta y_t^{(1)} \end{bmatrix}$ and report $corr\left(s_t^{(10)}, s_t'^{(10)} \right)$ and $var\left(s_t^{(10)} \right) / var\left(s_t'^{(10)} \right)$.
   Plot also the actual and theoretical yield spreads together in a figure.