Quantitative Financial Economics

Matlab tutorials

Problem set on time-varying risk premia

1. The purpose of this problem is to analyze the predictive ability of the Cyclically Adjusted Price-Earnings (CAPE) ratio for future stock returns. The CAPE ratio is also known as the price-smoothed-earnings ratio or as the Shiller P/E ratio and the variable is available for free download at Robert Shiller's website. The CAPE ratio is defined as the real stock price divided by average real earnings over a ten-year period. It has been used in a series of articles by John Campbell and Robert Shiller to examine long-horizon stock market predictability.

a) Estimate long-horizon predictive regressions:

$$r_{t\to t+k} = \alpha_k + \beta_k x_t + \varepsilon_{t\to t+k} \tag{1}$$

where $r_{t\to t+k}$ is the log excess return on the US stock market from time t to $t+k$ and x_t is the log CAPE ratio at time t.[1] Consider horizons in the range from one month up to ten years: $k = 1$, 6, 12, 24, 36, 48, 60, 72, 84, 96, 108, and 120. Report and compare the β_k coefficients and R^2 statistics across the forecast horizons. All necessary data to estimate (1) are available in the excel file "`TimeVaryingRiskPremia.xlsx`". The sample period is from 1926:m7 to 2020:m7.

b) The use of overlapping data in (1) leads to autocorrelation in the error term. To address this issue, we can use e.g. the Newey-West estimator to compute t-statistics across the different forecast horizons. To examine the effect of how standard errors are computed in long-horizon regressions, try with two different lag-length specifications in the Newey-West estimator. First, try to set the lag length in the Newey-West estimator equal to the forecast horizon and then afterwards try with no lags in the Newey-West estimator.

c) Similar to the price-dividend ratio, the CAPE ratio is highly persistent and slow to mean-revert, implying that forecasts build up over time. Make two scatter plots where you plot the time t log CAPE ratio against the one-month ahead log excess return ($r_{t\to t+1}$) and the ten-year ahead log excess return ($r_{t\to t+120}$), respectively.

d) In-sample evidence of time-varying expected excess returns does not imply that it is possible to predict returns out-of-sample. Use an out-of-sample period from 1990:m1 to 2020:m7 to check the out-of-sample predictive power of the log CAPE ratio by computing the out-of-sample R^2 and Clark and West test statistic for the $k = 1$, 6, and 12 horizons. In addition, plot the Goyal and Welch (2008) cumulative-squared-error-difference figure for $k = 1$.

[1] The conclusions from the regression output do not depend on whether one uses the CAPE ratio in levels or use a log version of the CAPE ratio.

2. Lettau and Ludvigson (2001) find that there is a cointegration relationship between consumption (c_t), financial asset wealth (a_t), and income (y_t). They show that the estimated cointegration residual $\widehat{cay}$ has the ability to capture time-varying expected returns on the US stock market. The excel file "`TimeVaryingRiskPremia.xlsx`" contains the log excess return on the S&P500 index as well as the original $\widehat{cay}$ data used by Lettau and Ludvigson (2001) with a sample period from 1952:q4 to 1998:q3.

a) Estimate the predictive regression model:

$$r_{t \to t+k} = \alpha_k + \beta_k \widehat{cay}_t + \varepsilon_{t \to t+k} \tag{2}$$

where $r_{t \to t+k}$ is the k-period ahead log excess return. Is β_k statistically significant across horizons? Compare your results with Table VI (row 2) in Lettau and Ludvigson (2001).[2]

b) It is important to take into account small sample bias in order to be able to conduct valid inference from predictive regressions. Small sample bias in predictive regressions is particularly severe for financial predictive variables such as the CAPE ratio, the price-dividend ratio and other predictive variables scaled by price, but often found to be less severe for macroeconomic predictive variables such as the $\widehat{cay}$ ratio. To judge the degree of small sample bias in the predictive regression in (2), conduct a bootstrap analysis where you bootstrap under the null hypothesis of no predictability and assume an AR(1) data-generating process for $\widehat{cay}$:

$$r_{t+1} = \alpha + \varepsilon_{t+1} \tag{3}$$

$$\widehat{cay}_{t+1} = \mu + \phi \widehat{cay}_t + \eta_{t+1} \tag{4}$$

Compute $N = 10,000$ artificial estimates of the slope coefficients under the null of no predictability and then compute the degree of bias in β_k as well as one-sided empirical p-values across the different forecast horizons. In addition, make a histogram of the bootstrapped slope coefficients for $k = 1$. Based on the output from your bootstrap analysis, do the predictive regression in (2) suffer from small sample bias?

Reference list

Goyal, A. and I. Welch (2008). A comprehensive look at the empirical performance of equity premium prediction. *Review of Financial Studies* 21, 1455-1508.

Lettau, M. and S. Ludvigson. (2001). Consumption, aggregate wealth and expected stock returns. *Journal of Finance* 56, 815-849.

[2] You should not expect to find results exactly identical to those reported in Lettau and Ludvigson (2001) as we use a different risk-free rate proxy. In addition, Lettau and Ludvigson (2001) do not specify how many lags they use in the Newey-West estimator.