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Infiltration into soils: Conceptual approaches and solutions

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[1] Infiltration is a key process in aspects of hydrology, agricultural and civil engineering, irrigation design, and soil and water conservation. It is complex, depending on soil and rainfall properties and initial and boundary conditions within the flow domain. During the last century, a great deal of effort has been invested to understand the physics of infiltration and to develop quantitative predictors of infiltration dynamics. Jean-Yves Parlange and Wilfried Brutsaert have made seminal contributions, especially in the area of infiltration theory and related analytical solutions to the flow equations. This review retraces the landmark discoveries and the evolution of the conceptual approaches and the mathematical solutions applied to the problem of infiltration into porous media, highlighting the pivotal contributions of Parlange and Brutsaert. A historical retrospective of physical models of infiltration is followed by the presentation of mathematical methods leading to analytical solutions of the flow equations. This review then addresses the time compression approximation developed to estimate infiltration at the transition between preponding and postponding conditions. Finally, the effects of special conditions, such as the presence of air and heterogeneity in soil properties, on infiltration are considered.

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1. General Introduction

[2] When a wetting fluid enters in contact with dry porous medium, flow is induced in the porous material due to capillarity. When this movement occurs in the absence of gravity forces (as in horizontal flow, for example), it is considered as a "sorption" process. When it occurs under the influence of gravity, it is considered as an "infiltration" process. In the following review, we will limit ourselves to that specific case of wetting under gravity, and infiltration will be defined, following *Brutsaert* [2005], as "the entry of water into the soil surface and its subsequent vertical motion through the soil profile."

[3] Infiltration is a complex process, which depends upon a large number of factors: water supply rate; the elapsed time since the onset of water application; soil and water chemical compositions; spatial variability and distribution of the hydraulic properties within the soil profile; initial and boundary conditions; topography; temperature; and probably additional factors linked to biological and microbiological activities in the soil. Wetting of a porous medium consists in a multitude of events where the wetting fluid invades fully or partially empty pores, crevices, or cavities. It is thus a phenomenon that involves a series of microscale processes that affect its macroscale behavior. However, most of our conceptual and quantitative tools regarding the infiltration process rely on observations and measurements pertaining to that macroscale reflection of the events occurring at the pore scale. This introduces an intrinsic "tension" between the phenomenon in reality and the conceptual approaches and models developed during the years to describe it. This tension is exacerbated in some specific cases when the conditions induce flow instability, and the models reach their limits of applicability.

[4] Different types of approaches were developed to provide quantitative tools able to describe and predict infiltration into porous media in general, and soils in particular. Based on a large body of experimental studies that provided the community with valuable data from laboratory and field experiments, empirical expressions were first proposed. Then, mathematical solutions were derived from the basic physical model expressed in terms of differential equations. With the apparition of computers, numerical models were also developed that could solve directly these differential equations, thus releasing some of the assumptions and constraints needed to reach amenable mathematical solutions.

[5] Infiltration can accommodate the entire water supply rate or only part of it. The "infiltration capacity," or the potential infiltration rate, of a soil [*Horton*, 1940] is the maximal rate at which the soil surface can absorb water. Therefore, when the water supply exceeds the infiltration capacity, only part of it infiltrates and the remaining part ponds on the soil surface or runs off according to local topography. The infiltration capacity function can be thus considered a soil characteristic with dependence on the initial soil water content profile. When infiltration is below its capacity, the infiltration function will also depend upon the temporal history of the application rate. Figure 1 illustrates

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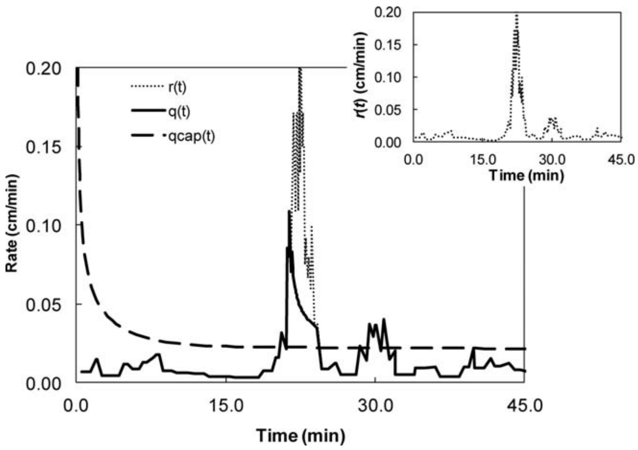

Figure 1. Infiltration capacity $q_{\mathrm{cap}}(t)$ (dashed curve) and actual simulated infiltration rate $q(t)$ (solid line) of a sandy loam soil profile exposed to a rainfall event with variable intensity $r(t)$ (dotted line). The rainfall intensity $r(t)$ is depicted in the inset to show that for low $r(t)$ values, $q(t) = r(t)$.

these different infiltration regimes. The dashed curve represents the infiltration capacity $q_{\mathrm{cap}}(t)$ of a sandy loam soil profile. The dotted line depicts the temporal variation of rainfall intensity, $r(t)$, measured during a natural rainfall event in the Coastal Plain, Central Israel, during winter 2003 (see also inset in Figure 1). The solid line shows the simulated infiltration rate $q(t)$ during that event (details can be found in *Assouline et al.* [2007]). At low rainfall intensities, all the rainfall can infiltrate into the soil, and $r(t)$ is similar to $q(t)$. At high rainfall intensities, only part of the rainfall could infiltrate, and the difference between $r(t)$ and $q(t)$ represents the excess rainfall rate that will produce runoff. There is no apparent simple relationship between $q_{\mathrm{cap}}(t)$ and runoff formation as the actual $q(t)$ depends on the specific temporal distribution of $r(t)$.

[6] Infiltration is a key component of the water budget equation. Accurate estimate of infiltration rate is crucial in hydrology, agricultural and civil engineering, irrigation design, and soil and water conservation. Infiltration is the vector for solutes into the soil profile and is a determining factor for their concentration in the runoff. These solutes can be nutrients indispensable to plant growth or pollutants decreasing the quality of soil and water resources. Therefore, infiltration rate is one of the principal variables in transport processes in soils and influences fertilization efficiency and environmental quality and conservation issues. One can thus easily understand the tremendous efforts invested in understanding the physics and developing mathematical and numerical tools capable of offering reliable and accurate quantitative descriptions of this process. These efforts produced a huge body of research, and only part of the published papers and reports will be reviewed in the following.

[7] Several reviews on different aspects of sorption and infiltration into soils were written in the past [*Gardner*, 1960; *Philip*, 1969, 1974; *Parlange*, 1980; *Skaggs*, 1982; *Parlange et al.*, 1999a; *Raats*, 2001; *Raats et al.*, 2002; *Clothier*, 2001; *Hopmans et al.*, 2007; *Barry et al.*, 2007], and these topics are key chapters in textbooks [*Childs*, 1969; *Bear*, 1972; *Chow et al.*, 1988; *Hillel*, 1998; *Warrick*, 2003; *Brutsaert*, 2005; *Delleur*, 2007]. The aim of this review is to retrace some of the main steps in building up our conceptual perception of the infiltration process and mathematical ability to describe it, while highlighting the seminal contributions of Parlange and Brutsaert to this field.

[8] This review will provide first a brief historical retrospective of the evolution of the basic physical model of infiltration and will present methods that lead to mathematical solutions of the flow equations. This review then addresses the time compression approximation (TCA) developed to estimate infiltration at the transition between preponding and postponding conditions. Finally, it will deal with effects of special conditions on infiltration, such as the presence of air and heterogeneity in soil properties.

2. Evolution of the Basic Physical Model of Infiltration: Brief Historical Retrospective

2.1. Darcy [1856]

[9] *Darcy* [1856], a French engineer of the city of Dijon, conducted experiments on water filtration through sand beds. Based on his observations, he formulated the first empirical quantitative description of flow through a saturated porous medium, known as *Darcy's law*:

$$f = \frac{F}{A} = \frac{V}{At} = -K_s \frac{\Delta\Phi}{L}, \tag{1}$$

where f is the flux of water (the discharge rate F flowing through a cross-sectional area A), V is the cumulative volume of water flowing at time t, K_s is a proportionality constant characterizing the medium and named "the saturated hydraulic conductivity," and $\Delta\Phi$ is the difference in total hydraulic head between two points separated by a distance L within the saturated porous medium (in saturated soils, Φ is the sum of pressure and elevation heads). For saturated conditions, the flux is thus conceptualized as the product of a constant which is a characteristic of the porous medium, and the prevailing hydraulic head gradient. It is well accepted that this proportionality is valid for flows characterized by a Reynolds number smaller than 1.0 [*Brutsaert*, 2005]. Under these conditions, Darcy's law resembles *Poiseuille's* [1846] *law*, which is a solution of the Navier-Stokes equation for an isothermal steady laminar flow of an incompressible Newtonian fluid in a straight cylindrical tube with a "no-slip" condition at the tube wall (see detailed discussion in *Philip* [1970]). A correction of equation (1) for Reynolds numbers larger than 1.0 was proposed by *Forchheimer* [1930].

2.2. Buckingham [1907]

[10] *Buckingham* [1907] proposed to extend Darcy's law to unsaturated water flow, where the actual water content in the porous medium, θ, is lower than its maximal value at saturation, θ_s. The main assumption is that the constant saturated hydraulic conductivity value K_s could be replaced by a function of soil water content θ or capillary potential ψ as the characteristic of the unsaturated porous medium. That function was named "the unsaturated hydraulic conductivity

function (HCF)" and annotated $K(\theta)$ or $K(\psi)$. Following the notation of equation (1), the resulting unsaturated flow equation resulting from Buckingham's assumptions is

$$f = -K(\theta)\frac{\partial \Phi}{\partial z} = -K(\psi)\frac{\partial(\psi + z)}{\partial z} = -K(\psi)\left(\frac{\partial \psi}{\partial z} + 1\right), \quad (2)$$

where z is the vertical coordinate being positive upward, $z = 0$ representing a prescribed reference level. Given the $K(\theta)$ or $K(\psi)$ function, equation (2) can be solved for the flux, f, or for the corresponding distribution of the capillary potential in the soil profile, $\psi(z)$. A review of Buckingham's central ideas can be found in *Narasimhan* [2007].

2.3. Richards [1931]

[11] The general equation for flow through porous media, presented by *Richards* [1931], combines the flow equation of *Buckingham* [1907] (equation (2)) and the principle of continuity. The 3-D mathematical expression of this principle in a Cartesian coordinate system is

$$\frac{\partial \theta}{\partial t} = \nabla \cdot f = \left(\frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z}\right), \quad (3)$$

where ∇ is the divergence of f, and f_x, f_y, and f_z denote the components of the flux at any spatial location in the x, y, and z directions, respectively. Consequently, the 3-D expression of Richards' equation for vertical infiltration under isothermal conditions in a homogeneous, isotropic, and rigid porous medium where air can escape freely is

$$\begin{aligned}\frac{\partial \theta}{\partial t} = \nabla \cdot [K(\psi)\nabla\Phi] = \frac{\partial}{\partial x}\left(K(\psi)\frac{\partial \psi}{\partial x}\right) + \frac{\partial}{\partial y}\left(K(\psi)\frac{\partial \psi}{\partial y}\right) \\ + \frac{\partial}{\partial z}\left(K(\psi)\frac{\partial \psi}{\partial z}\right) + \frac{\partial K(\psi)}{\partial z}.\end{aligned} \quad (4)$$

The well-known and widely used 1-D expression of equation (4) for vertical infiltration is

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left[K(\psi)\left(\frac{\partial \psi}{\partial z} + 1\right)\right]. \quad (5)$$

[12] Equations (4) and (5) are highly nonlinear parabolic partial differential equations. To be solved, they require the definition of an additional characteristic of the porous medium, beside $K(\psi)$, which relates θ to ψ. This is the soil water retention curve (WRC) $\theta(\psi)$. In some cases, it is advantageous to express equation (4) or (5) in terms of θ rather than ψ, and a new characteristic of the porous medium is introduced, based on $K(\theta)$ and the derivative of the WRC, $d\theta/d\psi$, also known as the specific water capacity of the porous medium, $C(\theta)$:

$$D(\theta) = \frac{K(\theta)}{C(\theta)}, \quad (6)$$

where $D(\theta)$ denotes the water diffusivity of the porous medium [*Klute*, 1952]. The alternative expression of equation (5) is thus

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left[D(\theta)\frac{\partial \theta}{\partial z} + K(\theta)\right], \quad (7)$$

which is also known as the nonlinear Fokker-Planck diffusion convection equation.

[13] The $\theta(\psi)$, $K(\theta)$, and $D(\theta)$ functions represent the hydraulic properties of the porous medium. A succinct presentation of some of the mathematical expressions used to quantify these properties is given in Appendix A. One should note that solving for ψ (equation (5)) can be appropriate for heterogeneous soil profiles since ψ is a continuous variable across textural discontinuities, while solving for θ (equation (7)) corresponds to homogeneous soil profiles solely [*Russo*, 1998; *LaBolle and Clausnitzer*, 1999; *Talbot et al.*, 2004].

[14] Richards' equation represents the actual physical model that illustrates our conception of the infiltration process in porous media. It addresses only the macroscale behavior and is valid for a representative volume for which the prescribed hydraulic properties could be applied. The solution of Richards' equation requires the definition of initial and boundary conditions. When water is ponding at the soil surface, infiltration is governed by a concentration type boundary condition, also known as a Dirichlet boundary condition. When the water application rate is below the soil infiltration capacity, a flux or Neuman boundary condition can be applied at the soil surface. Since the equation is highly nonlinear, analytical solutions can only be derived for specific initial and boundary conditions and soil hydraulic properties. Consequently, solutions to practical flow problems generally require the use of numerical schemes designed to solve partial differential equations.

3. Expressions for the Infiltration Function

3.1. Empirical Expressions

[15] The importance of infiltration and the need to describe it quantitatively on one hand and the high nonlinearity of the flow equation (equation (5)) on the other hand led to the development of empirical expressions relating infiltration rate q to time t.

[16] For a constant wetting rate, experimental data show that $q(t)$ gradually decreases during wetting and tends toward a steady final rate q_f. Consequently, the different forms of the suggested infiltration equations are all monotonically decreasing functions based on the exponential or power law decays. The parameters of these functions do not generally have a physical meaning and are evaluated by fitting to experimental data.

[17] The most widely used empirical infiltration equation in hydrology was suggested by *Horton* [1933, 1939]:

$$q(t) = q_f + (q_o - q_f)\exp(-\alpha t), \quad (8)$$

where q decreases exponentially from its initial value q_o to its final one q_f with a decay constant of α. For $t \to \infty$, q_f can be associated with the hydraulic conductivity of the wetted soil layer. *Eagleson* [1970] and *Raudkivi* [1979] have shown that equation (8) can be derived from equation (7) if K and D are constants independent of θ.

[18] A widely used empirical infiltration equation in surface irrigation is the Lewis-Kostiakov-Mezencev set of equations [*Kostiakov*, 1932; *Lewis*, 1937; *Mezencev*, 1948] presenting the following general form:

$$q(t) = q_f + n\beta t^{(\beta-1)}, \quad (9)$$

where n and β are fitting parameters. These parameters can vary depending on the prevailing initial and boundary conditions of the system. *Furman et al*. [2006] modified equation (9) to account for different initial and boundary conditions in the soil profile.

3.2. Analytical and Quasi-Analytical Solutions of the Flow Equation

3.2.1. Time Expansion Approach

[19] *Philip* [1957, 1969] presented the first analytical solution to Richards' equation. It is based on a time expansion method considering infiltration as a sorption process with a perturbation generated by the presence of gravity. Therefore, this method corresponds by definition to the first stages of infiltration into a relatively dry soil profile where gravity plays only a minor role. The resulting expression of $q(t)$ is

$$q(t) = \frac{1}{2}A_0 t^{-1/2} + (A_1 + K_s) + \frac{3}{2}A_2 t^{1/2} + 2A_3 t + \frac{5}{2}A_4 t^{3/2} + \ldots, \tag{10}$$

where A_i are the functions of θ. The function A_0 is defined as the soil sorptivity, also noted as S:

$$A_0 = S = \int_{\theta_i}^{\theta_s} \zeta \, \mathrm{d}\theta \tag{11}$$

where θ_i is the initial water content in the soil profile, θ_s is the saturated water content of the soil, and ζ is the independent variable resulting from the *Boltzmann* [1894] transformation:

$$\zeta = z t^{-1/2}. \tag{12}$$

A truncated version of equation (10) considering the first two terms of the series is widely applied:

$$q(t) = \frac{1}{2}A_0 t^{-1/2} + (A_1 + K_s), \tag{13}$$

which is equivalent to equation (9) when $\beta = 1/2$, $n = A_0$, and $q_f = (A_1 + K_s)$.

[20] An exact solution for vertical infiltration into a dry porous medium was derived by *Brutsaert* [1968b] for simple algebraic functions for the soil hydraulic properties. The solution, a two-term equation where the first term represents the solution for sorption [*Brutsaert*, 1968a] and the second one represents the effect of gravity, accurately reproduced the experimental data of *Davidson et al*. [1963]. Because the solution in equation (10) is basically a perturbation series around the solution for sorption, it fails to behave properly for large values of t, when the role of gravity is preponderant. In the case of ponded infiltration, $(A_1 + K_s)$ should tend to K_s for large values of t. However, *Philip* [1969] and *Talsma and Parlange* [1972] have shown that $1/3K_s \leq (A_1 + K_s) \leq 2/3K_s$. This issue was discussed recently by *Triadis and Broadbridge* [2012].

[21] *Philip* [1957] derived also the large time traveling wave solution, which in combination with equation (10), provides a good solution for all time. In order to provide a measure as to when to swap between the two solutions, *Philip* [1969] introduced the "time of gravity" $t_G = A_0^2/4A_1{}^2$ that defines the time when gravitational flow dominated over capillary flow and provides the time limit for which equation (10) is valid. Infiltration models based on the traveling and kinematic wave solutions of the flow equation were also proposed by *Basha* [1999, 2011].

[22] For some cases (narrow pore size distributions), the time expansion method can behave properly for both small and large t values [*Brutsaert*, 1977] and could be approximated quite accurately by

$$q(t) = K_s + \frac{1}{2}St^{-1/2}\left(1 + \frac{\beta_0 K_s t^{1/2}}{S}\right)^{-2}, \tag{14}$$

where β_0 is a constant that depends on the pore size distribution of the soil and that is of the order of 2/3 for most soils. The time expansion approach was also applied by *Warrick et al*. [1985] to develop a generalized solution of equation (7).

3.2.2. Integral Approach

[23] The integral approach was proposed by *Parlange* [1971, 1972]. This solution is based on the formulation of the flow equation that differed slightly from the conventional equation (7) and that was also used by *Philip* [1969]:

$$\frac{\partial z}{\partial t} + \frac{\partial}{\partial \theta}\left[\frac{D}{(\partial z/\partial \theta)}\right] = \frac{\mathrm{d}K}{\mathrm{d}\theta}. \tag{15}$$

Parlange [1971] introduced the double integration technique to solve equation (15) for the following initial and boundary conditions:

$$\begin{array}{lll} t = 0, & z > 0, & \theta = \theta_i \\ t \geq 0, & z = 0, & \theta = \theta_s \end{array} \tag{16a}$$

$$\begin{array}{lll} t = 0, & z > 0, & \theta = \theta_i \\ t \geq 0, & z = 0, & K - D\dfrac{\partial \theta}{\partial z} = q, \end{array} \tag{16b}$$

leading to the following relationship for ponded infiltration:

$$\int_{\theta_i}^{\theta_s} (\partial z/\partial t)\mathrm{d}\theta + D_s/(\partial z/\partial \theta)_s - D(\theta_i)/(\partial z/\partial \theta)_i = K_s - K(\theta_i) \tag{17a}$$

or to the simpler one in the case of an initially completely dry soil profile where $\theta_i = 0$ and $K(\theta_i) = 0$:

$$\int_{\theta_i}^{\theta_s} (\partial z/\partial t)\mathrm{d}\theta + D_s/(\partial z/\partial \theta)_s = K_s. \tag{17b}$$

[24] It was further applied to solve the more complicated case where a constant flux is imposed at the surface during infiltration [*Parlange*, 1972]. The integral approach was put on very strong foundation by *Parlange* [1975] by releasing the need for numerical iterations and by deriving

an analytical optimization technique that minimize a variational integral rather than a mass conservation integral. The combination between the integral approach and the double integration technique stimulated subsequent advances in the field of infiltration. It was applied by *Brutsaert* [1976] to derive a general equation for horizontal infiltration and by *Smith and Parlange* [1978] to develop a hydrologic infiltration model. It might have provided the inspiration to the flux-concentration method proposed by *Philip* [1973] and *Philip and Knight* [1974] as "it has affinities with the Parlange's method" [*Philip and Knight*, 1974, p. 1].

[25] The premise of the integral approach could be found in the model proposed by *Green and Ampt* [1911]. Although limited in its practical applications because of the very drastic physical assumptions imbedded in it, this model is physically based and captures the macroscale behavior of the water movement in soils during infiltration. One of its key simplifications is the assumption of a sharp infiltrating front, i.e., of piston flow. For ponding infiltration with a negligible ponded water depth at the surface, the following relationship was formulated:

$$\int_0^{I(t)} \left(1 - \frac{\psi_f \Delta\theta}{I + \psi_f \Delta\theta}\right) \mathrm{d}I = \int_0^t K_s \mathrm{d}t, \tag{18}$$

where $I(t)$ is the cumulative infiltration at time t, ψ_f is the capillary head at the wetting front, and $\Delta\theta$ is the difference between the saturated and the initial water content of the soil profile. The implicit expression for the cumulative infiltration $I(t)$ is

$$I(t) = K_s t - \psi_f \Delta\theta \ln\left(1 - \frac{I(t)}{\psi_f \Delta\theta}\right), \tag{19}$$

which need to be solved iteratively for I. Based on $I(t)$ from equation (19), the infiltration rate can be estimated from

$$q(t) = K_s\left(\frac{\psi_f \Delta\theta}{I(t)} + 1\right). \tag{20}$$

The capillary head at the wetting front, ψ_f, can be linked to soil properties through the relationship proposed by *Bouwer* [1964]:

$$\psi_f = K_s^{-1} \int_0^{\psi_i} K(\psi) \mathrm{d}\psi, \tag{21}$$

where ψ_i is the initial capillary head of the soil at the beginning of the wetting. For early stages of infiltration, ψ_f can also estimated using the expression of *Neuman* [1976]:

$$\psi_f = -\frac{S^2}{2K_s(\theta_s - \theta_i)}. \tag{22}$$

[26] The expressions in equations (19) and (20) assume an initially completely dry profile for which $K(\theta_i) = 0$ and a negligible water ponding depth $h_s = 0$. The generalized expression of $I(t)$ for $K(\theta_i) \neq 0$ and $h_s > 0$ is [*Swartzendruber*, 1987; *Ross et al.*, 1996]

$$I(t) = K_s t + \frac{\Delta\theta(h_s - \psi_f)K_s}{[K_s - K(\theta_i)]} \ln\left\{1 + \frac{[I(t) - K(\theta_i)t][K_s - K(\theta_i)]}{\Delta\theta(h_s - \psi_f)K_s}\right\}. \tag{23}$$

[27] *Parlange et al.* [1982] proposed a quasi-exact solution for equation (15) that is valid for the entire time range. The resulting expression assuming that $K(\theta_i) = 0$ is

$$t = \frac{S^2}{2K_s^2(1-\delta)}\left[\frac{2K_s}{S^2} I - \ln \frac{\exp(2\delta K_s I/S^2) + \delta - 1}{\delta}\right]. \tag{24}$$

An approximate value of $\delta = 0.85$ was suggested by *Parlange et al.* [1982] for a range of soil types. For the value of $\delta = 0$, equation (24) corresponds to the Green and Ampt solution and to soil types for which the diffusivity increases much more rapidly with θ than $\mathrm{d}K/\mathrm{d}\theta$. The value of $\delta = 1$ corresponds to soil types for which the diffusivity is practically proportional to $\mathrm{d}K/\mathrm{d}\theta$ (solution of *Talsma and Parlange* [1972]). The parameter δ was more rigorously defined by *Parlange et al.* [1985] and was related to the soil type via the $K(\theta)$ function:

$$\delta = \frac{1}{\theta_s - \theta_0} \int_{\theta_0}^{\theta_s} \frac{K_s - K(\theta)}{K_s} \mathrm{d}\theta. \tag{25}$$

For short time after the beginning of wetting, the expression for the cumulative infiltration is

$$I(t) = St^{1/2} + \left(\frac{2-\delta}{3}\right) K_s t. \tag{26}$$

For long time after the beginning of wetting, $I(t)$ is expressed as

$$I(t) = K_s t + \frac{S^2}{2K_s(1-\delta)} \ln\left(\frac{1}{\delta}\right). \tag{27}$$

The $q(I)$ relationship corresponding to equation (24) is [*Espinoza*, 1999]

$$q(I) = K_s - \delta K_s \left(1 - \exp\left(\frac{2\delta K_s I}{S^2}\right)\right)^{-1}. \tag{28}$$

[28] The theoretical scope of equation (24) is limited to nonponded conditions because it results from the integration of the water content based form of Richards' equation. A generalization of equation (24) that includes ponded conditions was introduced by *Parlange et al.* [1985] and *Haverkamp et al.* [1990], with an explicit form presented by *Barry et al.* [1995]. The implicit form of the Green and Ampt solution (equation (19)) can be inverted to obtain explicit infiltration equations by using the Lambert W-functions [*Parlange et al.*, 2002]. The properties of this function have been extensively discussed by *Barry et al.* [2005]. The infiltration function in equation (24) was

verified recently by *Triadis and Broadbridge* [2010]. Their solution relies on the nonlinear models of *Broadbridge and White* [1988] and *Sander et al*. [1988a] and was validated numerically by *Broadbridge et al*. [2009].

[29] The high performances of the different solutions resulting from the integral approach and their good accuracy in reproducing experimental data or numerical solutions of Richards's equation are illustrated in Figures 2a–2c. Figure 2a illustrates the agreement between the water content profile $\theta(z)$ resulting from the solution in *Parlange* [1972] for the case of vertical infiltration with a constant flux condition at the surface and the corresponding numerical solution of Richards's equation proposed by *Rubin and Steinhardt* [1963] for the Rehovot sand. The depicted profile corresponds to a relatively short time (40 min) for which discrepancies are expected to be large. For large infiltration times, the profile tends toward the "profile at infinity" [*Philip*, 1969; *Parlange*, 1971, 1972], which is practically indistinguishable from the profile resulting from the numerical solution. Figure 2b shows the good agreement between the infiltration function $q(t)$ predicted by the integral solution of *Talsma and Parlange* [1972] and corresponding experimental laboratory and field data for the Bungendore fine sand. Finally, Figure 2c illustrates the high predictive ability of the equation of *Haverkamp et al*. [1990] for vertical infiltration under ponded conditions in terms of the cumulative infiltration function $I(t)$ by comparison to experimental data for the coarse soil and a numerical solution of the flow equations for the clay soil presented by *Haverkamp et al*. [1988].

3.2.3. Space Expansion Approach: The Heaslet and Alksne Method

[30] The solution of the infiltration equation describes naturally the water content distribution with depth at any given time, $\theta(z, t)$, although it is easier to express it as $z(\theta, t)$ [*Parlange et al*., 1982, 1984a]. The *Heaslet and Alksne* [1961] technique was initially applied to solve the nonlinear diffusion equation with a power law diffusivity and obtained a solution for $\theta(z, t)$ by expansion around the wetting front position. It was found to coincide with the numerical solution of *Philip* [1955] by *Brutsaert and Weizman* [1970] and was later extended for arbitrary soil water diffusivities by *Prasad and Römkens* [1982] and *Parlange et al*. [1984a, 1992]. It was further generalized to include the effects of gravity by *Parlange et al*. [1997], resulting in

$$\int_0^{\theta_s} \frac{D\mathrm{d}\theta}{q(\theta/\theta_s) - K(\theta)} = z + M(t)z^2, \tag{29}$$

where M is an unknown function of time. The corresponding expression for the cumulative infiltration I is given by

$$I(t) = \int_0^{\theta_s} \frac{D\theta\mathrm{d}\theta}{q(\theta/\theta_s) - K(\theta)} - M(t)\int_0^{\theta_s} z^2\mathrm{d}\theta. \tag{30}$$

The solution of *Parlange* [1972] is used to estimate $\int_0^{\theta_s} z^2\mathrm{d}\theta$, and M is evaluated based on the boundary conditions of the case under interest [*Barry et al*., 2007]. For

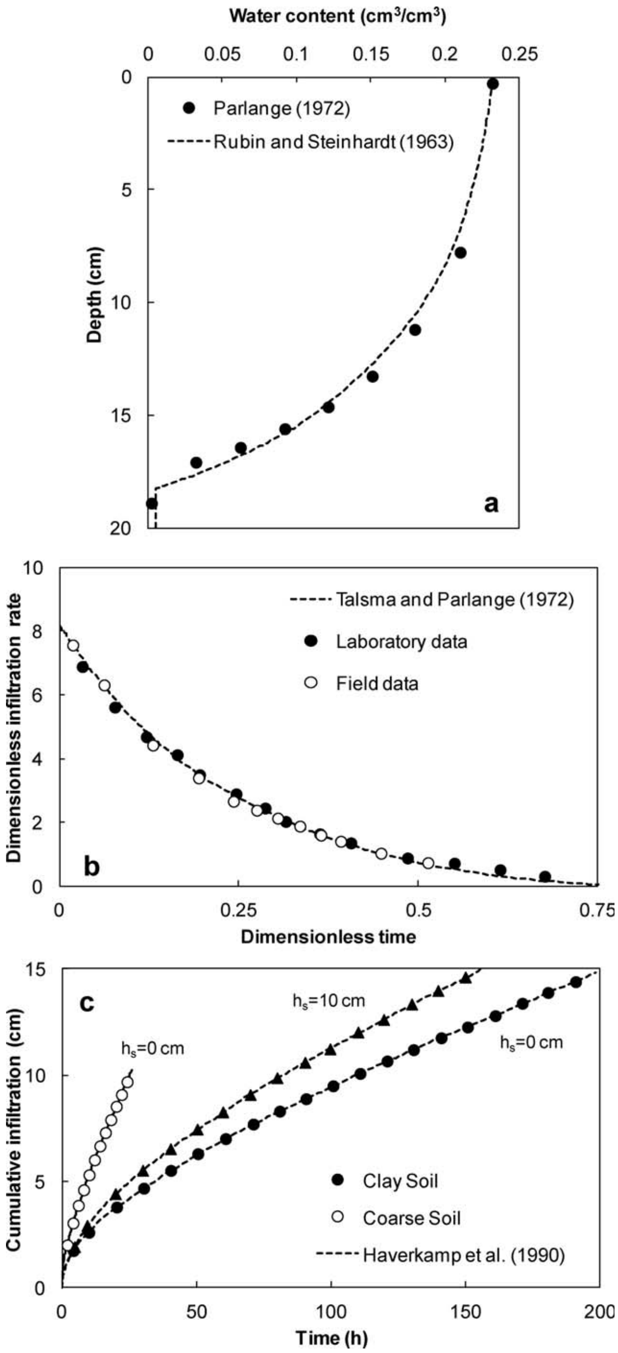

Figure 2. (a) Water content profile $\theta(z)$ resulting from the solution in *Parlange* [1972] for the case of vertical infiltration with a constant flux condition and the corresponding numerical solution of Richards's equation proposed by *Rubin and Steinhardt* [1963] for the Rehovot sand after a relatively short time (40 min); (b) dimensionless infiltration rate curve predicted by the integral solution of *Talsma and Parlange* [1972] and experimental laboratory and field data for the Bungendore fine sand; and (c) cumulative infiltration function $I(t)$ predicted by the equation of *Haverkamp et al*. [1990] for vertical infiltration under ponded conditions and experimental data for the coarse soil ($h_s = 0$ cm) and a numerical solution of the flow equations for the clay soil for $h_s = 0$ cm (black dots) and $h_s = 10$ cm (black triangles) [*Haverkamp et al*., 1988].

$M = 0$, the traveling wave solution of *Fleming et al*. [1984] and *Ross and Parlange* [1994] is obtained. A more detailed description of the method can be found in *Barry et al*. [2007].

3.3. Numerical Solutions of the Infiltration Equation

[31] The flow equations in equation (4) and (5) are highly nonlinear parabolic partial differential equations, and in most of the cases, analytical solutions are not possible. Therefore, application of numerical methods is often the only way to perform quantitative analysis of practical flow problems. Numerical methods emerged and became more sophisticated with the development of computers with increased computing speed and memory [*Philip*, 1955; *Rubin and Steinhardt*, 1963; *Hogarth et al*., 1989, 1992]. Because of a series of issues intrinsically related to the nature of the infiltration process, accurate numerical simulation of infiltration remains a challenge, and convergence and stability are continuing problems. Among these issues, we can list the presence of steep wetting fronts; the elliptic form of Richards' equation in saturated domains; the non-mass-conserving of algorithms solving for ψ; and the fact that θ-based algorithms cannot be applied to situations where parts of the domain are saturated [*Milly*, 1985, 1988; *Hills et al*., 1989; *Kirkland et al*., 1992]. Different finite difference algorithms were developed that deal with these issues [*Klute*, 1952; *Hanks and Bowers*, 1962; *Rubin*, 1968; *Brandt et al*., 1971; *Neuman*, 1972; *Vauclin et al*., 1979]. Mass conservation was significantly improved by *Celia et al*. [1990]. Different weighing techniques were proposed to compute intermodal hydraulic conductivity to improve the accuracy of finite difference schemes [*Haverkamp and Vauclin*, 1979; *Warrick*, 1991; *Zaidel and Russo*, 1992; *Miller et al*., 1998; *Brunone et al*., 2003]. Alternatively, finite element models were developed, but they also suffered from similar performance problems [*Cooley*, 1983; *Huyakorn et al*., 1984; *Allen and Murphy*, 1986; *Kool and van Genuchten*, 1991; *Šimunek et al*., 2008].

4. Transition From Preponding to Postponding Infiltration

4.1. Estimation of the Ponding Time t_p

[32] The initial infiltration capacity of a soil, especially if it is dry, is high (Figure 1) and is generally higher than the intensity of natural rainfall events. Consequently, in most cases, all the water infiltrates into the soil profile during the early stages of rainfall. During this period, the water content at the soil surface, θ_0, and its corresponding hydraulic conductivity, $K(\theta_0)$, increase gradually, while the total hydraulic head gradient decreases as the wetting front deepens. If the rainfall intensity, r, remains lower than the saturated hydraulic conductivity of the soil surface, K_s, the infiltration capacity of the soil profile will not be exceeded, θ_0 is determined from $K(\theta_0) = r$, and no water ponding will occur at the soil surface. However, if $r \geq K_s$, θ_0 will reach its maximum value θ_s, after a given period of time $t = t_p$. From that moment onward, the infiltration capacity of the soil profile will be exceeded, ponding will take place, and runoff will begin. This time t_p is identified as the *ponding time* and corresponds to the transition from a preponding infiltration regime (Neuman boundary condition) to a postponding one (Dirichlet boundary condition). The dynamics of the water content distribution with depth $\theta(z, t)$, described earlier, is depicted in Figure 3 for two different constant rainfall rates $r = 1/4K_s$ and $r = K_s$. Note the difference in $\theta(z, t)$ for the cases representing the same amount of cumulative rainfall [($r = 1/4K_s$, $t = 4$ h and $r = K_s$, $t = 1$ h) versus ($r = 1/4K_s$, $t = 1$ h and $r = K_s$, $t = 0.25$ h)], illustrating the impact of the wetting rate on the water content profile and consequently, on the distribution of the total head gradients with depth. This will play a role when infiltration under variable rainfall is considered. The ability to predict accurately when this transition occurs, namely, to predict t_p, is of great importance in watershed hydrology, agricultural engineering, irrigation design, erosion control, and soil and water conservation issues. At ponding, the cumulative rainfall equals the cumulative infiltration in the soil profile:

$$R(t_p) = \int_{t=0}^{t_p} r(t)\mathrm{d}t = I(t_p) = \int_{t=0}^{t_p} q(t)\mathrm{d}t, \qquad (31)$$

where $r(t)$ is the observed rainfall rate and $R(t)$ is the cumulative rainfall.

[33] Using the $q(t)$ solutions presented earlier, expressions for t_p can be derived for constant rainfall intensity $r > K_s$ from the solutions of Green and Ampt (equation (32)), Philip (equation (33)), or Horton (equation (34)) [*Mein and Larson*, 1973; *Parlange and Smith*, 1976; *Smith*

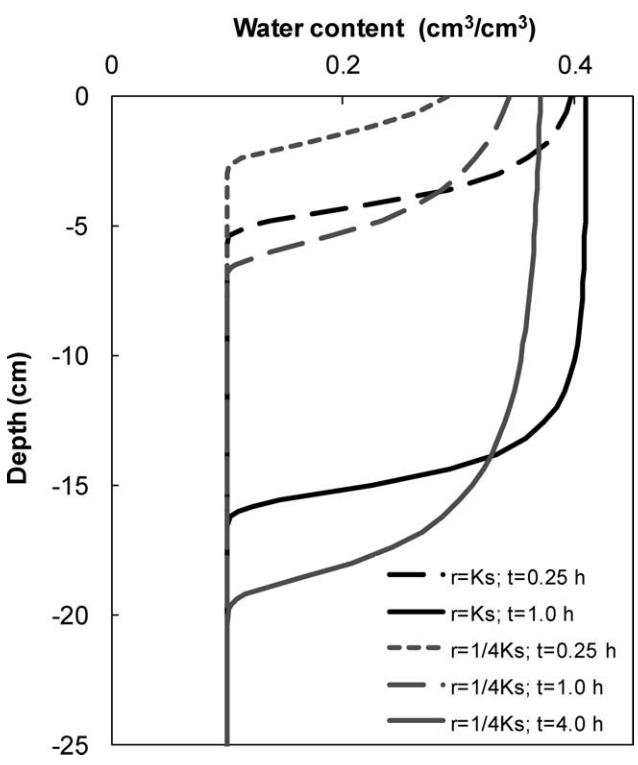

Figure 3. Dynamics of the water content distribution with depth, $\theta(z, t)$, of a sandy loam for two different constant rainfall rates, $r = 1/4K_s$ (gray lines) and $r = K_s$ (black lines).

and Parlange, 1977; *Broadbridge and White*, 1987; *Chow et al.*, 1988; *Brutsaert*, 2005]:

$$t_p = \frac{K_s \psi_f \Delta\theta}{r(r - K_s)}; \quad r > K_s \tag{32}$$

$$t_p = \frac{S^2 (r - K_s/2)}{2r(r - K_s)^2}; \quad r > K_s \tag{33}$$

$$t_p = \frac{1}{rK_s}\left[q_o - r + q_c \ln\frac{q_o - q_c}{r - q_c}\right]; \quad q_f < r < q_o. \tag{34}$$

Based on numerical calculations, *Smith* [1972] and *Smith and Chery* [1973] suggested an implicit expression of t_p:

$$\left(\frac{r_p}{K_s} - 1\right)^{\nu - 1} \int_{t=0}^{t_p} r(t)\mathrm{d}t = A, \tag{35}$$

where $r(t)$ is required to be at most slowly varying close to t_p; r_p is the rainfall rate at t_p; A is a linear function of the initial water content assumed constant with depth in the soil profile; and ν is a parameter found to be close to 2. *Parlange and Smith* [1976] proposed an alternative expression without the parameter ν, applicable for any rainfall temporal pattern for which $r_p < K_s$:

$$\frac{\int_{t=0}^{t_p} r(t)\mathrm{d}t}{\ln\left[r_p / \left(r_p - K_s\right)\right]} = \frac{S^2}{2K_s}, \tag{36}$$

where S is the soil sorptivity. Similar expressions were derived by *Basha* [2011] based on the kinematic wave solution of Richards' equation. Assuming a constant rainfall rate prior to ponding, equation (36) can be written as [*Brutsaert*, 2005]

$$t_p = \frac{S^2}{2rK_s} \ln\left[\frac{r}{(r - K_s)}\right]. \tag{37}$$

Broadbridge and White [1987] developed an expression similar to equation (37) for t_p for the case of rainfall events characterized by a linear increase of rainfall intensity with time. An expression of t_p that is very close to equation (37) resulted from the application of the model presented in equation (30) to solve the infiltration equation [*Parlange et al.*, 1999b].

4.2. Postponding Infiltration: TCA

[34] Ponding time and postponding infiltration can also be explored in terms of the relationship between infiltration rate and cumulative infiltration, $q(I)$. For practically constant rainfall rates, $q(I)$ is independent of r [*Skaggs*, 1982; *Smith et al.*, 2002], and infiltration at any given time depends only on the cumulative infiltration volume, regardless of the previous rainfall history. This is illustrated in Figure 4, where the upper plot represents $q(t)$ for two constant rainfall intensities, while the lower plot represents the corresponding $q(I)$ functions. After ponding, the $q(I)$ curves for the two rainfall intensities merge into a unique one. Merging of $q(I)$ is inherent to the expressions for short and long time infiltration (equations (26) and (27)) stemming from the solution of *Parlange et al.* [1982] and the related flux expressions [*Espinoza*, 1999]. Figure 5 depicts the

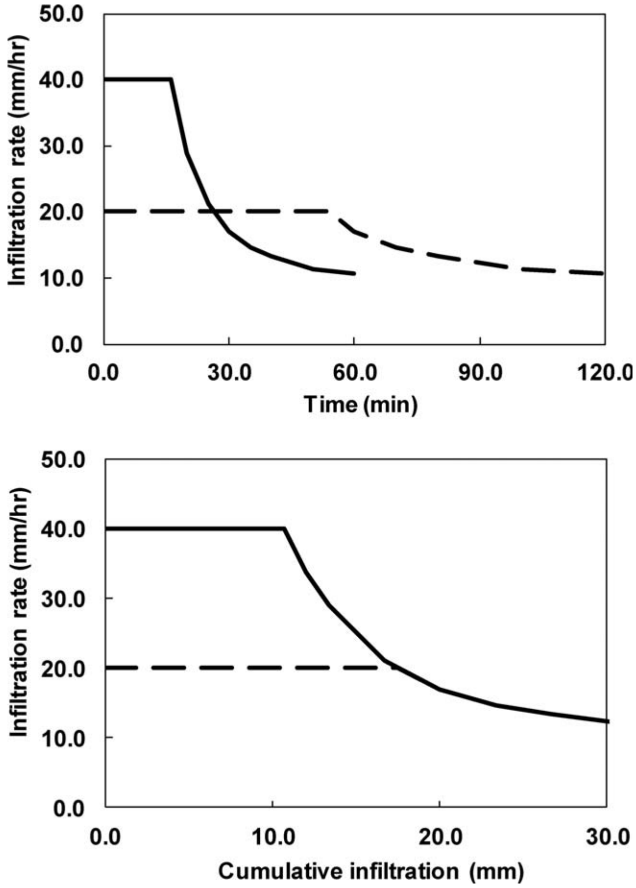

Figure 4. (upper plot) Infiltration curves versus time, $q(t)$ of a sandy loam, and (lower plot) the corresponding infiltration curves versus cumulative infiltration, $q(I)$ for two constant rainfall intensities, $r = 20$ mm/h (dashed line) and $r = 40$ mm/h (solid line).

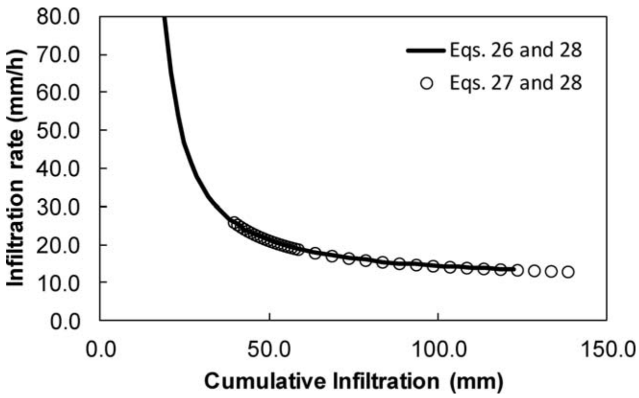

Figure 5. Infiltration versus cumulative infiltration relationships, $q(I)$, for the short and long time infiltration expressions based on the solution of *Parlange et al.* [1982] (equations (26) and (27)) and the related flux expression (equation (28)).

respective $q(I)$ relationships based on equations (26) and (28) and equations (27) and (28), indicating that they are both similar. The time invariance of $q(I)$ holds true also when a layered profile or a sealed soil surface is considered [*Smith*, 1990; *Mualem and Assouline*, 1996]. Based on the uniqueness of $q(I)$, the condition for ponding is

$$t = t_p; \quad r(R) = q(I) = q_{\text{cap}}\left(I_{\text{cap}}\right). \tag{38}$$

Consequently, the definition of the ponding time t_p is the time in the inverse cumulative rainfall relationship $t(R)$, for which $r(R) = q(I)$ [*Brutsaert*, 2005]. In the cases where intensity varies within the rainfall event, it can be approximated by the time in $t(R)$ for which $r(R) = q_{\text{cap}}(I_{\text{cap}})$ [*Assouline et al.*, 2007].

[35] Once t_p is evaluated, the second important need is the evaluation of the postponding infiltration curve $q(t)$, a key function for predicting runoff-related processes such as erosion at the local scale and floods at the regional one. Postponding infiltration is similar to infiltration capacity in that sense that both present the same upper boundary condition ($z = 0$; $t \geq 0$; $\theta = \theta_s$) at the soil surface. However, the difference between the two results from the different initial conditions (initial distribution with depth of the water content). When dealing with infiltration capacity (ponded infiltration problems), the initial conditions can be generally prescribed beforehand, while in the case of postponding infiltration the initial conditions are the result of the preponding infiltration stage, which depends on the specific rainfall characteristics (duration, intensity). Consequently, accurate definition of the initial conditions for the postponding phase requires the solution of the flow equation (equation (4)), a task that is not always possible or practical. Therefore, approximations were developed. The most widely used one stems from the concept of time compression [*Sherman*, 1943; *Holtan*, 1945; *Reeves and Miller*, 1975; *Sivapalan and Milly*, 1989; *Salvucci and Entekhabi*, 1994; *Kim et al.*, 1996]. The core of these approximations is the uniqueness of $q(I)$ (Figures 4 and 5). In other words, after ponding, this robust property of the infiltration process can be exploited, and $q(I)$ can be directly derived from $q_{\text{cap}}(I_{\text{cap}})$. However, before ponding, the actual cumulative infiltration, I, is smaller than the cumulative infiltration capacity, I_{cap}, and especially at the transition between preponding and postponding infiltration, $I(t_p) < I_{\text{cap}}(t_p)$. Therefore, to provide a similar cumulative infiltration under both conditions, a time shift t_c, with $t_c < t_p$, is introduced so that $I(t_p) = I_{\text{cap}}(t_c)$. Once t_p and t_c are known, postponding infiltration $q(t)$ can be evaluated as

$$q(t) = q_{\text{cap}}\left[t - \left(t_p - t_c\right)\right]. \tag{39}$$

[36] Similar to t_p, several expressions were proposed to estimate t_c based on the solutions of Green and Ampt (equation (40)), Philip (equation (41)), and Horton (equation (42)) [*Chow et al.*, 1988; *Brutsaert*, 2005]:

$$t_c = \frac{1}{K_s}\left[I\left(t_p\right) - \psi_f \Delta\theta \ln\left(1 + \frac{\psi_f \Delta\theta}{I\left(t_p\right)}\right)\right] \tag{40}$$

$$t_c = \frac{1}{4{K_s}^2}\left(\sqrt{S^2 + 4K_s I\left(t_p\right)} - S\right)^2 \tag{41}$$

$$t_c = \frac{1}{K_s}\ln\left[\frac{q_o - q_c}{r - q_c}\right]. \tag{42}$$

[37] The graphical representation of the TCA method is provided in Figure 6, for the case of constant rainfall intensity. The upper plot depicts the infiltration capacity curve $q_{\text{cap}}(t)$ (dashed line) characterizing the soil profile at the given initial condition and the actual infiltration curve $q(t)$ (solid line) corresponding to the prescribed rainfall rate $r(t)$

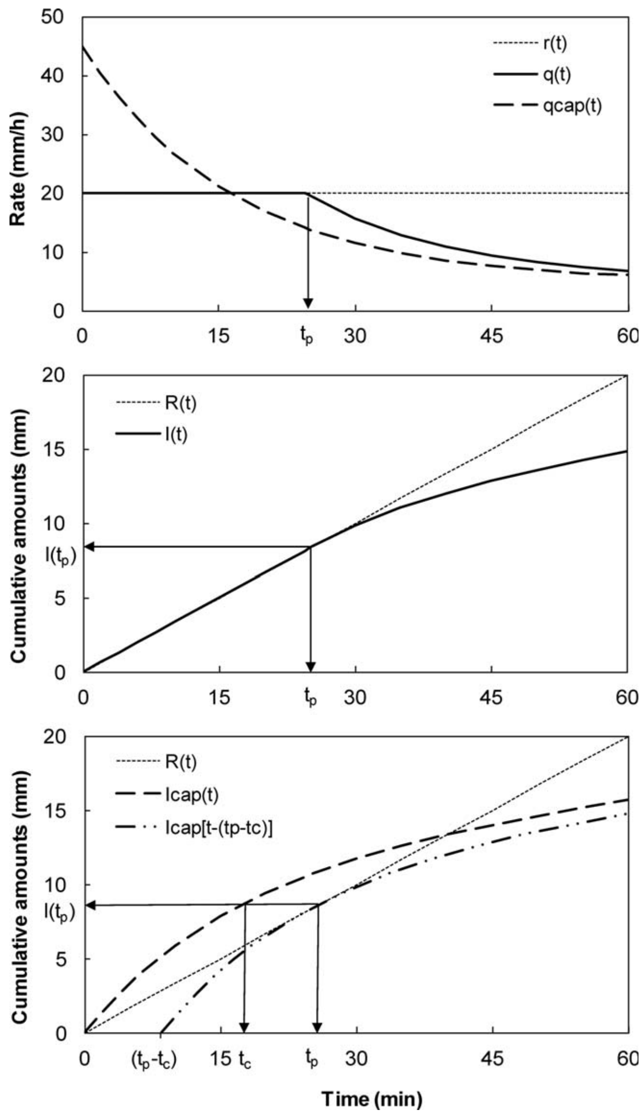

Figure 6. Graphical representation of the TCA method for the case of constant rainfall intensity: (upper plot) infiltration capacity curve $q_{\text{cap}}(t)$ (dashed line) and the actual infiltration curve $q(t)$ (solid line) for the prescribed rainfall rate $r(t)$ (dotted line); (middle plot) corresponding cumulative rainfall $R(t)$ and cumulative infiltration $I(t)$ curves; (lower plot) cumulative infiltration capacity $I_{\text{cap}}(t)$ and rainfall $R(t)$ curves and the cumulative infiltration capacity curve translated in time, $I_{\text{cap}}[t - (t_p - t_c)]$.

(dotted line). After t_p, $q(t)$ is smaller than $r(t)$ and ponding occurs. The middle plot expresses this situation in terms of cumulative rainfall $R(t)$ and cumulative infiltration $I(t)$, indicating that t_p corresponds to the moment beyond which $I(t)$ is smaller than $R(t)$. The lower plot illustrates the definition of t_c and the translation in time of the cumulative infiltration capacity curve so that t_p is now the moment beyond which $I_{cap}[t-(t_p-t_c)]$ is smaller than $R(t)$. Note that for $t>t_p$, $I(t)=I_{cap}[t-(t_p-t_c)]$ (equation (39)).

[38] As for t_p, the expressions for t_c assume either constant or time-averaged wetting rate [*Brutsaert*, 2005]. Such assumption could lead to some error when the TCA is applied under natural rainfall conditions where $r(t)$ can vary significantly with time [*Assouline et al.*, 2007]. Associated problems to temporal variability in rainfall intensity, such as redistribution and hysteresis, could introduce significant errors in the TCA estimates [*Reeves and Miller*, 1975; *Smith et al.*, 1993; *Corradini et al.*, 1994; *Agnese and Bagarello*, 1997; *Govindaraju et al.*, 2006]. In fact, when the rainfall intensity varies significantly with time, the time invariance of $q(I)$ is lost. This is illustrated in Figure 7 for the case of the rainfall event presented in Figure 1 (dotted line; see also inset), where it is shown that $q_{cap}(I_{cap})$ and $q(I)$ are no longer similar after ponding, thus weakening the basis of the TCA method.

[39] Modification of the TCA that does not assume that the average flux before ponding is the flux at ponding but rather uses a priori information on t_p or $I(t_p)$ was proposed by *Liu et al*. [1998], *Parlange et al*. [2000], *and Hogarth et al*. [2011]. This modification reduced by half the error in the cumulative infiltration estimates resulting from the original TCA.

5. Infiltration Under Special Conditions

[40] Richards' equation (4) corresponds to infiltration under isothermal conditions in an isotropic and rigid porous medium where air can escape freely. These constraints might not always fit natural as well as experimental conditions. Deviations from such constraints can alter significantly the accuracy of the estimates and even make the whole approach inadequate. Different causes to such deviations can be identified. High clay content in the soil could lead to soil swelling during wetting and thus induce dynamic changes in soil porosity and hydraulic properties and consequently affect infiltration [*Raats and Klute*, 1969; *Philip*, 1970; *Smiles*, 1974; *Sposito*, 1975; *Giraldez and Sposito*, 1985]. Soil sodicity and electrolyte concentration of the wetting fluid could also have a significant impact on soil hydraulic properties and on the infiltration process [*Quirk and Schofield*, 1955; *McNeal and Coleman*, 1966; *Bresler et al*., 1982; *Jury et al*., 1991; *Russo*, 2005; *Assouline and Narkis*, 2011]. Temporal variability of intensity within the same rainfall event could induce drainage and redistribution to occur simultaneously in the same soil profile. Consequently, hysteresis in the $\psi(\theta)$ relationship [*Haines*, 1930; *Everett*, 1955; *Poulovassilis*, 1962; *Topp*, 1971; *Mualem*, 1974; *Mualem and Dagan*, 1975; *Parlange*, 1976; *Hogarth et al*., 1988; *Nimmo*, 1992; *Huang et al*., 2005; *Mualem and Beriozkin*, 2009] would play a crucial role in the accurate description of the flow processes within that soil profile [*Ibrahim and Brutsaert*, 1968; *Hanks et al*., 1969; *Scott et al*., 1983; *Glass et al*., 1989]. The capability to describe the ensemble of scanning curves within the hysteresis loop is necessary to achieve any reliable quantitative treatment of the flow process. In such case, expression of the infiltration process in terms of the diffusion equation (equation (7)) will not be appropriate since different points within the soil profile will follow different scanning curves, and there will be no unique relationship between gradients of θ and gradients of ψ. Thermal effects will also induce significant changes in the estimated fluxes as temperature was found to affect soil hydraulic properties [*Gardner*, 1955; *Philip and de Vries*, 1957; *Nimmo and Miller*, 1986; *Hopmans and Dane*, 1986; *Grant and Salehzadeh*, 1996; *Parlange et al*., 1998; *Grant and Bachman*, 2002].

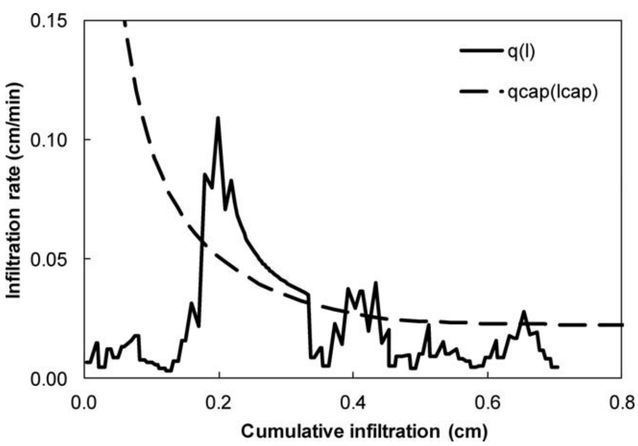

Figure 7. Infiltration capacity and actual infiltration curves versus corresponding cumulative infiltration, $q_{cap}(I_{cap})$ (dashed line) and $q(I)$ (solid line) for a variable rainfall case $r(t)$ (presented in the dotted line in the inset of Figure 1).

[41] Wetting front instability occurring under certain flow regimes can also affect significantly the infiltration process [*Raats*, 1973; *Philip*, 1975; *Parlange and Hill*, 1976; *Jury et al*., 2003; *DiCarlo*, 2004; *Or*, 2007]. Key contributions on this issue were made by Parlange's group [*Glass et al*., 1989; *Selker et al*., 1992; *Nieber et al*., 2000]. A detailed review of this topic is part of the Special Section.

[42] In the following, we will discuss in more details the impact of two factors on infiltration: (a) vertical and spatial heterogeneity in soil hydraulic properties and (b) air movement in the soil.

5.1. Infiltration in Heterogeneous Soil Profiles

[43] Natural soil profiles are seldom homogeneous with depth and present generally successive distinct layers of soil with different hydraulic properties. In some cases, a continuous variation in soil hydraulic properties can be observed due to overburden, natural soil reconsolidation, or mechanical compaction. Several studies have proposed solutions for infiltration in layered soil systems [*Colman and Bodman*, 1945; *Hanks and Bowers*, 1962; *Philip*, 1967; *Childs and Bybordi*, 1969; *Miller and Gardner*, 1962; *Zaslavsky*, 1964; *Raats*, 1983; *Warrick and Yeh*, 1990]. *Chu and Marino* [2005] presented a solution for determining ponding conditions and simulating infiltration into a layered soil profile

based on the Green and Ampt approach for unsteady rainfall. *Beven* [1984] and *Selker et al*. [1999] also extended the Green and Ampt model for infiltration into soil profiles where pore size varied with depth. A review of the applications of the Green and Ampt model to heterogeneous conditions was provided by *Kale and Sahoo* [2011].

[44] A special case of layered soil profile is when a seal layer resulting from the destructive action of raindrop impacts on the soil is formed at the surface. A review of the approaches proposed to model infiltration into sealed soils can be found in *Ahuja and Swartzendruber* [1992], *Mualem and Assouline* [1992, 1996], and *Assouline* [2004]. The direct effect of the presence of the impeding seal layer at the soil surface is to reduce ponding time and infiltration rate during rainfall [*Römkens et al*., 1986a, 1986b]. Solving the infiltration equation in the case of sealed (also identified as crusted) soils was first addressed by *Hillel and Gardner* [1969, 1970]. They presumed that a sealed soil can be modeled as a uniform soil profile capped with a saturated thin layer of low permeability with constant prescribed physical properties such as the saturated hydraulic conductivity. Their simplified solution was based on the Green and Ampt model, assuming a constant water content (or suction) at the interface between the seal and the soil beneath. It was further applied in different studies [*Ahuja*, 1974, 1983; *Moore*, 1981a; *Parlange et al*., 1984b]. Variations and extensions of this basic approach included the simulation of infiltration with time-dependent seal HCFs [*Farrell and Larson*, 1972; *Whisler et al*., 1979; *Moore*, 1981b; *Ahuja*, 1983; *Brakensiek and Rawls*, 1983; *Chu et al*., 1986; *Vandervaere et al*., 1998]. *Philip* [1998] concluded that the Green and Ampt solution is ill-fitted to the analysis of infiltration into crusted soils and suggested that the flux-concentration method [*Philip*, 1973] should be more appropriate. An additional conceptual model, based on the model of *Corradini et al*. [1997], was suggested by *Smith et al*. [1999]. *Römkens and Prasad* [1992] applied the solution of *Prasad and Römkens* [1982] based on the spectral series approach to solve the infiltration equation in soils topped by a constant or transient crust. In all these studies, the hydraulic properties of the seal layer were arbitrarily chosen. *Mualem and Assouline* [1989], *Baumhardt et al*. [1990], and *Assouline and Mualem* [1997] have addressed the problem of infiltration into sealed and sealing soils by attributing to the seal layer hydraulic functions that evolved from those of the undisturbed soil and that were related to the specific rainfall kinetic energy and intensity involved in the seal formation.

5.2. Effect of Spatial Variability in Soil Hydraulic Properties on Infiltration

[45] Under natural conditions, rainfall-infiltration relationships at the field scale are mostly determined by the spatial variability of soil hydraulic properties. Significant spatial variability of these properties was reported by *Nielsen et al*. [1973], *Warrick and Nielsen* [1980], *Peck* [1983], and *Logsdon and Jaynes* [1996]. In most of the cases, the field saturated hydraulic conductivity K_s was lognormally distributed [*Reynolds and Elrick*, 1985; *White and Sully*, 1992; *Russo et al*., 1997]. However, steady-state infiltration fluxes and soil surface water content distributed either normally or lognormally [*Vieira et al*., 1981; *Sisson and Wierenga*, 1981; *Loague and Gander*, 1990; *Kutilek et al*., 1993; *Cosh et al*., 2004]. Several studies have dealt with modeling the effect of spatial variability of soil hydraulic properties on infiltration [*Smith and Hebbert*, 1979; *Warrick and Nielsen*, 1980; *Dagan and Bresler*, 1983; *Sivapalan and Wood*, 1986; *Milly and Eagleson*, 1988; *Woolhiser et al*., 1996; *Assouline and Mualem*, 2002; *Govindaraju et al*., 2006]. *Smith and Hebbert* [1979] have shown that accounting for spatial variability leads to shorter ponding times and to a more gradual decrease of the infiltration flux with time.

[46] *Govindaraju et al*. [2006] suggested a semianalytical model to compute the space-averaged infiltration at the hillslope scale when spatial variability in both soil property and rainfall intensity are accounted for. The soil spatial heterogeneity is characterized by a lognormal distribution of the saturated hydraulic conductivity, while the rainfall spatial heterogeneity is simulated by a uniform distribution between two extreme rainfall intensities. At each location, the soil saturated hydraulic conductivity and the rainfall intensity were assumed to remain constant during the rainfall event. The model was validated against Monte Carlo simulations. The main result is that ponding time decreases with the increase in the coefficient of variation of K_s. The results of this model are in agreement with those of *Smith and Hebbert* [1979] who applied the infiltration model of *Smith and Parlange* [1978].

[47] The combined effect of soil surface sealing and spatial variability in soil properties on infiltration and runoff was presented by *Assouline and Mualem* [2002, 2006]. The dynamic model of seal formation [*Assouline and Mualem*, 1997] was applied to simulate the effect of spatial variability in soil and seal properties on the infiltration curve in the cases of a sealed and an unsealed soil surface. These results are depicted in Figure 8 in terms of the $q(t)$ and $q(I)$ curves corresponding to homogeneous and heterogeneous fields exposed to constant rainfall for the unsealed and the soil surface sealing cases. For the unsealed case, spatial variability of the soil properties has a considerable effect on infiltration (Figure 8a). The ponding time is drastically reduced, and the infiltration curve decreases more slowly compared to a homogeneous soil, in agreement with *Smith and Hebbert* [1979] and *Govindaraju et al*. [2006]. Similar trends are obtained when a seal develops at the soil surface, with some differences. The reduction of the ponding time when spatial variability is accounted for is not as drastic as it is in the case of the unsealed soil (Figure 8a). The interesting point here is that while the soil surface sealing significantly reduces the ponding time in the homogeneous field as expected, it does not affect the ponding time of the heterogeneous one. Consequently, runoff will appear in the heterogeneous field at about the same time. This results from the fact that part of the heterogeneous field has much lower hydraulic conductivity than that of the homogeneous one, and this generates the early runoff. This reason remains valid also when surface sealing is accounted for. It is worth noting that independently of the soil surface condition, the time invariance of $q(I)$ is lost in the case of the heterogeneous field (Figure 8b). This indicates that the TCA method might not be applicable to systems presenting a significant spatial variability in soil properties.

6. Two-Phase Flow in Porous Media

[48] When water infiltrates into an initially dry porous medium, the infiltrating water replaces air in the voids.

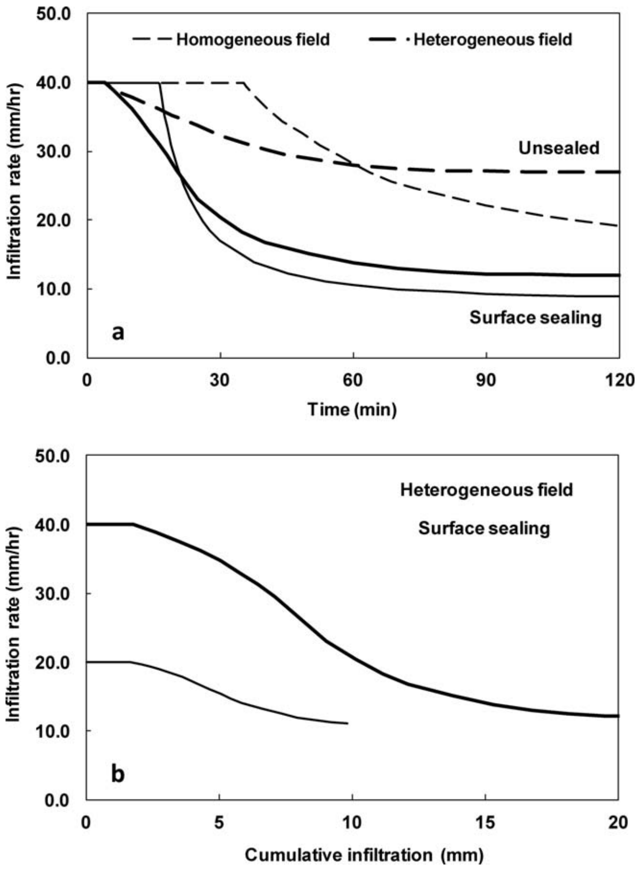

Figure 8. (a) Infiltration curves versus time, $q(t)$, corresponding to homogeneous and heterogeneous fields exposed to constant rainfall for the unsealed and the surface sealing cases and (b) infiltration curves versus cumulative infiltration, $q(I)$, corresponding to the heterogeneous field-surface sealing case under two constant rainfall intensities.

Consequently, infiltration is basically a problem of immiscible movement of water and air. Under natural conditions, the movement of air is generally small [*Parlange and Hill*, 1979]. Therefore, all the solutions of the flow equation presented here earlier, which assume that air can move freely within the porous medium and remain practically at atmospheric pressure so that its impact on the movement of water can be neglected, can be applied. However, under conditions of flood irrigation, intense rainfall, and soil column experiments, air can be compressed at the wetting front and beyond and reduce significantly the infiltration rate until it could find a way to escape and release that pressure buildup [*Peck*, 1965; *McWhorter*, 1971; *Dixon and Linden*, 1972; *Vachaud et al*., 1974; *Touma et al*., 1984; *Wang et al*., 1998]. Solving the problem of two-phase flow in porous medium presented an increased interest since oil can replace air in the two-phase definition and thus address practical issues related to the oil industry. Different approaches were developed [*Youngs and Peck*, 1964; *McWhorter*, 1971; *Morel-Seytoux*, 1973; *Wooding and Morel-Seytoux*, 1976; *Sander and Parlange*, 1984; *Sander et al*., 1988a, 1988b, 1988c; *Weir and Kissling*, 1992; *Celia and Binning*, 1992; *Sander et al*., 1993; *Weeks et al*., 2003]. Following *McWhorter* [1971], the basic equation describing the flux of water, f_w, affected by the flux of air, f_a, that was solved by *Sander et al*. [1988c, 1993] is

$$f_w(t) = [1 - \kappa(\theta)]D(\theta)\frac{\partial\theta}{\partial z} + [1 - \kappa(\theta)]\left(1 - \frac{\rho_a}{\rho_w}\right)K(\theta) + \kappa(\theta)(f_w + f_a), \tag{43}$$

where ρ_a and ρ_w are the densities of the air and water phases, respectively; $D(\theta)$ and $K(\theta)$ are the diffusivity function and HCF of water, respectively, as defined previously, and $\kappa(\theta)$ is an increasing function of θ between 0 and 1 that was defined by *McWhorter* [1971] as

$$\kappa(\theta) = \frac{K(\theta)}{K(\theta) + K_a(\theta)}, \tag{44}$$

with $K_a(\theta)$ being the air conductivity function.

[49] *Culligan et al*. [2000] used the general equation of *Haverkamp et al*. [1990] to describe quantitatively the infiltration when air could escape through capillary glass tubes of different diameters (controlled air escape condition) and show that air pressure measurements could be used to estimate the water flux and the cumulative infiltration into the column.

7. Conclusion

[50] Richards's equation captures the physics of flow in variably saturated media at low Reynolds number, which corresponds to infiltration into a wide range of porous media and soils. This highly nonlinear equation has proven challenging to solve, though empirical, numerical, and mathematical methods have been developed providing solutions for the very wide variety of flow problems issued from the fields of hydrology and agricultural, environmental, and civil engineering. Parlange and Brutsaert played leadership roles in the establishment of infiltration theory and in the development of analytical solutions to the flow equations. One of the major breakthroughs was the introduction of the integral approach with the double integration technique [*Parlange*, 1971, 1972], which stimulated subsequent advances in the field of infiltration. Parlange and his colleagues were also actively involved in developing mathematical solutions to multiphase flow, flow in heterogeneous media with special attention to infiltration during soil surface sealing, and flow at a range of spatial and temporal scales. Infiltration is a key component of the hydrological response of natural systems. In that context, special interest is given to the accurate prediction of the transition between preponding and postponding infiltration as it determines the appearance of runoff. Key contributions in this area have been made by Parlange and Brutsaert. Expressions for accurate prediction of the time-of-ponding and in-depth investigation of the widely used TCA were provided. The TCA relies on the assumption that the infiltration rate is a unique function of the cumulative infiltration rate. If such assumption is generally valid for homogeneous and even layered porous media, it might not remain valid in systems characterized by spatial and/or temporal variability of soil and rainfall properties.

[51] Richards' equation addresses only the macroscale behavior of infiltration and requires that the flow domain

could be characterized by well-defined hydraulic functions. However, wetting of a porous medium consists in a multitude of events where the wetting fluid invades fully or partially empty pores or cavities, thus involving a series of microscale processes that affect the overall macroscale behavior. In some specific cases, the flow regime can induce flow instability, and the Richards equation can reach its limit of applicability. Parlange and his group have made seminal contributions to infiltration under unstable flow conditions.

[52] The transition between the macroscale behaviors expressed in terms of continuum models like Richards's equation and the microscale nature of the wetting process illustrates the remaining gaps in our ability to describe flow processes in unsaturated porous media. Filling these gaps will be one of the challenges of soil science in the next decades. The scientific legacies of Brutasert and Parlange will serve as a solid foundation to deal with those challenges.

Appendix A: Definition of the Soil Hydraulic Properties

[53] The solutions of the flow equations require quantitative expressions of two soil hydraulic characteristics, the WRC and the HCF. The WRC describes the relationship between the soil capillary head ψ and the volumetric water content θ. The HCF describes the relationship between the unsaturated hydraulic conductivity K and θ or ψ. Sometimes, θ is expressed in terms of the effective saturation degree S_e, which is defined as

$$S_e(\theta) = \left(\frac{\theta - \theta_r}{\theta_s - \theta_r}\right), \tag{A1}$$

where θ_s and θ_r are the soil saturated and residual volumetric water contents, respectively.

[54] Mathematical and numerical solutions of the flow equations require smooth, easy-to-derive, functions for the soil hydraulic properties. *Brooks and Corey* [1964] proposed a power function of ψ to model $S_e(\psi)$:

$$\begin{aligned} S_e(\psi) &= \left(\frac{\psi}{\psi_c}\right)^{-\lambda}; \quad \psi < \psi_c \\ S_e(\psi) &= 1; \qquad\qquad \psi \geq \psi_c, \end{aligned} \tag{A2}$$

where ψ_c and λ are the fitting parameters. The parameter ψ_c is related to the air entry value, and the parameter λ is related to the pore size distribution of the porous medium. One of the shortcomings of the Brooks and Corey equation is the sharp discontinuity in its first derivative at ψ_c. *Brutsaert* [1966] reviewed several expressions for $S_e(\psi)$ based on probability laws, among which he proposed to use

$$S_e(\psi) = \left(\frac{a}{a + |\psi|^b}\right), \tag{A3}$$

where a and b are empirical constants. A similar expression with a third parameter m was introduced by *van Genuchten* [1980]:

$$S_e(\psi) = [1 + (\alpha|\psi|)^n]^{-m}, \tag{A4}$$

where $\alpha = a^{-1}$. Unlike the Brooks and Corey's model (equation (A2)), equations (A3) and (A4) do present an inflection point.

[55] Additional mathematical expressions are available, which increase the possibility to fit accurately measured data of specific soils or porous media [*Campbell*, 1974; *Clapp and Hornberger*, 1978; *Kosugi*, 1994; *Assouline et al.*, 1998].

[56] Comprehensive reviews of the different models developed to predict the HCF are available [*Brutsaert*, 1967; *Mualem*, 1986; *Brutsaert*, 2000]. The most widely used model was proposed by *Mualem* [1976]:

$$K(\theta) = K_s S_e^{0.5} \left[\frac{\int_0^{\theta} d\theta/\psi}{\int_0^{\theta_s} d\theta/\psi}\right]^2, \tag{A5}$$

where K_s is the saturated hydraulic conductivity of the soil. Closed-form mathematical expressions could be obtained from equation (A5) for most of the WRC models. A model which releases to the need to assume constant power values of 0.5 and 2 in equation (A5) has been proposed by *Assouline* [2001]. In many analytical solutions of the flow equations, it was convenient to apply the exponential function suggested by *Gardner* [1958]:

$$K(\psi) = K_s e^{(-\alpha\psi)}, \tag{A6}$$

where α is a soil characteristic.

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