TUTORIAL 2.1

1. Bob, John, Luke and Tim play a tennis tournament. The rules of the tournament are such that at the end of the tournament a ranking will be made and there will be no ties. How many different rankings can there be?

2. There is a basket of fruit containing an apple, a banana and an orange and there are five girls who want to eat one fruit. How many ways are there to give three of the five girls one fruit each and leave two of them without a fruit to eat?

3. How many ways are there to order the letters of the word INDESCREETNESS?

4. There are 12 hospitals in a town. How many different ways can 7 patients are sent to the hospitals so that no 2 patients may be in the same hospital.

5. A computer scientist is trying to discover the keyword for a financial account. If the keyword consists only of 10 lower case characters (e.g., 10 characters from among the set: a, b, c, ..., y, z) and no character can be repeated, how many different unique arrangements of characters exist?

6. Given 3 sets of integers. $A = \{1,3,5\}$, $B = \{4,6\}$, $C = \{0,2,7,9\}$. How many ways are there to choose one integer from set *A, B,* or *C?*

7. A Thai restaurant offers 5 fish dishes, 3 meat dishes, 3 vegetables dishes and 2 rice dishes for dinner service.

   a) How many different dinner meals of 5 dishes are possible if any dish can be chosen only once?

   b) How many different dinner meals are possible if each meal can consists of 1 fish dish, 1 meat dish, 1 vegetable dish and 1 rice dish.

   c) How many different dinner meals are possible if each meal can consists of 2 fish dishes, 2 meat dishes and 1 vegetable dish but each dish can be chosen only once?

d) How many different dinner meals of four dishes are possible if the meal must consists only 1 fish dish, 1 meat dish, 1 vegetable dish and 1 rice dish or the meals can consists only 1 fish dish, 1 meat dish, and 2 different vegetable dishes?

8. A bracelet is made from 5 alphabet beads as shown in Figure 1.

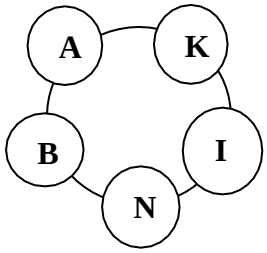

Figure 1

a) How many designs of the bracelet can be made from the 5 beads in any possible arrangement?

b) How many designs of the bracelet can be made if the bead with alphabet **A** and **N** should be side by side?

c) How many designs of the bracelet can be made if the bracelet should consists the sequence of beads **A**, **I**, **N** in any arrangement?

d) If the 5 beads are cut loose from the bracelet and rearranged in linear sequence, how many initials of 3 alphabet beads can be made?

9. A bagel shop has 3 union bagels, 3 poppy seed bagels, 3 egg bagels, 4 salty bagels and 4 pumpernickel bagels. How many way are there to choose

a) a dozen bagels

b) seven bagels of egg bagel, pumpernickel bagels and salty bagels with three egg bagels and no more than two salty bagels?

10. A linear algebra class consists of 10 mathematics majors and 12 computer science majors. A team of 12 has to be selected from this class. Find the number of ways of selecting a team if

a) The team has 6 from each discipline

b) The team has a majority of computer science majors

Tutorial 2.1

1. Bob, John, Luke and Tim play a tennis tournament. The rules of the tou
Are such that at the end of the tournament a ranking will be made
and there will be no ties. How many different ranking can there be?

$P_4 = 4! = 4 \times 3 \times 2 \times 1$

$= 24$

2. There is a basket of fruit containing apple, a banana and an orange
and there are five girls who want to eat one fruit. How many ways
are there to give three of the five girls one fruit each and leave
of them without a fruit to eat?

$P(5,3) = \frac{5!}{(5-3)!}$

$= \frac{5!}{2!} = \frac{5.4.3.2.1}{2.1} = 5.4.3 = 60$

3. How many ways are there to order the letters of the word INDESCR

$\frac{14!}{2!3!4!} = 302702400$ ways

4. There are 12 hospitals in a town. How many different ways can 7 pat
are sent to the hospitals so that no 2 patients may be in the sa
hospital.

$12P_7 = \frac{12!}{(12-7)!} = \frac{12!}{5!} = 3991680$

5. A computer scientist is trying to discover the keyword for financial account
If the keyword consists only of 10 lower case characters (e.g. 10 chara
from among the set: a, b, c, ... y, z) and no character can be repeated
how many different unique arrangement of character exist?

$n = 26$ (choosing from 26 possibilities (eg: a, b, c, ..., y, z))

$r = 10$ characters

$26P_{10} = \frac{26!}{(26-10)!} = \frac{26!}{16!} = 19275223968000$

8. A bracelet is made from 5 alphabet beads as shown in figure 1

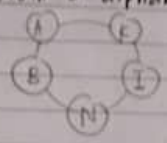

figure 1

a) How many designs of bracelet can be made from the 5 beads in any possible arrangement?

$(5-1)! = 4! = 24$

b) How many designs of the bracelet can be made if the bead with alphabet A and N should be side by side?

AN, B, K, I $(4-1)! = 3!$

AN switch places $= 3! \cdot 2! = 12$

c) How many designs of the bracelet can be made if the bracelet should consists the sequence of beads A, I, N in any arrangement?

AIN, B, K $= (3-1) = 2!$

AIN in any arrangement $= 2! \cdot 3!$

$= 12$

d) If the 5 beads are cut loose from the bracelet and linear sequence, how many triffals of 3 alphabet beads can be made?

$= {}_5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = 60$

9. A bagel shop has 3 onion bagels, 3 poppy seed bagels, 3 egg bagels, 4 salty bagels and 4 pumpernickel bagels. How many way are there to choose

a) a dozen bagels

$n = 3+3+3+4+4 = 17$

$r = 12$

$C(n+r-1, r) = (17+12-1, 12) = C(28, 12) = \frac{28!}{12!(17-1)!}$

$= \frac{28!}{12!\,16!} = 30421755$ ways

DATE:

9. b) seven bagels of egg bagel, pumpernickel bagels and salty bagels with three egg bagels and no more than two salty bagels?

$n = 17$, $r = 7$

No salty bagels

$n = 17 - 4 = 13$ (exclude salty bagels) $r = 7 - 3 = 4$ (7 - 3 egg bagels)

$C(n+r-1, r) = C(13+4-1, 4) = C(16,4) = \frac{16!}{4!(13-1)!} = \frac{16!}{4!\,12!} = 1820$ ways

One salty bagel

$n = 13$ $r = 7 - 3 - 1 = 3$ (7 - 3 egg bagels - 1 salty bagel)

$C(n+r-1, r) = C(13+3-1, 3) = C(15,3) = \frac{15!}{3!(13-1)!} = \frac{15!}{3!\,12!} = 455$ ways

Two salty bagels

$n = 13$ $r = 7 - 3 - 2 = 2$ (7 - 3 egg bagels - 2 salty bagels)

$C(n+r-1, r) = C(13+2-1, 2) = C(14,2) = \frac{14!}{2!(13-1)!} = \frac{14!}{2!\,12!} = 91$ ways

$C(16,4) + C(15,3) + C(14,2) = 1820 + 455 + 91 = 2366$ ways

10. A linear algebra class consists of 10 mathematics majors and 12 computer science majors. A team of 12 has to be selected from this class. Find the number of ways of selecting a team if

a) The team has 6 from each discipline

$$^{10}C_6 \times {}^{12}C_6$$

$$= \frac{10!}{6!\,4!} \times \frac{12!}{6!\,6!}$$

$$= 210 \times 924$$

$$= 194040 \text{ ways}$$

10. b) The team has a majority of computer science majors

mathematics major		Computer science majors	
$^{10}C_0$	x	$^{12}C_{12}$	$= 1$
$^{10}C_1$	x	$^{12}C_{11}$	$= 120$
$^{10}C_2$	x	$^{12}C_{10}$	$= 2970$
$^{10}C_3$	x	$^{12}C_9$	$= 26400$
$^{10}C_4$	x	$^{12}C_8$	$= 103950$
$^{10}C_5$	x	$^{12}C_7$	$= 199584$

$1 + 120 + 2970 + 26400 + 103950 + 199584 = 333025$ ways