Sections 8.1 and 8.2

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"Learning is not attained by chance, it must be sought for with ardor and diligence."

- Abigail Adams

Xylitol Gum and Ear Infections

Xylitol is a food sweetener that may also have antibacterial properties. A Finland study was designed to test if chewing gum containing xylitol could reduce the risk of middle ear infections in children in daycare centers. The researchers divided a representative group of 533 daycare children into three groups. Group 1 regularly chewed gum containing xylitol, group 2 took xylitol lozenges, and group 3 regularly chewed gum without xylitol. Subjects were followed for three months and for each child the researchers recorded whether or not the child had an ear infection.

The results are shown in the table below.

Ear Inf?	Lozenge	Xylitol Gum	Placebo	Total
Yes	39 (22.2%)	29 (16.2%)	49 (27.6%)	117 (22%)
No	137 (77.8%)	150 (83.8%)	129 (72.5%)	416 (78%)
Total	176	179	178	533

Observational Units

(1) What are the observational units?

A Gum and lozenges

B Ear infections

C Finish children in daycare centers

D Whether or not xylitol can reduce the number of ear infections in Finnish daycare children

E None of the above

(2) What is the response variable?

A Whether or not a subject developed an ear infection.

B The proportion of children that developed an ear infection.

C The treatment given, xyltiol gum, xylitol lozenge, or placebo gum.

D Whether or not xylitol can reduce the number of ear infections in Finnish daycare children.

E The mean absolute difference in proportions.

(3) If one wanted to summarize the overall proportion of subjects that developed an ear infection, which of the following would be the best statistic to use?

A $\hat{p}_{placebo}$, since this group really did not receive any treatment.

B $\hat{p}_{lozenge}$, since it is in between the other two conditional proportions.

C The average proportion, $\dfrac{\hat{p}_{lozenge} + \hat{p}_{gum} + \hat{p}_{placebo}}{3} = 0.22$.

D The total proportion, $\hat{p} = \frac{117}{533} = 0.2195$.

(4) What is the value of the MAD statistic for the xylitol gum study?

A 0.22

B 0.075

C 0.226

D 0.038

E 0.66

F None of the above.

(5) What would the null hypothesis be if we wanted to test all three proportions to see if they are equal?

A $H_0 : \hat{p}_{gum} = \hat{p}_{lozenge} = \hat{p}_{placebo}$

B $H_0 : MAD = 0.076$

C $H_0 : \pi_{earInfection} = 0.5$

D $H_0 : \pi_{earInfection} - \pi_{noInfection} = 0$

E $H_0 : \pi_{gum} - \pi_{lozenge} - \pi_{placebo} = 0$

F None of the above

(6) What would the alternative hypothesis be if we wanted to test all three proportions to see if they are equal?

A H_a : there is an association between the treatments and ear infections.

B $H_a : \pi_{gum} \neq \pi_{lozenge} \neq \pi_{placebo}$

C $H_a : \pi_{earInfection} - \pi_{noInfection} \neq 0$

D H_a : at least two of $\pi_{gum}, \pi_{lozenge}$ and $\pi_{placebo}$ are different.

E Two of the above are correct.

(7) True or False, the expected number of ear infections in the xylitol gum group is $\hat{p}_{earInfection} \times n_{gum} = \frac{117}{533} \times 179 \approx 39.29$ (all the arithmetic is correct).

A True and I am confident

B True but I am not too confident

C False and I am confident

D False but I am not too confident

(8) Values of the counts *Observed* − *Expected* are shown in the table below. The Multiple Proportions applet produced CIs for $\pi_{lozenge} - \pi_{gum}, \pi_{gum} - \pi_{placebo}$ and $\pi_{lozenge} - \pi_{placebo}$ to be (in random order) $(-0.198, -0.028), (-0.144, 0.036)$ and $(-0.022, 0.141)$, which is the CI for $\pi_{lozenge} - \pi_{gum}$?

A $(-0.198, -0.028)$

B $(-0.144, 0.036)$

C $(-0.022, 0.141)$

	observed - expected		
	Lozenge	**Gum**	**Placebo**
Yes	0.365854	-10.2927	9.926829
No	-0.36585	10.29268	-9.92683

(9) The study had $MAD \approx 0.076$; 500 simulated values of the MAD statistic are shown in the image below, what can you say about the p–value for this study?

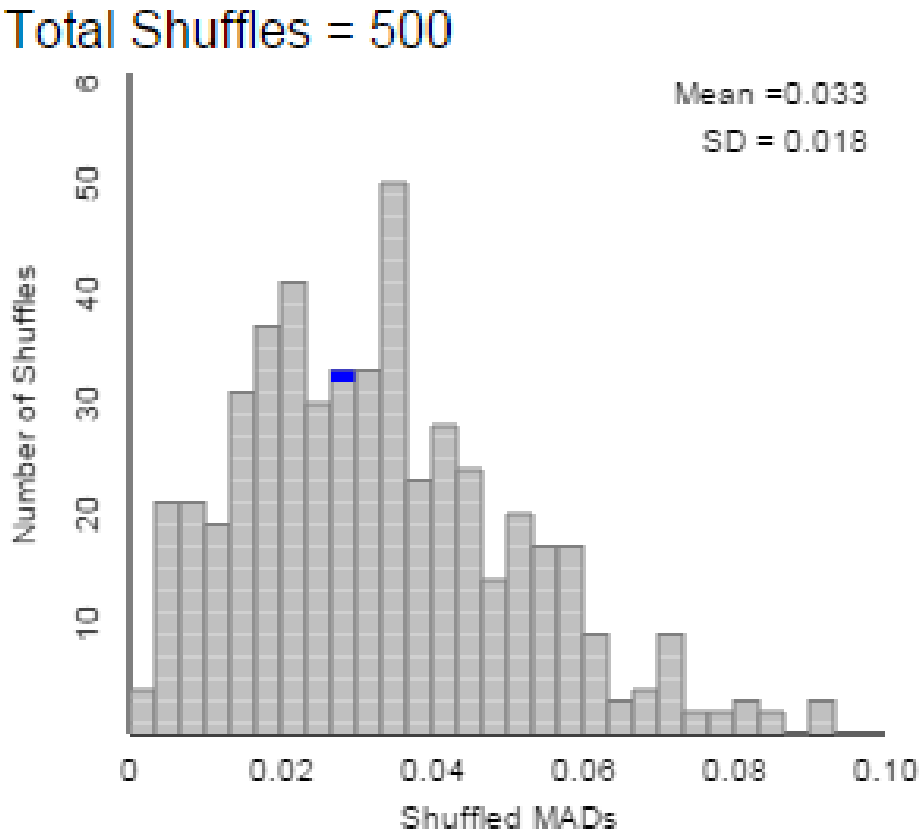
A $p < 0.01$

B $0.01 < p < 0.05$

C $0.05 < p < 0.1$

D $0.1 < p < 0.5$

E $p > 0.5$



(10) The chi-squared contributions $\left(\frac{(observed-expected)^2}{expected}\right)$ for the xylitol gum group ear infections, and no ear infections are 2.70 and 0.76 respectively, which is the best interpretation of these values?

A Both are much larger than 0.05 and therefore represent no evidence against our null hypothesis.

B Both values are larger than expected and the ear infection contribution of 2.70 is more significant than the no infection contribution of 0.76.

C Both values are quite small and are essentially negligible in terms of the total chi-square statistic.

D The smaller the contribution, the more evidence against the null hypothesis, so 0.76 is more significant than 2.70.

Key Terms and Ideas to Understand in Chapter 8

- Chi-square distribution
- Chi-square statistic
- MAD derived statistic
- Validity conditions for chi-square test
- Pairwise confidence intervals