

ROBUST FLIGHT CONTROL SYSTEM DESIGN

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Abstract. The short period longitudinal mode of an F4-E aircraft with additional horizontal canards is unstable in subsonic flight and unsufficiently damped at supersonic speed. The control system has to provide acceptable pole locations according to military specifications for flying qualities. A fixed gain controller is designed, such that the pole region requirements are met in four typical flight conditions simultaneously. For robustness of pole region requirements with respect to one or two sensor failures it turns out, that it is advantageous not to use accelerometers. A controller with three parallel gyros and dynamic output feedback is designed. The design is performed by a novel parameter space design method.

Keywords. Aircraft control; Pole region assignment; Robust control; Parameter space method.

INTRODUCTION

Redundancy management in control systems is usually viewed separately from the control algorithm. The control system is designed under the assumption, that sensors do not fail. Then redundancy management has to provide the required measurements with only very short interruptions by failures of individual sensors. If the plant is for example an unstable aircraft, this means that failure detection is vital for stabilization, it has to operate fast and this requirement is in conflict with the requirement of low probability of false alarms.

In this paper a hierarchical concept is proposed. Its basic level is a fixed gain control system, which is designed such, that pole region requirements are robust with respect to changing flight conditions and component failures. All more sophisticated tasks like failure detection and redundancy management, plant parameter identification and controller parameter adaptation or gain scheduling are assigned to higher levels, if they are required for best performance. The higher levels process more information and are operating in a slower time scale than the basic level. Since the higher levels are not vital for stabilization they can make their decisions without panic haste.

This paper deals with the design of the robust basic level control system. The particular example is an F4-E aircraft, which is destabilized by horizontal canards, see Fig. 1. Only the short period longitudinal mode is considered, i.e. second order dynamics. The actuator is modelled as a first order low pass with transfer function

$14/(s + 14)$, its state variable is δ_e , the deviation of the elevator deflection from its trim position. δ_e is not fed back, because this would require an estimate of the trim position.

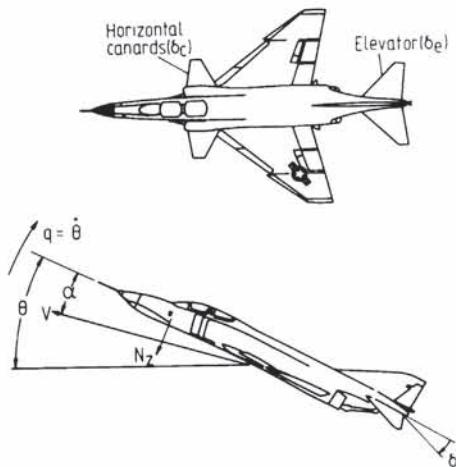


Fig. 1 F4-E aircraft with additional horizontal canards.

In a previous study Franklin (1980, 1981) assumed measurement of normal acceleration N_z and pitch rate q and the linearized state equations were written in sensor coordinates with the state vector $\underline{x}^T = [N_z \quad q \quad \delta_e]$. Thus

$$\begin{aligned}\dot{\underline{x}} &= \underline{A} \underline{x} + \underline{b} u \\ \underline{A} &= \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & -14 \end{bmatrix} \quad \underline{b} = \begin{bmatrix} b_1 \\ 0 \\ 14 \end{bmatrix} \quad (1)\end{aligned}$$

Data for four typical flight conditions were taken from Berger, Hess and Anderson (1973) and are given in the appendix. The eigenvalue locations of the short period mode are given in Table 1.

TABLE I OPEN-LOOP EIGENVALUES

FLIGHT CONDI-	MACH	ALTITUDE	OPEN LOOP SHORT PERIOD EIGENVALUES
1	0.5	5000'	-3.07
2	0.85	5000'	-4.90
3	0.9	35000'	-1.87
4	1.5	35000'	-0.87 \pm j4.3

The aircraft is unstable in subsonic flight and unsufficiently damped in supersonic flight, such that adequate handling properties must be provided by the control system. Note that in stationary flight the elevator and canard are not used independently. The commanded deflections are coupled as $\delta_{\text{ecom}} = u$, $\delta_{\text{ccom}} = -0.7u$, where the factor -0.7 was chosen for minimum drag. Thus the short period mode stabilization is a single-input problem.

The required closed loop eigenvalue locations are given by military specifications for flying qualities of piloted airplanes (1969). For the short period mode described by

$$s^2 + 2\zeta_{\text{sp}}\omega_{\text{sp}}s + \omega_{\text{sp}}^2 = 0 \quad (2)$$

the restricted range of damping ζ_{sp} and natural frequency ω_{sp} is

$$0.35 \leq \zeta_{\text{sp}} \leq 1.3 \quad (3)$$

$$\omega_a \leq \omega_{\text{sp}} \leq \omega_b$$

where ω_a and ω_b depend on the flight condition and are given in the appendix for the four conditions considered here.

Fig. 2 shows the nominal region Γ_j , eq.(3) together with the open loop eigenvalues for a subsonic flight condition j . Damping greater than one in eq.(3) corresponds to two real eigenvalues. Eq.(3) would admit some real pairs of poles with one of them outside the region Γ_j . In the following no use is made of this possibility. For all real pairs inside Γ_j condition (3) is satisfied. We require, that the closed loop short period poles of each flight condition $j = 1, 2, 3, 4$ are located in the respective region Γ_j .

The military specifications do not contain requirements for the location of additional closed loop poles originating from actuator or feedback dynamics. Quick response is essential for a fighter, therefore the non short

period eigenvalues should not unnecessarily slow the dynamic response. In order to keep them fast enough and separate from short period eigenvalues an additional region to the left of Γ_j is prescribed. The damping requirement $\zeta \geq 0.35$ is kept from eq.(3) and a natural frequency range $\omega_b \leq \omega \leq \omega_d$, $\omega_d = 70$ rad/sec is chosen in order to maintain a bandwidth limitation below the first structural mode frequency. The extended region is also shown in Fig. 2.

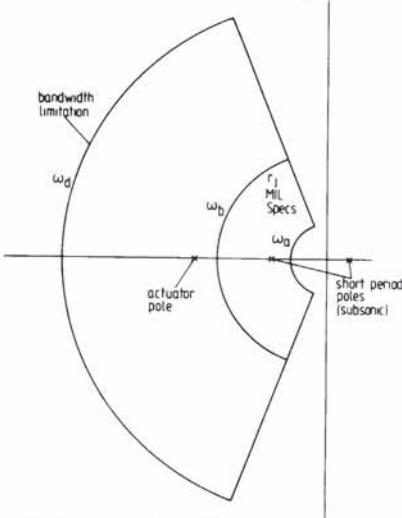


Fig. 2 Required pole region

The assumed type of sensor failure is that the nominal gain $v = 1$ is reduced to some value $0 \leq v < 1$. As far as eigenvalue location is concerned, only this multiplicative error is important. There may be an additive bias or noise term, which should be removed by a failure detection system at a higher hierarchical level.

The objective of this paper is to design the basic level control system such that the pole region requirements of Fig. 2 are robust with respect to changing flight conditions and sensor failures. This is an example for the application of a novel parameter space design technique and generalized D-decomposition, see Ackermann and Kaesbauer (1980, 1981). It will be reviewed briefly in the following paragraph. In application to the example it is then shown, how robustness with respect to changing flight conditions can be achieved by appropriate choice of k_{Nz} and k_q in an output feedback control law

$$u = -[k_{Nz} \quad k_q \quad 0] x \quad (4)$$

For robustness with respect to sensor failures Franklin (1980, 1981) studied a configuration with two gyros and one accelerometer and dynamic feedback. It showed the disadvantage of using the accelerometer. Therefore here a different solution with three gyros and dynamic feedback is given. For this solution the responses in C^∞ for a pilot step input are given, where

$$C^\infty = (N_z + 12.43q)/C_\infty \quad (5)$$

The stationary value C_∞ is used for normalization.

POLE REGION ASSIGNMENT

If a tradeoff with other design requirements has to be made it is not satisfactory to find one solution, for which all eigenvalues are in their respective regions in s -plane, e.g. by pole placement or root locus techniques. It is desirable to find all such solutions. This is achieved by mapping the region Γ in s -plane first into a region P_Γ in the parameter space \mathcal{P} of coefficients of the desired characteristic polynomial:

$$P(s) = [\underline{p}^T \ 1] [1s \dots s^n]^T = \prod_{i=1}^n (s - s_i)$$

$$\underline{p}^T \in P_\Gamma \Leftrightarrow s_i \in \Gamma \text{ for } i = 1, 2 \dots n \quad (6)$$

In the second step a controller structure is assumed, e.g. state feedback

$$u = -\underline{k}^T \underline{x}, \quad \underline{k}^T = [k_1 \ k_2 \ \dots \ k_n] \quad (7)$$

and the region P_Γ is mapped into a region K_Γ in the controller parameter space \mathcal{K} with coordinates $k_1, k_2 \dots k_n$ such that $\underline{k}^T \in K_\Gamma$ if and only if $\underline{p}^T \in P_\Gamma$. It was shown by Ackermann (1980), that for state feedback, eq.(7), this is accomplished by an affine mapping

$$\underline{k}^T = [\underline{p}^T \ 1] \underline{E} \quad (8)$$

where the pole assignment matrix \underline{E} describes the plant A , b . For each pair A_j, b_j , i.e. for each flight condition, a different mapping \underline{E}_j results and the solution set is the intersection of the regions K_{Γ_j} in \mathcal{K} -space. In the aircraft example for each flight condition the respective region Γ_j and P_{Γ_j} has to be mapped. For the aircraft $n = 3$ and $k_3 = 0$, i.e. we are looking at a two dimensional cross-section of the three-dimensional region K_Γ .

By eq.(8) a fixed gain k_3 implies a linear relationship $k_3 = [\underline{p}^T \ 1] \underline{n}_3$, where \underline{n}_3 is the third column of \underline{E} . Eq.(8) can be used to express the two free gains by coefficients of a second order factor of the characteristic polynomial, which is varied along the boundary in s -plane to produce the boundary in the plane of the two free gains, see Franklin (1980). Alternatively the generalized D-decomposition technique described by Kaesbauer (1981) may be used. This tool will be applied to the aircraft example in the next section.

ROBUSTNESS WITH RESPECT TO FLIGHT CONDITION

The first design objective will be to design an output feedback controller, eq.(4), which meets the nominal pole region requirements at all four flight conditions.

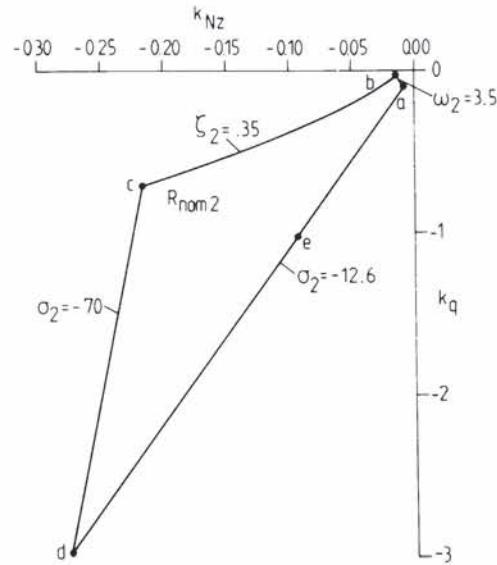


Fig. 3 Admissible gain region for flight condition 2

The boundary for flight condition 2 is shown in Fig. 3. On a-b eigenvalues are on the lower natural frequency boundary $\omega_{sp} = 3.5$, on b-c they are on the damping 0.35 lines. At c a real root boundary takes over: on c-d the actuator eigenvalue is at $\sigma = -70$. On d-e a real short period eigenvalue is at the upper natural frequency limit $\sigma = -12.6$ and for e-a the actuator eigenvalue is at $\sigma = -12.6$. The condition for having no real root $\sigma = -3.5$ is satisfied in the total region. This region R_{nom2} is bounded by two straight lines c-d and d-a resulting from real root conditions and by the two complex boundary curves a-b and b-c. Note that the boundaries in s -plane are conic sections and thus a-b and b-c are segments of conic section also.

The regions $R_{nom1} - R_{nom4}$ for the other flight conditions were found by mapping the eigenvalue constraints for each flight condition into the $k_{Nz} - k_q$ -plane. These four regions have the intersection R_{nom} shown in Fig. 4. Thus robustness with respect to changing flight conditions can be achieved by static output feedback of the accelerometer and gyro signals. More precisely: All eigenvalues at all four flight conditions are in their prescribed regions in s -plane if and only if the pair k_{Nz}, k_q is chosen in the region R_{nom} .

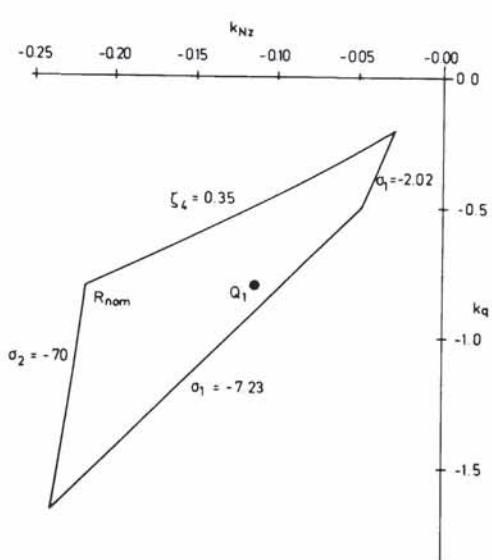


Fig. 4 Intersection of admissible gain regions for four flight conditions

As an example choose the design point Q_1 , i.e. $k_{Nz} = -0.115$, $k_q = -0.8$. The closed loop eigenvalues are given in Table 2.

TABLE 2 CLOSED-LOOP EIGENVALUES FOR Q_1

FC	Short period eigenvalues damping	natural frequency	Actuator eigen- value
1	0.94	4.68	- 18.31
2	0.61	9.18	- 37.29
3	0.79	4.63	- 17.78
4	0.55	8.11	- 27.04

The selection of a design point in R_{nom} is a tradeoff, in which the designer learns, which requirements are conflicting. E.g. structural vibrations are critical in flight condition 2 (high speed, low altitude). They can be reduced by avoiding the vicinity of the $\sigma_2 = -70$ boundary. Low damping is most critical at the supersonic flight condition 4. Damping can be increased by avoiding the vicinity of the $\zeta_4 = 0.35$ boundary. Sluggish responses in landing approach would occur in the vicinity of the $\sigma_1 = -2.02$ boundary. The $\sigma_1 = -7.23$ boundary is only necessary in order to separate actuator and short period poles, the design point may be chosen close to this boundary.

ROBUSTNESS WITH RESPECT TO SENSOR FAILURES

As far as stability is concerned, a failure of the accelerometer (gyro) is equivalent to a reduction of k_{Nz} (k_q) from the nominal value to zero or some value in between.

Fig. 4 shows that the nominal region does not intersect the axes, thus in the assumed output feedback structure it is not possible to maintain nominal specifications after either failure.

Fig. 4 shows however that a considerable gain reduction inside R_{nom} is admissible, if $|k_{Nz}|$ and $|k_q|$ are reduced simultaneously. This can be achieved by replacing the accelerometer measurement N_z by an estimate \hat{N}_z , which is produced by a filter from q , see Fig. 5. It is not necessary that this is a true estimate, e.g. generated by an adaptive observer. It is sufficient, that this is a constant filter connected to q such that the frequency response from u to the filter output \hat{N}_z is an approximation to the frequency response from u to N_z for an average over the four flight conditions. Of course separation does not hold, i.e. we can not take the same pair of feedback gains k_{Nz} , k_q as in the case of accelerometer measurement. However this consideration leads to a structure of the feedback system with a two dimensional signal basis q and \hat{N}_z , and a new exact determination of admissible regions in the plane of the two feedback gains k_{Nz} , k_q , can be made.

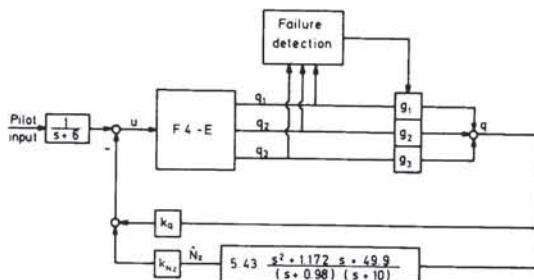


Fig. 5 Gyro feedback control system

Both transfer functions from u to N_z and to q have the same denominator, thus the filter has to cancel approximately the zeros in the q -channel and to replace them by the averaged zeros of the N_z channel. Table 3 shows the zeros and gain ratios of the transfer functions at the four flight conditions.

TABLE 3 OPEN LOOP ZEROS AND GAIN RATIO

FC	q-ZERO	Nz-ZERO	K_N/K_Q
1	-0.884	-0.542+j5.33	0.527
2	-1.57	-0.929+j9.12	0.536
3	-0.637	-0.392+j5.67	0.537
4	-0.826	-0.481+j8.05	0.577
AVERAGED VALUES	-0.98	-0.586+j7.04	0.543

Fortunately the gain ratio is almost constant. The filter is then

$$\hat{\frac{N_z}{q}} = 0.543 \frac{s^2 + 1.172s + 49.9}{(s+0.98)} \cdot \frac{10}{s+10} \quad (9)$$

The term $10/(s+10)$ was included to make the filter realizable. The pole at $s = -0.98$ approximately cancels the q -zero and is therefore weakly controllable from u , i.e. the corresponding closed loop pole will remain in the vicinity of -0.98 . This however has little effect on the C^∞ step responses and is exempted from the pole region requirements.

Note that the corresponding idea to use the accelerometer only and to omit the gyro leads to the inverse filter of eq.(9) without the term $10/(s+10)$. Here the approximate cancellation occurs for a complex pair in the vicinity of $s = -0.586 \pm j7.04$, i.e. close to the imaginary axis and no robustness with respect to changing flight condition can be achieved, see Franklin (1980).

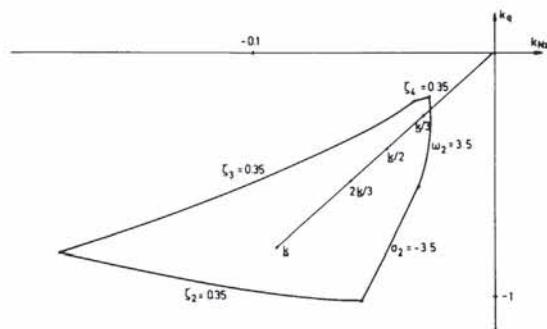


Fig. 6 Intersection of admissible gain regions. \underline{k} admits gain reduction to one third.

Fig. 6 shows the intersection of the admissible regions for the four flight conditions. It is seen, that flight conditions 2 and 3 are the critical ones now. We choose

$$\underline{k} = \begin{bmatrix} k_{Nz} \\ k_q \end{bmatrix} = \begin{bmatrix} -0.09 \\ -0.8 \end{bmatrix} \quad (10)$$

such that the pole region requirements are satisfied for $2k/3$, $k/2$ and $k/3$.

The failure detection logic in Fig. 5 decides as follows

- a) Three gyros unfailed: $g_1 = g_2 = g_3 = 1/3$
- b) Gyro i failed: $g_i = 0$, $g_j = 1/2$ $j \neq i$.

Before the decision b) has been made, we have a case between k and $2k/3$. If a second gyro fails after decision b) has been made, we have a case between k and $k/2$. Only in the unlikely case that a second gyro fails before the first failure has been detected, the gain may be reduced to $k/3$. For the four typical cases k , $2k/3$, $k/2$ and $k/3$ the eigenvalues are given in the appendix. They meet all pole region requirements. Also the C^∞ step responses have been simulated for the open loop $k_{Nz} = k_q = 0$, Fig. 7, and for the closed loop $k_{Nz} = k_q = \underline{k}$ with \underline{k} and $\underline{k}/2$, Fig. 8.

Figure 9 shows the corresponding elevator deflections δ_e .

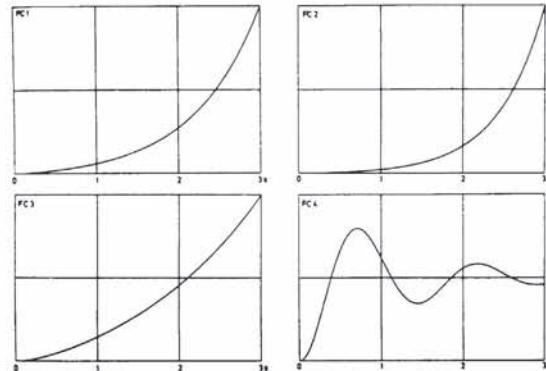


Fig. 7 Open loop step responses for four flight conditions

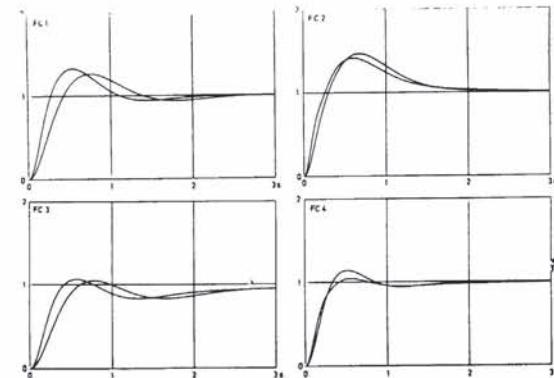


Fig. 8 Closed loop step responses for nominal gain (steeper rise) and for 50 % gain reduction

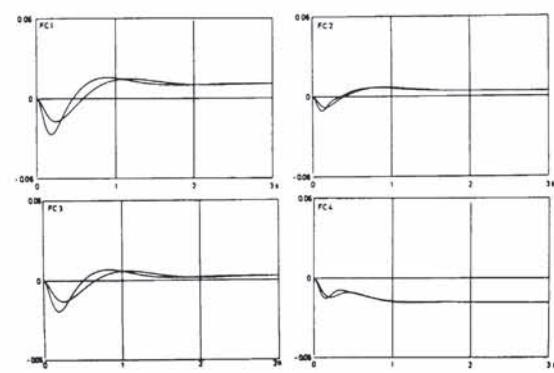


Fig. 9 Elevator deflections for nominal gain and for 50 % gain reduction

CONCLUSIONS

It has been demonstrated, that an integrated view of redundancy and handling quality requirements for the control of the short period mode of an unstable fighter plane can lead to a simple fault tolerant control system. It is interesting that the measurement by gyros alone not only saves the cost

of additional accelerometers but even has advantages for the control system. It is also interesting that a constant controller can control the aircraft at very different flight conditions. Here it must be noted of course that nice stabilization for several typical stationary flight conditions, i.e. different linearizations of a nonlinear system is only a necessary, not a sufficient condition for the stability of the nonlinear system. The nonlinear system is usually tested in simulations. Also for these simulations it is an advantage if the control system is simple, i.e. does not require gain scheduling and different sensor types and if failure detection is not vital for stabilization. Also in the nonlinear case, Fig.6 offers a set of promising controller candidates for further investigation.

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APPENDIX

1) Aerodynamic data for eq.(1)
for four flight conditions (FC)

	FC 1	FC 2	FC 3	FC 4
Mach	0.5	0.85	0.9	1.5
Altitude	5000'	5000'	35000'	35000'
a_{11}	-0.9896	-1.702	-0.667	-0.5162
a_{12}	17.41	50.72	18.11	26.96
a_{13}	96.15	263.5	84.34	178.9
a_{21}	0.2648	0.2201	0.08201	-0.6896
a_{22}	-0.8512	-1.418	-0.6587	-1.225
a_{23}	-11.39	-31.99	-10.81	-30.38
b_1	-97.78	-272.2	-85.09	-175.6

2) Military specifications for flying qualities, see eq.(3)

Natural frequ. (rad/sec)	FC 1	FC 2	FC 3	FC 4
ω_a	2.02	3.50	2.19	3.29
ω_b	7.23	12.6	7.86	11.8

3) Closed-loop eigenvalues

Complex eigenvalues $s^2 + 2\zeta\omega_s + \omega^2$ are written (ζ, ω) . The short period eigenvalues are listed first.

Gain	FC 1	FC 2	FC 3	FC 4
\underline{k}	(0.60, 4.30) (0.60, 17.2) -0.87	(0.68, 4.63) (0.38, 26.4) -1.63	(0.57, 4.38) (0.64, 16.2) -0.62	(0.65, 5.34) (0.45, 20.9) -0.86
$2\underline{k}/3$	(0.57, 3.86) (0.71, 15.3) -.86	(0.66, 4.41) (0.47, 22.0) -1.67	(0.52, 3.95) (0.74, 14.5) -0.61	(0.60, 5.47) (0.55, 17.6) -0.87
$\underline{k}/2$	(0.55, 3.47) (0.77, 14.4) -.85	(0.64, 4.17) (0.54, 19.6) -1.72	(0.50, 3.59) (0.80, 13.8) -0.60	(0.56, 5.54) (0.62, 1.57) -0.88
$\underline{k}/3$	(0.56, 2.86) (0.84, 13.5) -0.83	(0.61, 3.69) (0.64, 17.0) -1.84	(0.48, 3.05) (0.87, 13.1) -0.59	(0.48, 5.53) (0.74, 13.9) -0.90
0 open loop	1.23 -3.07 -14 -10 -0.98	1.78 -4.90 -14 -10 -0.98	0.56 -1.87 -14 -10 -0.98	(0.20, 4.4) -14 -10 -0.98

For Discussion see page 1201