KOM 501E
Control of Systems with Parametric Uncertainty

Final Project
Robust Flight Control System Design

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ROBUST FLIGHT CONTROL SYSTEM DESIGN

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Abstract. The short period longitudinal mode of an F4-E aircraft with additional horizontal canards is unstable in subsonic flight and unsufficiently damped at supersonic speed. The control system has to provide acceptable pole locations according to military specifications for flying qualities. A fixed gain controller is designed, such that the pole region requirements are met in four typical flight conditions simultaneously. For robustness of pole region requirements with respect to one or two sensor failures it turns out, that it is advantageous not to use accelerometers. A controller with three paralieled gyros and dynamic output feed-back is designed. The design is performed by a novel parameter space design method.

Keywords. Aircraft control; Pole region assignment; Robust control; Parameter space method.

- Problem Definition
- F4-E at Different Flight Conditions (FCs)
- Required Pole Regions from Military Specifications
- Γ-Stability
- D-Decomposition Admissible Gain Regions (R_{nom-i})
- Robust Design Point and Results
- Robustness w.r.t. Sensor Failures

Problem Definition

The short period longitudinal mode of an F4-E aircraft with additional horizontal canards is

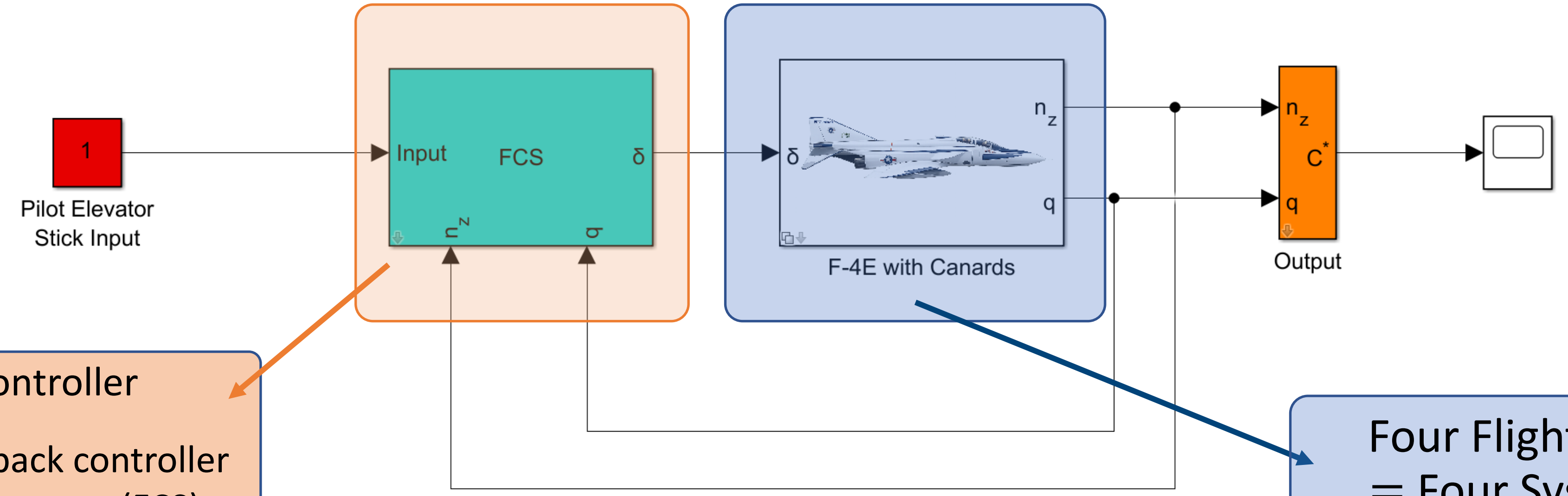
- unstable in subsonic flight
- insufficiently damped in supersonic flight

Horizontal canards destabilizes the aircraft

YF-4E: First Fighter Aircraft with Analog Fly-by-Wire

Design (fixed gain) controller that provide acceptable pole locations according to military specifications for flying qualities in four typical flight conditions simultaneously.

Problem Definition (continued)



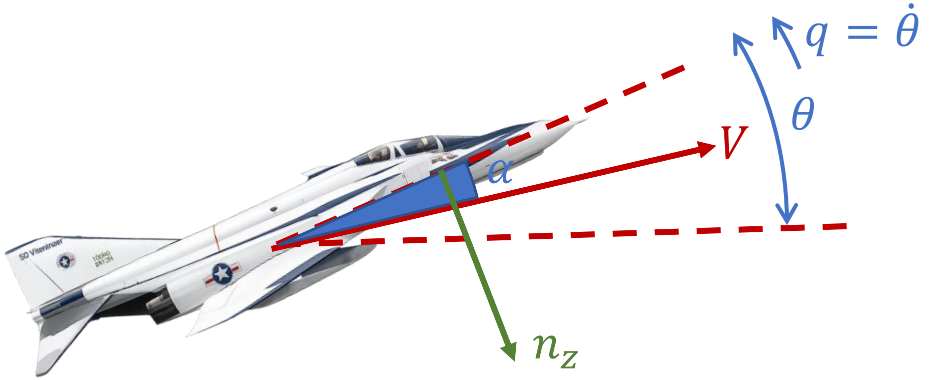
Fixed-Gain Controller

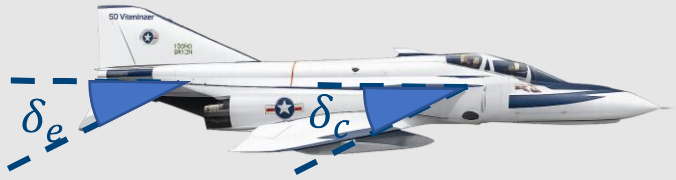
Basically state feedback controller as a flight control system (FCS)

Four Flight Condition = Four System Model

$$\dot{x}(t) = A^{(j)}x(t) + B^{(j)}u(t)$$

$$j = 1, 2, 3, 4$$





$$\delta_{e-com} = u, \delta_{c-com} = -0.7\,u$$

Single Input Problem

$$x = \begin{bmatrix} n_z \\ q \\ \delta_e \end{bmatrix}$$

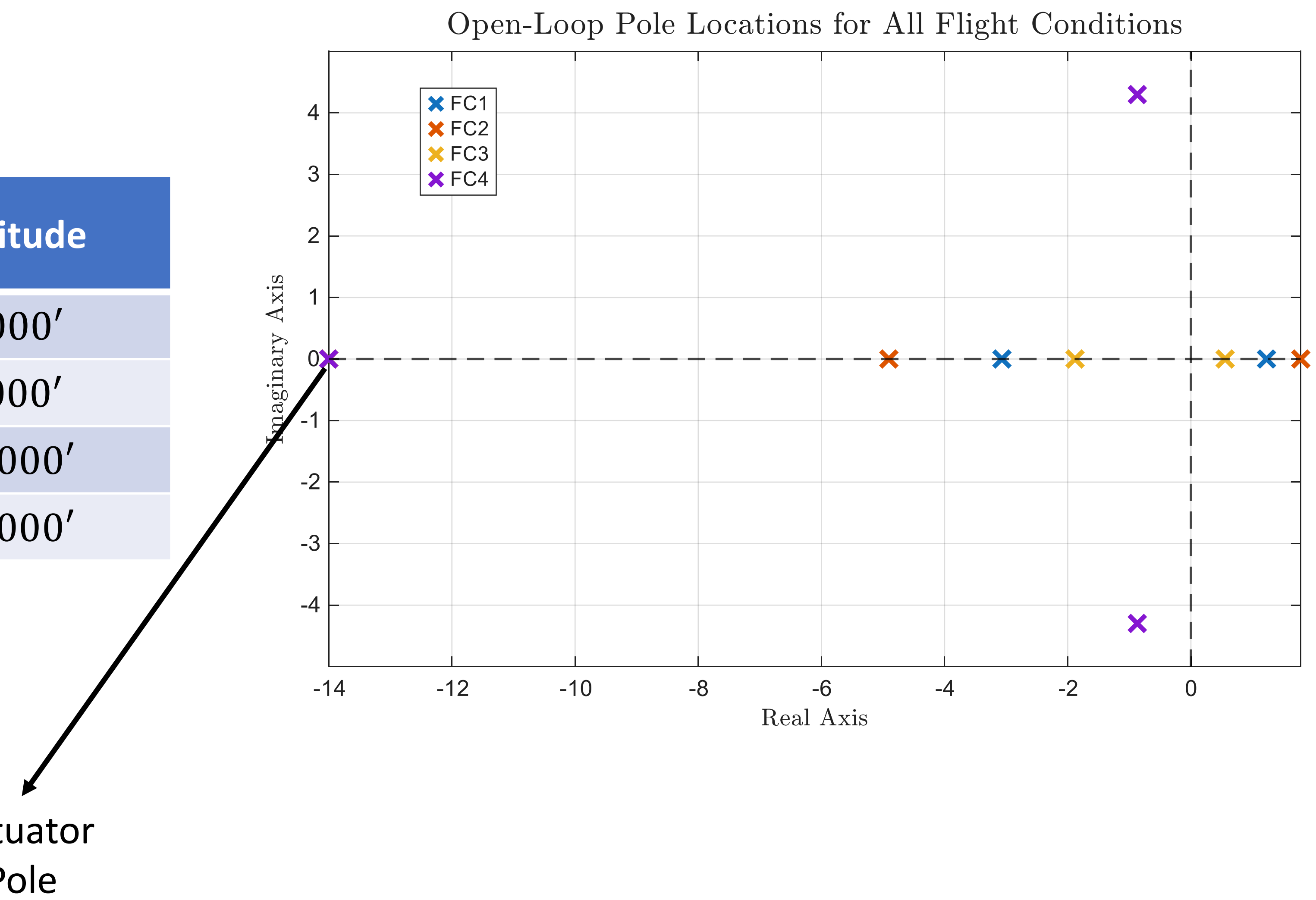
n_z: normal acc.

q: pitch rate

δ_e: elev. deflection (is not fed back)

F4-E at Different Flight Conditions

Flight Condition	Mach	Altitude
1	0.5	$5000'$
2	0.85	$5000'$
3	0.9	$35000'$
4	1.5	$35000'$



Required Pole Regions from Military Specs.

According to military specifications (MIL-F-8785 C) for flying qualities of piloted airplanes

Short Period Mode: $\boxed{s^2 + 2\zeta_{sp}\omega_{sp} + \omega_{sp} = 0}$

Restricted Ranges

$$0.35 \leq \zeta_{sp} \leq 1.3$$

$$\omega_a \leq \omega_{sp} \leq \omega_b$$

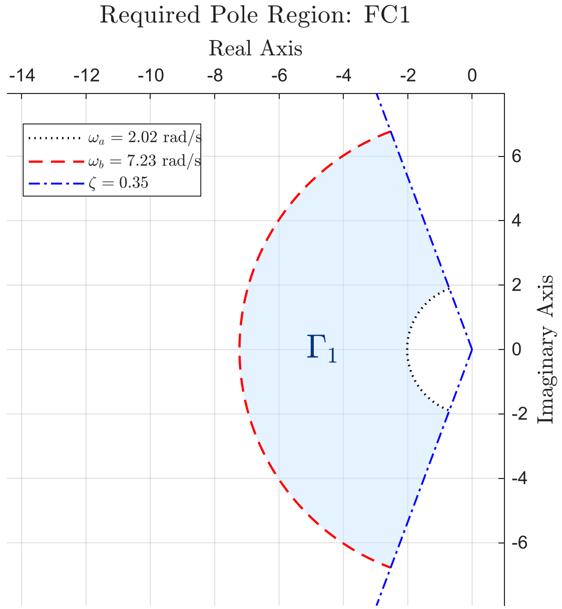
ω_a and ω_b depends on FC

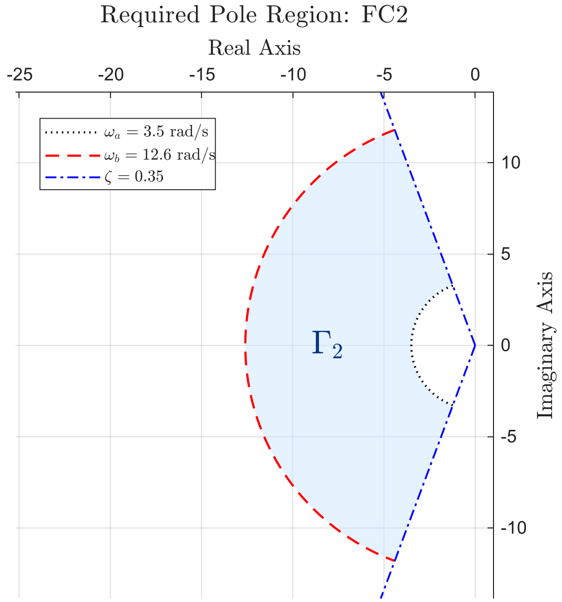
Natural Frequency $[rad/sec]$	FC 1	FC 2	FC 3	FC 4
ω_a	2.02	3.50	2.19	3.29
ω_b	7.23	12.6	7.86	11.8

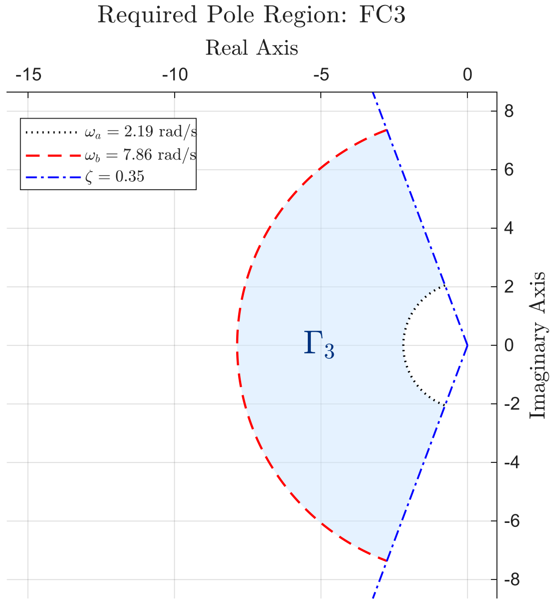
There are no requirements for the location of additional poles from actuator or feedback dynamics. Quick response is essential for fighter. However, triggering the structural mode frequency should be avoided.

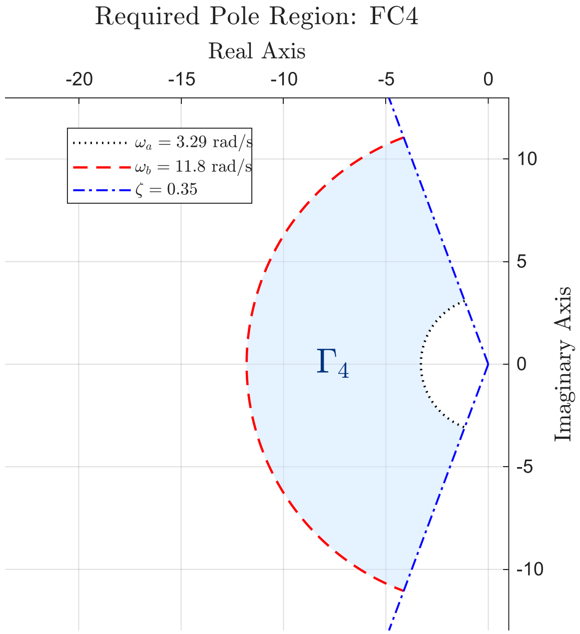
$$\omega_d = 70\, rad/sec \text{ and } \omega_b \leq \omega \leq \omega_d$$

Required Pole Regions from Military Specs. (ctd.)









Γ: Desired Pole Region

Gamma–Stability (Γ-Stability)

In the case of the left half plane, the boundary is described by

$$\partial\Gamma \triangleq \{s \mid s = j\omega, \qquad 0 \le \omega < \infty\}$$

The boundary $\partial\Gamma$ of the desired root region Γ can be described by

$$\partial\Gamma \triangleq \{s \mid s = \sigma(\alpha) + j\omega(\alpha), \qquad \alpha^{-} \le \alpha \le \alpha^{+}\}$$

$\sigma(\alpha)$: Real Part of the Boundary
$\omega(\alpha)$: Ima. Part of the Boundary
α: Generalized Frequency

Gamma–Stability (Γ-Stability)

Lines of Constant Dampings

D: Damping

Let $\alpha = \sigma$

$$s = \sigma + j\sigma\frac{\sqrt{1-D^2}}{D}, \qquad \sigma \leq 0$$

Interval of Two Real Root Boundaries

Let $\alpha = \sigma$

$$s = \sigma, \qquad -b \leq \sigma \leq -a$$

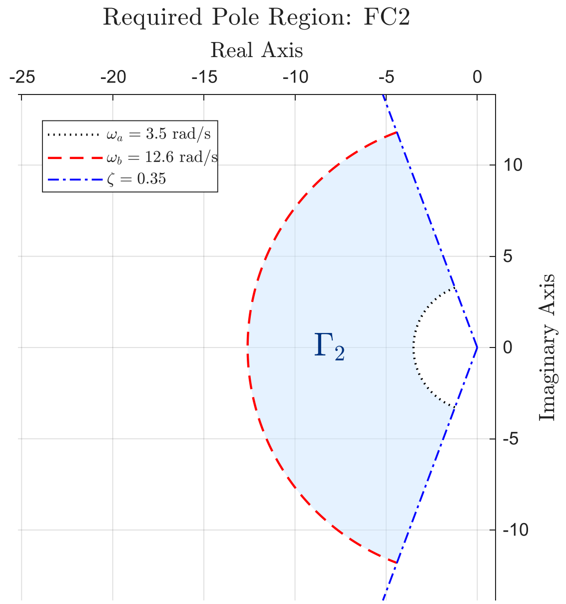
Circle

Center: $s = 0$ Radius: R

$$R^2 = \sigma^2 + \omega^2$$

Let $\alpha = \sigma$

$$s = \sigma + j\sqrt{R^2 - \sigma^2}, \qquad -R \leq \sigma \leq R$$

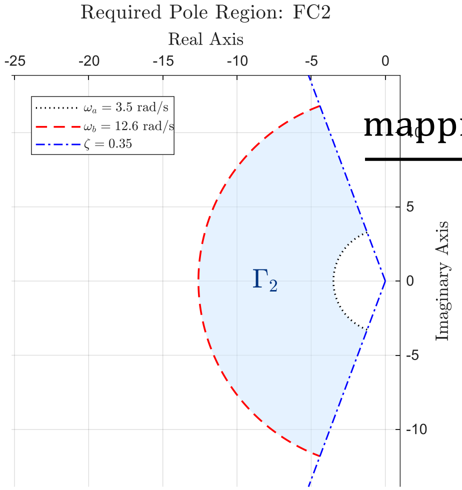


$$s = \begin{cases} \alpha + j\sqrt{12.6^2 - \alpha^2} & \text{for } \alpha \in [-12.6; -12.6 \cdot 0.35] \\ \beta + j\beta\dfrac{\sqrt{1-0.35^2}}{0.35} & \text{for } \beta \in [-12.6 \cdot 0.35; -3.5 \cdot 0.35] \\ \gamma + j\sqrt{3.5^2 - \gamma^2} & \text{for } \gamma \in [-3.5 \cdot 0.35; -3.5] \end{cases}$$

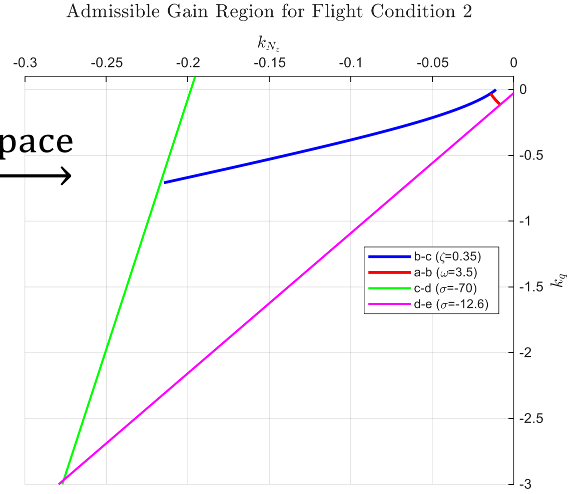
Pole Region Assignment and D-Decomposition

We know what we want. How do we get it?

$$\begin{bmatrix}\Gamma_1\\ \Gamma_2\\ \Gamma_3\\ \Gamma_4\end{bmatrix} \xrightarrow{\text{mapping into } k-\text{space}} \begin{bmatrix}K_\Gamma^{(1)}\\ K_\Gamma^{(2)}\\ K_\Gamma^{(3)}\\ K_\Gamma^{(3)}\end{bmatrix} = \begin{bmatrix}R_{nom-1}\\ R_{nom-2}\\ R_{nom-3}\\ R_{nom-4}\end{bmatrix}$$



$\xrightarrow{\text{mapping into } k-\text{space}}$



Our Control Law

$$u = -Kx$$

$$K = [k_{n_z} \quad k_q \quad 0]$$

$$A_{cl} = A - bK$$

For a fixed boundary point s, the condition $\det(sI - (A - bK)) = 0$ becomes a linear equation in terms of gains.

D-Decomposition (continued)

$$\det(sI - (A - bK)) = 0$$

Matrix Determinant Lemma

$$\det((sI - A) + bK)) = \det(sI - A) \cdot (1 + K(sI - A)^{-1}b) = 0$$

$$\boxed{\Phi(s) = (sI - A)}$$

$$\det(\Phi(s) + bK) = \det\big(\Phi(s)\big) \cdot (1 + K\Phi(s)^{-1}b) = 0$$

$$\boxed{\Phi(s)^{-1} = \frac{\operatorname{adj}\big(\Phi(s)\big)}{\det\big(\Phi(s)\big)}}$$

$$\det\big(\Phi(s)\big) + K \cdot \operatorname{adj}\big(\Phi(s)\big) \cdot b = 0$$

$$\boxed{v(s) = \operatorname{adj}\big(\Phi(s)\big) \cdot b = \begin{bmatrix} v_1(s) \\ v_2(s) \\ v_3(s) \end{bmatrix}}$$

$$\det\big(\Phi(s)\big) + K \cdot v(s) = 0$$

$$K = [k_{n_z} \quad k_q \quad 0]$$

$$\boxed{\det\big(\Phi(s)\big) + k_{n_z} \cdot v_1(s) + k_q \cdot v_2(s) = 0}$$

D-Decomposition (continued)

$$\boxed{\det(\Phi(s)) + k_{n_z} \cdot v_1(s) + k_q \cdot v_2(s) = 0}$$

$$\begin{bmatrix} \mathrm{Re}\{v_1(s)\} & \mathrm{Re}\{v_2(s)\} \\ \mathrm{Im}\{v_1(s)\} & \mathrm{Im}\{v_2(s)\} \end{bmatrix} \begin{bmatrix} k_{n_z} \\ k_q \end{bmatrix} = \begin{bmatrix} -\mathrm{Re}\{\det(\Phi(s))\} \\ -\mathrm{Im}\{\det(\Phi(s))\} \end{bmatrix}$$

Flying Qualities

Complex Root Boundaries

To calculate ζ and ω borders

$$\begin{bmatrix} k_{n_z} \\ k_q \end{bmatrix} = \begin{bmatrix} \mathrm{Re}\{v_1(s)\} & \mathrm{Re}\{v_2(s)\} \\ \mathrm{Im}\{v_1(s)\} & \mathrm{Im}\{v_2(s)\} \end{bmatrix}^{-1} \begin{bmatrix} -\mathrm{Re}\{\det(\Phi(s))\} \\ -\mathrm{Im}\{\det(\Phi(s))\} \end{bmatrix}$$

produces curve

Physical and Stability Limits

Real Root Boundaries

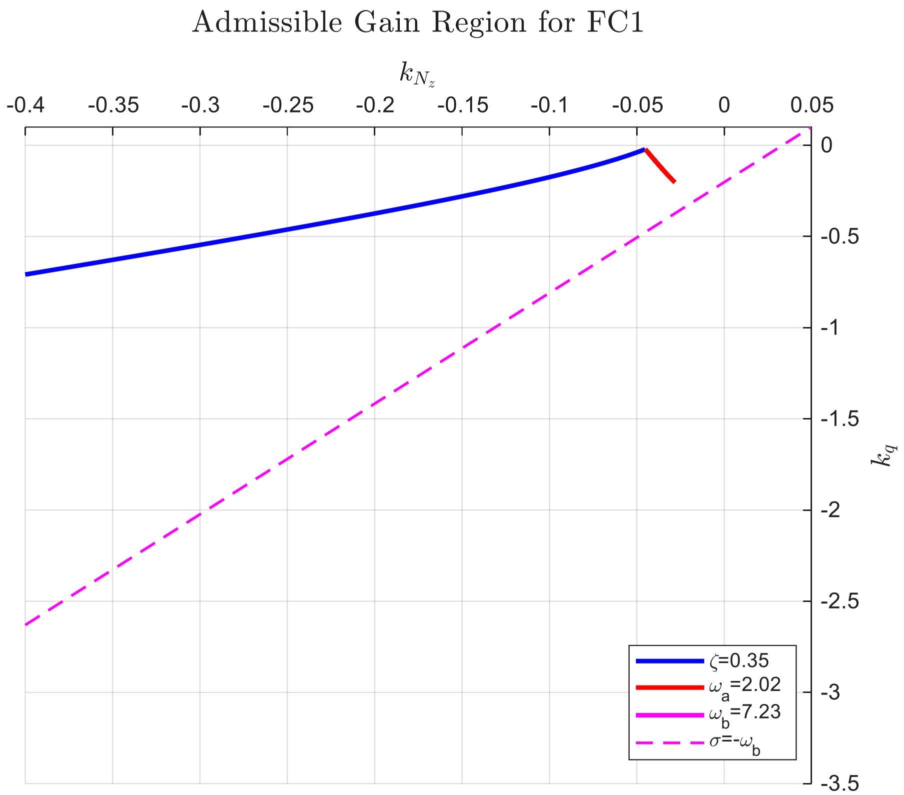
To calculate σ borders

$$s = \sigma$$

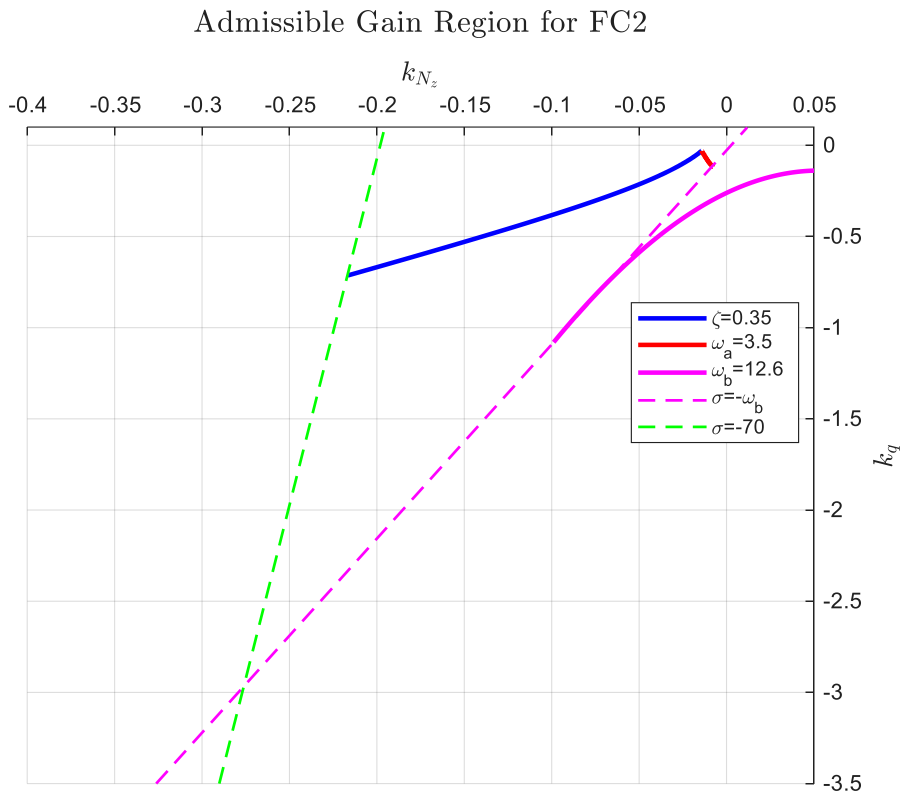
$$\det(\sigma I - A) + k_{n_z} \cdot v_1(\sigma) + k_q \cdot v_2(\sigma) = 0$$

produces straight line

Admissible Gain Regions

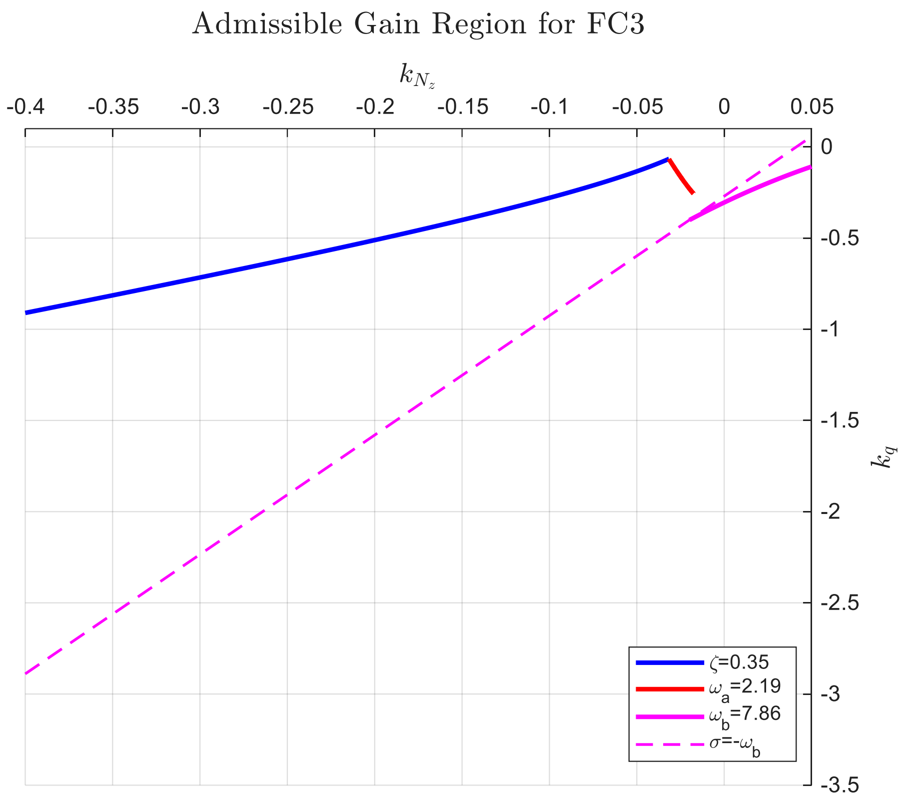


R_{nom-1}

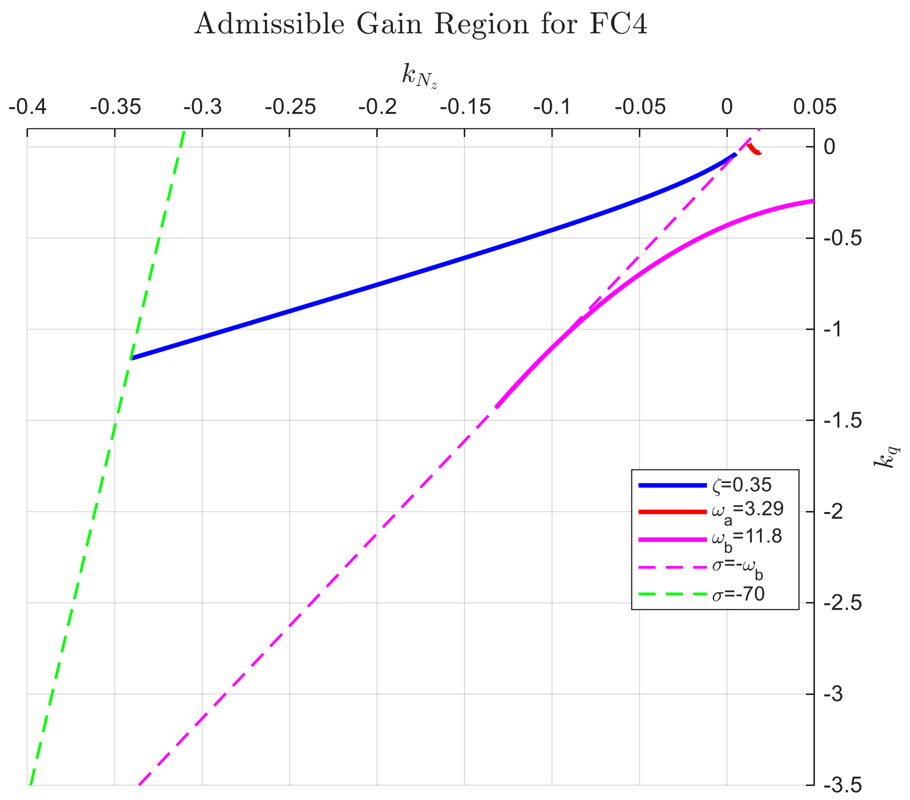


R_{nom-2}

Admissible Gain Regions (continued)

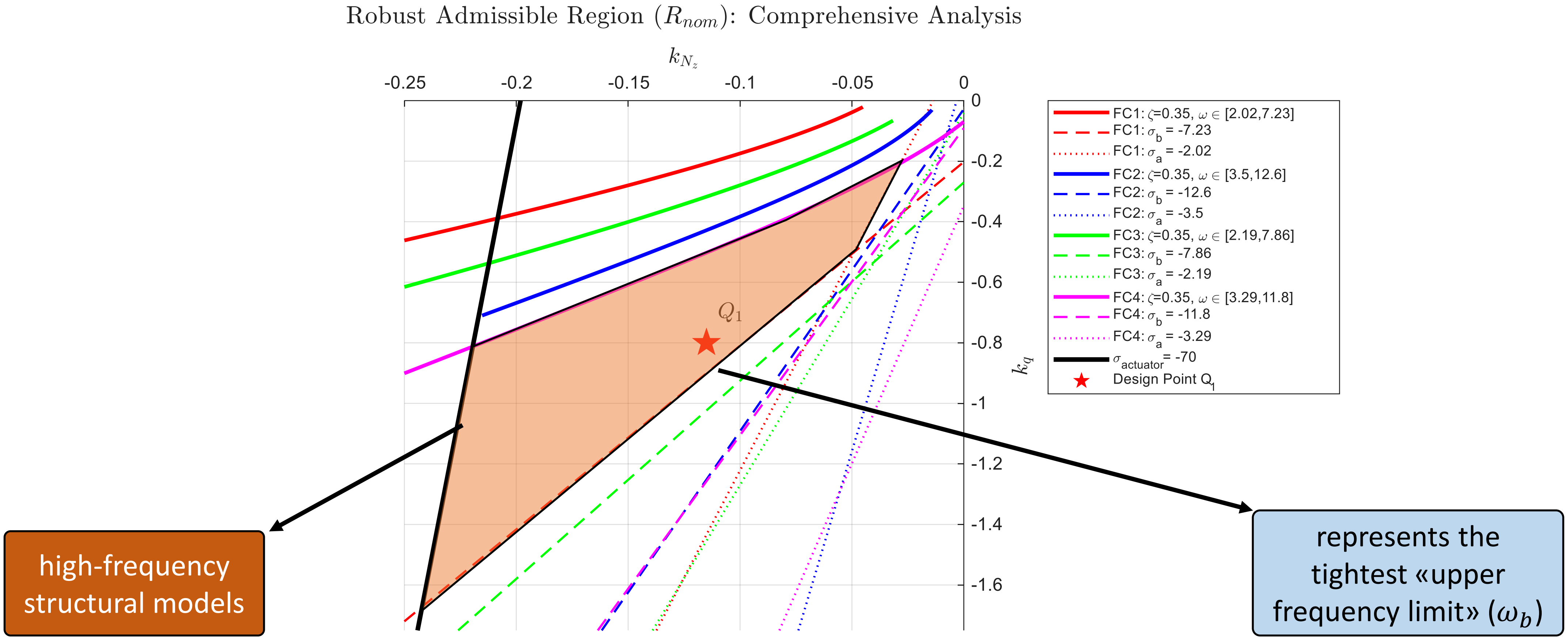


R_{nom-3}



R_{nom-4}

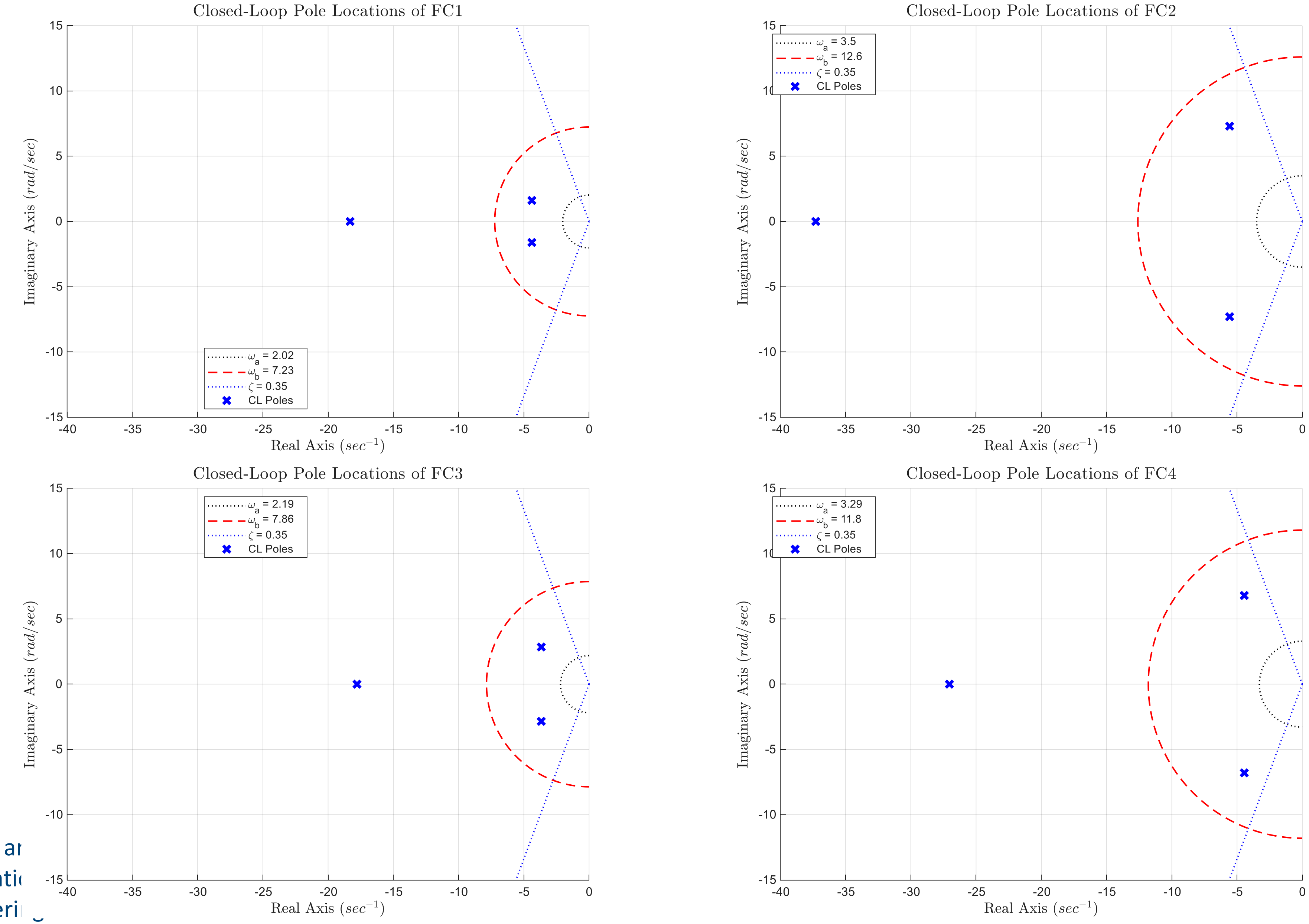
Admissible Gain Regions (continued)



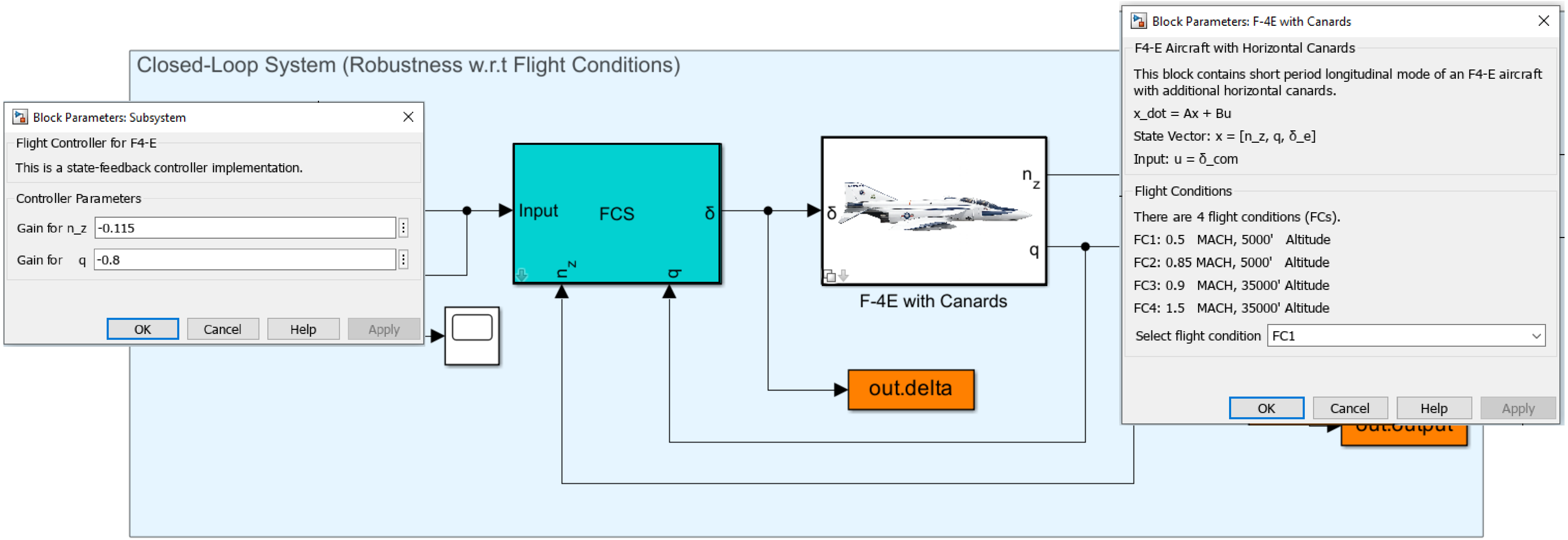
$$R_{nom} = R_{nom-1} \cap R_{nom-2} \cap R_{nom-3} \cap R_{nom-4}$$

$$Q_1 = [-0.115 \quad -0.8]$$

Robust Design Point (Q_1) and Results

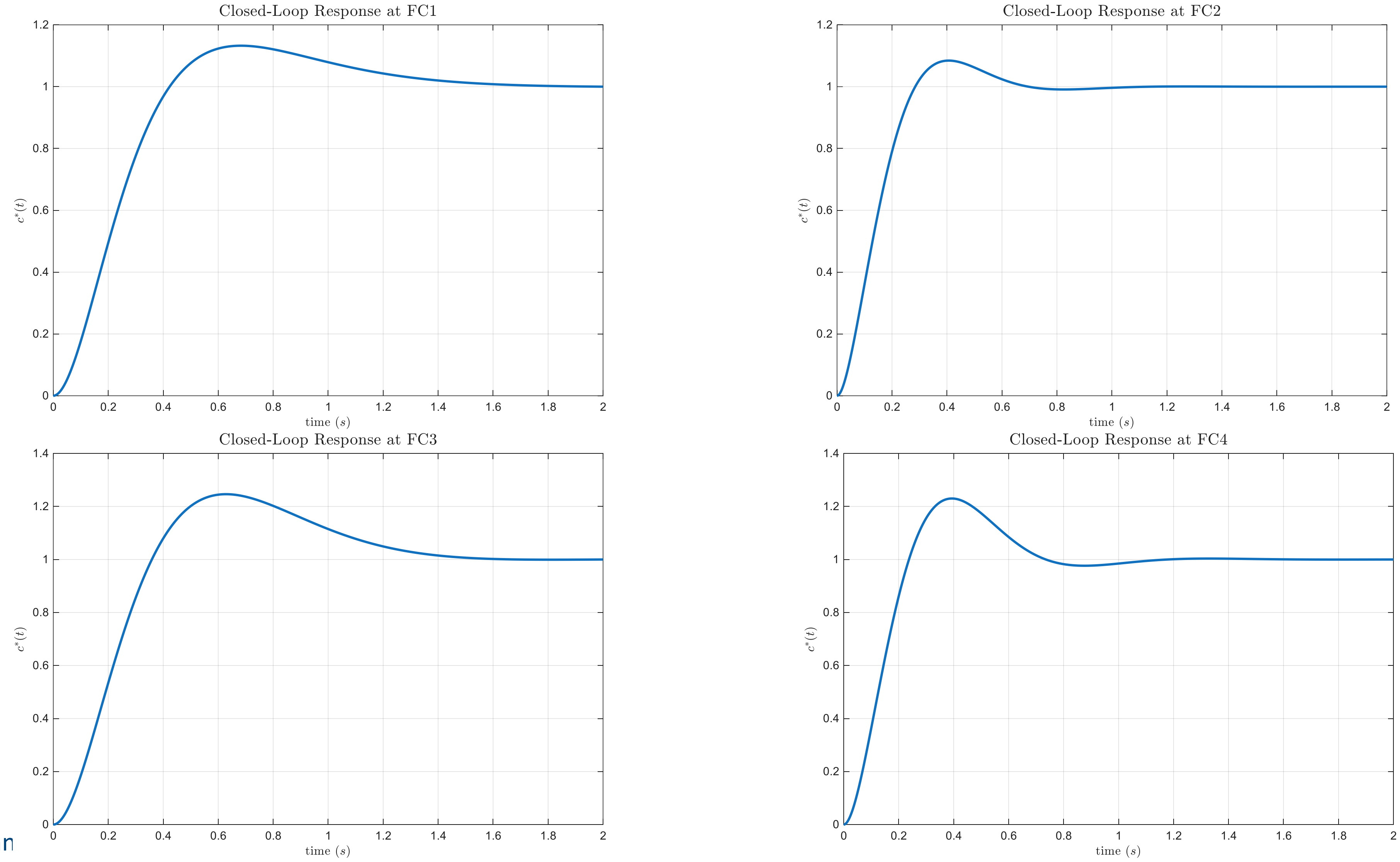


Robust Design Point (Q_1) and Results (ctd.)



Evaluation of the Step Response: $c^* = (n_z + 12.43q)/c_\infty$

Robust Design Point (Q_1) and Results (ctd.)

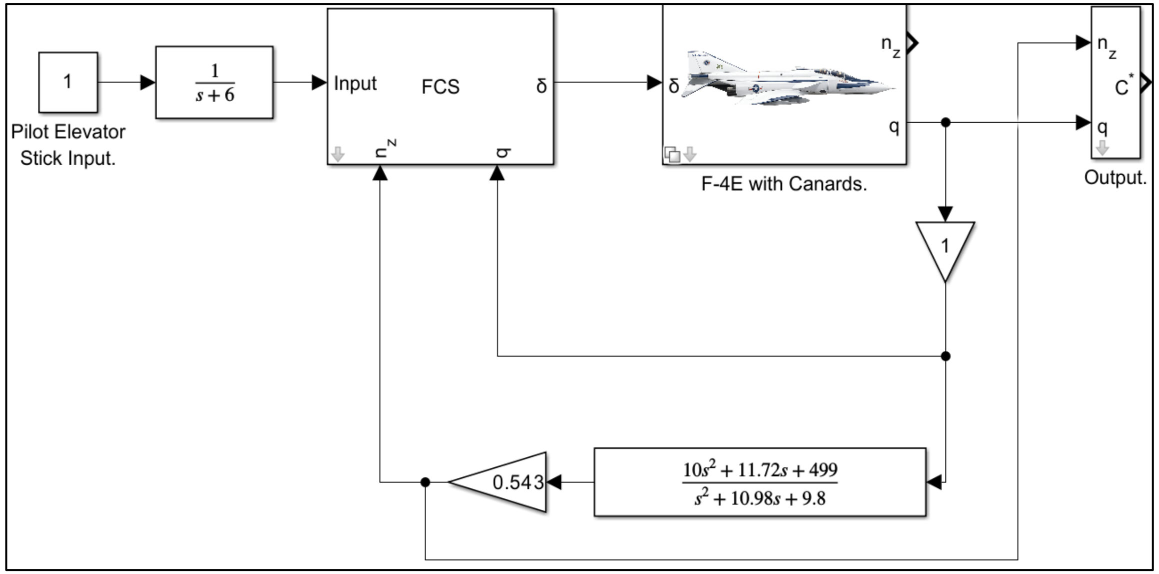


Robustness w.r.t. Sensor Failures

In the nominal case, both pitch rate (q) and normal acceleration (n_z) are used for feedback. In practice, accelerometer sensors may degrade or partially fail.

This degradation is modeled as a reduction in sensor gain:

- Nominal sensor: $k = 1$
- Degraded sensor: $k = \frac{1}{2}, \frac{2}{3}, \frac{1}{3}$



Problem

Direct use of the measured n_z under sensor degradation reduces robustness and leads to poor closed-loop performance.

Idea

Do not directly use the measured N_z. Estimate N_z from pitch rate q, which is more reliable.

Robustness w.r.t. Sensor Failures (ctd.)

Model-Based Estimation of N_z

The transfer functions from elevator input (u) to pitch rate (q) and to normal acceleration (N_z) are derived from the same aircraft model. Therefore:

- Both transfer functions have the **same denominator**
- They share the **same aircraft dynamics and poles**

The difference is only in the **numerators**. Taking the ratio eliminates the common dynamics:

$$\frac{N_z(s)}{q(s)} = \frac{N_z(s)/u(s)}{q(s)/u(s)}$$

Using the aircraft model, the estimation transfer function is obtained as:

$$\frac{\hat{N}_z(s)}{q(s)} = 0.543\ \frac{s^2 + 1.172s + 49.9}{s + 0.98}$$

This transfer function represents the dominant dynamic relation between q and N_z.

Robustness w.r.t. Sensor Failures (ctd.)

Realizability and Simulation Results

The estimation transfer function is **not realizable** due to its numerator order. To make it realizable, a first-order filter is added:

$$\frac{10}{s+10}$$

The final estimation transfer function becomes:

$$\boxed{\frac{\hat{N}_z(s)}{q(s)} = 0.543\ \frac{s^2 + 1.172s + 49.9}{s + 0.98} \cdot \frac{10}{s+10}}$$

Closed-loop simulations are performed for four flight conditions (FC1–FC4). Two cases are compared:

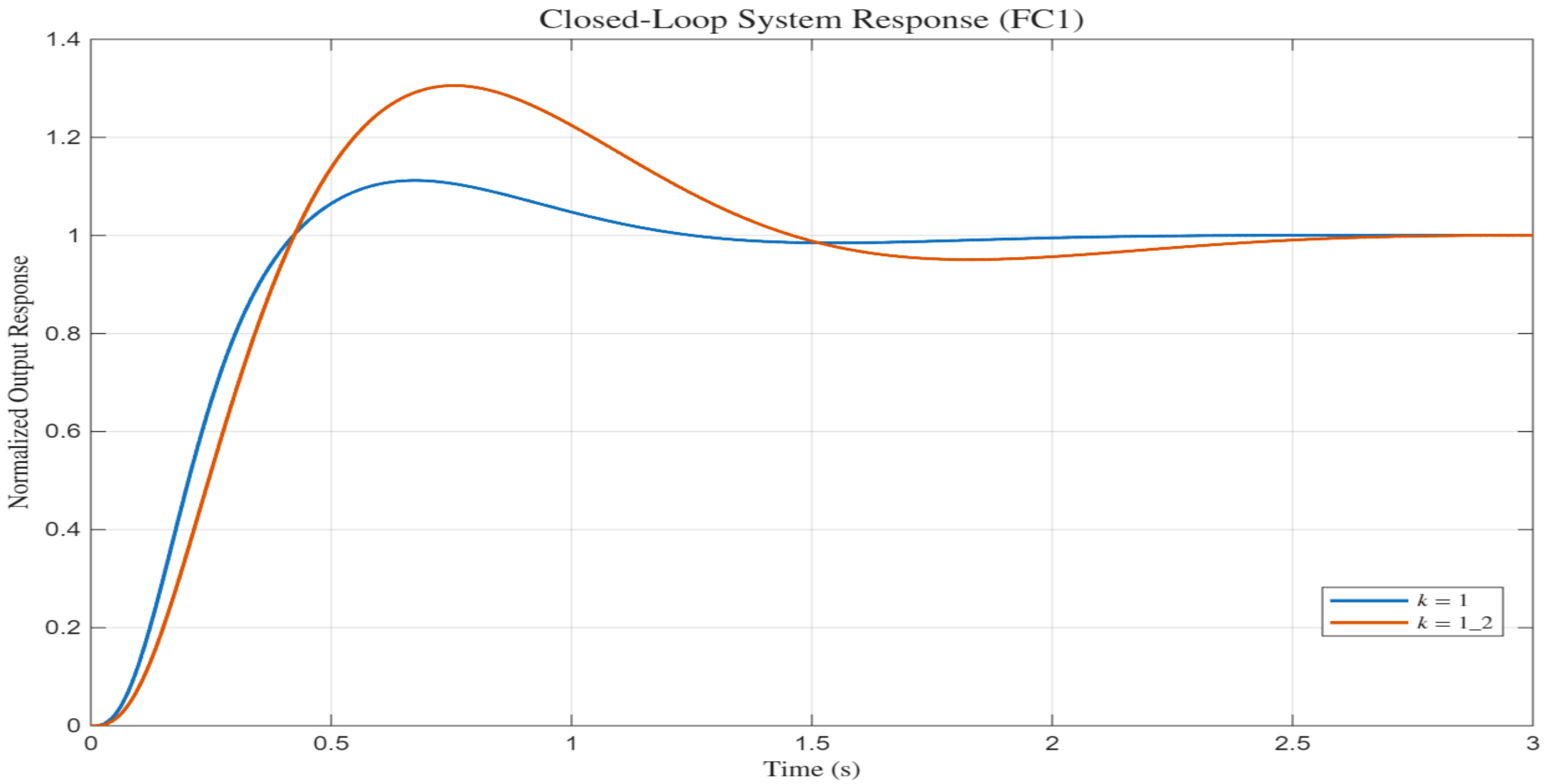
- Nominal sensor: $k = 1$
- Degraded sensor: $k = \frac{1}{2}$

Results

Although sensor degradation leads to slower response and lower damping, the system remains stable and satisfies the required pole-region constraints.

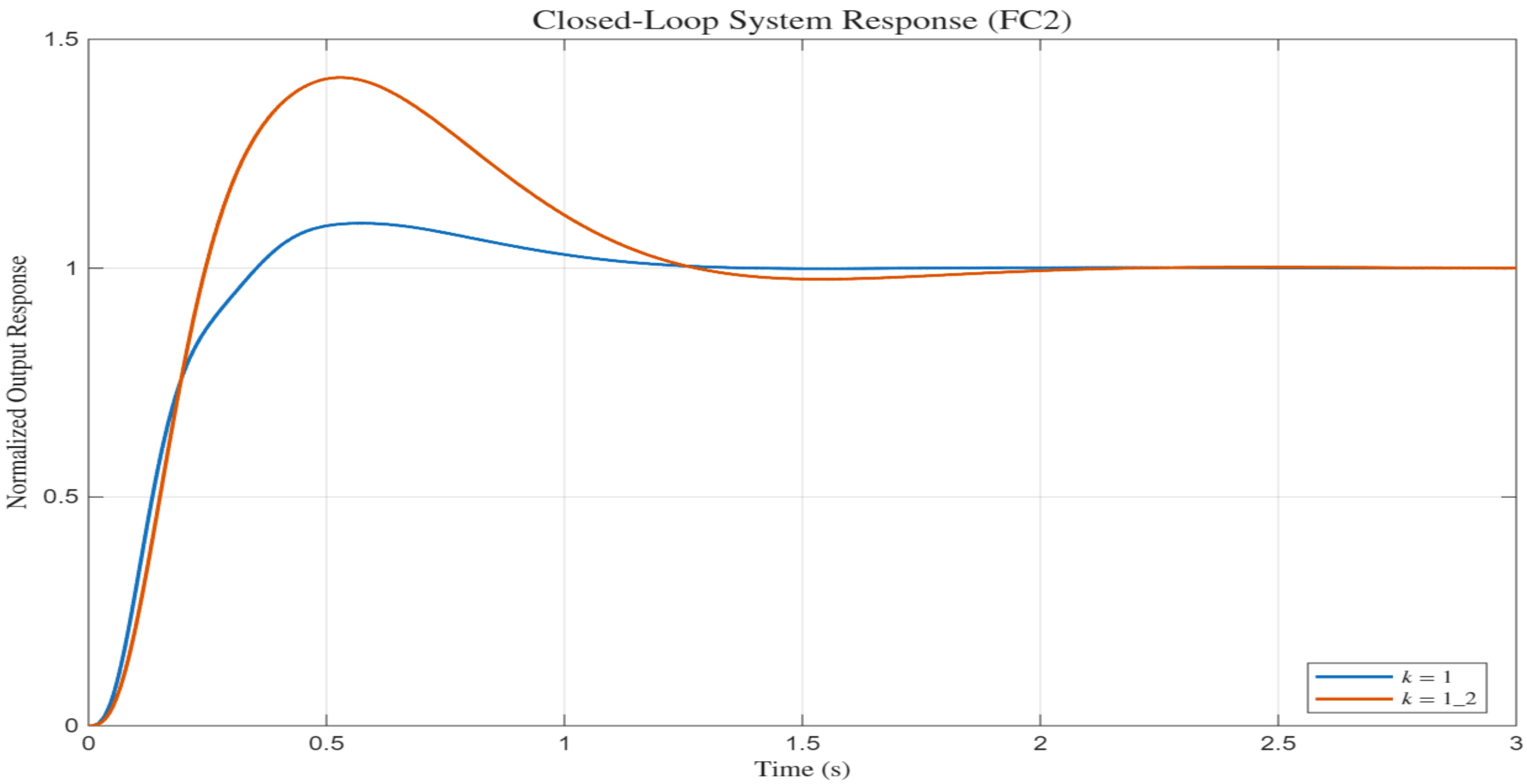
Robustness w.r.t. Sensor Failures (ctd.)

Closed-Loop Simulation Results FC-1



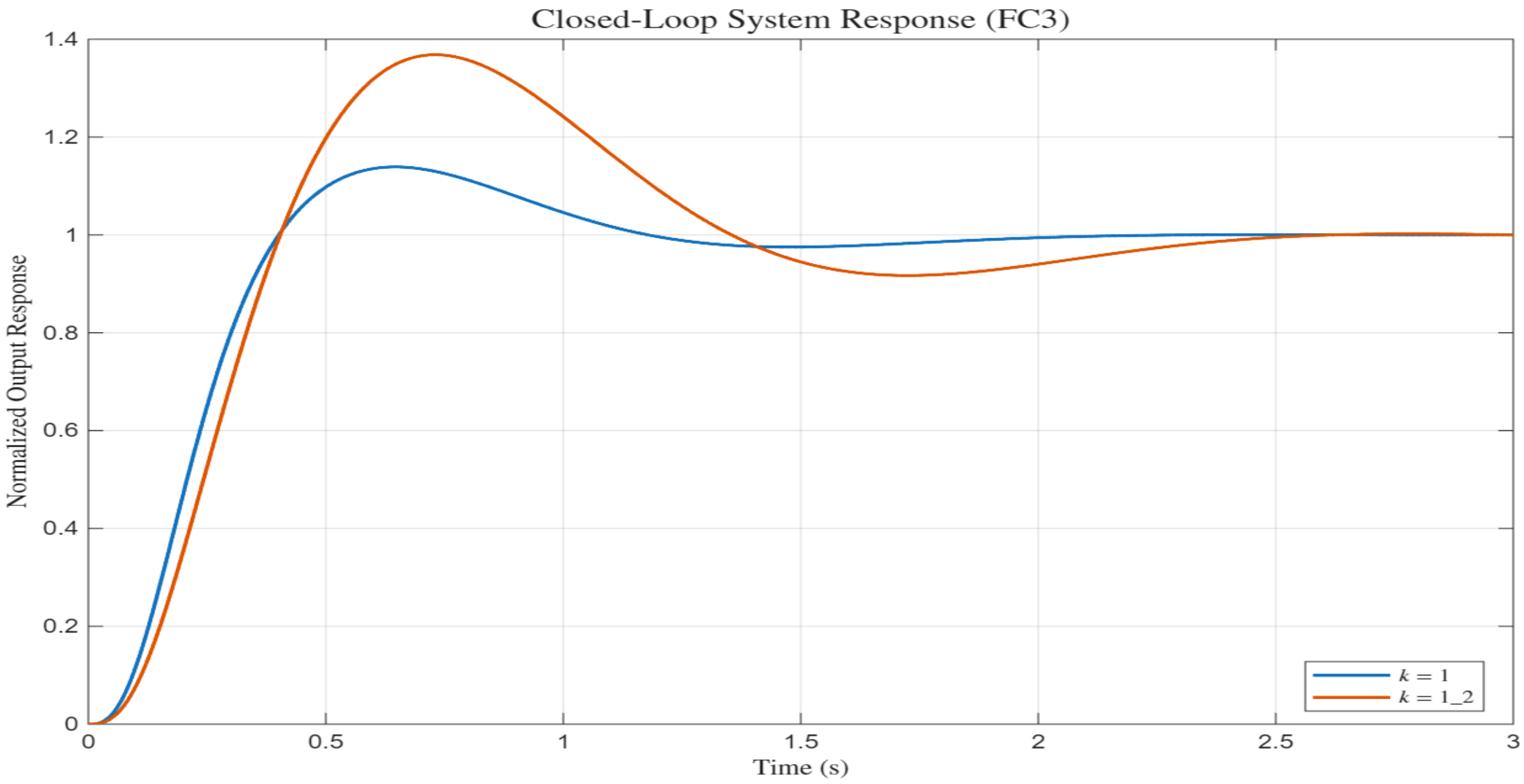
Robustness w.r.t. Sensor Failures (ctd.)

Closed-Loop Simulation Results FC-2



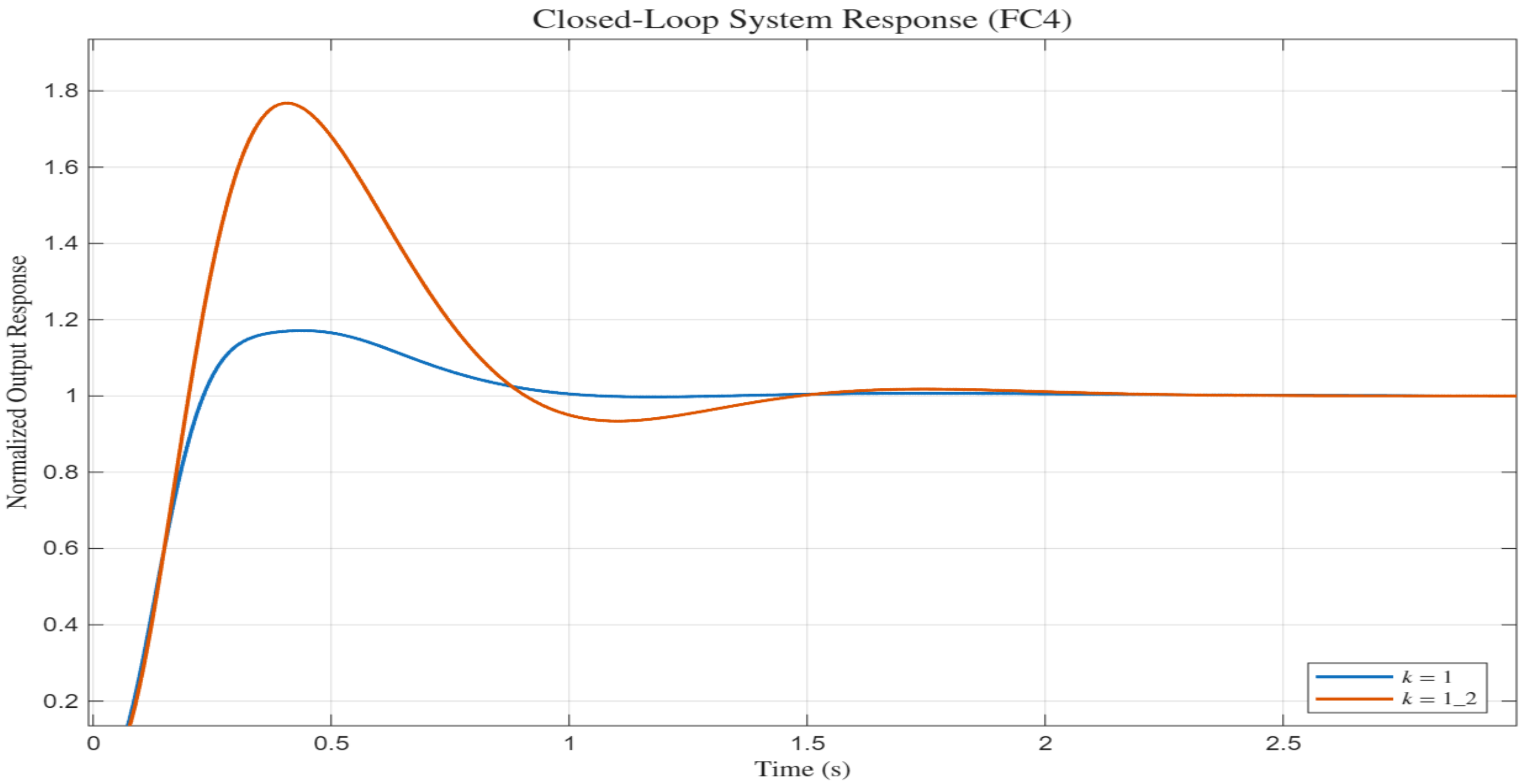
Robustness w.r.t. Sensor Failures (ctd.)

Closed-Loop Simulation Results FC-3



Robustness w.r.t. Sensor Failures (ctd.)

Closed-Loop Simulation Results FC-4



Conclusion

It has been demonstrated that a **fixed-gain controller** exhibits successful performance across significantly diverse **flight conditions**.

It has also been demonstrated that measurements by **gyros alone** are not only more cost-effective but actually provide some advantages for the control system stability.

Although results for the linearized models are robust, we must acknowledge that **linear stability does not always imply nonlinear stability**. Therefore, the next vital step is to perform comprehensive **nonlinear simulations** to confirm

Cem Doğut's Award-Winning Photograph (2024), From the 50th Anniversary Celebration of the Turkish Air Force's F4-E Phantom II

Thank You for Your Attention!

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