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Comparison of LQG and Adaptive PID Controller for USV Heading Control

Tahiyatul Asfihani, Didik Khusnul Arif, Subchan, Firdaus Priyatno Putra, Moch. Ardi Firmansyah

Mathematics Department, Institut Teknologi Sepuluh Nopember (ITS), Indonesia

E-mail: t_asfihani@matematika.its.ac.id

Abstract. This paper considers USV heading controller using Adaptive PID and LQG controller. PID is a conventional controller and the most industry controller. However, PID parameters are selected by trial and error. That problem proposed Adaptive PID how to tune PID parameters. In this paper, PID parameters are estimated by RLS method. The second controller for comparison is LQG. LQG controller is robust controller. A gain regulator estimation method, Kalman Filter, is used as LQG controller. The computational results show the LQG controller has better performance than Adaptive PID.

1. Introduction

USV is unmanned surface vehicle. A USV uses in the military for rescue, offshore exploration, coast guard, etc [1]. A USV controller is a most important thing in an unmanned system. A USV heading controller is one of the focus of ship controller. Many methods for design ship heading controller was proposed by many researchers [2], [3], [4]. One of the conventional controller is PID controller [5].

PID is the most industry controller. However, the PID parameters are selected by trial and error [6]. Therefore, many researchers are studied about how to tune PID parameters (Adaptive PID) [6], [7], [8]. PID parameters are a proportional gain (K_p), integration gain (K_i), and derivative gain(K_d).

The research of tuning PID parameters use least square method. The proposed adaptive PID was applied on second order system [7]. Rania, et. al. proposed Adaptive PID using Recursive Least Square (RLS) method for unstable system. RLS method is used to estimate PID parameters. RLS method is estimation method for linear system. Based on simulations, the proposed method has good performance on stable and unstable system [6].

Linear Quadratic Gaussian (LQG) is robust controller which can reject disturbance. LQG controller consists of a gain regulator obtained from the design LQR and gain estimator obtained from the Kalman filter [9].

Carlucho, et. al., were studied about PID and LQG controller for an autonomous underwater vehicle [10]. In that research, PID combine with Kalman filter for state estimation and LQG controller consits Kalman filter for gain estimator. The methods are tested on Springer AUV. Based on experiment and simulation show that LQG has better performance than PID. LQG controller can save energy more than PID.

Based on that research background, this paper focus on comparison of LQG and Adaptive PID for USV heading controller. The objective of this USV controller is following the desired heading angle. This problem is solved using Adaptive PID and LQG. This work gives an alternative method which is better than previous ones in controlling the surface vehicle. The rest of this paper is organized as follows. Section 2 presents mathematical model of USV motion. Section 3 presents Design of LQG Controller. Section 4 presents Adaptive PID Controller. Section 5 presents simulation results. Section 6 presents the conclusion.

2. Mathematical Model of USV Motion

A normalization form of a mathematical model of ship motion is given as follow [1]

$$M'\dot{v}' + N'(u_0')v' = b'\delta_R' \tag{1}$$

where $v' = [v', r']^T$ and

$$M' = \begin{bmatrix} m' - Y_{\dot{v}}' & m'x_g' - Y_{\dot{r}}' \\ m'x_g' - N_{\dot{v}}' & I_z' - N_{\dot{r}}' \end{bmatrix} \tag{2}$$

$$N'(u_0') = \begin{bmatrix} -Y_v' & m'u_0' - Y_r' \\ -N_v' & m'x_g'u_0' - N_r' \end{bmatrix} \tag{3}$$

$$b' = \begin{bmatrix} Y_\delta' \\ N_\delta' \end{bmatrix} \tag{4}$$

A notation (') is a non-dimensional variables. Normalization use Prime System I [11]. A mathematical model of motion on Eq. (1) consider 2 Degree of Freedom, that are sway and yaw.

In this paper, a mathematical model of USV motion use 1 degree of freedom (yaw) for ship heading controller. A mathematical model was approached with the Nomoto model as follow [2]

$$\frac{r(s)}{\delta_R(s)} = \frac{K_R(1+T_3S)}{s(1+T_1S)(1+T_2s)} \tag{5}$$

The parameters of the transfer functions are related to the hydrodynamic derivatives as follow

$$T_1T_2 = \frac{det(M')}{det(N')} \tag{6}$$

$$T_1T_2 = \frac{n_{11}'m_{22}' + n_{22}'m_{11}' - n_{12}'m_{21}' - n_{21}'m_{12}'}{det_{\mathbf{N}'}} \tag{7}$$

$$K_R = \frac{n_{21}'b_1' - n_{11}'b_2'}{det(N')} \tag{8}$$

$$K_RT_3 = \frac{m_{21}'b_1' - m_{11}'b_2'}{det(N')} \tag{9}$$

where $m_{i,j}'$ $n_{i,j}'$ and b_i' are element of matrices/vector defined in Eq. (2)-(4).

In this paper, the USV is SSV Ship BRP TARLAC (LD-601). The transfer function of the USV motion is shown in the following equation

$$\frac{r(s)}{\delta_R(s)} = \frac{244.2s + 484.6}{0.1203s^2 + 0.7312s + 1} \tag{10}$$

where r is a yaw rate and δ_R is a rudder angle as the input system. Based on Eq. (10), the state space of the USV motion is given bellow

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} \tag{11}$$

where $\mathbf{x} = [\psi, r]^T$, ψ is yaw angle (heading angle),

$$A = \begin{bmatrix} 0 & 1 \\ -8.31 & -6.08 \end{bmatrix}$$

,

$$B = \begin{bmatrix} 0 \\ 8.31 \end{bmatrix}$$

.

The output system is yaw angle given as follow

$$\mathbf{y} = C\mathbf{x} \tag{12}$$

where $\mathbf{y}$ is the outputs system and

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

.

3. Design of LQG Controller for USV Heading Controller

LQG is a control method for linear system with Gaussian noise. LQG is an optimal control where minimizing the quadratic objective function. LQG has Linear Quadratic Regulator (LQR) and Linear Quadratic Sate Estimator (LQE) i.e. Kalman Filter. Linear system with Gaussian noise is given on the following equation

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t) + G\mathbf{w}(t) \tag{13}$$
$$\mathbf{y}(t) = C\mathbf{x}(t) + \mathbf{v}(t) \tag{14}$$

where $\mathbf{x}$ is the vector of state variables of the system, $\mathbf{y}$ is the vector of outputs system, $\mathbf{u}$ is the vector of control inputs, $\mathbf{w}$ is white Gaussian system noise, $\mathbf{v}$ is white Gaussian measurement noise, A, B, G and C are the time invariant matrices.

LQR method is used to find the gain regulator with the state space model equations as follows:

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t) \tag{15}$$
$$\mathbf{y}(t) = C\mathbf{x}(t) \tag{16}$$

The first step is to find the value of Q_c and R_c. Q_c is a weighting matrix of variables state system that is symmetric and positive semi-definite, and R_c is a weighting matrix of inputs system which is symmetric and positive definite. Based on trial and error, the optimal value of Q_c and R_c are given as follows:

$$Q_c = \begin{bmatrix} 75 & 0 \\ 0 & 75 \end{bmatrix}$$

and

$$R_c = \begin{bmatrix} 0.001 \end{bmatrix}$$

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and substituted into the following the Riccati algebra equation

$$A^T P_c + P_c A + Q_c - P_c B R_C^{-1} B^T P_C = 0 \tag{17}$$

from Eq. (17) will be obtained P_c which is used to find the gain regulator K_c, according to the equation

$$K_c = R_c^{-1} B^T P_c. \tag{18}$$

Based on Eqs. (17) and (18), we obtain

$$P_c = \begin{bmatrix} 2.227 & 0.032 \\ 0.032 & 0.0008 \end{bmatrix}$$

and the gain regulator is as follows

$$K_c = \begin{bmatrix} 272.86 & 7.405 \end{bmatrix}$$

.

The control input $u(t)$ is obtained by:

$$u(t) = -K_c \widehat{x}(t). \tag{19}$$

where $\widehat{x}(t)$ is variables state estimation which will be obtained from the Kalman filter.

The variables state estimator is according to the equation below

$$\widehat{\dot{x}}(t) = (A - K_f C)\widehat{x}(t) + Bu(t) + K_f y(t) \tag{20}$$

where K_f is the gain Kalman given as follows

$$K_f = P_f C^T R_f^{-1}. \tag{21}$$

Based on Eq. (21), P_f is the error covariance obtained from Algebra Riccati equation as follows

$$P_f + P_f A^T + G Q_f G^T - P_f C^T R_f^{-1} C P_f = 0 \tag{22}$$

where Q_f and R_f are a covariances of process noise and measurement noise, respectively. That matrices are a positive definite. We determine

$$Q_f = \begin{bmatrix} 0.001 \end{bmatrix}$$

and

$$R_f = \begin{bmatrix} 0.001 \end{bmatrix}$$

. From Eq. (22), we obtaine as follows

$$P_f = \begin{bmatrix} 0.000541 & 0.000146 \\ 0.000146 & 0.000547 \end{bmatrix}$$

and a gain Kalman filter is derived from Eq. (21) as follows

$$K_f = \begin{bmatrix} 0.541 & 0 \\ 0.146 & 0 \end{bmatrix}$$

4. Design of Adaptive PID controller for USV Heading Controller

The discrete form of system (11) using forward difference equation is obtained as follows, by taking time sampling 0.1 second

$$\mathbf{x}(k+1) = A_d\mathbf{x}(k) + B_d\mathbf{u}(k) \tag{23}$$

where

$$A_d = \begin{bmatrix} 1 & 0.1 \\ -0.831 & 0.392 \end{bmatrix} \qquad ; B_d = \begin{bmatrix} 0 \\ 0.831 \end{bmatrix}$$

The continue PID controller is given as follows

$$u(t) = K_p\left[e(t) + \frac{1}{T_i}\int e(\tau)d\tau + T_d\frac{de(t)}{dt}\right] \tag{24}$$

where K_p is proportional gain, $e(t) = r(t) - y(t)$ is error between output system and reference output, T_i is integration time, T_d is derivative time. And the discrete PID controller using backward difference equation is obtained as follows

$$u(k) = K_p\left[\ e(k) + \tfrac{T_S}{T_i}\Sigma e(i) + T_d\tfrac{e(k)-e(k-1)}{T_s}\ \right]. \tag{25}$$

By subtracting $u(k)$ with $u(k-1)$, we obtain the linear regression equation for PID

$$u(k) - u(k-1) = a_0e(k) + a_1e(k-1) + a_2e(k-2) \tag{26}$$

where the parameters are

$$a_0 = K_p(1 + \frac{T_s}{T_i} + \frac{T_d}{T_s}); a_1 = K_p(-1 - 2\frac{T_d}{T_s}); a_2 = \frac{K_pT_d}{T_s}.$$

Eq. (26) can be written

$$u(k) = u(k-1) + \varphi^T(k)\theta \tag{27}$$

where $\theta = [a_0, a_1, a_2]^T$ and $\varphi^T(k) = [e(k), e(k-1), e(k-2)]$.

In this paper, the online process tuning PID parameters use recursive least squares with several process

a. Initialization θ_0 control parameters using conventional PID control parameters and $P(0)$ using the equation $P(0) = \gamma I$ where γ is constant ($\gamma > 0$) and I is identity matrix.

b. Obtain $\hat{\theta}$ using equation

$$\begin{aligned} \widehat{\theta} &= \widehat{\theta}(t-1) + P(t)\varphi(t-1)\varepsilon(t) \\ \varepsilon(t) &= y(t) - \varphi^T(t)\widehat{\theta}(t-1) \\ K(t) &= P(t-1)\varphi(t)(\lambda I + \varphi^T(t)P(t-1)\varphi(t))^{-1} \\ P(t) &= (I - K(t)\varphi^T(t))\frac{P(t-1)}{\lambda} \end{aligned}$$

where ε is prediction error, λ is forgetting factor ($0 < \lambda \leq 1$).

c. Results of the estimation ($\hat{\theta}$) in step (b) will be applied to the PID control parameters.

Furthermore, we determine parameters for RLS scheme $\gamma = 2 \times 10^3$,

$$P_0 = \begin{bmatrix} 2 \times 10^3 & 0 & 0 \\ 0 & 2 \times 10^3 & 0 \\ 0 & 0 & 2 \times 10^3 \end{bmatrix},$$

$$\Theta_0 = \begin{bmatrix} 6.546 \\ -8.958 \\ 2.979 \end{bmatrix}$$

and forgetting factor parameter $\lambda = 0.9998$.

5. Simulation

There are five scenarios in this simulation. The scenarios are shown in Table 1. The result

Table 1: The Simulation Scenarios

Scenario	Initial Condition (rad)	Reference (rad)
I	0	0.03
II	0.03	0.09
III	0.09	0.18
IV	-0.03	0.03
V	-0.03	0

of simulation I is shown in Fig. 1-2. Fig. 1 shows that LQG control response had a lower overshoot than the adaptive PID control response. But, the value of settling time capable of faster adaptive PID, comparison transient response of the adaptive PID control, conventional PID and LQG are shown in Table 2. The simulation results for the rudder angle of Scenario I is given in Fig. 2.

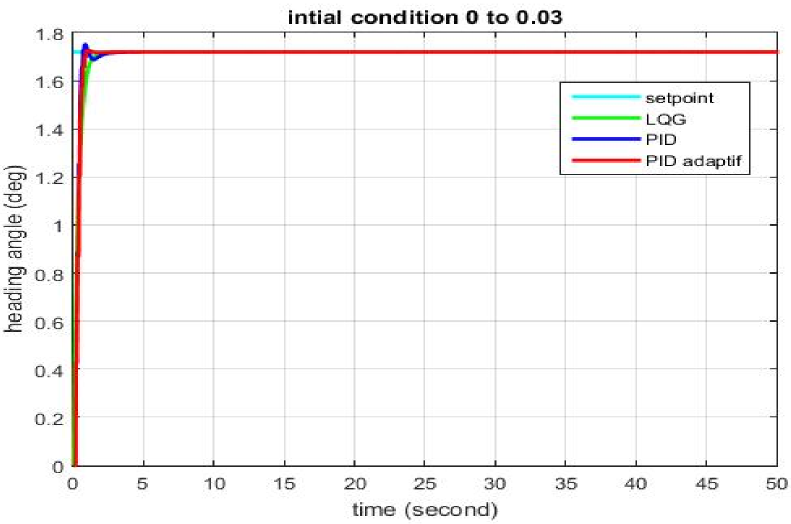

Figure 1: Ship Heading in Scenario I

Four Scenarios are shown in Fig. 3-6. Furthermore, the results of the comparison settling time of 5 Scenarios are shown in Table 3.

Table 2: Transient Response of LQG and Adaptive PID

Specification	LQG	Adaptive PID
t_d	0.3	0.3
t_p	1.4	1.1
t_r	1.4	1
M_p	0.03	0.3
t_s	1.4	1

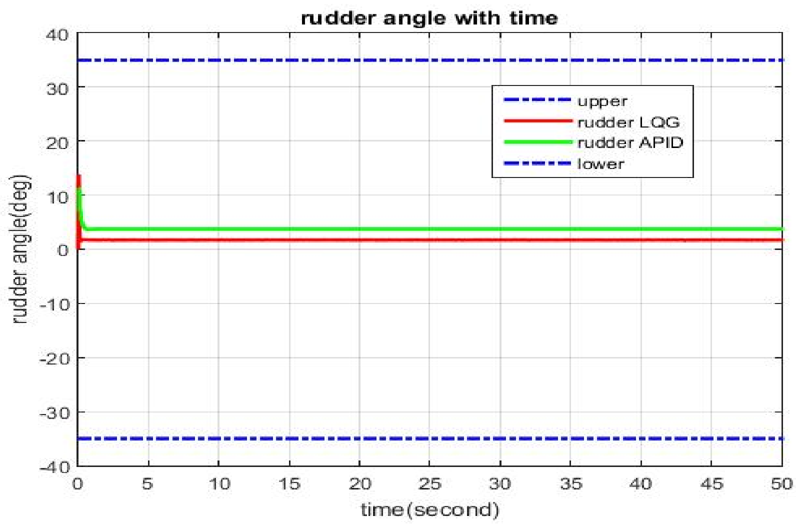

Figure 2: Rudder Angle in Scenario I

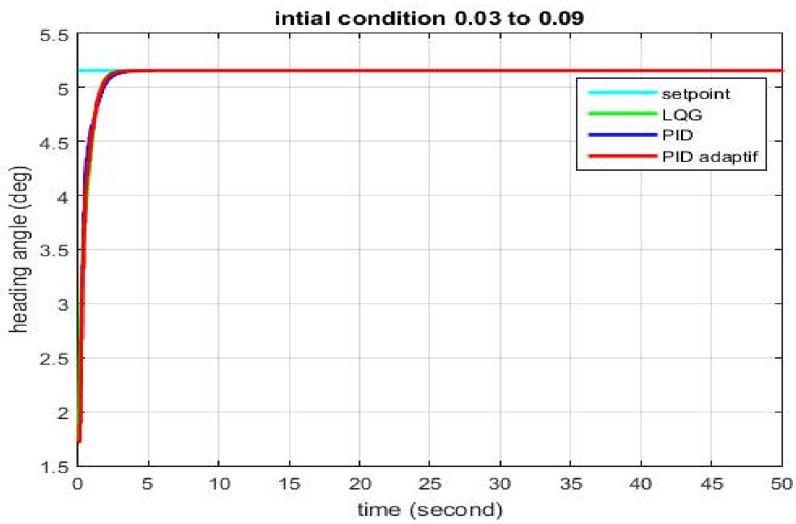

Figure 3: Ship Heading in Scenario II

6. Conclusion

In this paper, the LQG and adaptive PID controller are designed to control ship heading angle move to desired angle. The computational results show that LQG, conventional PID and

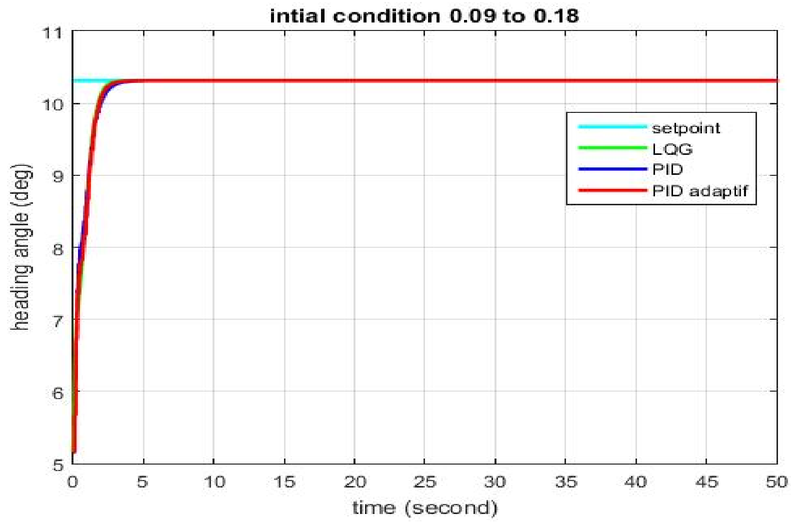

Figure 4: Ship Heading in Scenario III

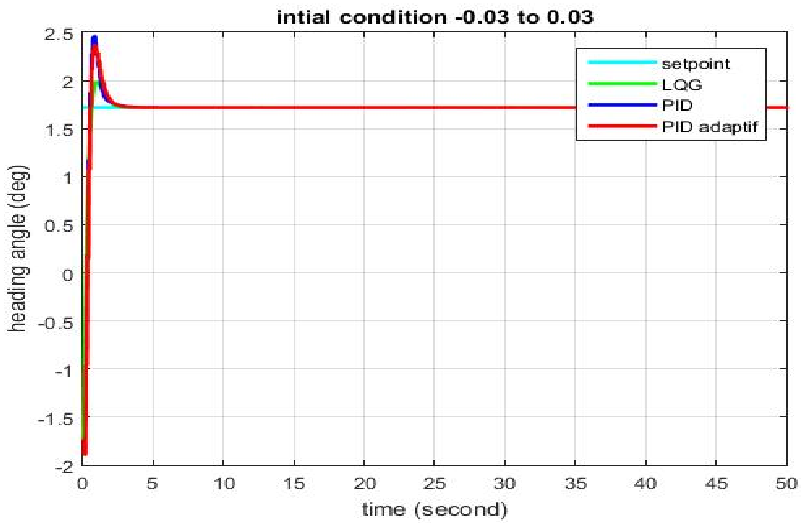

Figure 5: Ship Heading in Scenario IV

Table 3: Settling Time of LQG and Adaptive PID

Scenario	LQG	Adaptive PID
I	1.4	1
II	2.4	2.5
III	2.8	2.9
IV	2.4	2.6
V	2.3	2.4
Mean	2.26	2.28

adaptive PID can derive the USV move to desired heading angle. And, the LQG controller has better performance than conventional PID and Adaptive PID in settling time response.

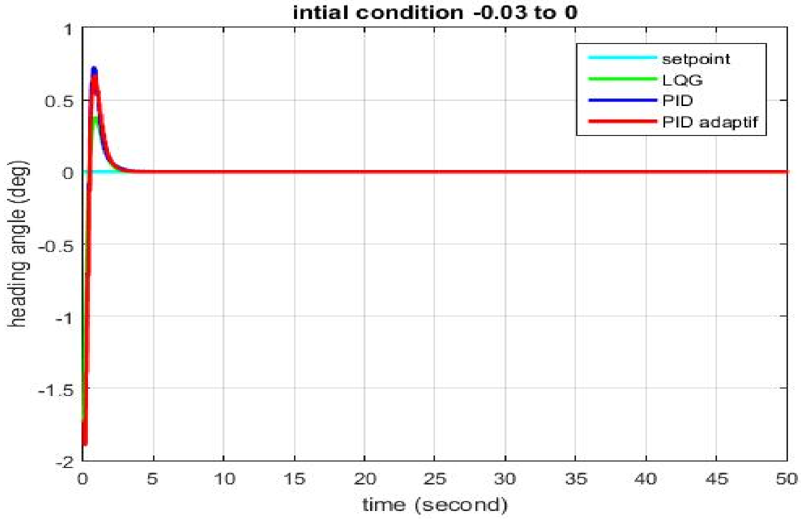

Figure 6: Ship Heading in Scenario V

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