MAT 451: Mathematical Modeling (Assignment)

Instructions:

- Answer all questions.
- You may use **Python, MATLAB, or R** to compute solutions, generate graphs, and verify numerical results.
- Clearly label all plots, explain your modeling steps, and justify assumptions.
- Submit both **written solutions** and **code as pdf file**.

Question 1: Modeling Change with Difference Equations (Dynamical System)

The population of insects grows according to the logistic difference equation:

$$P_{n+1} = P_n + rP_n(1 - \frac{P_n}{K})$$

where $r = 0.7$and carrying capacity $K = 500$.

Tasks:
a. For initial populations $P_0 = 50$, $P_0 = 200$, and $P_0 = 600$, simulate the system for 30 time steps.
b. Plot all three trajectories on the same graph.
c. Describe and compare the long-term behavior for different initial values.
d. Comment on sensitivity to initial conditions.

Question 2: Model Fitting and Least Squares

The table below contains experimental measurements of bacterial growth.

Time (hours)	**0**	**2**	**4**	**6**	**8**	**10**
Population (×100)	5	7	11	20	34	57

Tasks:
a. Fit an **exponential model** of the form $P(t) = Ce^{kt}$ using **least squares**.
b. Fit a **cubic spline model** using any numerical software.
c. Plot both fitted models along with the raw data.
d. Briefly compare which model fits better and why.

Question 3: Modeling with Differential Equations (ODE System)

A predator–prey system is described by

$$\frac{dx}{dt} = ax - bxy,$$
$$\frac{dy}{dt} = -cy + dxy,$$

where $a = 0.8$, $b = 0.02$, $c = 0.6$, $d = 0.01$.

Tasks:
a. Solve the system numerically for initial values $x(0) = 40$, $y(0) = 9$over $0 \le t \le 200$.
b. Plot:
- $x(t)$and $y(t)$vs. time
- Phase portrait (trajectory in the x–y plane)

c. Identify equilibrium points and classify their type (center, saddle, etc.).
d. Discuss the ecological interpretation of your results.

Question 4: Model Fitting and Approximation

Given the following data:

x	**1.2**	**2.5**	**3.1**	**4.0**	**5.2**
y	3.1	4.9	7.4	9.8	13.6

Tasks

(a) Fit the approximate equations:

(i) $y = ax + b$

(ii) $y = ax^2$

(iii) $y = ax^3$

For each model:

- Determine the best-fit parameter(s) using least squares.
- Build a comparison table showing:
  - Original data
  - Estimated (fitted) values
  - Absolute errors

Draw all three fitted curves on the same graph and identify which model fits best.