Exoplanet Transits

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March 13, 2024

Abstract

The radial velocity of the star was analysed and the radial velocity curve of the star was plotted and fit, providing an estimate for the mass of its exoplanet companion. The estimate for the mass was constrained by fitting a transit light curve and finding the inclination of the planet's orbit. From the transit light curve the planet's semi-major axis and its radius, and therefore its density were found also. From the semi-major axis the flux received at the planet was also calculated. From all of these values it was determined that the planet is a hot Saturn, with a mass and radius of 1.066(14) and 0.974(3) that of Saturn respectively, orbiting at a distance of 0.01984(17) au from its host star.

1 Introduction

Exoplanets are planets which orbit stars other than the Sun. There are over 5,500 confirmed exoplanets and over 10,000 more candidate exoplanets which are as yet unconfirmed. [1]

One of the most common methods for exoplanet detection is the transit method in which the light of host star is blocked by a planet passing in front of the star, causing a dip in its brightness for the duration of the transit. [2]

If limb darkening is ignored the greatest dip in the star's brightness, which occurs when all of the planet's disk is between the detector and the star's disk, the transit depth, Δf, is governed by the ratio of the radii of the planet and star,

$$\Delta f = \left(\frac{R_P}{R_\star}\right)^2 = \rho^2 \tag{1.1}$$

where ρ is the ratio of the planet's and star's radii so the flux f at this point is

$$f(\rho) = 1 - \rho^2 \ . \tag{1.2}$$

Where star's flux is normalised to 1 outside of the transit.

When the planet is only partially in front of the star only a portion of the star's light is blocked, the exact amount depending on the amount of the planet's disk in front of the star. If

$$z = \frac{d}{R_\star} \tag{1.3}$$

where d is the apparent distance between the centre of the planet and star, then the flux during the period where only part of the star is occluded by the planet, which corresponds to $1-\rho \leq z \leq 1+\rho$, is given by

$$f(z,\rho) = 1 - \frac{1}{\pi}\left(\rho^2\kappa_0 + \kappa_1 - \sqrt{\frac{4z^2 - (1+z^2-\rho^2)^2}{4}}\right) \tag{1.4}$$

where

$$\kappa_0 = \arccos\left(\frac{\rho^2 + z^2 - 1}{2\rho z}\right) \tag{1.5}$$

and

$$\kappa_1 = \arccos\left(\frac{1 - \rho^2 + z^2}{2z}\right) \ . \tag{1.6}$$

And of course when the planet is not occluding the star the flux is 1. [2]

The value of z as a function of time, t, must be found next. As can be seen in figure 1.1 the value of z is simply given by

$$z = \sqrt{x^2 + y^2} \ . \tag{1.7}$$

The value of x and y depend on the orbital phase of the planet, ϕ, given by

$$\phi = \frac{2\pi}{P}(t - T_0) \tag{1.8}$$

where P is the orbital period of the planet and T_0 is the central transit time.

The value of x, in units of the radius of the star, is given by

$$x = \frac{a}{R_\star}\sin\phi \tag{1.9}$$

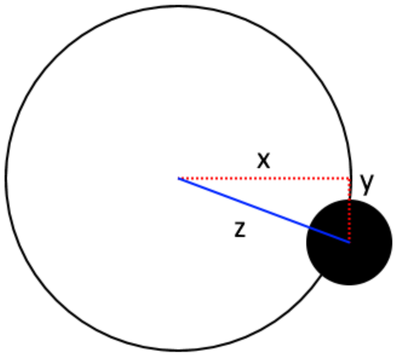


Figure 1.1: The planet (black circle) passing in front of the star (white circle) and orbital coordinates from the perspective of the Earth shown.

where a is the semi-major axis of the planet and ϕ is the orbital phase of the planet. And y is given by

$$y = \frac{a}{R_\star} \cos\phi \cos i \tag{1.10}$$

where i is the orbital inclination of the planet (where $i = \frac{\pi}{2}$ is an edge-on orbit from the perspective of the observer). This then gives

$$z = \frac{a}{R_\star} \sqrt{\sin^2\phi + \cos^2 i \cos^2\phi}\ . \tag{1.11}$$

The light curve of a transit then is given by (after rescaling to have the flux out of the transit, f_{oot}, not be exactly 1

$$f(z,\rho) = \begin{cases} f_{oot} \\ f_{oot} - \frac{1}{\pi}\left(\rho^2\kappa_0 + \kappa_1 - \sqrt{\frac{4z^2-(1+z^2-\rho^2)^2}{4}}\right) \\ f_{oot} - \rho^2 \end{cases} \tag{1.12}$$

for the cases of $z > 1+\rho$, $1-\rho < z \leq 1+\rho$ and $z \leq 1-\rho$ respectively.

The radial velocity method is another method used to detect exoplanets and to determine other properties of exoplanets discovered by other methods such as transits. As a planet orbits a star it and the star actually orbit the star-planet barycentre. [2]

The Doppler shift of the star's spectrum with respect to a model spectrum for a star of that spectral type can be determined and then used to determine the star's radial velocity towards or away from the Earth. If there is a periodic variation to the shift of the star's spectrum that is an indication that there is a companion causing the star to orbit around the mutual barycentre. From the period of this variation the orbital period and distance of the companion as well as its mass can be found (given knowledge of the star's mass). [2], [3]

From Kepler's Third Law the exoplanet's orbital period, P, and so the period of oscillation of the Doppler shift, is given by,

$$G(M_\star + M_P) = \omega_\star^2 a^3 \tag{1.13}$$

where $M_\star$ is the mass of the star, M_P is the mass of the exoplanet, $\omega_\star = \frac{2\pi}{P}$ is the orbital frequency, with P the period of the orbit, and a is the semi-major axis about the barycentre. If

$$a = a_P + a_\star \tag{1.14}$$

where a_P and $a_\star$ are the distances to the barycentre from the planet and star respectively then from the definition of the centre of mass,

$$M_\star a_\star = M_P a_P \tag{1.15}$$

therefore

$$\begin{aligned} a = a_\star + a_P &= a_\star\left(1 + \frac{a_P}{a_\star}\right) \\ &= a_\star\left(1 + \frac{M_P}{M_\star}\right) = \frac{a_\star(M_\star + M_P)}{M_P}\ . \end{aligned} \tag{1.16}$$

If this is inserted into equation 1.13,

$$G(M_\star + M_P) = \omega_\star^2 \frac{a_\star^3 (M_\star + M_P)^3}{M_P^3} \tag{1.17}$$

then this is rearranged to give

$$\begin{aligned} a_\star &= \left(\frac{G(M_\star + M_P)}{\omega_\star^2}\right)^{1/3} \frac{M_P}{(M_\star + M_P)} \\ &= \left(\frac{G}{\omega_\star^2}\right)^{1/3} \frac{M_P}{(M_\star + M_P)^{2/3}}\ . \end{aligned} \tag{1.18}$$

The amplitude of the radial velocity curve, K depends on the inclination angle of the orbit, i, and is given by

$$K_\star = v_\star \sin i = \omega_\star a_\star \sin i \tag{1.19}$$

when the eccentricity of the orbit is 0, where $v_\star$ is the star's velocity with respect to the barycentre.

Inserting the expression for $a_\star$ we get

$$K_\star = \frac{M_P \omega_\star \sin i}{(M_\star + M_P)^{2/3}} \left(\frac{G}{\omega_\star^2}\right)^{1/3}$$
$$= \frac{M_P \sin i}{(M_\star + M_P)^{2/3}} \left(G\omega_\star\right)^{1/3}$$

$$K_\star = \frac{M_P \sin i}{(M_\star + M_P)^{2/3}} \left(\frac{2\pi G}{P}\right)^{1/3} \tag{1.20}$$

if it is assumed that $M_P \ll M_\star$ then

$$K_\star = \frac{M_P \sin i}{M_\star^{2/3}} \left(\frac{2\pi G}{P}\right)^{1/3} \tag{1.21}$$

so

$$M_P = \frac{K_\star M_\star^{2/3}}{\sin i} \left(\frac{P}{2\pi G}\right)^{1/3} . \tag{1.22}$$

The Stefan-Boltzmann law gives the luminosity of a star for a given radius and temperature,

$$L_\star = 4\pi R_\star \sigma T_\star . \tag{1.23}$$

and the flux, F, at a distance d from a star is given by

$$F = \frac{L_\star}{4\pi d^2} = \frac{R_\star \sigma T_\star}{d^2} . \tag{1.24}$$

2 Methods

2.1 Task 1

The data for the radial velocity was downloaded and imported. The wavelength, velocity and model spectrum were separated into their own arrays. The spectra and time data were separated into arrays of the values stored under file names with the appropriate string in their file name ('spectrum' for the spectral data and 'time' for the time stamps). The time data had the first value subtracted from it, to remove the offset. This was done as it allowed the fitting to be carried out easier later in the program, and the offset could easily be readded later.

The model spectrum, one of the measured spectra and the model and measured spectrum together were plotted.

Each of the spectra were cross-correlated with the model spectrum using the `numpy correlate` function. The mean values of the model and the spectrum were subtracted from each dataset before the correlation began. The centre of the correlation curve was then fit to a Gaussian function, the peak of which occurred at the velocity shift of the spectrum. The velocity shift and the uncertainty in this value were saved to an array. This uncertainty was taken as the uncertainty in the velocity shift for the time being.

An example of one of these correlation curves with the Gaussian function fit to it was plotted.

2.2 Task 2

The data for the transit was downloaded and imported. The flux and time arrays were separated into their own variables. The flux was then plotted against time to produce the light curve of the transit.

A function was defined to model the transit function shown in equation 1.12. This was then fit to the data and it was plotted with the original data. The residuals of the fit were plotted also.

The radius of the planet, its orbital inclination and its semi-major axis were then found.

2.3 Task 3

A sin function was fit to the RV curve date found in section 2.1. The amplitude of this curve is equal to $K_\star$, and was passed into equation 1.22 to solve for the planet's mass, under the assumption that $M_P \ll M_\star$. The mass of the planet was also found numerically using `fsolve` in the `scipy.optimize` module. The associated uncertainties were all found also.

The phase-folded version of this curve was also produced to show the radial velocity of the star across one orbit of the planet. The uncertainty used in these plots is initially taken as the uncertainty in the position of the peaks of the Gaussian fits to the correlation curves in section 2.1.

2.4 Task 4

The relative radial velocity of the planetary system to the Earth was found from the fit of the RV curve. Its radius, mass, orbital inclination and semi-major axis had also been found previously. Its density was also found, as was the flux it receives from its star, based on the luminosity of the star which was calculated using the Stefan-Boltzmann law as shown in equations1.23 and 1.24.

2.5 Eclipse

An attempt was made at fitting the eclipse data, however an adequate function which would give the correct shape was not found.

2.6 Details of some functions used

2.6.1 fitting

The function `fitting` is used to fit a function to data, it was used to fit the Gaussian and sin functions and to fit the transit curve. It uses the `least_squares` from `scipy.optimize` to do so. The uncertainties in the coefficients of the function are then found by taking the diagonals of the covariance matrix found from the Hessian matrix. They are then rescaled and the χ^2 value is found also.

2.6.2 find_Mp

This is used to numerically find the mass of the planet and the associated uncertainty. Using `fsolve` from `scipy.optimize` the value of M_P in equation 1.20 is found such that $K_\star$ is equal to the value found by fitting the radial velocity curve to a sin function. The same is done for the uncertainty by solving equation A.3 for $\Delta K_\star$ equal ot the value found from the fit.

3 Results

3.1 Task 1

The model spectrum, one of the measured spectra and the model and measured spectrum together are shown in figure 3.1.

The relative shift in the spectra in the lower image shows that the actual spectra are shifted to longer wavelengths than the model spectrum, i.e. they are red shifted and the system was moving away from us at this time. This is in fact the case for all the spectra, and the system is consistently moving away from the Earth.

When cross correlating it was unclear form the documentation whether the model or observed spectrum should be first in the function, however from trial and error it was found that putting the model spectrum first and the observed spectrum second gave negative values for the velocity shift, as would be expected from the system moving away from the Earth.

An example of the correlation curve, both the entire curve as well as the centre of the curve with the fit of the Gaussian curve plotted for one observed spectrum is shown in figure 3.2. The peak is clearly centred on a negative value.

3.2 Task 2

The light curve of the transit is shown in figure 3.3.

The fitted curve to the transit data and the residuals of this are plotted in figure 3.4.

The transit parameters and any planetary data calculated are shown in table 3.1.

Table 3.1: The transit parameters and any data calculated about the planet from these parameters.

Parameter	Value
T_0	2459637.24772(14) BJD 2022-02-26 17:56:41
a	6.38(5) $R_\star$ 2.97(3) $\times 10^9$ m 0.01984(17) au
R_P	0.1261(4) $R_\star$ 5.87(17) $\times 10^7$ m 9.20(3) $R_\oplus$ 0.821(2) $R_{♃}$ 0.974(3) $R_{♄}$
i	1.4510(18) rad 83.1(1) $^\circ$
f_{oot}	0.99834(2)

Where subscripts containing 'P' refer to the exoplanet, '$\oplus$' refer to the Earth, '♃' refer to Jupiter and '♄' refer to Saturn. (I am aware that quantities are not usually given with reference to Saturn, however in this case I feel it is justified as the planet's properties are very similar that of Saturn.)

3.3 Task 3

The radial velocity curve, both the normal and phase folded are shown in figure 3.5.

From this fit it can be seen that the system's velocity is 7340.0(7) m s^{-1} away from the Earth. This fits with the observed Doppler shift in the spectra as shown in figure 3.1. The spectra are red-shifted and the system is moving away from the Earth.

3.4 Task 4

Table 3.2 shows all the values either given or derived for the system as a whole, the star and the planet. In the table $R_\star$, $M_\star$, $T_\star$ and $L_\star$ are the radius, mass, temperature and luminosity of the star (all of which except $L_\star$ are given), a is the planet's semi-major axis, i is the planet's orbital inclination, R_{planet}, M_{planet} and ρ_{planet} are the radius, mass and density of the planet respectively, with 'min' and 'approx' referring to the minimum value calculated when assuming $i = \frac{\pi}{2}$ and the approximation of the mass calculated using equation 1.22 respectively, and

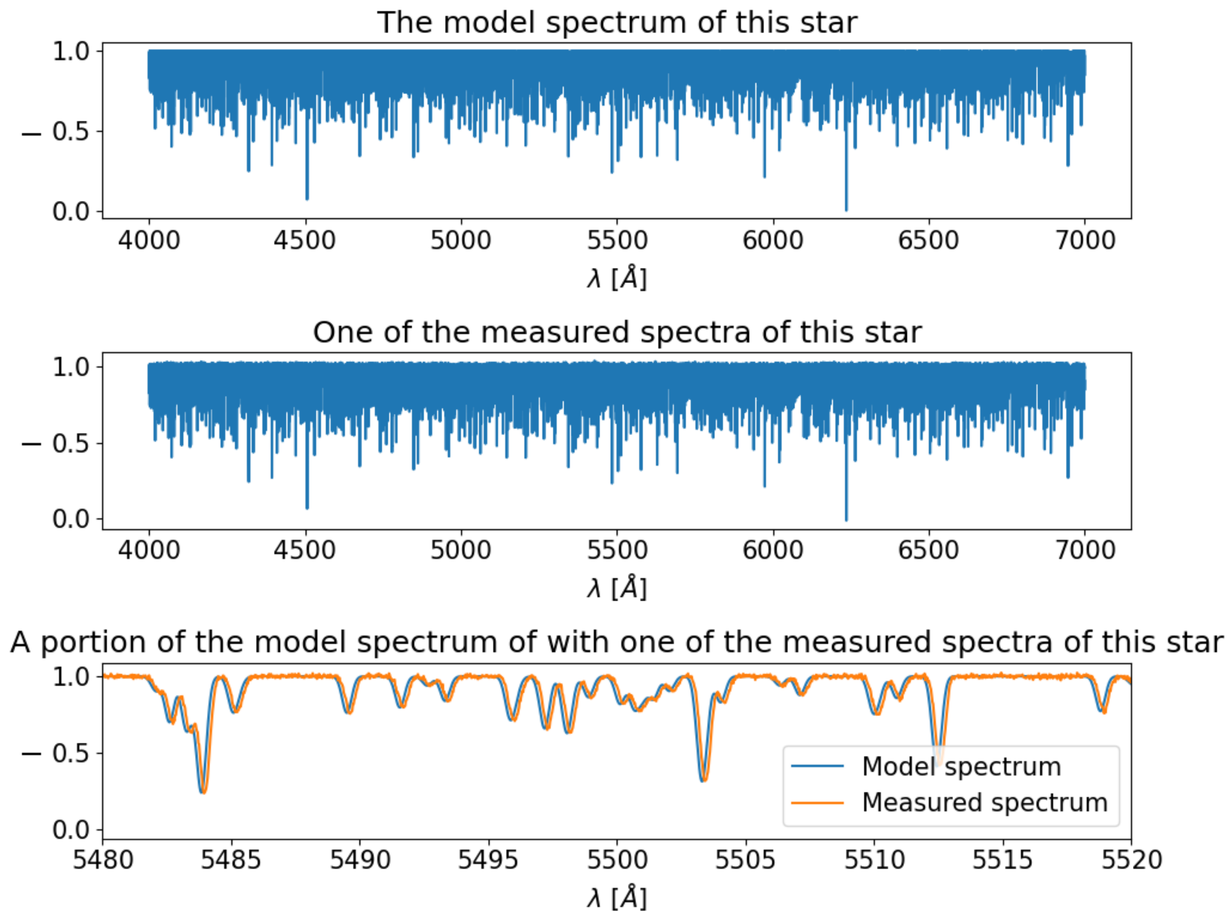


Figure 3.1: The model spectrum, one of the measured spectra and the model and measured spectrum together.

F_{planet} is the flux received at the planet from its star, where $F_{\oplus}$ is the flux received on Earth from the Sun.

From these values we see that the planet is a hot Jupiter with a mass and density a little over those of Saturn, and a radius a little less than Saturn's. Its temperature could also be calculated from the eclipse light curve.

4 Discussion and Conclusions

In all of this, it was assumed that the spectra were given with respect to the Solar System's barycentre and not with respect to the Earth. This is supported by a lack of a component with a period of one day in the RV curve, and by the fact that the values in the RV curve are not noticeable higher at one end of the records than the other. Although the data was taken over only a period of about 2.5 days so this may not be visible in the data over such a short timescale. A lack of this correction would affect the magnitude of the systemic velocity more so than the other quantities.

This planet is a hot Saturn, a massive gas giant which orbits very close to its host star with a very low density. As would be expected from its orbital period of approximately 1.2 days its semi-major axis is very small, 0.01984(17) au (compare this to Mercury's semi-major axis of 0.39 au [2]).

It was assumed when finding the equation for $K_{\star}$ (equation 1.20) that the planet's orbital eccentricity was close to 0, this is supported by the shape of the radial velocity curve, as seen in figure 3.5. Exoplanets with a small eccentricity produce more sinusoidal radial velocity curves. [2] So the values calculated for the mass of the planet, and any subsequent values were likely close to accurate for the planet.

If the eclipse data had been fit then the temperature of the planet could be estimated from its emitted flux. The depth of the eclipse curve would give an indication of the flux emitted by the planet and so the temperature could be estimated.

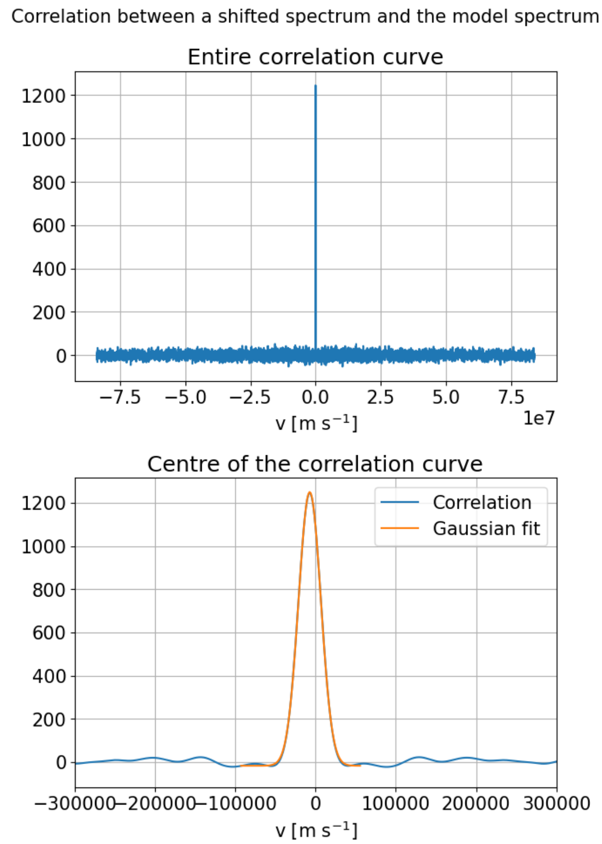


Figure 3.2: The curve of the correlation between one of the observed spectra and the model spectrum.

The transit spectra of the planet could, if they were measured with high enough sensitivity, give an indication of the make-up of the planet's atmosphere by comparing spectra obtained when the planet is transiting the star and when it is eclipsed by the star, any spectral lines which are present or stronger when the planet is transiting than when it is eclipsing are likely due to the planet and not the star or the interstellar medium between the Earth and the star.

The values calculated for the mass of the planet numerically and using the approximation and equation 1.22 are the same once rounded to account for the uncertainty. This, and the fact that the mass was calculated to be approximately 0.0004 that of its host star show that the approximation was appropriate to make initially.

A Propagation of Uncertainty

This shows the equations used to calculate the uncertainty in some of the more complicated cases, i.e. not for simple addition/subtraction, exponentiation,

Table 3.2: The values given for and calculated about the system as a whole, the star and the planet from the results of the code.

Parameter	Value
Systemic velocity	-7340.0(7) m/s
$R_\star$	4.65×10^8 m 0.669 $R_\odot$
$M_\star$	1.4×10^{30} kg 0.69 $M_\odot$
$T_\star$	4300 K
$L_\star$	5.3×10^{25} W 0.14 $L_\odot$
a	6.38(5) $R_\star$ 2.97(3) $\times 10^9$ m 0.01984(17) au
i	1.4510(18) rad 83.1(1) °
R_P	0.1261(4) $R_\star$ 5.87(17) $\times 10^7$ m 9.20(3) $R_\oplus$ 0.821(2) $R_{♃}$ 0.974(3) $R_{♄}$
$M_{P_{min}}$	6.02(8) $\times 10^{26}$ kg 100.8(1.3) $M_\oplus$ 0.317(4) $M_{♃}$ 1.059(14) $M_{♄}$
$M_{P_{approx}}$	6.06(8) $\times 10^{26}$ kg 101.5(1.3) $M_\oplus$ 0.319(4) $M_{♃}$ 1.066(14) $M_{♄}$
M_P	6.06(8) $\times 10^{26}$ kg 101.5(1.3) $M_\oplus$ 0.319(4) $M_{♃}$ 1.066(14) $M_{♄}$
ρ_P	716(11) kg m^{-3} 0.130(2) $\rho_\oplus$ 0.577(9) $\rho_{♃}$ 1.154(18) $\rho_{♄}$
F_P	477(8) $\times 10^3$ W m^{-2} 350(6) $F_\oplus$

multiplication and division of two values, etc.

It was assumed for the sake of simplicity that there was no uncertainty in any of the values given in the assignment.

These all make use of Gauss's law of error propagation. If there is a function $f(x_1, x_2, ...)$ then the uncertainty in f, Δf, is,

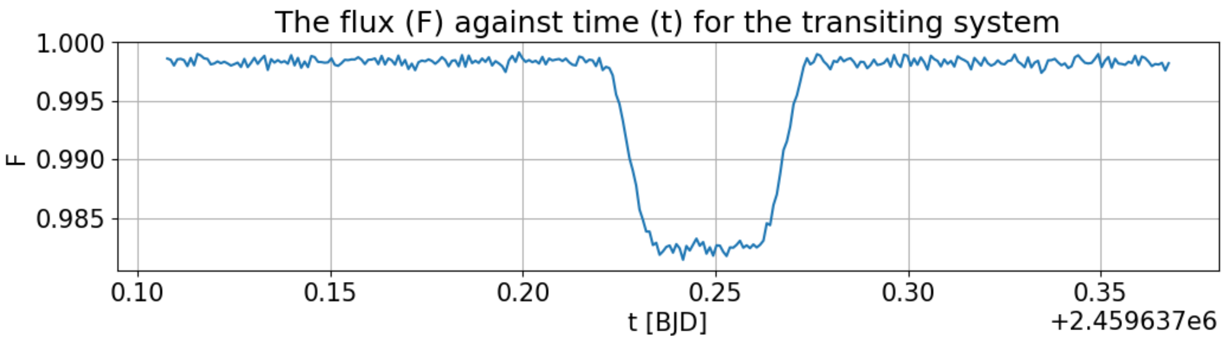


Figure 3.3: The light curve of this transiting system.

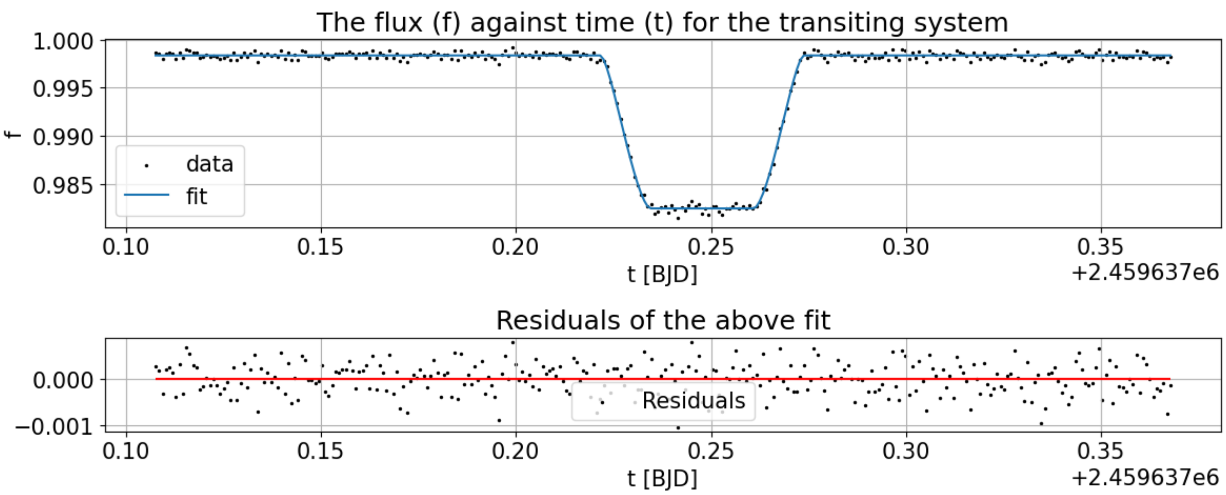
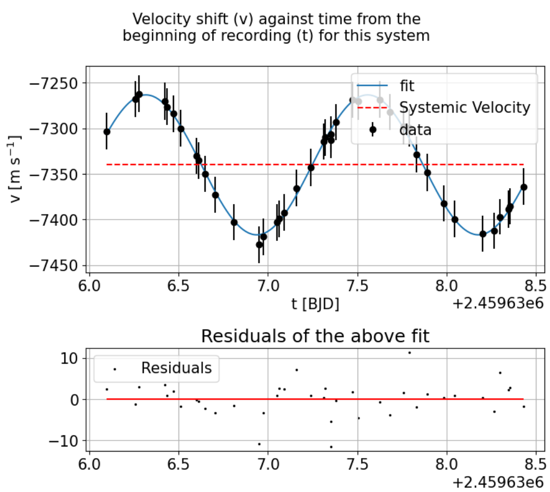
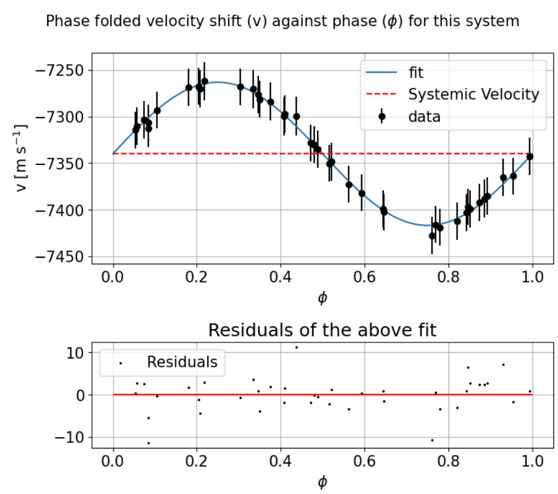


Figure 3.4: The light curve of this transiting system with the curve fit to it and the residuals.



(a) The radial velocity curve.



(b) The phase folded radial velocity curve.

Figure 3.5: The radial velocity curve for the exoplanet with a sin curve fit to the data and the residuals shown below the curve.

$$\Delta f = \sqrt{\left(\frac{\partial f}{\partial x_1}\Delta x_1\right)^2 + \left(\frac{\partial f}{\partial x_2}\Delta x_2\right)^2 + ...} \quad (A.1)$$

where Δx_i is the uncertainty in x_i.

A.1 Uncertainty in $K_\star$

The uncertainty in the value of $K_\star$ as calculated in equation 1.20, repeated here for reference,

$$K_\star = \frac{M_P \sin i}{(M_\star + M_P)^{2/3}} \left(\frac{2\pi G}{P}\right)^{1/3} \quad (A.2)$$

is given by,

$$\Delta K_\star = \frac{(M_P + 3M_\star) \sin i}{3(M_P + M_\star)^{5/3}} \left(\frac{2\pi G}{P}\right)^{1/3} \Delta M_P \ . \quad (A.3)$$

A.2 Uncertainty in the calculation of density

The density, ρ, of a sphere is given by

$$\rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi R^3} \ . \quad (A.4)$$

From Gauss' law then the uncertainty in the density given uncertainty in the mass and radius is

$$\Delta \rho = \sqrt{\left(\frac{\Delta M_P}{\frac{4}{3}\pi R^3}\right)^2 + \left(\frac{-3M_P}{\frac{4}{3}\pi R^4}\Delta R_P\right)^2} \quad (A.5)$$

References

[1] NASA, *Exoplanet Exploration: Planets Beyond our Solar System* (Accessed: 2024, Mar. 8). Available: https://exoplanets.nasa.gov/

[2] M. Perryman, *The Exoplanet Handbook*, 2nd ed. Cambridge: Cambridge University Press, 2018.

[3] H. Karttunen, P. Kröger, H. Oja, M. Poutanen, K. J. Donner, *Fundamental Astronomy*, 6th ed. Springer, 2017.