Computer Simulation 2

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1 Plot of cases against time

The dataset was downloaded and the number of cases per day against the number of days since the 3^{rd} of March 2020 were plotted. The graph in figure 1 shows four obvious waves of infection with peaks visible on the graph on 25 April 2020, 20 October 2020, 10 January 2021 and 23 August 2021 respectively.

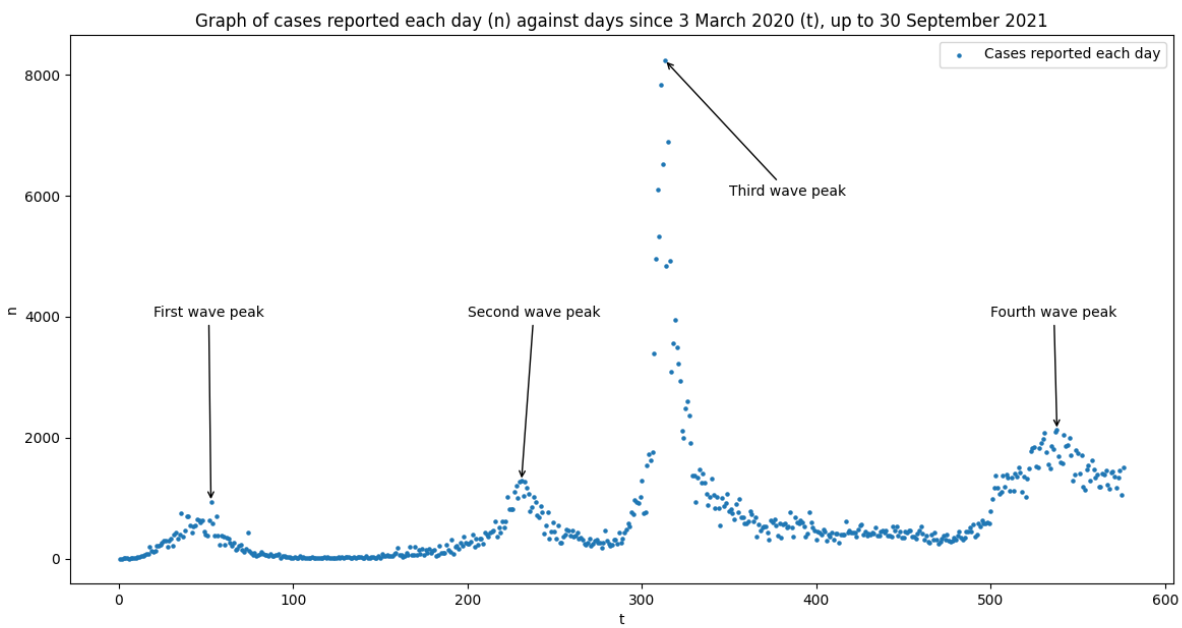

Figure 1: The graph of cases per day against the number of days since the 3^{rd} of March 2020 (t). The peaks of four distinct waves of infection are visible in the graph at approximately t = 53, t = 231, t = 313 and t = 538.

The first three waves appear to show exponential rises and decreases in the numbers of cases reported each day. The exponential parts of the graph should be described by

$$n = n_0 e^{\lambda(t-t_0)} \tag{1}$$

where n is the case number, t is the time in days since the start of the respective wave, t_0 is the number of days between 3 March 2020 and the start of that wave, n_0 is the number of cases on that day (i.e. $n(t_0) = n_0$) and λ is a constant which must be determined from fitting. It is positive at the rise of a wave and negative in the decaying portion.

2 Plot of natural log of cases against time and fitting

The natural log of the daily reported cases was plotted against the number of days since the 3^{rd} of March 2020 as shown in figure 2. The same peaks are visible, as are the almost straight-line rises and declines in the number of reported cases as the waves begin and end - this suggests that the waves are indeed exponential in nature.

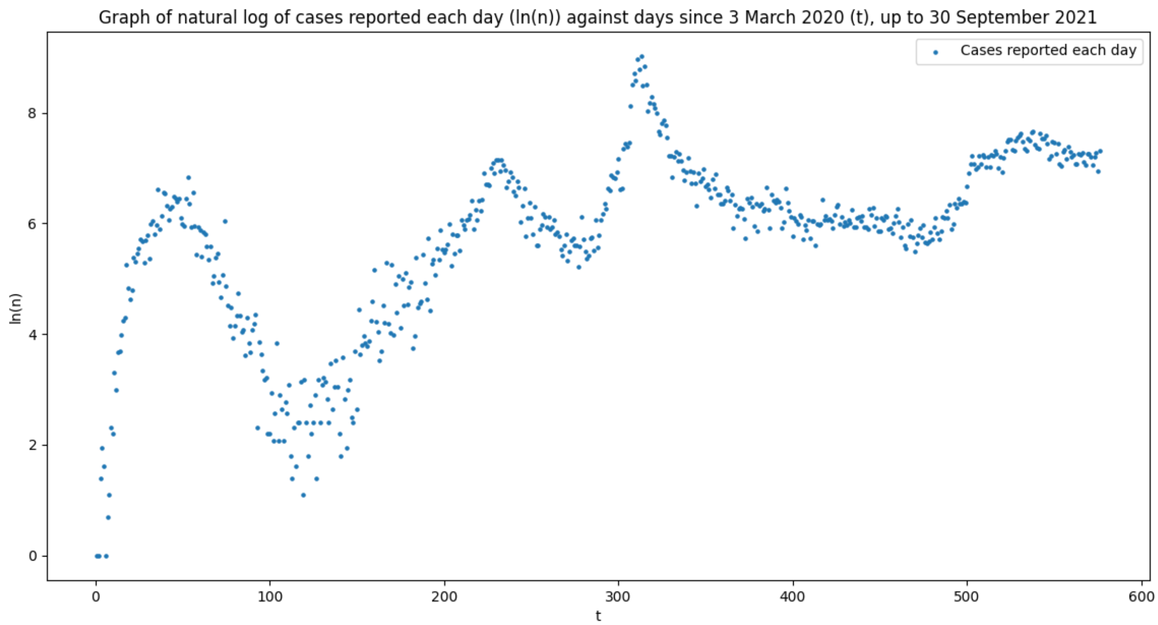

Figure 2: The graph of the natural log of the cases per day against the number of days which have passed since the 3^{rd} of March 2020 (t).

Taking the natural logarithm of equation 1 gives

$$\ln n = \ln n_0 + \lambda(t - t_0) = \ln n_0 - \lambda t_0 + \lambda t \tag{2}$$

In this equation, the only term dependent on t is λt, meaning that for a line fit to this data the slope is given by λ and $\ln n - \lambda t_0$ gives the intercept. Meaning this has the functional form

$$\ln n = a + bt \tag{3}$$

where

$$a = \ln n_0 - \lambda t_0 \tag{4}$$

and

$$b = \lambda \tag{5}$$

The values of a and b can be found through linear regression on the data allowing λ and n_0 for the particular fit to be found.

Appropriate values of t_0 were found from studying the graph in figure 2 and then using the scipy optimize package the data was fit to the form of equation 3 and plotted with the reported cases as shown in figure 3.

From equation 4, it can be see that for the functions as calculated by the fitting, $n_0 = e^{a+\lambda t_0}$.

The values of t_0, a, b (which is the same as λ) and n_0 as calculated by python are shown in table 1.

Table 1: The parameters of the fit found by python for the rising and decaying portions of different waves of infection. Each wave of infection begins at the respective value of t_0, and ends the day before the subsequent wave starts. a and b are coefficients of the linear fit (equation 3), while λ and n_0 are part of the exponential fit (equation 1).

Wave	t_0	a	b, λ	n_0
1 Rise	1	1.689	0.111	4.98
1 Decay	54	9.716	-0.0662	627.45
2 Rise	120	-2.868	0.0422	251.25
2 Decay	234	14.032	-0.0312	867.42
3 Rise	278	-23.325	0.1018	132.42
3 Decay	314	19.370	-0.0359	3242.10

It should be noted that, as seen in figure 2 there are two distinct slopes in the decay of the third wave, with the change being at approximately t = 372, so the fitting for the decay of the third wave was stopped there.

This may be partially explained as the date of this was the 10^{th} of March 2021 which is about one week after 6^{th} years, half of primary school students and all students in special schools returned to in-person teaching.[1] As such, a decrease in the decay rate is understandable as more people are interacting with each other and potentially coming into contact with people infected with the virus than before this.

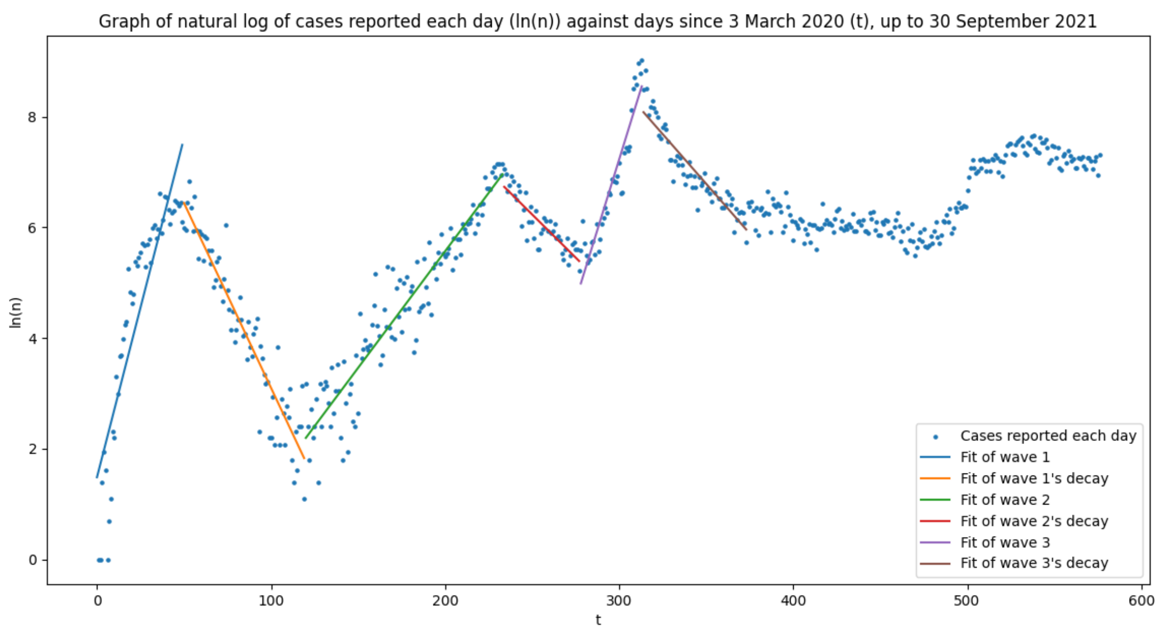

Figure 3: The graph of the natural log of the cases per day against the number of days which have passed since the 3^{rd} of March 2020 (t) with the fits plotted on the same axes.

3 Fitting to linear graph

The parameters λ and n_0 in table 1 were used to fit exponential curves on the linear graph for the rise and decay of the first three waves as seen in figure 4.

As can be seen in figure 4 the fits stick very close to the data in most cases. The fit of the rise of the first wave does reach much higher than the data does. The end date of this fit was chosen as 52 days after 3 March 2020, or 24 April 2020, well into the first lockdown in the country.[2] This would have

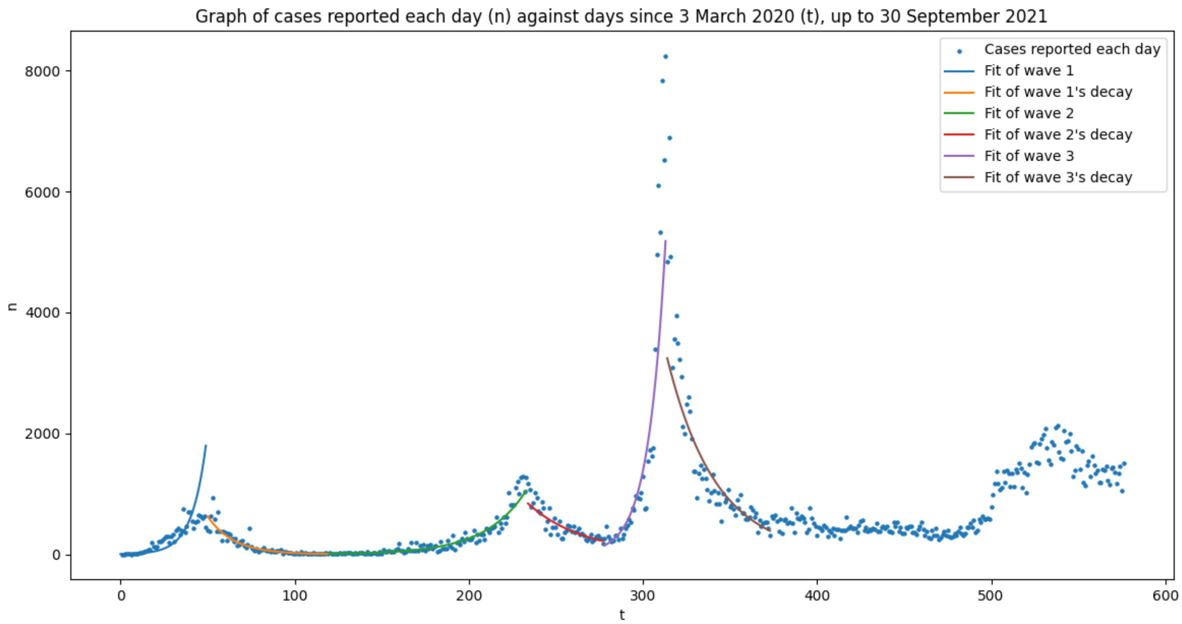

Figure 4: The graph of the cases per day against the number of days since the 3^{rd} of March 2020 (t) with the fits plotted on the same axes.

decreased people's exposure to the virus and might have meant that the initial high growth rate might have affected the fitting function. No credible choice of end date for the first wave was able to remove the large overestimation of the peak of the first wave.

The peak of the third wave is also slightly problematic, neither the rising nor falling functions reach the highest point of the data. However in general throughout the majority of the third wave, both the rise and the decay, the two fits stick very close to the data and tend to reflect it very well.

4 Fourth wave

The fourth wave is different to the previous waves in that there is a long period of relative stability in the case numbers after the decay of the third wave followed by a quick rise resembling a 'hump' shape - the rise of which does not appear to resemble an exponential function. This appears to resemble a Gaussian curve. Indeed, if the scipy optimize package is used to fit this data to a curve of the form

$$n = a \exp\left(-\left(\frac{x-b}{c}\right)^2\right) + d \tag{6}$$

where a, b, c, and d are fitting parameters which can be initially estimated through trial and error and from studying the graph to be 2000, 538, 30, 300 respectively. This was necessary as python encounters a run time error and the parameters were unfitable if the initial estimation is left blank. Through the scipy optimize package the values of the fitting parameters were found and are shown in table 2.

These values for these parameters produce the graph in figure 5. As can be seen, the Gaussian curve appears to fit the fourth wave well.

Table 2: The initial estimates and the found values for the fitting parameters of the Gaussian curve, as given in equation 6.

Parameter	Initial estimate	Fit value
a	2000	1393.3
b	538	539.80
c	30	36.489
d	300	419.58

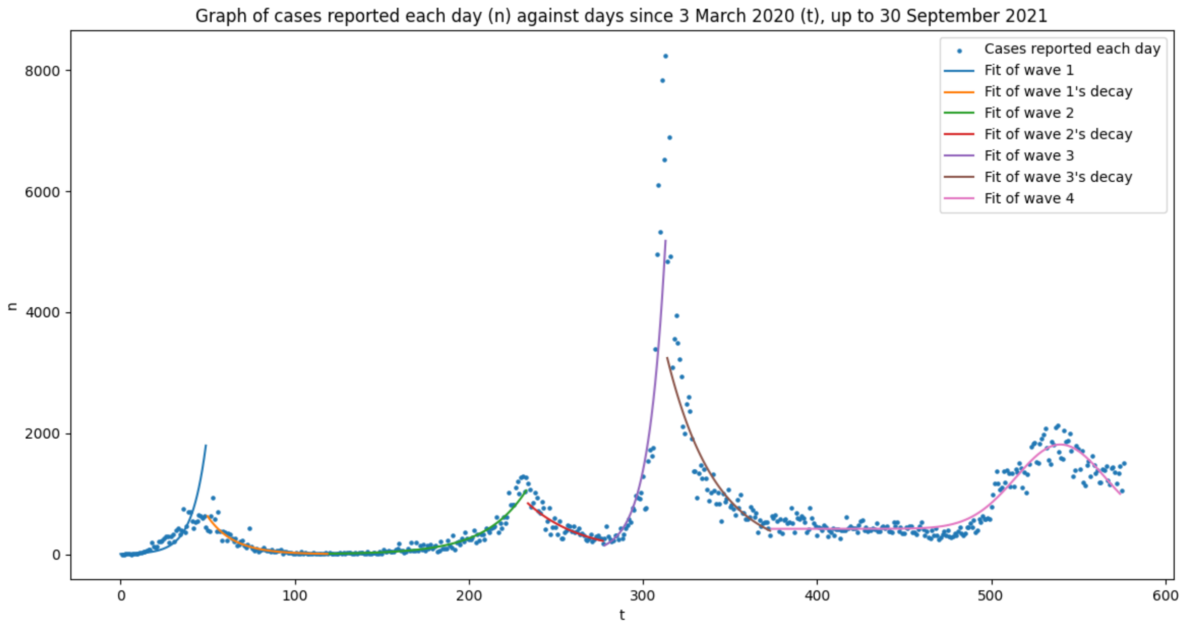

Figure 5: The graph of the cases per day against the number of days since the 3^{rd} of March 2020 (t) with the of the fits plotted on the same axes.

The reasons for the odd shape of the fourth wave are the circumstances of the world at the time. As previously mentioned, the third wave's deacy slowed at approximately the 10^{th} of March 2021 ($t = 372$) and more or less plateaued from there until around $t = 489$ or the 5^{th} of July 2021 when it began to increase greatly.

The slow in the decay of the third wave could be due to some schools returning a week before that as mentioned above, and a large increase in case numbers likely didn't happen because at that point over half a million COVID-19 vaccines had been administered with the number passing one million in early April.[3], [4]

The sharp increase is likely due to a combination of easing of some restrictions which were occurring at that point, as the vaccine uptake had been high, and the introduction of the Delta variant into Ireland.[5] The peak of this increase occurs around $t = 537$, or the 22^{nd} of August 2021. At this point vaccinations had be opened to everyone over the age of 12 and nearly seven million doses of vaccines had been administered.[6],[7] These vaccinations likely helped to stop the case numbers rising faster and to higher numbers, as well as causing the rise to stop relatively quickly.

These are all potential explanations or parts of explanations as to the shape of the fourth wave's data.

References

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