Galactic Structure Continuous Assessment 1

ismisebrendan

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1 Introduction

Adding 3000 to the last four digits of my student number (3451), the temperature I get for my star is $3000 + 3451 = 6451$ K.

2 Peak wavelength using Wien's Law

Wien's Law is

$$\lambda_{max} T = 0.002897755 \, m \, K \tag{1}$$

where λ_{max} is the peak wavelength due to blackbody emission, and T is the temperature of the blackbody.[1] As such the peak wavelength is given by

$$\lambda_{max} = \frac{0.002897755 \, m \, K}{T} = \frac{0.002897755 \, m \, K}{6451K} = 4.492 \times 10^{-7} m \tag{2}$$

converting to angstroms (1 m = 10^{10} Å) the peak wavelength is λ_{max} = 4492 Å.

3 Flux per wavelength of the blackbody

The flux per wavelength in units of ergs s^{-1} cm^{-3} can be calculated from the following equation

$$F_\lambda(T) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda k_B T} - 1} \tag{3}$$

where h is Planck's constant, c is the speed of light λ is the wavelength of light, k_B is the Boltzmann constant and T is the temperature of the blackbody (in this case the star).

The graph in figure 1 was generated by plotting the results of equation 3 taking wavelength inputs in terms of centimetres equivalent to 200 Å to 20000 Å in steps equivalent to 0.01 Å. When plotted the wavelengths were converted to angstroms by multiplying by 10^8.

The peak wavelength was also found and marked on the plot.

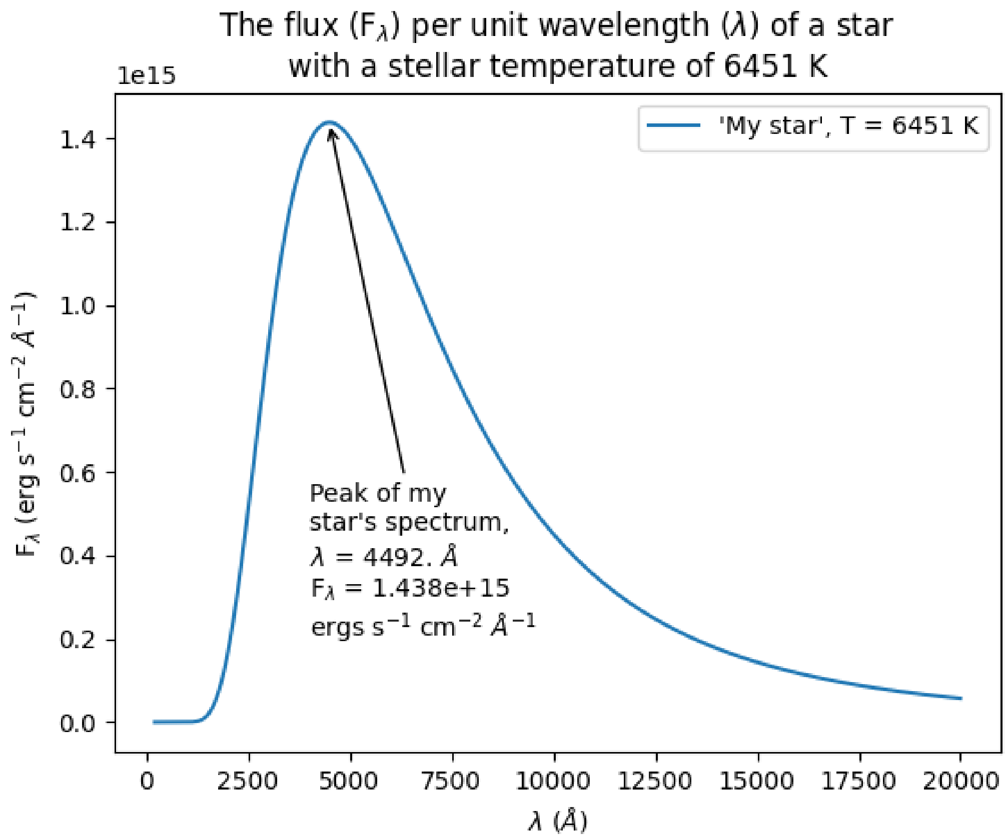


Figure 1: The flux per unit wavelength for a star with a temperature of 6451 K plotted against wavelength between 200 Å and 20000 Å. The wavelength of the peak flux and the peak flux is also marked on the graph.

4 Comparison between the peak wavelengths from Wien's Law and the Blackbody Flux

The peak wavelength of the flux from the plot is 4492 Å and the peak calculated from Wien's Law was also calculated to be 4492 Å. These show that the two formulae are in mutual agreement.

The peak from the plot was found by finding the index of the element in the array of fluxes which was the highest. The corresponding wavelength at this index was then simply found and outputted.

5 Comparison between Star and Proxima Centauri

The flux per unit wavelength of Proxima Centauri was calculated in a similar manner as that of the fake star. The plot of the two spectra together can be seen in figure 2 and the plot of the spectrum of Proxima Centauri can be seen on its own in figure 3 as its spectrum is much lower in flux than that of the fake star with a temperature of 6451 K so it must be represented separtely to see any details of the spectrum.

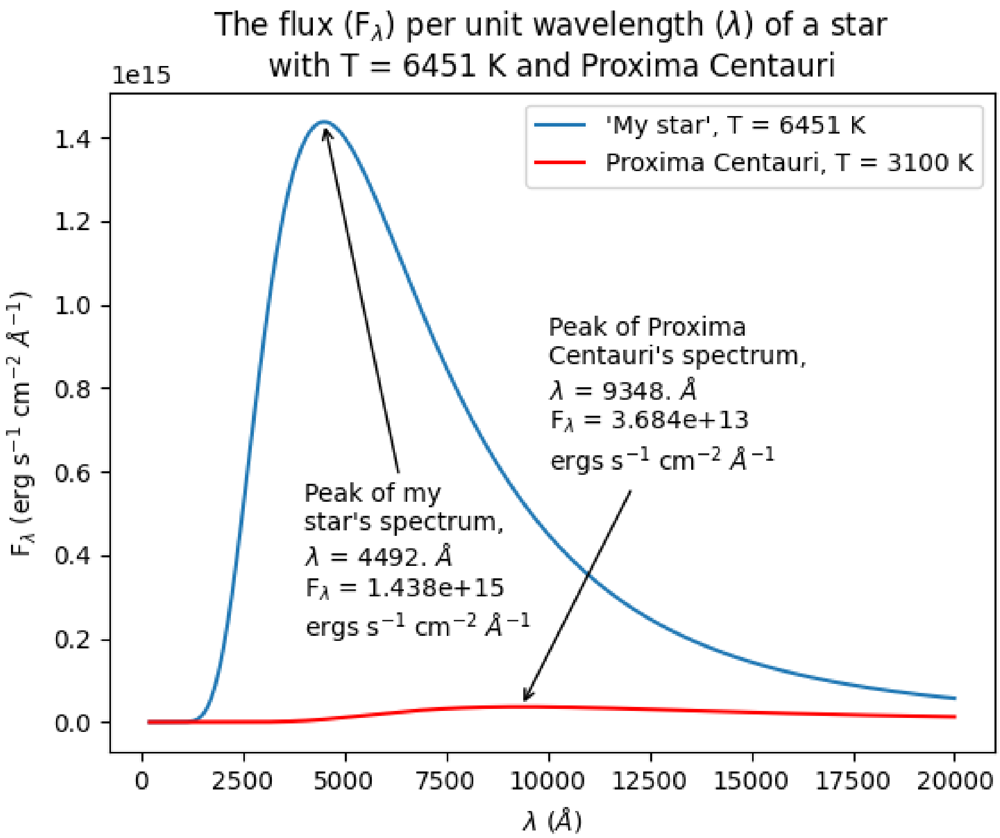


Figure 2: The flux per unit wavelength for a star with a temperature of 6451 K and Proxima Centauri (T = 3100 K) plotted against wavelength between 200 Å and 20000 Å. The wavelengths of the peak fluxes and the peak fluxes are also marked on the graph.

As can be seen in the graph in figure 2 the peak and the flux in general of Proxima Centauri is much lower than that of the fake star ($F_{\lambda,peak}$ = 3.68 × 10^{13} for Proxima Centauri and $F_{\lambda,peak}$ = 1.44 × 10^{15} for the fake star). This is as expected, as Proxima Centauri is much cooler and therefore redder than the fake star, with red light corresponding to longer wavelengths of light.

It can be seen more clearly in figures 1 and 3 that the spectrum of Proxima Centauri remains very low until longer wavelengths than is the case for the fake star (increasing sharply at approximately 3000 Å as opposed to at approximately 1250 Å for the fake star). The graphs also show that the spectrum of Proxima Centauri is thicker and that the flux rises slower and falls off much slower than does the flux in the spectrum of the fake star, despite the flux of the fake star reaching a higher peak at shorter wavelengths.

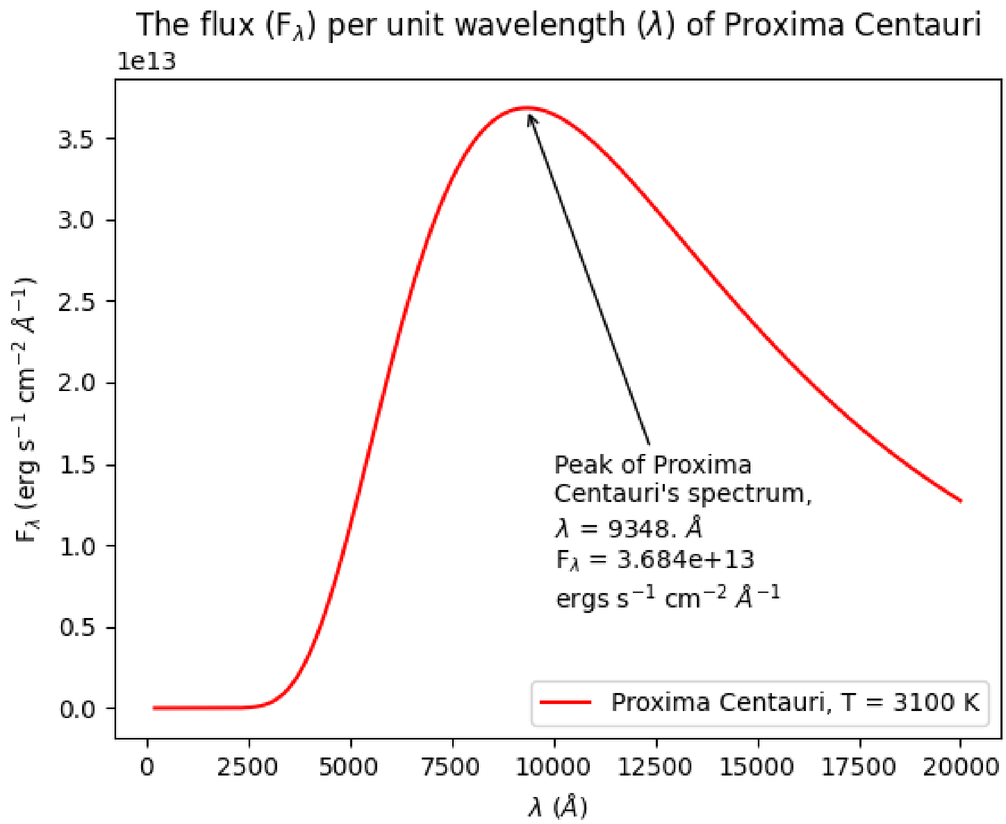


Figure 3: The flux per unit wavelength for Proxima Centauri (T = 3100 K) plotted against wavelength between 200 Å and 20000 Å. The wavelength of the peak flux is also marked on the graph as is the peak flux itself.

6 Main sequence radius

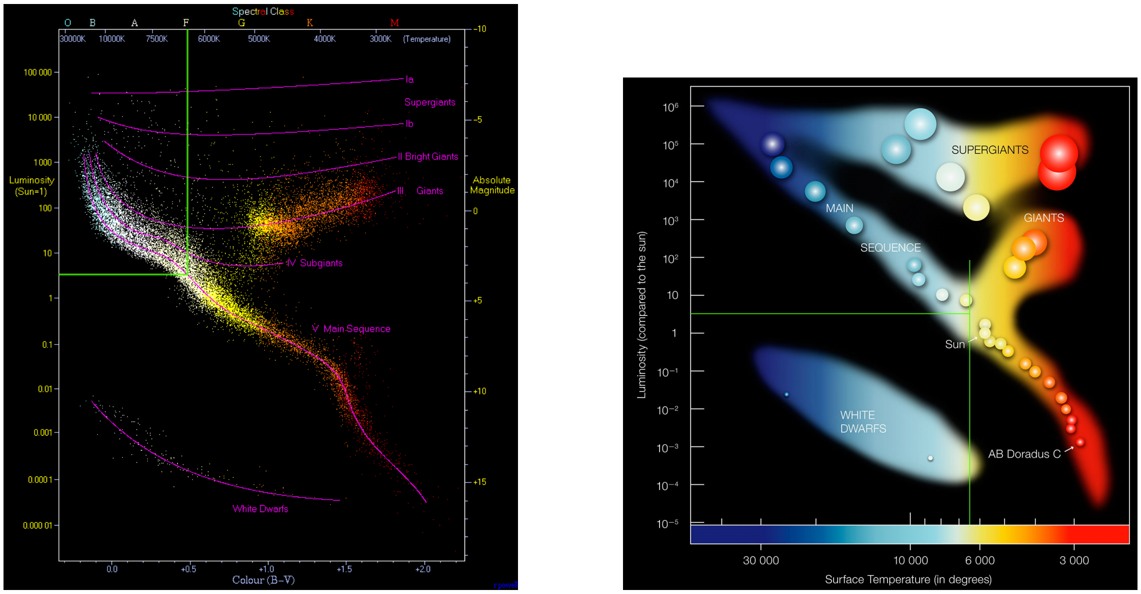
The radius of a star can be calculated from the Stefan-Boltzmann Law and Hertzsprung–Russell diagram. The Stefan-Boltzmann law states

$$L = 4\pi R^2 \sigma T^4 \tag{4}$$

where L is the luminosity of the star, R is the radius of the star, T is the temperature of the star and σ is the Stefan-Boltzmann constant. This can be rearranged to give

$$R = \sqrt{\frac{L}{4\pi\sigma T^4}} \tag{5}$$

The temperature of the star is known (6451 K), and so its luminosity can be found from the Hertzsprung-Russell diagram. On the Hertzsprung–Russell the temperature and luminosities of stars are plotted on a log-log plot. This is not the most precise method so this method was carried out for two different versions of the Hertzsprung-Russell diagram, as seen in figures 4a and 4b.



(a) HR diagram used to find the luminosity and subsequently the radius of the star. Here the horizontal line corresponds to a luminosity of approximately 3.40 $L_\odot$[2].

(b) HR diagram used to find the luminosity and subsequently the radius of the star. Here the horizontal line corresponds to a luminosity of approximately 3.24 $L_\odot$[3].

Figure 4: The two Hertzsprung-Russell diagrams used to find the luminosity and subsequently the radius of the star. Both diagrams feature a line coming vertically from the axes at approximately 6451 K and a line going horizontally from the intersection of the T = 6451 K line with the main sequence.

Diagram A

The diagram in figure 4a gives a luminosity of approximately 3.40 $L_\odot$, inserting this and the temperature of 6451 K into equation 5 we obtain

$$R = \sqrt{\frac{3.40\ L_\odot}{4\pi\sigma(6451\ K)^4}} = 1.03 \times 10^{11}\ cm \tag{6}$$

so the star could have a radius of approximately 1.03 $\times 10^{11}$ cm.

Diagram B

The diagram in figure 4b gives a luminosity of approximately 3.24 $L_\odot$, inserting this and the temperature of 6451 K into equation 5 we obtain

$$R = \sqrt{\frac{3.24\ L_\odot}{4\pi\sigma(6451\ K)^4}} = 1.01 \times 10^{11}\ cm \tag{7}$$

so the star could have a radius of approximately 1.01 $\times 10^{11}$ cm.
Taking the mean of these two values

$$\frac{(1.03\ cm + 1.01\ cm) \times 10^{11}}{2} = 1.02 \times 10^{11}\ cm \tag{8}$$

the radius of the star if it was on the main sequence would be approximately 1.02 $\times 10^{11}$ cm

7 Code

The code used in this assignment is found below.

```python
# -*- coding: utf-8 -*-
import numpy as np
import matplotlib.pyplot as plt

#################
#               #
#   Question 3  #
#               #
#################

# Physical constants
c = 2.99792458e10 # cm/s
h = 6.62607015e-27 # erg s
kb = 1.380649e-16 # erg/K

# Flux given wavelength (l) and temperature (T) in cgs units.
def flux(l, T):
    return (2*np.pi*h*c**2)/(l**5) * 1/(np.exp((h*c)/(l*kb*T)) - 1) # erg s^-1 cm^-3
     equivalent to erg s^-1 cm^-2 angstrom^-1

wavelengths = np.arange(200e-8, 20000e-8, 1e-10) # in cm
fluxes = flux(wavelengths, 6451)

# Find index of maximum flux
max_flux_loc = np.where(fluxes == max(fluxes))[0]

# Wavelength at this index
peak_wavelength = wavelengths[max_flux_loc-1].item() * 10**8 # convert to angstrom

# Plot
plt.plot(wavelengths * 10**8, fluxes, label="'My star', T = 6451 K") # Convert to
    angstroms while plotting
plt.xlabel("$\lambda$ ($\AA$)")
plt.ylabel("F$_{\lambda}$ (erg s$^{-1}$ cm$^{-2}$ $\AA^{-1}$)")
plt.title("The flux (F$_{\lambda}$) per unit wavelength ($\lambda$) of a star\nwith
    a stellar temperature of 6451 K")
plt.legend()
plt.annotate("Peak of my\nstar's spectrum,\n$\lambda$ = " + str(np.rint(
    peak_wavelength))[0:-1] + " $\AA$\nF$_{\lambda}$ = "+str(np.
    format_float_scientific(max(fluxes),3))+"\nergs s$^{-1}$ cm$^{-2}$ $\AA^{-1}$",
    xy=(peak_wavelength, max(fluxes)), xytext=(4000,0.2e15), arrowprops=dict(
    arrowstyle="->"))
plt.savefig("blackbody.png")
plt.close()
```

```python

##################
#                #
#   Question 4   #
#                #
##################

# Find index of maximum flux
max_flux_loc = np.where(fluxes == max(fluxes))[0]

# Wavelength at this index
peak_wavelength = wavelengths[max_flux_loc-1].item() * 10**8 # convert to angstrom

print("The peak wavelength of the star's emission is " + str(peak_wavelength) + " \u00C5")

##################
#                #
#   Question 5   #
#                #
##################

pc_fluxes = flux(wavelengths, 3100)

# Find index of maximum flux for PC
pc_max_flux_loc = np.where(pc_fluxes == max(pc_fluxes))[0]

# Wavelength at this index for PC
pc_peak_wavelength = wavelengths[pc_max_flux_loc-1].item() * 10**8 # convert to angstrom

# Plot both blackbody curves
plt.plot(wavelengths * 10**8, fluxes, label="'My star', T = 6451 K")
plt.plot(wavelengths * 10**8, pc_fluxes, label="Proxima Centauri, T = 3100 K", color="red")
plt.xlabel("$\lambda$ ($\AA$)")
plt.ylabel("F$_{\lambda}$ (erg s$^{-1}$ cm$^{-2}$ $\AA^{-1}$)")
plt.title("The flux (F$_{\lambda}$) per unit wavelength ($\lambda$) of a star\nwith T = 6451 K and Proxima Centauri")
plt.legend()
plt.annotate("Peak of my\nstar's spectrum,\n$\lambda$ = " + str(np.rint(peak_wavelength))[0:-1] + " $\AA$\nF$_{\lambda}$ = "+str(np.format_float_scientific(max(fluxes),3))+"\nergs s$^{-1}$ cm$^{-2}$ $\AA^{-1}$", xy=(peak_wavelength, max(fluxes)), xytext=(4000,0.2e15), arrowprops=dict(arrowstyle="->"))
plt.annotate("Peak of Proxima\nCentauri's spectrum,\n$\lambda$ = " + str(np.rint(pc_peak_wavelength))[0:-1] + " $\AA$\nF$_{\lambda}$ = "+str(np.format_float_scientific(max(pc_fluxes),3))+"\nergs s$^{-1}$ cm$^{-2}$ $\AA^{-1}$", xy=(pc_peak_wavelength, max(pc_fluxes)), xytext=(10000,0.6e15), arrowprops=dict(arrowstyle="->"))
plt.savefig("blackbody_both.png")
plt.close()

# Plot blackbody curve of Proxima Centauri on its own
plt.plot(wavelengths * 10**8, pc_fluxes, label="Proxima Centauri, T = 3100 K", color="red")
plt.xlabel("$\lambda$ ($\AA$)")
plt.ylabel("F$_{\lambda}$ (erg s$^{-1}$ cm$^{-2}$ $\AA^{-1}$)")
```

```python
plt.title("The flux (F$_{\lambda}$) per unit wavelength ($\lambda$) of Proxima Centauri")
plt.legend()
plt.annotate("Peak of Proxima\nCentauri's spectrum,\n$\lambda$ = " + str(np.rint(pc_peak_wavelength))[0:-1] + " $\AA$\nF$_{\lambda}$ = "+str(np.format_float_scientific(max(pc_fluxes),3))+"\nergs s$^{-1}$ cm$^{-2}$ $\AA^{-1}$", xy=(pc_peak_wavelength, max(pc_fluxes)), xytext=(10000,0.6e13), arrowprops=dict(arrowstyle="->"))
plt.savefig("blackbody_PC.png")
plt.close()

print("The maximum flux of Proxima Centauri is " + str(np.format_float_scientific(max(pc_fluxes))) + " ergs s\u207B\u00B9 cm\u207B\u00B2 \u00C5\u207B\u00B9")
print("The maximum flux of 'my star' is " + str(np.format_float_scientific(max(fluxes))) + " ergs s\u207B\u00B9 cm\u207B\u00B2 \u00C5\u207B\u00B9")
```

References

[1] B. W. Carroll, D. A. Ostile, *An Introduction to Modern Astrophysics*, 2nd ed. Harlow, England: Pearson: 2014

[2] Wikimedia Commons. Accessed: 2023, Oct. 3. *File:HRDiagram.png - Wikimedia Commons* [Online]. Available: https://commons.wikimedia.org/wiki/File:HRDiagram.png

[3] European Southern Observatory. Accessed: 2023, Oct. 3. *Hertzsprung-Russell Diagram — ESO Ireland* [Online]. Available: https://www.eso.org/public/ireland/images/eso0728c/