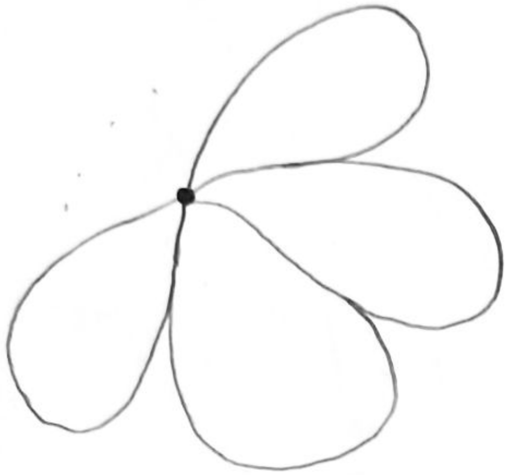
№2



f петель длины N

$d=3$

$$R_g^2 = \frac{1}{2N^2f^2}\sum_{i,j=1}^{f}\int_0^N\int_0^N d\tau_1\, d\tau_2 \left(r_i(\tau_1)-r_j(\tau_2)\right)^2$$

1) $P(R_g^2)$ - ?

$$P(R_g^2) = \int D_r \exp\left[\sum_{i=1}^{f} -\frac{3}{2b^2}\int_0^N d\tau\, r_i^2(\tau)\right]\delta\left(R_g^2 - \frac{1}{2N^2f^2}\int_0^N d\tau_1\int_0^N d\tau_2 \cdot \right.$$

$$\left. \cdot \sum_{i,j=1}^{f}\left(r_i(\tau_1) - r_j(\tau_2)\right)\right) = \left\{ s = \tau/N \right\} =$$

$$= \int D_r \exp\left[\frac{-3}{2b^2N}\cdot\sum_{i=1}^{f}\int_0^1 r_i^2(s)\,ds\right]\delta\left[R_g^2 - \frac{1}{2f^2}\sum_{i,j=1}^{f}\int_0^1\int_0^1 ds_1\, ds_2 \left(r_i(s_1)-r_j(s_2)\right)^2\right]$$

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2) $$\delta(x) = \int_{-\infty}^{\infty}\frac{d\xi}{2\pi}e^{i\xi x}$$

$$P(R_g^2) = \int\frac{d\xi}{2\pi}\cdot e^{-i\xi R_g^2}\cdot \underbrace{\int D_r \exp\left[\frac{-3}{2b^2N}\sum_{i=1}^{f}\int_0^1 r_i^2(s)\,ds + \frac{i\xi}{2f^2}\int_0^1\int_0^1 ds_1\, ds_2 \cdot \sum_{i,j=1}^{f}\left(r_i(s_1)-r_j(s_2)\right)\right]}_{k(\xi)}$$

$$K(\xi) = \int D_r \exp\left[\frac{-3}{2b^2N}\sum_{i=1}^{f}\int_0^1 r_i(s)\,ds + \frac{i\xi}{2f^2}\int_0^1\int_0^1 ds_1\, ds_2 \sum_{i,j=1}^{f}\left(r_i(s_1)-r_j(s_1)\right)^2\right.$$

$$\sum_{ij}\left(r_i(s_1)-r_j(s_2)\right)^2 = \sum_{i,j=1}\left(r_i^2(s_1)+r_j^2(s_2) - 2r_i(s_1)\,r_j(s_2)\right)$$

$$\int_0^1\int_0^1 ds_1 ds_2 \sum_{i,j=1}^{f} \left(r_i^2(s_1) + r_j^2(s_2) - 2 r_i(s_1) r_j(s_2) \right) =$$

$$= f\int_0^1 ds \sum_{i=1}^{f} \left(2 r_i^2(s)\right) - 2\left(\int_0^1 ds \sum_{i=1}^{f} r_i(s)\right)^2 = 2\left\{ \int_0^1 ds \sum_{i=1}^{f} r_i^2(s) - 2\left(\int_0^1 ds \sum_{i=1}^{f} r_i(s)\right)^2 \right\}$$

$$e^{-b^2/4a} = \left(\frac{a}{\pi}\right)^{3/2} \int dx\, e^{-ax^2 + ibx} \qquad b = \int_0^1 \sum_{i=1}^{f} r_i\, ds$$

$$\omega^2 = \frac{2 N b^2 i \xi}{3 f} \qquad a = \frac{f^2}{4 i \xi}$$

$$K(\xi) = \int D_{\vec r} \exp\left[\frac{-3}{2b^2 N} \sum_{i=1}^{f} \int_0^1 \left(\dot r_i^2(s) - \omega^2 r_i^2(s) \right) ds + \frac{i\xi}{f^2} \left(\int_0^1 \sum_{i=1}^{f} r_i(s) ds \right)^2 \right]$$

$$= K(\xi) = \int dx\, e^{-\frac{f^2 x^2}{4 i \xi}} \cdot \left(\frac{f^2}{4 i \xi \pi}\right)^{3/2} \cdot \int_{r(0)=0}^{r(1)=0} D_{\vec r} \exp\left[\sum_{i=1}^{f} \left\{ \frac{-3}{2b^2 N} \int_0^1 ds \left(\dot r_i^2(s) - \omega^2 r_i^2(s) \right) \right.\right.$$

$$\left.\left. + ix \int_0^1 ds\, r_i(s) \right\} \right] = \left(\frac{f^2}{4 i \pi \xi}\right)^{3/2} \int dx\, e^{-\frac{f^2 x^2}{4 i \xi}} \int_{r(0)=0}^{r(1)=0} D_{\vec r} \exp\left[\sum_{i=1}^{f} \left\{ -\frac{3}{2b^2 N} \right\} \int_0^1 ds \,\cdot \right.$$

$$\left. \cdot \left\{ \dot r_i^2(s) - \omega^2 r_i^2(s) + \frac{2 i x b^2 N}{3} r_i(s) \right\} \right]$$

$$r(s) = v_{sp}(s) + \rho(s) \qquad \rho(0) = 0 \quad \rho(1) = 0$$

$$S = \frac{3}{2b^2 N} \sum_{i=1}^{f} \int_0^1 ds \left\{ \dot r_i^2(s) - \omega^2 r_i^2(s) + \frac{2 i x b^2 N}{3} r_i(s) \right\}$$

$$K(\xi) \propto \xi^{-3/2}\, G(\xi) \int dx \cdot e^{i x^2 f^2 / 4\xi} \cdot e^{-S[v_{sp}]}$$

$$G = \int_{\rho(0)=0}^{\rho(1)=0} D_\rho \exp\left[-\frac{3}{2Nb^2} \sum_{i=1}^{f} \int_0^1 ds \left(\dot\rho_i^2(s) - \omega^2 \rho_i^2(s) \right) \right] =$$

$$= \int_{\rho(0)=0}^{\rho(1)=0} D_\rho \exp\left[-\frac{3f}{2Nb^2} \int_0^1 ds \left(\dot\rho^2(s) - \omega^2 \rho^2(s) \right) \right] = \left(\left(\frac{\omega}{\sin\omega} \right)^{3/2} \right)^f = \left(\frac{\omega}{\sin\omega} \right)^{3f/2}$$

$$F\left(s, r(s), \dot{r}(s)\right) = \dot{r}^2(s) - \omega^2 r^2(s) + \frac{2ix b^2 N}{3} r(s)$$

$$\left.\frac{\partial F}{\partial r} - \frac{d}{ds}\frac{\partial F}{\partial \dot{r}}\right|_{r=V_{sp}} = 0 \Rightarrow \ddot{V}_{sp} - \omega^2 V_{sp} = -\frac{ixb^2N}{3} \qquad V_{sp}(0)=0$$

$$V_{sp}(1)=0$$

$$V_{sp}(s) = \frac{-ixNb^2}{3\omega^2}\left[1 - \cos\omega s - \frac{\sin(\omega s)(1-\cos\omega)}{\sin\omega}\right]$$

$$S[V_{sp}] = \frac{3f}{2b N^2}\int_0^1 ds\left\{\dot{V}_{sp}^2 - \omega^2 V_{sp}^2 + \frac{2ixb^2N}{3}V_{sp}\right\} =$$

$$= -\frac{b^2 f N x^2}{6\omega^3}\left(\omega - 2\operatorname{Tan}\left[\frac{\omega}{2}\right]\right)$$

$$K(\zeta) \propto \zeta^{-3/2}\left(\frac{\omega}{\sin\omega}\right)^{3f/2}\cdot\int dx\, \exp\left[\frac{ix^2f^2}{4\zeta} + \frac{b^2fNx^2}{6\omega^3}\left(\omega - 2\operatorname{Tan}\left[\frac{\omega}{2}\right]\right)\right]$$

$$\frac{ix^2f^2}{4\zeta} = \frac{ix^2f^2\, 2Nb^2}{-4\cdot i3\omega^2 f} = \frac{-b^2fNx^2}{6\omega^2} \Rightarrow$$

$$K(\zeta) \propto \omega^{-3}\cdot\left(\frac{\omega}{\sin\omega}\right)^{3f/2}\cdot\int \underbrace{dx}_{d^3x}\cdot\exp\left[-\frac{b^2fNx^2}{3\omega^3}\operatorname{Tan}\left[\frac{\omega}{2}\right]\right] =$$

$$= \omega^{-3}\cdot\left(\frac{\omega}{\sin\omega}\right)^{3f/2}\int dx\; x^2 \exp\left[-\frac{b^2fN\operatorname{Tan}(\frac{\omega}{2})}{3\omega^3}x^2\right] \propto$$

$$\propto \omega^{-3}\left(\frac{\omega}{\sin\omega}\right)^{3f/2}\cdot\left(\operatorname{Tan}\left(\frac{\omega}{2}\right)\right)^{-3/2}\cdot\omega^{9/2} =$$

$$= \omega^{3f/2+3/2}\cdot\left(\frac{1}{2\sin\frac{\omega}{2}\cos\frac{\omega}{2}}\right)^{3f/2}\cdot\left(\frac{\cos\frac{\omega}{2}}{\sin\frac{\omega}{2}}\right)^{3/2} \propto$$

$$\propto \left(\frac{\omega}{\sin\frac{\omega}{2}}\right)^{\frac{3}{2}(f+1)} \cdot \left(\cos\frac{\omega}{2}\right)^{\frac{3}{2}(1-f)} \cdot C$$

$$K(\omega \to 0) = 1 \Rightarrow K\Big|_{\omega=0} = 2^{3/2(f+1)} \cdot C \Rightarrow C = \left(\frac{1}{2}\right)^{3/2(f+1)}$$

$$K(\xi) = \left(\frac{\omega}{2\sin\frac{\omega}{2}}\right)^{3/2(f+1)} \cdot \left(\cos\frac{\omega}{2}\right)^{3/2(1-f)}$$

3) $$K(\omega) = 1 + (2f-1)\cdot\frac{1}{8}\,\omega^2 + \frac{(60f^2-44f+1)}{1920}\,\omega^4 + O(\omega^6)$$

$$K(\xi) = \sum_{k=0}^{\infty} \frac{\mu_k}{k!}(i\xi)^k \qquad \mu_1 = \langle R_g^2 \rangle \qquad \mu_2 = \langle R_g^4 \rangle$$

$$K(\xi) = 1 + \underbrace{(2f-1)\cdot\frac{1}{8}\cdot\frac{2Nb^2}{3f}}_{\mu_1}\cdot(i\xi)^1\cdot\frac{1}{1!} +$$

$$+ \underbrace{\frac{(60f^2-44f+1)}{1920}\,\frac{2\cdot 4N^2b^4}{9f^2}}_{\mu_2}\frac{(i\xi)^2}{}\cdot\frac{1}{2!}$$

$$\langle R_g^2 \rangle = \frac{Nb^2(2f-1)}{12f}$$

$$\langle R_g^4 \rangle = \frac{N^2b^4(60f^2-44f+1)}{2160f^2}$$