N4

$$G_p(\tau_1,\tau_2)=\begin{cases} A\cosh(\omega(\tau_1-\tau_2)+\varphi_1) & \tau_1<\tau_1 \\ B\cosh(\omega(\tau_1-\tau_2)+\varphi_2) & \tau_1\geq\tau_2 \end{cases}$$

$$G_p\left(-\frac{\beta}{2},\tau\right)=G_p\left(\frac{\beta}{2},\tau\right)$$

$$G_p\left(\tau,-\frac{\beta}{2}\right)=G_p\left(\tau,\frac{\beta}{2}\right)$$

$$(\mathrm{I})\ A\cosh\left(\omega\left(-\beta/2-\tau_2\right)+\varphi_1\right)=B\cosh\left(\omega\left(\frac{\beta}{2}-\tau_2\right)+\varphi_2\right)$$

$$(\mathrm{II})\ A\cosh\left(\omega\left(\tau-\frac{\beta}{2}\right)+\varphi_1\right)=B\cosh\left(\omega\left(\tau+\frac{\beta}{2}\right)+\varphi_2\right)$$

$$\mathrm{I}/\mathrm{II}:\quad \cosh\left(-\omega\left[\beta/2+\tau\right]+\varphi_1\right)\cosh\left(\omega\left[\tau+\frac{\beta}{2}\right]+\varphi_2\right)\Leftrightarrow$$

$$\Leftrightarrow \cosh\left(\omega\left[\frac{\beta}{2}-\tau\right]+\varphi_2\right)\cosh\left(-\omega\left[\frac{\beta}{2}-\tau\right]+\varphi_1\right)\Rightarrow$$

$$\cancel{1+\cosh(\varphi_1+\varphi_2)=\cosh(\varphi_1+}$$

$$\cancel{\cosh(\varphi_1+\varphi_2)}+\cosh\left(2\omega\left[\tau+\frac{\beta}{2}\right]+\varphi_2-\varphi_1\right)=$$

$$=\cancel{\cosh(\varphi_1+\varphi_2)}+\cosh\left(2\omega\left[\frac{\beta}{2}-\tau\right]+\varphi_2-\varphi_1\right)$$

$$2\omega\left[\tau+\frac{\beta}{2}\right]+\varphi_2-\varphi_1=-2\omega\left[-\tau+\frac{\beta}{2}\right]+\varphi_1-\varphi_2$$

$2\omega \beta/2 \cdot 2 = 2\varphi_1 - 2\varphi_2 \Rightarrow \varphi_1 = \omega\beta + \varphi_2$

(I): $A\cosh(\omega[-\beta/2 - \tau] + \varphi_1) = B\cosh(\omega[\beta/2 - \tau] + \varphi_2)$

$\varphi_1 = \omega\beta + \varphi_2 \qquad \Rightarrow A = B$

~~Из~~ из непрерывности ф. Грина:

$\cosh(\omega(\tau_1 - \tau_2) + \varphi_1) = \cosh(\omega(\tau_1 - \tau_2) + \varphi_2)$

при $\tau_1 = \tau_2$

$\cosh(\omega\beta + \varphi_2) = \cosh(\cancel{\omega\beta}\ \varphi_2) \Rightarrow \varphi_2 = -\omega\beta/2$

$\varphi_1 = \frac{\omega\beta}{2}$

$$A\,\omega m\left[-\sinh\left(-\frac{\omega\beta}{2}\right) + \sinh\left(\frac{\omega\beta}{2}\right)\right] = 1$$

$\cancel{\sinh\frac{\omega\beta}{2} = 1/A}$ $\qquad A = \frac{1}{2\omega m \sin\left(\frac{\omega\beta}{2}\right)}$

$$G_p = \frac{1}{2m\omega\sin\left(\frac{\omega\beta}{2}\right)}\begin{cases}\cosh(\omega(\beta/2 - (\tau' - \tau))) & \tau < \tau' \\ \cosh(\omega(\beta/2 - (\tau - \tau'))) & \tau > \tau'\end{cases}$$

$$\boxed{G_p = \frac{1}{2m\omega}\,\frac{\cos\left[\frac{\omega\beta}{2} - \omega|\tau - \tau'|\right]}{\sin\left[\frac{\omega\beta}{2}\right]}}$$

N5

$$\lim_{\beta\to\infty} G_P = \frac{1}{2m\omega}\lim_{\beta\to\infty}\frac{e^{-\frac{\omega\beta}{2}+|\tau-\tau'|\omega}+e^{\frac{\omega\beta}{2}-|\tau-\tau'|\omega}}{-e^{-\frac{\omega\beta}{2}}+e^{\frac{\omega\beta}{2}}} = \frac{1}{2m\omega}\exp\left[-\omega|\tau-\tau'|\right]$$

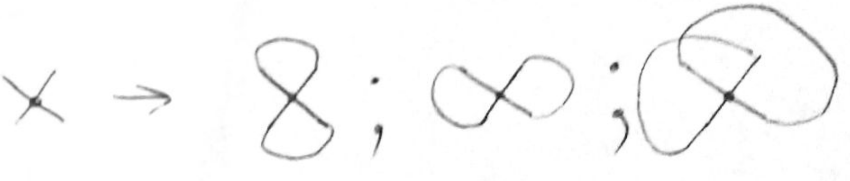
$$\lim_{\beta\to\infty} G_D = \frac{1}{2m\omega}\lim_{\beta\to\infty}\frac{\left[e^{\omega\beta/2+\tau_<\omega}-e^{-\omega\beta/2-\tau_<\omega}\right]\cdot\left[e^{\omega\beta/2-\tau_>\omega}-e^{-\omega\beta/2+\tau_>\omega}\right]}{-e^{-\omega\beta}+e^{+\omega\beta}} =$$

$$= \frac{1}{2m\omega}\exp\left[\omega(\tau_<-\tau_>)\right] = \frac{1}{2m\omega}\exp\left[-\omega|\tau-\tau'|\right]$$

N6

C:  ← (I) и (II) не пересекаются (т.е. эквивал)

I II

рассм (I): 

3 варианта

$\Rightarrow N = 3\cdot 3 = 9$

B: 

I II

рассм (I): 

6 вариантов

\+ еще остается свобода в соединении 2 оставшихся концов между (I) и (II)

2 варианта

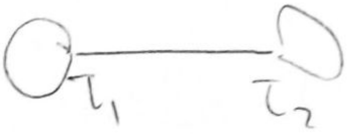
$$\Rightarrow \underline{N = 6 \cdot 6 \cdot 2 = 72}$$

N7

$$Z(\beta) = Z^0(\beta)\exp\left(-\hbar\lambda\int d\tau\left(\frac{\delta}{\delta J}\right)^3\right)e^{\frac{1}{2} J G_p J}\Big|_{J=0}$$

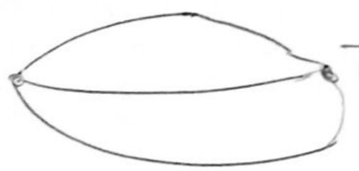
$$\approx Z^0(\beta)\left(1+\frac{\hbar^2\lambda^2}{2}\left(\int d\tau\left(\frac{\delta}{\delta J(\tau)}\right)^3\right)^2\right)e^{\frac{1}{2} J G_p J}\Big|_{J=0}$$

$$= Z^0(\beta)\left[1+\frac{\hbar^2\lambda^2}{2}\iint d\tau_1 d\tau_2 \left(\frac{\delta}{\delta J(\tau_1)}\right)^3\left(\frac{\delta}{\delta J(\tau_2)}\right)^3 e^{\frac{1}{2} J G_p J}\right]_{J=0}$$

1) 

— 9 вариантов

$G_p(\tau_1,\tau_2)G_p(\tau_1,\tau_1)G_p(\tau_2,\tau_2)$

2) 

— 6 вариантов

$G_p^3(\tau_1,\tau_2)$

$$Z(\beta) = Z^0(\beta)\left[1+\frac{1}{2}\hbar^2\lambda^2\int d\tau_1 d\tau_2\left\{6G_p^3(\tau_1,\tau_2)+9G_p(\tau_1,\tau_2)\cdot G_p(\tau_1,\tau_1)G_p(\tau_2,\tau_2)\right\}\right]$$

$$Z(\beta) = Z^0(\beta)\left[1+\frac{1}{2}\hbar^2\lambda^2\int\left[6\,\frac{\cosh^3\left[\omega\left(\beta/2-|\tau_1-\tau_2|\right)\right]}{8m^3\omega^3\,\sinh^3(\beta\omega/2)}+\right.\right.$$

$$\left.\left.+\,9\,\frac{\cosh^2(\beta\omega/2)\,\cosh\left[\frac{\beta}{2}\omega-\omega|\tau_1-\tau_2|\right]}{8m^3\omega^3\,\sinh^3\frac{\beta}{2}\omega}\right]d\tau_1 d\tau_2\right.$$

$$Z(\beta) = Z^0(\beta)\left[1 + \frac{3\,\hbar^2 J^2}{16 m^3 \omega^3 \sinh^3(\omega\beta/2)} \cdot \left\{ \int_{-\beta/2}^{\beta/2}\int_{-\beta/2}^{\beta/2} d\tau_1 d\tau_2 \right.\right.$$

$$\cdot \left\{ 2\cosh^3\left(\omega\left\{\beta/2 - |\tau_1 - \tau_2|\right\}\right) + 3\cosh^2\left(\frac{\beta\omega}{2}\right) \cdot \cosh\left(\frac{\beta}{2}\omega - \right.\right.$$

$$\left.\left. - \omega|\tau_1 - \tau_2|\right)\right\} = Z^0\left[1 + \frac{3\,\hbar^2 J^2}{16 m^3 \omega^3 \sinh^3\left(\frac{\omega\beta}{2}\right)} \cdot I\right]$$

$$I = 2 I_3 + 3\cosh^2\left(\frac{\beta\omega}{2}\right) I_1$$

$$I_1 = \int_{-\frac{\beta}{2}}^{\beta/2}\int_{-\beta/2}^{\beta/2} d\tau_1 d\tau_2 \cosh\left(\omega\left(\beta/2 - |\tau_1 - \tau_2|\right)\right) =$$

$$= \{\tau_1 \to \tau_1 + \tau_2\} = \int_{-\beta/2}^{\beta/2} d\tau_2 \int_{-\beta/2+\tau_2}^{\beta/2+\tau_2} d\tau_1 \left\{ \cosh\left(\omega\left[\beta/2 + |\tau_1|\right]\right)\right.$$

$$= \int d\tau_2 \cdot \left[\int_{-\beta/2+\tau_2}^{0} d\tau_1 \cosh\left[\omega\left[\beta/2 \mp \tau_1\right]\right] + \int_0^{\beta/2+\tau_2} d\tau_1 \cosh\left[\omega\left[\beta/2 \pm \tau_1\right]\right]\right]$$

$$= \frac{1}{\omega}\int d\tau_2 \cdot \left[-\sinh\left[\frac{\beta\omega}{2}\right] + \sinh\left[\omega(\beta - \tau_2)\right]\right] +$$

$$+ \sinh\left[\omega(\beta + \tau_2)\right]\Big]\Big) = \frac{2\sinh\left(\frac{\beta\omega}{2}\right)}{\omega^2}\left[-\beta\omega + 2\sinh\beta\omega\right.$$

$$I_3 = \frac{1}{4}\left(3I_1 + I_1\big|_{\omega \to 3\omega}\right)$$

$$I_1 = \int_{-\beta/2}^{\beta/2} d\tau_2 \int_{-\beta/2+\tau_2}^{\beta/2+\tau_2} d\tau_1 \cosh\left[\omega\left\{\beta/2 - |\tau_1|\right\}\right] =$$

$$= \int_{-\beta/2}^{\beta/2} d\tau_2 \left[\int_{-\beta/2+\tau_2}^{0} \cosh\left[\omega\left\{\beta/2 + \tau_1\right\}\right] + \int_{0}^{\beta/2+\tau_2} d\tau_1 \cosh\left[\omega\left\{\beta/2 - \tau_1\right\}\right]\right] = \int_{-\beta/2}^{\beta/2} d\tau_2 \frac{\sinh\frac{\beta\omega}{2}}{\omega} = \frac{\beta}{\omega}\sinh\frac{\beta\omega}{2}$$

$$I_3 = \frac{1}{4}\left(\frac{3\beta}{\omega}\sinh\frac{\beta\omega}{2} + \frac{\beta}{3\omega}\sinh\frac{3\beta\omega}{2}\right)$$

$$Z_0 = \frac{1}{2\sinh\left[\frac{\omega\beta}{2\hbar}\right]}$$

$$Z = \frac{1}{2\sinh\left[\frac{\omega\beta}{2\hbar}\right]}\left[1 + \frac{3\hbar^2\lambda^2}{\cancel{8}\,16\, m^3\omega^3\sinh^3\frac{\omega\beta}{2}} \cdot \right.$$

$$\cdot\left\{\frac{3\beta}{2\omega}\sinh\frac{\beta\omega}{2} + \frac{\beta}{8\omega}\left(3\sinh\frac{\omega\beta}{2} + 4\sinh^3\frac{\omega\beta}{2}\right) + \right.$$

$$\cancel{2}\ \left.\frac{3\beta}{\omega}\cosh^2\frac{\beta\omega}{2}\sinh\frac{\beta\omega}{2}\right\}$$

$$Z = \frac{1}{2\sinh\left(\frac{\omega\beta}{2\hbar}\right)} \underbrace{\left[1 + \frac{\hbar^2\lambda^2 3\beta}{\cancel{48}\,16\, m^3\omega^4}\left(\frac{2}{\sinh^2\frac{\beta\omega}{2}} + \frac{2}{3} + 3\coth^2\frac{\beta\omega}{2}\right)\right]}_{Z'}$$

$$E_0 = F\Big|_{\beta\to\infty} = -\frac{\hbar}{\beta}\ln Z = -\frac{\hbar}{\beta}\cdot\left[-\ln Z_0 + \ln Z'\right] =$$

$$\overset{\beta\to\infty}{\simeq} -\frac{\hbar}{\beta}\left[-\underbrace{\ln\left(2\sinh\frac{\omega\beta}{2\hbar}\right)}_{-\frac{\omega\beta}{2\hbar}} + \ln\left[1 + \frac{3\hbar^2\lambda^2}{16 m^3\omega^4}\beta\left(3\underbrace{\coth^2\frac{\beta\omega}{2}}_{3} + \frac{2}{3} + \underbrace{\frac{2}{\sinh^2\frac{\beta\omega}{2}}}_{0}\right)\right]\right] = \frac{\hbar\omega}{2} + \left(-\frac{\hbar}{\beta}\right)\cdot\frac{3\hbar^2\lambda^2\beta}{2\cdot 8 m^3\omega^4}\,\frac{11}{3} =$$

$$= \frac{\hbar\omega}{2} - \frac{11}{2\cdot 8}\,\frac{\hbar^3\lambda^2}{m^3\omega^4} = \frac{\hbar\omega}{2}\left[1 - \frac{11}{8}\,\frac{\hbar^2\lambda^2}{m^3\omega^5}\right]$$