N_2

$$\langle x_f | \hat{U}(T)\, T\left\{\frac{\left(\hat{x}(t+\tau) + - \hat{x}(t)\right)^2}{\tau^2}\right\} | x_i \rangle$$

$$= \langle x_f | \hat{U}(T)\left\{\hat{x}(t+\tau)\hat{x}(t+\tau) + \hat{x}(t)\hat{x}(t) - 2\hat{x}(t+\tau)\hat{x}(t)\right\} | x_i \rangle \equiv \left(U_{fi} \cdot \left\{ \frac{x_{free}}{\tau^2}(t+\tau) \right.\right.$$

$$\equiv U_{fi} \cdot \left\{ \frac{x^2_{free}(t+\tau)}{\tau^2} - \frac{ih}{\tau^2} G(t+\tau, t+\tau) + \frac{x^2_{free}(t)}{\tau^2} \right.$$

$$\left. - \frac{ih}{\tau^2} G(t,t) + \frac{2ih}{\tau^2} G(t+\tau, t) \pm \frac{2 x_{free}(t+\tau) x_{free}(t)}{\tau^2} \right\} =$$

$$= U_{fi} \cdot \left\{ F_{free} + \frac{ih}{m\tau^2} \left[2(t+\tau-T)t - t(t-T) - (t+\tau)(t+\tau-T) \right] \right\} = U_{fi} \cdot \left\{ F_{free} + \frac{ih}{m\tau^2} \left[2t^2 + 2\tau t - 2Tt - t^2 + tT - t^2 - t\tau + \tau T \right.\right.$$

$$\left. - Tt - \tau^2 + \tau T \right] = U_{fi} \cdot \left\{ F_{free} + \frac{ih}{m\tau^2} \left[\tau T - \tau^2 \right] \right\} \equiv$$

$$\cancel{x_f}\ F_{free} = \frac{1}{\tau^2 T^2}\Big[\big(t x_f + (T-t)x_i\big)^2 + \big((t+\tau)x_f + (T-t-\tau)x_i\big)^2 -$$

$$- \big(t x_f + (T-t)x_i\big)\big((t+\tau)x_f + (T-t)x_i\big)\Big] =$$

$$= \frac{1}{\tau^2 T^2}\Big[\big(t^2 + \tau t + \tau^2\big)x_f^2 + \big((T-2\tau)\tau + 2t(T-\tau) - 2t^2\big)x_f x_i +$$

$$+ \big((t-T)^2 + (t-T)\tau + \tau^2\big)x_i^2\Big]$$

$$\Rightarrow U_{fi}\Big\{\frac{1}{\tau^2 T^2}\Big[x_f^2\big(t^2 + t\tau + \tau^2\big) + x_f x_i\big((T-2\tau)\tau + 2t(T-\tau) - 2t^2\big) + x_i^2 \cdot$$

$$\cdot \big((t-T)^2 + (t-T)\tau + \tau^2\big)\cancel{x_i^2}\Big] + \frac{i\hbar}{m\tau^2 T}\big[T\tau - \tau^2\big]\Big\}$$

$$U_{fi} = \langle x_f | \hat{U}(T) | x_i \rangle$$
