$$-\frac{1}{2}\frac{\partial^2}{\partial x_0^2}\tilde{P}(s|x_0) + s x_0 \tilde{P}(s|x_0) = 0 \qquad \tilde{P}(s|0) = 1$$

$$\tilde{P}(s|\infty) \to 0$$

$$\frac{\partial^2}{\partial x_0^2}\tilde{P}(s|x_0) - 2 s x_0 \tilde{P}(s|x_0) = 0$$

$$x_0 \to \frac{\tilde{x}_0}{\sqrt[3]{2s}} \Rightarrow \left(\frac{\partial^2}{\partial \tilde{x}_0^2} - \tilde{x}_0\right)\tilde{P}(s|x_0) = 0$$

$$\tilde{P}(s|\tilde{x}_0) = A_i(\tilde{x}_0)\cdot C_1 + B_i(\tilde{x}_0)\cdot C_2$$

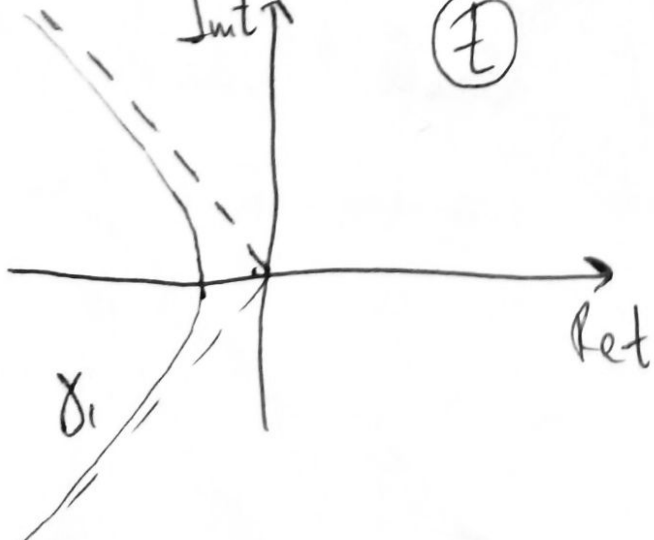
$$\tilde{x}_0 = x_0 \sqrt[3]{2s} \qquad A_i(\infty) = 0;$$

$$B_i \xrightarrow[x\to\infty]{} \infty \Rightarrow C_2 = 0 \qquad A_i(0) = \frac{1}{3^{2/3}\Gamma(2/3)}$$

$$\tilde{P}(s|x_0) = \overbrace{3^{2/3}\Gamma(2/3)}^{C_1} A_i\left(x_0 \sqrt[3]{2s}\right)$$

$$\tilde{P}(s|x_0) = C_1 \frac{1}{2\pi i}\int_{\gamma_1} dt\, \exp\left((2s)^{1/3} x_0 t - \frac{t^3}{3}\right) =$$

$$p = t\cdot(2s)^{1/3}$$

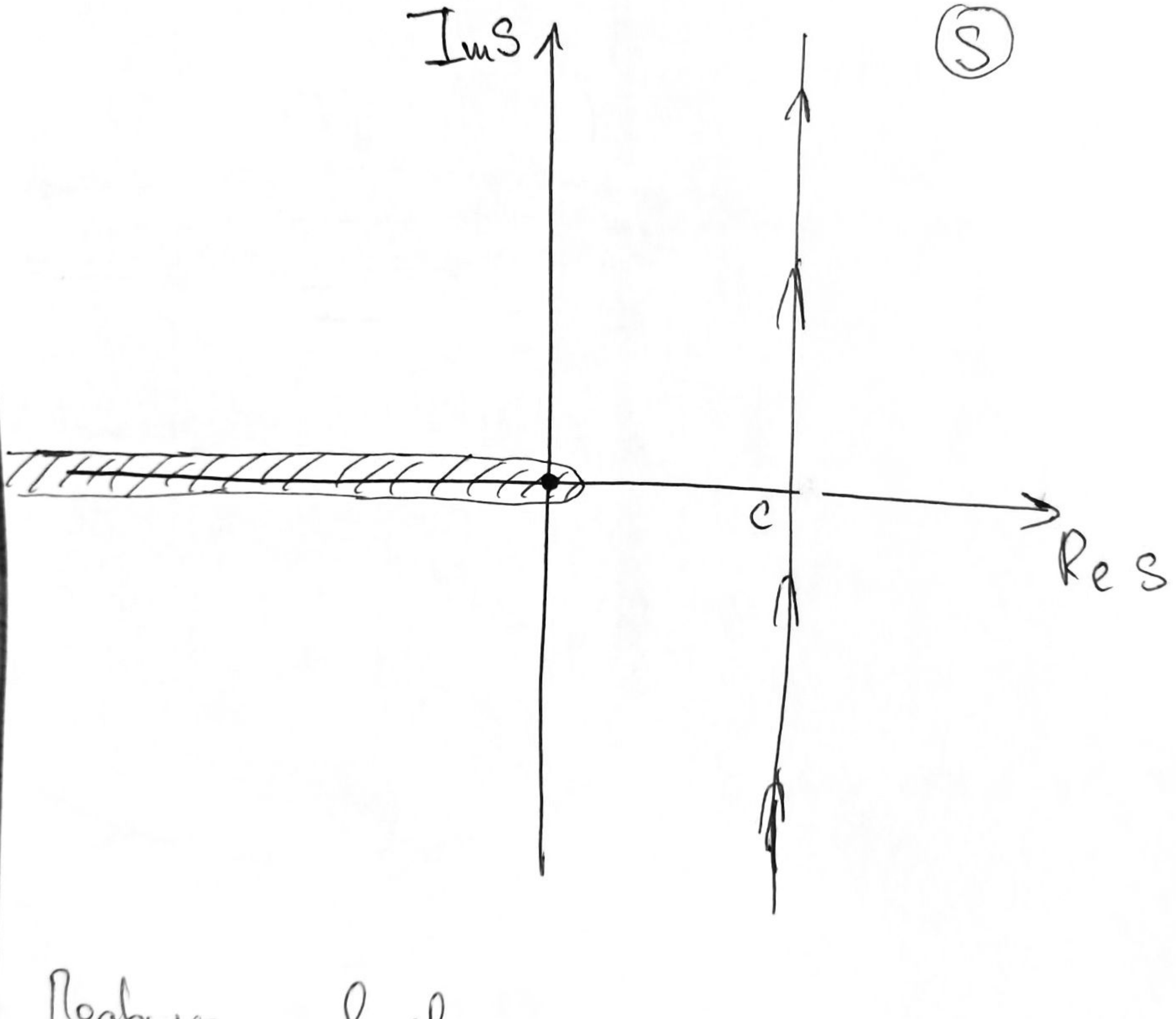


$$= \frac{C_1}{2\pi i}\int_{\gamma_1^p} dp\, \frac{1}{(2s)^{1/3}} \exp\left(x_0 p - \frac{p^3}{6s}\right)$$

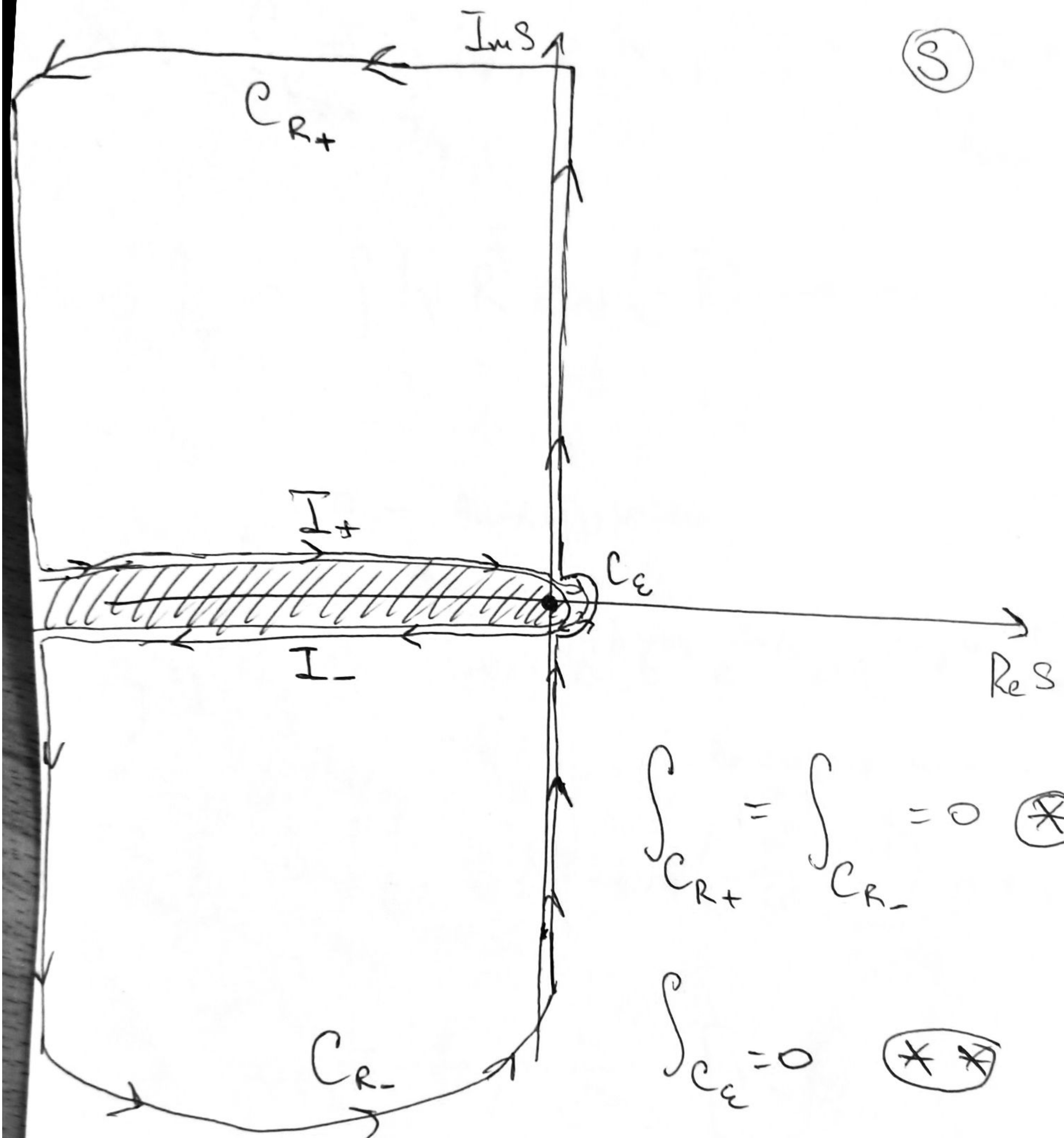
$$P(T|x_0) = \frac{C_1}{2\pi^2 i^2 2^{1/3}} \int_{\gamma_1^p} dp \exp(x_0 p) \underbrace{\int_{c-i\infty}^{c+i\infty} ds\, s^{-1/3} \exp\left(sT - \frac{p^3}{6s}\right)}_{I}$$

$$\beta = \frac{p^3}{6}$$

I: s=0 точка ветвл.



Подвинем влево до нуля и замкнем контур на левую полуокр.



$$\int_{C_{R+}} = \int_{C_{R-}} = 0 \quad (*)$$

$$\int_{C_\varepsilon} = 0 \quad (**)$$

$$I + I_+ + I_- = 0$$

$$I = -\int_{-\infty}^{0} f_+ - \int_{0}^{-\infty} f_- = \int_{0}^{-\infty} (f_+ - f_-)$$

$$(*):\quad \int_{C_{R+}} = \int_{\pi/2}^{\pi} d\varphi \cdot Re^{i\varphi} \cdot R^{-1/3} \cdot e^{-i\varphi/3} \cdot \exp\Big[\underbrace{RTe^{i\varphi}}_{Re<0} - \frac{p^3}{6s}\Big]$$

$$\Rightarrow \int_{C_{R+}} \sim \int d\varphi\, R^{\frac{2}{3}} \exp(-R) \to 0$$

$$\int_{C_{R-}} = 0 \text{ — АНАЛОГИЧНО}$$

$$(**):\quad \int_{C_\varepsilon} = \int_{\pi/2}^{-\pi/2} d\varphi\, \varepsilon e^{i\varphi} \varepsilon^{-1/3} e^{-i\varphi/3} \exp\Big(\varepsilon T e^{i\varphi} - \frac{p^3}{6\varepsilon} e^{-i\varphi}\Big)$$

Re < 0, т.к. интегрируем по γ_1 дальше

$$\int_{C_\varepsilon} \sim \int_{\pi/2}^{-\pi/2} d\varphi\, \varepsilon^{2/3} \cdot \exp\Big(\overbrace{-\frac{p^3}{6\varepsilon} \underbrace{e^{-i\varphi}}_{Re<0}}\Big) \to 0$$

$$I = \int_0^{-\infty} \exp\Big(sT - \frac{p^3}{6s}\Big) \cdot \frac{ds}{s^{1/3}} \cdot \Big[1 - e^{-2\pi i/3}\Big] =$$

$$= \Big(\frac{3}{2} + i\frac{\sqrt{3}}{2}\Big) \cdot \int_0^{-\infty} \exp\Big(sT - \frac{p^3}{6s}\Big) \frac{1}{s^{1/3}} ds$$

$$\cdot \Big(\frac{3}{2} + i\frac{\sqrt{3}}{2}\Big) \cdot e^{-i\pi/3} \int_0^{\infty} \exp\Big(-sT + \frac{p^3}{6s}\Big) \frac{1}{s^{1/3}} ds \ominus$$

$$\ominus\left(\frac{3}{2}+i\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)\int_0^\infty s^{-1/3}ds\exp\left[-sT+\frac{p^3}{6s}\right]=-i\cdot\int_0^\infty s^{-(-2/3)-1}\exp\left(-T\cdot s+\frac{p^3}{6}\frac{1}{s}\right)$$

$$=-i2\left(\frac{6T}{-p^3}\right)^{-1/3}K_{-2/3}\left(2\sqrt{\frac{T}{6}}\cdot e^{i\pi/2}\cdot p^{3/2}\right)$$

$$K_\alpha=\frac{\pi}{2}\frac{I_{-\alpha}-I_\alpha}{\sin\alpha\pi}\qquad K_{-\alpha}=\frac{\pi}{2}\frac{I_\alpha-I_{-\alpha}}{-\sin\alpha\pi}\Rightarrow K_\alpha=K_{-\alpha}$$

$$I=-i\cdot2\cdot(6T)^{-1/3}\cdot p\cdot e^{+i\pi/3}\cdot K_{2/3}\left(2e^{i\pi/2}\sqrt{\frac{T}{6}}\,p^{3/2}\right)$$

$$P(T|X_0)=\frac{-C_1}{2^{1/3}\cdot4\pi^2}\cdot\int dp\exp(x_0p)\int ds\,s^{-1/3}\exp\left(sT-\frac{p^3}{6s}\right)$$

$$Ai'(x)=-\frac{x}{\pi}K_{2/3}\left(\frac{2}{3}x^{3/2}\right)$$

$$\frac{x^{3/2}}{3}=\sqrt{T/6}\,e^{i\pi/2}p^{3/2};\quad x=p\left(\frac{3T}{2}\right)^{+1/3}e^{+i\pi/3}$$

$$x^{3/2}=\sqrt{\frac{3T}{2}}\,e^{i\pi/2}p^{3/2}$$

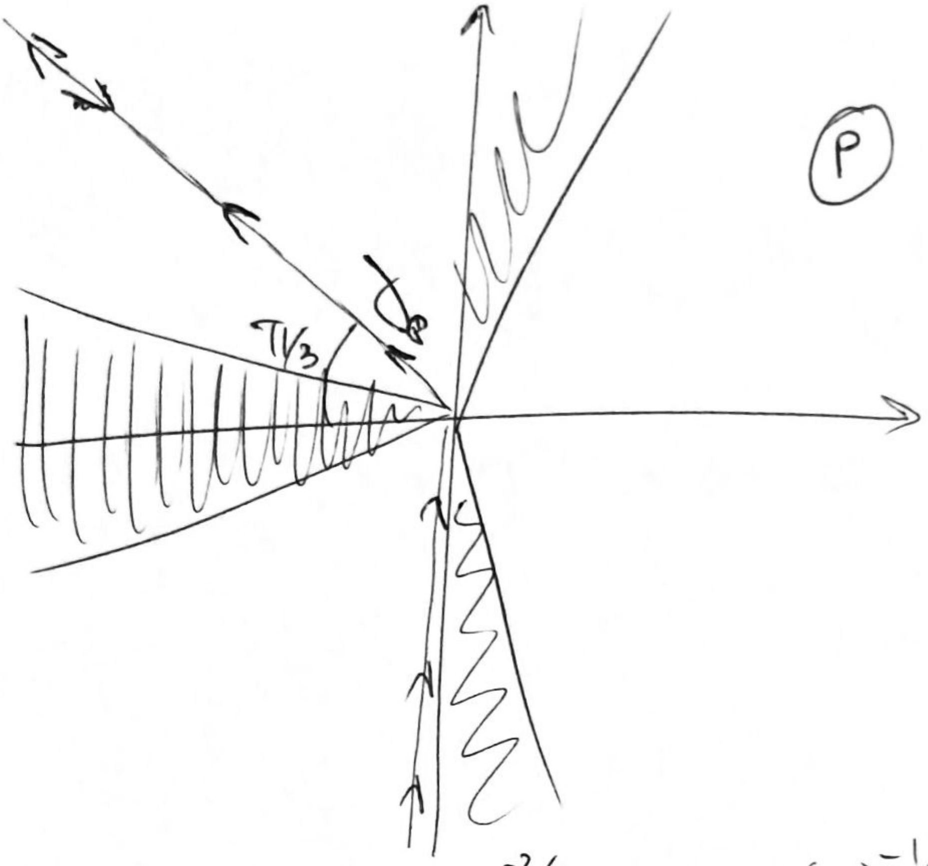
$$p=x\cdot\left(\frac{3T}{2}\right)^{-1/3}e^{-i\pi/3}$$

$$I=2\frac{(6T)^{-1/3}}{p^{-1}}e^{i\pi/3}K_{2/3}\Big(\underbrace{2i\sqrt{\frac{p^3T}{6}}}_{\frac{2}{3}x^{3/2}}\Big)=2(6T)^{-1/3}x\left(\frac{3T}{2}\right)^{-1/3}\cdot\frac{-\pi\sqrt{3}}{-\pi\sqrt{3}}\cdot$$

$$\cdot K_{2/3}\left(\frac{2}{3}x^{3/2}\right)=-\frac{6}{\sqrt{3}}\cdot(3T)^{-2/3}\cdot\pi\cdot Ai'\left(p\cdot e^{i\pi/3}\cdot\left(\frac{3T}{2}\right)^{+1/3}\right)=$$

$$=-\frac{6\pi}{\sqrt{3}}(3T)^{-2/3}Ai'\left(pe^{i\pi/3}\cdot\left(\frac{3T}{2}\right)^{1/3}\right)$$

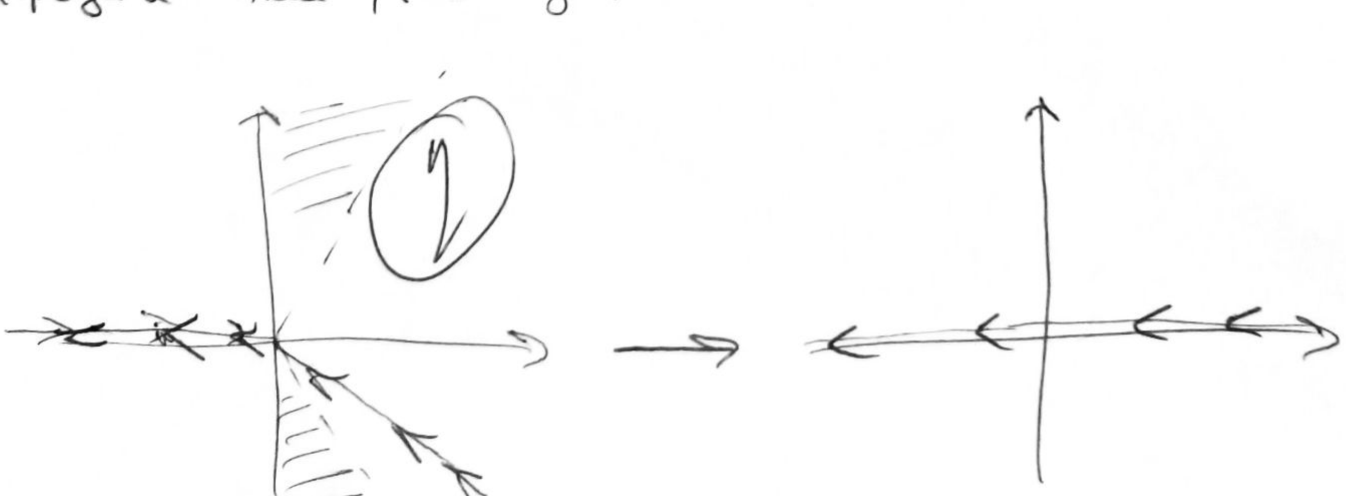
$$P(T|x_0)=\frac{6C_1(3T)^{-2/3}}{\sqrt{3}\,2^{1/3}\cdot4\pi}\int_{\gamma_1^p}\exp(x_0p)\,Ai'\left(pe^{i\pi/3}\cdot\left(\frac{3T}{2}\right)^{1/3}\right)dp$$



$$P(T|x_0) = \underbrace{\frac{2\sqrt{3}\, C_1 (3T)^{-2/3}}{2^{1/3}\cdot 4\pi} \cdot e^{-i\pi/3} \cdot \left(\frac{3T}{2}\right)^{-1/3}}_{C_0} \int_{\gamma'_1} d\eta\, e^{\eta \cdot x_0 e^{-i\pi/3} \cdot \left(\frac{3T}{2}\right)^{-1/3}} Ai'(\eta) =$$

$$= C_0 \Big[\underbrace{Ai(\eta)\, e^{x_0 e^{i\pi/3} \left(\frac{3T}{2}\right)^{-1/3}} \Big|_{e^{3\pi/2} e^{i\pi/3}\infty}^{e^{2\pi/3}\cdot e^{i\pi/3}\infty}}_{= 0 \text{ по опр. } Ai(\eta)} + \underbrace{x_0 e^{i\pi/3} \left(\frac{3T}{2}\right)^{-1/3}}_{\ominus} \cdot \int_{-\infty}^{\infty} Ai(\eta)\, e^{\eta x_0} d\eta$$

Пределы такие, т.к. деформируем контур:



$$\lim Ai(\eta) \sim \frac{1}{\eta^{1/4}} \exp(-\#\, \eta^{3/2}) \Rightarrow \varphi \in \left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$$

$$P(T|x_0) = C_0\,\theta \int_{-\infty}^{\infty} Ai(y)\, e^{\theta y}\, dy = C_0\,\theta \exp\left(\frac{\theta^3}{3}\right) =$$

$$= \frac{3^{2/3} \cdot \Gamma(2/3)}{2^{1/3} \cdot 4\pi}\,(3T)^{-2/3}\,\cancel{e^{-i\pi/3}} \cdot \left(\frac{3T}{2}\right)^{-1/3} \cdot x_0 \cdot \cancel{e^{i\pi/3}}\,\left(\frac{3T}{2}\right)^{-1/3} \cdot \cancel{\int_{-\infty}^{\infty}} \exp\left(\frac{-2x_0^3}{9T}\right) =$$

$$= \frac{\sqrt{3}\,3^{-2/3}\,2\pi}{\Gamma(1/3)\,2\pi\sqrt{3}} \cdot 2^{1/3} \cdot x_0 \cdot T^{-4/3} \cdot \exp\left(-\frac{2x_0^3}{9T}\right) = \frac{2^{1/3}}{3^{2/3}\,\Gamma(1/3)} \cdot x_0\, T^{-4/3} \cdot e^{-\frac{2x_0^3}{9T}}$$