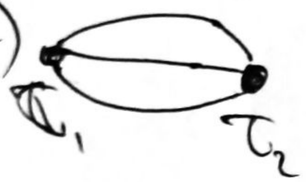
$$Z(\beta) = Z^0(\beta)\exp\left(-\hbar\lambda\int d\tau\left(\frac{\delta}{\delta J}\right)^3\right) e^{1/2\, J G_p J}\Big|_{J=0}$$

$$\approx Z^0(\beta)\left[1+\frac{\hbar^2\lambda^2}{2}\left(\int d\tau\left(\frac{\delta}{\delta J(\tau)}\right)^3\right)^2\right] e^{1/2\, J G_p J}\Big|_{J=0}$$

$$= Z^0(\beta)\left[1+\frac{\hbar^2\lambda^2}{2}\left(\int d\tau_1\left(\frac{\delta}{\delta J(\tau_1)}\right)^3\left(\frac{\delta}{\delta J(\tau_2)}\right)^3 e^{1/2\, J G_p J}\right]\Big|_{J=0}$$

1) 

– 9 вариантов

2) 

– 6 вариантов

$$Z(\beta) = Z_0^0(\beta)\left[1+\frac{1}{2}\hbar^2\lambda^2\int d\tau_1\, d\tau_2\left[6G_p^3(\tau_1,\tau_2)+9G_p(\tau_1,\tau_2)\cdot G_p(\tau_1,\tau_1)\,G_p(\tau_2,\tau_2)\right]\right]$$

$$Z(\beta) = Z^0(\beta)\left[1+\frac{1}{2}\hbar^2\lambda^2\int d\tau_1\, d\tau_2\left[\frac{6\cosh^3(\omega(\beta/2-|\tau_1-\tau_2|))}{(2m\omega)^3\sinh^3(\omega\beta/2)}+\frac{9}{(2m\omega)^3}\frac{\cosh^2(\omega\beta/2)\cosh(\omega(\beta/2-|\tau_1-\tau_2|))}{\sinh^3(\omega\beta/2)}\right]\right]$$

$$Z(\beta) = Z^0(\beta)\left[1+\frac{\hbar^2\lambda^2}{16m^3\omega^3\sinh^3(\omega\beta/2)}\int d\tau_1\, d\tau_2\left[6\cosh(\omega(\beta/2-|\tau_1-\tau_2|))+9\cosh^2(\omega\beta/2)\cosh(\omega(\beta/2-|\tau_1-\tau_2|))\right]\right]$$

$$Z(\beta) = Z^0(\beta)\left[1 + \frac{3\hbar^2\lambda^2}{16 m^3 \omega^3 \sinh^3(\omega\beta/\hbar)} \int d\tau_1 d\tau_2 \cdot\right.$$

$$\cdot \left[\underbrace{2\cosh^3\left(\omega\left(\beta/2 - |\tau_1-\tau_2|\right)\right)}_{=2C_3} + \underbrace{3\cosh^2\left(\frac{\omega\beta}{2}\right)\cosh\left(\omega\left(\beta/2 - |\tau_1-\tau_2|\right)\right)}_{=3\cosh^2\left(\frac{\omega\beta}{2}\right) C_1}\right]\Bigg]$$

$$I = \int\left(2C_3 + 3\cosh^2\frac{\omega\beta}{2}C_1\right)d\tau_1 d\tau_2$$

$$C_3 = \frac{1}{4}\left(3C_1 + C_1\Big|_{\omega\to3\omega}\right)$$

$$\int C_1 d\tau_1 d\tau_2 = \int_{-\beta/2}^{\beta/2}\int_{-\beta/2}^{\beta/2} d\tau_1 d\tau_2 \cosh\left(\omega\left(\beta/2 - |\tau_1-\tau_2|\right)\right) =$$

$$= \{\tau_1 \to \tau_1 + \tau_2\} = \int_{-\beta/2}^{\beta/2} d\tau_2 \int_{-\beta/2+\tau_2}^{\beta/2+\tau_2} d\tau_1 \cosh\left(\omega\left(\beta/2 + |\tau_1|\right)\right) =$$

$$= \int_{-\beta/2}^{\beta/2} d\tau_2 \left[\int_{-\beta/2+\tau_2}^{\beta/2+\tau_2} d\tau_1 \cosh\left[\omega\left(\beta/2 - \tau_1\right)\right] + \int_0^{\beta/2+\tau_2} d\tau_1 \cosh\left(\omega\left(\beta/2+\tau_1\right)\right)\right] =$$

$$= \frac{\beta}{\omega}\sinh\frac{\beta\omega}{2}$$

$$\int C_3 d\tau_1 dt_2 = \frac{1}{4}\left(\frac{3\beta}{\omega}\sinh\frac{\beta\omega}{2} + \frac{\beta}{3\omega}\sinh\frac{3\beta\omega}{2}\right)$$

$$Z_0 = \frac{1}{2\sinh\left(\frac{\omega\beta}{2\hbar}\right)} \qquad Z = \frac{1}{2\sinh\left(\frac{\omega\beta}{2\hbar}\right)}$$

$$Z = \frac{1}{2\sinh\left(\frac{\omega\beta}{2}\right)} \underbrace{\left[1 + \frac{3\hbar^2\lambda^2\beta}{8m^3\omega^3}\left(\frac{2}{\sinh^2\frac{\beta\omega}{2}} + \frac{2}{3} + 3\coth^2\frac{\beta\omega}{2}\right)\right]}_{Z'}$$

$$E_0 = F\Big|_{\beta\to\infty} = -\frac{\hbar}{\beta}\ln Z = -\frac{\hbar}{\beta}\left[-\ln Z_0 + \ln Z'\right] =$$

$$= \frac{\hbar\omega}{2} + -\frac{\hbar}{\beta}\,\frac{3\,\hbar^2\lambda^2\beta}{2.4\,m^3\omega^4}\,\frac{11}{3} = \frac{\hbar\omega}{2}\left[1 - \frac{11}{4}\,\frac{\hbar^2\lambda^2}{m^3\omega^5}\right]$$

$$H = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2} + \lambda x^3$$

$$E_0^{(1)} = V_{00} = 0 \qquad E_0^{(2)} = \sum_n \frac{|V_{n0}|^2}{E_0^{(0)} - E_m^{(0)}}$$

$$x^3 = \lambda x_0^3 Q^3$$

$$Q^3_{m0} = \langle m | Q^3 | 0 \rangle = \frac{1}{2\sqrt{2}} \cdot \left[ 2\langle m|1\rangle + \right.$$

$$\left. + \sqrt{6}\langle m|3\rangle + \langle m|1\rangle \right] = \frac{1}{2\sqrt{2}}\left[3\langle m|1\rangle + \langle m|3\rangle\sqrt{6}\right]$$

$$E_0^{(2)} = \frac{1}{8}\lambda^2 x_0^6 \cdot \left[ \frac{9}{\frac{\hbar\omega}{2} - \hbar\omega(\frac{1}{2}+1)} + \frac{6}{\frac{\hbar\omega}{2} - \hbar\omega(\frac{1}{2}+3)} \right] =$$

$$= -\frac{1}{8}\frac{\lambda^2 x_0^6}{\hbar\omega} \cdot \left[ 9 + 2 \right] = -\frac{11}{8}\lambda^2 x_0^6 / \hbar\omega \qquad x_0 = \sqrt{\frac{\hbar}{m\omega}}$$

$$E_0^{(2)} = -\frac{11}{8}\lambda^2 \hbar^2 / m^3\omega^4 = \frac{\hbar\omega}{2} \cdot \left[ -\frac{11}{4} \frac{\hbar \lambda^2}{m^3\omega^5} \right]$$