N4

$$U(q_a, q_B, T) - ? \qquad H = \frac{p^2}{2m}$$

$$U(q_a, q_b, T) = \int D_q(t)\, D_p(t) \exp\left[i \int_0^T dt \left\{p\dot{q} - \frac{p^2}{2m}\right\}\right]$$

$$= \int D_q(t)\, D_p(t) \exp\left[i \sum_{k=0}^{N-1} \left\{p_k(q_{k+1} - q_k) - \varepsilon H\left(\frac{q_{k+1}+q_k}{2}, p_k\right)\right\}\right]$$

$$= \int D_q(t)\, D_p(t) \exp\Big[\underbrace{i \sum_{k=0}^{N-1} \left\{p_k(q_{k+1} - q_k) - \varepsilon \frac{p_k^2}{2m}\right\}}_{iS}\Big] \; =$$

$$S = q_N p_{N-1} + \sum_{k=1}^{N-1} q_k(p_{k-1} - p_k) - q_0 p_0 - \frac{\varepsilon}{2m} \sum_{k=0}^{N-1} p_k^2$$

$$\int D_q(t)\, D_p(t) = \frac{dp_0}{2\pi} \prod_{k=1}^{N-1} \frac{dq_k\, dp_k}{2\pi}$$

$$= \int \frac{dp_0}{2\pi} \prod_{k=1}^{N-1} \frac{dp_k}{2\pi} \cdot \prod_{k=1}^{N-1} q_k \exp\left[i\left(q_N p_{N-1} - \sum_{k=1}^{N-1} q_k(p_{k-1} - p_k) - q_0 p_0 - \frac{\varepsilon}{2m} \sum_{k=0}^{N-1} p_k\right)\right]$$

$$= \int \prod_{k=0}^{N-2} \frac{dp_k}{2\pi} \cdot \frac{dp_{N-1}}{2\pi} \cdot \prod_{k=1}^{N-1} \delta(p_{k-1} - p_k)\, (2\pi)^{N-1} \cdot$$

$$\cdot \exp\left[i\left(p_{N-1}(\overset{q_b}{q_N} - \overset{q_a}{q_0}) - \frac{p_{N-1}^2}{2m} T\right)\right] =$$

$$= \frac{1}{2\pi} \int dp \, \exp\left[i \left\{ p(q_B - q_a) - \frac{p^2}{2m} T \right\} \right] =$$

$$= \frac{1}{2\pi} \int dp \, \exp\left[\frac{-T}{2m} i \left\{ p^2 + \frac{2m p \cdot 2(q_a - q_B)}{2T} + \frac{4m^2 (q_a - q_B)^2}{4T^2} - \frac{4(q_a - q_B)^2 \cdot m^2}{4T^2} \right\} \right]$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} dp \, \exp\left(-\frac{iT}{2m} \left\{ \left(p + \frac{4m(q_a - q_B)}{4T} \right)^2 \right\} \right] \cdot \exp\left[\frac{im}{2T} (q_B - q_a)^2 \right]$$

$$= \frac{1}{2\pi} \cdot \sqrt{\frac{\pi 2m}{iT}} \cdot \exp\left[\frac{im}{2T} (q_B - q_a)^2 \right] =$$

$$= \sqrt{\frac{m}{2\pi i T}} \exp\left[\frac{im}{2T} (q_B - q_a)^2 \right] = \sqrt{\frac{m}{2\pi T}} \exp\left[\frac{im}{2T} (q_B - q_a)^2 - \frac{i\pi}{4} \right]$$