N4

$$P(R_g^2) = \int_{-\infty}^{\infty} \frac{d\xi}{2\pi} e^{-W(\xi)} \qquad W(\xi) = \frac{\alpha^2 \omega^2(\xi)}{4} - \frac{3}{2}\ln\left[\frac{\omega(\xi)}{\sin\omega(\xi)}\right]$$

$$W'(\xi) = \frac{\partial\omega}{\partial\xi}\cdot W'(\omega) = \frac{\partial\omega}{\partial\xi}\cdot\left\{\frac{\alpha^2\omega(\xi)}{4} - \frac{3}{2}\,\frac{\sin(\omega(\xi))}{\omega(\xi)}\cdot\left[\frac{1}{\sin(\omega(\xi))} - \frac{\omega(\xi)\cdot\cos(\omega(\xi))}{\sin^2(\omega(\xi))}\right]\right\}$$

$$= \frac{\alpha^2\omega(\xi)^2 - 3 + 3\cot[\omega(\xi)]\,\omega(\xi)}{2\omega(\xi)}\cdot\omega'(\xi) = 0$$

1) $\alpha \gg 1$: $\xi = \alpha^2\omega^2 + 3\cot(\omega)\cdot\omega - 3 = 0$

$\xi \approx \alpha^2\omega^2 + 3\cot(\omega)\cdot\omega \Rightarrow \xi \approx \omega(\omega\alpha^2 + 3\cot(\omega))$

$\alpha \gg 1 \Rightarrow \omega\alpha^2 \gg 1 \Rightarrow -3\cot(\omega) \gg 1 \Rightarrow \omega \approx \pi$.

Тогда $\omega\left[\alpha^2\pi + 3\cot(\pi-\delta)\right]_{\delta\to 0} \approx 0 \Rightarrow \frac{\alpha^2\pi}{3} \approx \frac{1}{\delta} \Rightarrow$

$\delta = \frac{3}{\pi\alpha^2} \Rightarrow \omega_0 = \pi - 3/\pi\alpha^2$

$W'(\xi) = W'(\omega)\frac{\partial\omega}{\partial\xi} \qquad W'(\omega_0) \approx 0$

$W \approx W_0(\omega_0) + \frac{1}{2}W''(\omega_0)(\xi-\xi_0)^2$

$$W_0(\omega_0) = \frac{1}{4}\left[-6 + \cancel{\frac{9}{\pi^2\alpha^2}}^{\,0} + \pi^2\alpha^2 - 6\log\left\{\left[\pi - \frac{3}{\pi\alpha^2}\right]\cdot\csc\left[\frac{3}{\pi\alpha^2}\right]\right\}\right]$$

$$\approx -\frac{3}{2} + \frac{\pi^2\alpha^2}{4} - \frac{3}{2}\log\left[\left(1 - \frac{3}{\pi^2\alpha^2}\right)\cdot\pi\cdot\left(\frac{3}{\pi\alpha^2}\right)^{-1}\right] \approx$$

$$\approx -\frac{3}{2} + \frac{\pi^2 \alpha^2}{4} + \frac{3}{2}\log\left[\frac{3}{\pi^2\alpha^2}\right]$$

$$W''(\omega_0) = \frac{1}{2}\left[\omega'^2(\zeta_0) + \left(\pi - \frac{3}{\pi\alpha^2}\right)\cdot \omega''(\zeta_0)\right]\alpha^2$$

$$\zeta_0 = \frac{3\omega_0^2}{2Nb^2 i} \;;\quad \zeta = \frac{3\omega^2}{2Nb^2 i} \qquad \omega' = \frac{i b^2 N}{(i\cdot 6\cdot b^2 N \zeta)^{1/2}} \Rightarrow$$

$$\omega'(\zeta_0) = \frac{i b^2 N}{\left(\frac{3}{2}\omega^2\cdot 6\right)^{1/2}} = \frac{i b^2 N}{3\omega_0} \;,\quad \omega'' = \frac{b^4 N^2}{2\sqrt{6}\,(i b^2 N \zeta)^{3/2}}$$

$$\omega''(\zeta_0) = \frac{b^4 N^2}{2\sqrt{6}\cdot\left(\frac{3}{2}\omega_0^2\right)^{3/2}} = \frac{b^4 N^2}{\cancel{2}\cdot\cancel{6^{1/2}}\,\cancel{2^{1/2}}\cdot 3^{1/2}\cdot 3^{3/2}\,\cancel{2^{3/2}}\cdot\omega_0^3} = \frac{b^4N^2}{9\omega_0^3}$$

$$W''(\omega_0) = \frac{1}{2}\cdot\left[-\frac{b^4N^2}{9\omega_0^2} + \omega_0\cdot\frac{b^4N^2}{9\omega_0^3}\right]\alpha^2 = 0$$ надо искать

следующ. порядок.

$$W'' = \frac{1}{2\omega_0^2}\Big(\left(3 - 3\,\mathrm{Csc}[\omega]^2\,\omega^2\right)\left(\omega'^2 + \omega\left(-3 + 3\,\mathrm{Cot}[\omega]\,\omega\right)\left(\omega''\right)\right)\Big) \underset{\alpha\to\infty}{\approx} \frac{1}{54} b^4 N^2 \alpha^4$$

$$\Rightarrow W \approx -\tfrac{3}{2} + \frac{\pi^2\alpha^2}{4} + \frac{3}{2}\log\left(\frac{3}{\pi^2\alpha^2}\right) + \frac{1}{2}\cdot\frac{b^4N^2\alpha^4}{54}(\zeta-\zeta_0)^2$$

$$\Rightarrow P(R_g^2) \approx \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{3}{\pi^2\alpha^2}\right)^{-3/2}\cdot\exp\left[+\tfrac{3}{2} - \frac{\pi^2\alpha^2}{4} - \frac{1}{2}\,\frac{b^4N^2\alpha^4}{54}(\zeta-\zeta_0)^2\right]d\zeta$$

$$= \frac{1}{2\pi}\cdot\pi^{+3}\cdot 3^{-3/2}\cdot\alpha^{+3}\cdot e^{3/2}\cdot e^{-\pi^2\alpha^2/4}\cdot\pi^{1/2}\cdot\left(3^{-3/2}\cdot 2^{-2/2}\cdot b^2 N\alpha^2\right)^{-1} =$$

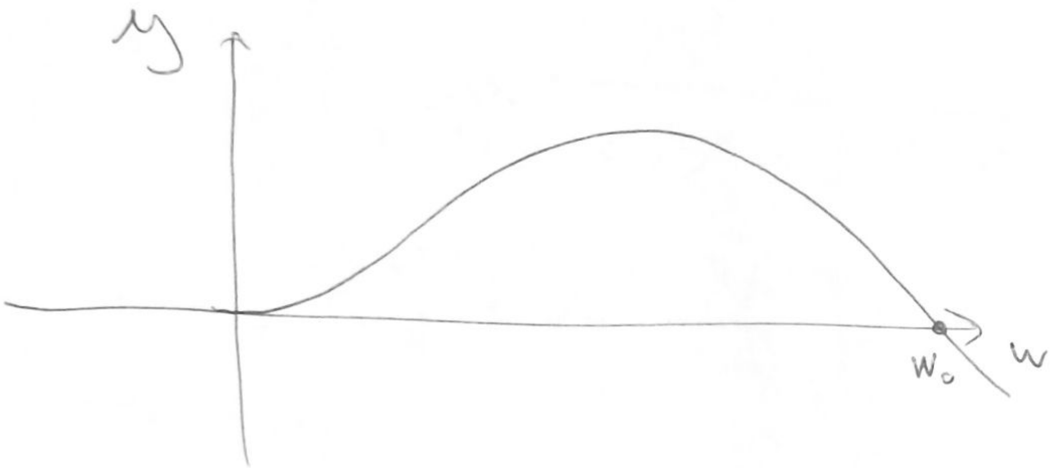
$= \pi^{5/2} e^{3/2} N^{-1} b^{-2} \alpha \exp[-\alpha^2 \pi^2/4]$

2) $\alpha \ll 1$: $y = \alpha^2 \omega^2 - 3 + 3\omega \cot(\omega) = 0$

На мнимой оси имеется корень. Найдем его:

$$w = \mathrm{Im}\,\omega: \quad -\alpha^2 w^2 - 3 + 3iw \cot(iw) = 0$$

$$-\alpha^2 w^2 - 3 + 3w \coth(w) = 0$$



$$3w \coth(w) - \alpha^2 w^2 - \underbrace{3}_{\text{мало}} = 0$$ по сравнению с $w_0, \frac{1}{\alpha}$

$$3w \coth(w) = \alpha^2 w^2 \Rightarrow w = \frac{3\coth(w)}{\alpha^2}$$

в нулевом приближ $\coth(w_0) = 1 \Rightarrow w_0' = \frac{3}{\alpha^2}$

$$\Rightarrow \omega = \frac{3i}{\alpha^2} \Rightarrow W_0 = \frac{\alpha^2}{4} \cdot \frac{-9}{\alpha^4} - \frac{3}{2} \ln\left[\frac{3}{\alpha^2 \cdot \sinh[3/\alpha^2]}\right]$$

$$= \frac{-9}{4} \frac{1}{\alpha^2} - \frac{3}{2} \ln\left[\frac{3}{\alpha^2} \cdot \exp[-3/\alpha^2] \cdot \frac{2}{1 - e^{-6/\alpha^2}}\right] =$$

$$= -\frac{9}{4} \frac{1}{\alpha^2} - \frac{3}{2} \cdot \frac{-3}{\alpha^2} - \frac{3}{2} \ln\left[\frac{6}{\alpha^2}\right] = +\frac{9}{4} \frac{1}{\alpha^2} - \frac{3}{2} \ln\left[\frac{6}{\alpha^2}\right]$$

$$W'' = \frac{\left(3+\left(\alpha^2-3\csc(\omega)^2\right)\omega^2\right)(\omega')^2 + \cancel{4}\,\omega\left(-3+3\cot(\omega)\cdot\omega+\alpha^2\omega^2\right)\cdot\omega''}{2\omega^2}$$

$$\omega_0 = \frac{3i}{\alpha^2}$$

$$\omega_0' = \frac{ib^2N}{\left(27i\cdot 6i\cdot\frac{1}{2}\alpha^{-4}\right)^{1/2}} = \frac{ib^2N}{9\alpha^{-2}(-1)^{1/2}}$$

$$\omega_0'' = \frac{N^2b^4}{3^5\alpha^{-6}(-1)^{3/2}}$$

$$W'' = \frac{\left(3+\left(\alpha^2-3\csc\left(\frac{3i}{\alpha^2}\right)\right)\left(\frac{3i}{\alpha^2}\right)^2\right)\left(\frac{ib^2n}{9\alpha^{-2}(-1)^{1/2}}\right) + \frac{3i}{\alpha^2}\left(-3+3\cot\left(\frac{3i}{\alpha^2}\right)\frac{3i}{\alpha^2}+\alpha^2\left(\frac{3i}{\alpha^2}\right)^2\right)\frac{n^2b^4}{3^5\alpha^{-6}(-1)^{3/2}}}{2\cdot 9\cdot(i)^2\cdot\alpha^{-4}}$$

$\csc\left(\frac{3i}{\alpha^2}\right) \longrightarrow 0$ экспоненциально

$\cot\frac{3i}{\alpha^2} \rightarrow -i$

$$W'' \approx \frac{1}{486}b^4N^2\alpha^4\left(-2\alpha^4+3\alpha^2\right)n \approx \frac{1}{162}b^4N^2\alpha^6$$

$$\Rightarrow W \approx \frac{9}{4}\frac{1}{\alpha^2} - \frac{3}{2}\ln\left[\frac{b}{\alpha^2}\right] + \frac{1}{324}N^2b^4\alpha^6\left(\xi-\xi_0\right)^2$$

$$P(R_g^2) \approx \frac{1}{2\pi}\left(\frac{b}{\alpha^2}\right)^{3/2}\cdot\exp\left(-\frac{9}{4}\frac{1}{\alpha^2}\right)\cdot\frac{\sqrt{\pi}}{\left(2^{-1}\cdot3^{-4}\cdot N^2b^4\alpha^6\right)^{\frac{1}{2}}} =$$

$$= \frac{1}{2\pi}\left(\frac{b}{\alpha^2}\right)^{3/2}\pi^{1/2}\cdot 2\cdot 3^2\cdot N^{-1}\cdot b^{-2}\cdot\alpha^{-3}\cdot\exp\left(-9/4\ 1/\alpha^2\right) =$$

$$= \pi^{-1/2}\cdot N^{-1}\cdot\alpha^{-6}\cdot b^{-2}\cdot\cancel{2}b^{1/2}\cdot 6\cdot 3^2\cdot\exp\left(-9/4\cdot 1/\alpha^2\right) =$$

$$= 54\left(b/\pi\right)^{1/2}\left(1/Nb^2\alpha^6\right)\cdot\exp\left[-9/4\alpha^2\right]$$

На семинаре получили

$$H = N\left(\frac{4b^2 v \sigma}{\pi^2}\right)^{1/3}$$

$$\frac{\pi}{2}(1-u) = \int_u^1 \frac{f(t)\,dt}{\sqrt{t-u}} \qquad f(1)=0$$

$$\frac{\pi}{2}(1-u) = \int_{u-1}^{0} \frac{\overbrace{f(t+1)}^{g(t)}\,dt}{\sqrt{t+1-u}} \Rightarrow \frac{\pi}{2}(-1+u) = \int_{0}^{u-1} \frac{f(t+1)\,dt}{\sqrt{t+1-u}} =$$

$$= \int_{0}^{\tilde{u}} \frac{g(t)\,dt}{\sqrt{t-\tilde{u}}} = \frac{\pi}{2}\{\tilde{u}\} = \cancel{\int_{0}^{\tilde{u}} \frac{\tilde{g}(-t)\,dt}{\sqrt{t-\tilde{u}}}} \int_{0}^{\tilde{u}} \frac{g(t)\,dt}{i\sqrt{\tilde{u}-t}}$$

$$\frac{\pi i}{2}\tilde{u} = \int_{0}^{\tilde{u}} \frac{g(t)\,dt}{\sqrt{\tilde{u}-t}} \Rightarrow \mathcal{L}\left\{\frac{\pi i \tilde{u}}{2}\right\} = \mathcal{L}\left\{\int_{0}^{\tilde{u}} \frac{g(t)\,dt}{\sqrt{\tilde{u}-t}}\right\}$$

$$= \mathcal{L}\{g(\tilde{u})\}\,\mathcal{L}\left\{\frac{1}{\sqrt{\tilde{u}}}\right\} \Rightarrow \mathcal{L}\{g(\tilde{u})\} =$$

$$= \frac{i\pi}{2s^2} \cdot \frac{\sqrt{s}}{\sqrt{\pi}} \Rightarrow g(\tilde{u}) = \mathcal{L}^{-1}\left\{\frac{i\sqrt{\pi}}{2s^{3/2}}\right\} = i\sqrt{\tilde{u}} = \sqrt{1-u}$$

$$f(\tilde{u}+1) = g(\tilde{u}) \Rightarrow f(\tilde{u}) = g(\tilde{u}-1)\ ; \quad u = \frac{x^2}{h^2} \qquad f(u) = \frac{g'(\sqrt{u})}{3\sqrt{u}}$$

$$g'\left(\frac{u}{h}\right) = h\,\psi(h)$$

$$\Psi(h) = g'\left(\frac{h}{H}\right)\cdot\frac{1}{H} \qquad g'(\sqrt{t}) = 3\sqrt{t}\cdot f(t) = 3\sqrt{t}\cdot g(t-1)$$

$$g'(\sqrt{t}) = 3\sqrt{t}\cdot\sqrt{1-(t-1)} = 3t \Rightarrow \cancel{g\,\Psi(h) = 3\frac{h}{H}H}$$

$$g'(t) = 3t^2 \Rightarrow \Psi(h) = \frac{3h^2}{H^3}$$