N2.2

$V = \frac{m\omega^2 x^2}{2} + \gamma x^3$ - разобрано в д.з. В.М. N4.2

$V = \frac{m\omega^2 x^2}{2} + \lambda x^4$

$$Z(\beta) = Z^0(\beta) \exp\left[-\lambda \int d\tau \left(\frac{\delta}{\delta J(\tau)}\right)^4\right] \exp\left[\frac{1}{2} J G_0 J\right]\Big|_{J=0} \approx$$

$$\approx Z^0(\beta)\left(1 - \lambda \int d\tau \left(\frac{\delta}{\delta J(\tau)}\right)^4\right) e^{\frac{1}{2} J G_0 J}\Big|_{J=0}$$

$$\frac{\delta}{\delta J(\tau_1)} \frac{\delta}{\delta J(\tau_2)} \frac{\delta}{\delta J(\tau_3)} \frac{\delta}{\delta J(\tau_4)} =$$

$$= G(\tau_1,\tau_2)G(\tau_3,\tau_4) + G(\tau_1,\tau_3)G(\tau_2,\tau_4) + G(\tau_1,\tau_4)G(\tau_2,\tau_3)$$

$$-\lambda \int d\tau \left(\frac{\delta}{\delta J(\tau)}\right)^4 \exp\left[\frac{1}{2} J G_0 J\right]\Big|_{J=0} = -3\lambda \int_{-\beta/2}^{\beta/2} d\tau\, G_0^2(\tau,\tau)$$

$3G(\tau,\tau)$: 

$$Z(\beta) = Z^0(\beta)\left[1 - 3\lambda \underbrace{\int_{-\beta/2}^{\beta/2} d\tau\, G_0^2(\tau,\tau)}_{I}\right]$$

$$I = \int_{-\beta/2}^{\beta/2} d\tau\, G_0^2(\tau,\tau) = \int_{-\beta/2}^{\beta/2} d\tau \cdot \frac{1}{(2m\omega)^2} \frac{\cosh^2(\beta\omega/2)}{\sinh^2(\beta\omega/2)} = \frac{\beta}{4m^2\omega^2} \coth^2\left(\frac{\beta\omega}{2}\right) \Rightarrow$$

$$\Rightarrow Z(\beta) = Z^{\circ}(\beta)\left[1 - \frac{3\hbar\lambda\beta}{4m^2\omega^2}\coth^2\left(\frac{\omega\beta}{2}\right)\right]$$

$$F = -\frac{1}{\beta}\ln Z = \frac{\hbar\omega}{2}\left[1 + \frac{1}{\omega\beta}\ln\left[1 - \exp(-\beta\omega)\right] + \frac{3\hbar\lambda}{2m^2\omega^3}\coth\left(\frac{\omega\beta}{2}\right)\right]$$

$$E_0 = \lim_{\beta\to\infty} F = \frac{\hbar\omega}{2}\left(1 + \frac{3\hbar\lambda}{2m^2\omega^3}\right)$$