N1

1) $K(\varphi_f, \varphi_i, t) = \langle \varphi_f | \sum |\psi_l\rangle\langle\psi_l| e^{iHt/\hbar} |\varphi_i\rangle =$

$$= \sum \psi_l(\varphi_f)\, e^{-iEt/\hbar}\, \psi_l^*(\varphi_i)$$

$$H = -\frac{\hbar^2}{2mR^2}\frac{\partial^2}{\partial\varphi^2} \qquad L_z = -i\frac{\partial}{\partial\varphi} \qquad [H, L_z] = 0 \Rightarrow$$

$$\Rightarrow \psi_l = e^{il\varphi}/\sqrt{2\pi} \qquad E_l = \frac{\hbar^2 l^2}{2mR^2}$$

$$K(\varphi_f, \varphi_i, t) = \frac{1}{2\pi}\sum_l \exp[il(\varphi_f - \varphi_i)]\exp\left(i\frac{\hbar l^2 t}{2mR^2}\right.$$

2) $$K'(\varphi_f, \varphi_i, t) = \langle \varphi_f | e^{iH\Delta t} | \varphi_i\rangle = \int_{\varphi(-\frac{\Delta t}{2}) = \varphi_i}^{\varphi(\frac{\Delta t}{2}) = \varphi_f} D\varphi \exp\left(-\frac{i\hbar}{2mR^2}\int_{-\frac{\Delta t}{2}}^{\Delta t/2}\dot\varphi^2 dt\right)$$

$$K'(\varphi_f, \varphi_i, t) = \underbrace{\int_{\varphi'(-\frac{\Delta t}{2})=0}^{\varphi'(\frac{\Delta t}{2})=0} D\varphi' \exp\left[\frac{imR^2}{2\hbar}\int_{-\frac{\Delta t}{2}}^{\Delta t/2}\dot\varphi' dt\right]}_{\sqrt{\frac{mR^2}{2\pi\hbar\Delta t\, i}}} \cdot \sum_n \exp\left[\frac{i mR^2}{2\hbar\Delta t}(\varphi_f - \varphi_i + 2\pi n)^2\right]$$

$\sqrt{\frac{mR^2}{2\pi\hbar\Delta t\, i}}$ - рез-т со 2 семинара

$$= \sqrt{\frac{mR^2}{2\pi\hbar\Delta t i}} \sum \exp\left[-\frac{mR^2}{2\hbar t}(\varphi_f - \varphi_i + 2\pi n)^2\right]$$

3) $$K(\varphi_f, \varphi_i, t) = \frac{1}{2\pi}\int_0^{2\pi}\sum_{l=-\infty}^{\infty} e^{il(\varphi_f - \varphi_i)} d\varphi_i \sqrt{\frac{mR^2}{2\pi i\hbar\Delta t}} \sum \exp\left[\frac{imR^2}{2\hbar\Delta t}(\varphi_f - \varphi_i)^2\right]$$

$$= \frac{1}{2\pi}\sqrt{\frac{mR^2}{2\pi i\hbar\Delta t}} \sum_{l=-\infty}^{\infty}\sum^{\infty} \int_0^{2\pi} d\delta\varphi \exp\left[i\frac{mR^2}{2\hbar t}(\delta\varphi + 2\pi n)^2 - il\delta\varphi\right] =$$

$$= \frac{1}{2\pi}\sqrt{\frac{mR^2}{2\pi i\hbar\Delta t}}\sum_{l=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\int_0^{2\pi} \exp\left[i\frac{mR^2}{\hbar t}\left(\delta\varphi-2\pi n\right)^2 - il\delta\varphi\right]d\delta\varphi =$$

$$\int_{-\infty}^{\infty}$$

$$= \frac{1}{2\pi}\sqrt{\frac{mR^2}{2\pi i\hbar\Delta t}}\sum_{l=-\infty}^{\infty}\int_{-\infty}^{\infty}\exp\left[i\frac{mR^2}{2\hbar t}x^2 - ilx\right]dx =$$

$$= \frac{1}{2\pi}\sum_{l=-\infty}^{\infty} e^{il(\varphi_f-\varphi_i)}\exp\left[\frac{-i\hbar l^2}{2mR^2}t\right]$$

$$\boxed{K' = K}$$

N/2

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}\left(|\psi_l\rangle + |\psi_r\rangle\right)$$

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}\left(|\psi_l\rangle - |\psi_r\rangle\right)$$

$$|\psi_l\rangle = \frac{1}{\sqrt{2}}\left(|\psi_0\rangle + |\psi_1\rangle\right)$$

$$|\psi_r\rangle = \frac{1}{\sqrt{2}}\left(|\psi_0\rangle - |\psi_1\rangle\right)$$

$$|\psi\rangle = \left(a_l|\psi_l\rangle + a_r|\psi_r\rangle\right) e^{-i\omega t/2}$$

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} a_l \\ a_r \end{pmatrix} = H\begin{pmatrix} a_l \\ a_r \end{pmatrix}$$

$$i\hbar\frac{\partial}{\partial t}\left[\left(a_l|\psi_l\rangle + a_r|\psi_r\rangle\right)e^{-i\omega t/2}\right] = H\cdot\left[\left(a_l|\psi_l\rangle + a_r|\psi_r\rangle\right)\right]e^{-\frac{i\omega t}{2}}$$

$$-\frac{i\hbar i\omega}{2}\cdot|\psi\rangle + i\hbar\frac{\partial a_l}{\partial t}|\psi_l\rangle e^{-\frac{i\omega t}{2}} + i\hbar\frac{\partial a_r}{\partial t}|\psi_r\rangle e^{-\frac{i\omega t}{2}} = H\cdot\left[\frac{a_l}{\sqrt{2}}\left(|\psi_0\rangle + |\psi_1\rangle\right) + \right.$$

$$\left. + \frac{a_r}{\sqrt{2}}\left(|\psi_0\rangle - |\psi_1\rangle\right)\right]e^{-i\omega t/2}$$

$$\frac{\hbar\omega}{2}|\psi\rangle + i\hbar\frac{\partial}{\partial t}\left(a_l|\psi_l\rangle + a_r|\psi_r\rangle\right) =$$

$$H|\psi_0\rangle\cdot\frac{1}{\sqrt{2}}\left(a_l + a_r\right) + H|\psi_1\rangle\frac{1}{\sqrt{2}}\left(a_l - a_r\right)$$

$$\frac{\hbar\omega}{2}\left(a_l|\psi_l\rangle + a_r|\psi_r\rangle\right) + i\hbar\frac{\partial}{\partial t}\left(a_l|\psi_l\rangle + a_r|\psi_r\rangle\right) = \frac{1}{\sqrt{2}}\left(E_0 - \frac{\Delta E}{2}\right)\left(a_l + a_r\right)|\psi_0\rangle$$

$$+\frac{1}{\sqrt{2}}\left(E_0 + \frac{\Delta E}{2}\right)\left(a_l - a_r\right)|\psi_1\rangle$$

$$i\hbar\frac{\partial}{\partial t}\left(a_l|\psi_l\rangle + a_r|\psi_r\rangle\right) = -\frac{\hbar\omega}{2}\left(a_l|\psi_l\rangle + a_r|\psi_r\rangle\right) + \frac{1}{2}\left(E_0 - \frac{\Delta E}{2}\right)\left(a_l + a_r\right)\cdot$$

$$\cdot\left(|\psi_l\rangle + |\psi_r\rangle\right) + \frac{1}{2}\left(E_0 + \frac{\Delta E}{2}\right)\left(a_l - a_r\right)\left(|\psi_l\rangle - |\psi_r\rangle\right)$$

$$i\hbar\frac{\partial}{\partial t}a_l = -\frac{\hbar\omega}{2}a_l + \frac{1}{2}\left(E_0 - \frac{\Delta E}{2}\right)\left(a_l + a_r\right) + \frac{1}{2}\left(E_0 + \frac{\Delta E}{2}\right)\left(a_l - a_r\right)$$

$$i\hbar\frac{\partial}{\partial t}a_l = -\cancel{\frac{\hbar\omega}{2}}a_l + \cancel{\frac{E_0}{2}}a_l + \cancel{\frac{E_0}{2}}a_r - \cancel{\frac{E_0}{2}}a_r - \frac{\Delta E}{2}a_r$$

аналогично:

$$i\hbar\frac{\partial}{\partial t}a_r = -\frac{\Delta E}{2}a_l \quad\Rightarrow\quad i\hbar\frac{\partial}{\partial t}\begin{pmatrix}a_l\\ a_r\end{pmatrix} = \begin{pmatrix}0 & -\frac{\Delta E}{2}\\ -\frac{\Delta E}{2} & 0\end{pmatrix}\begin{pmatrix}a_l\\ a_r\end{pmatrix}$$

N3

$$\Psi = \sum_n a_n \Psi_n$$

$$\Psi_n = \pi^{-1/4} \exp\left[\frac{-\left(x - \frac{\pi}{2}(2n+1)\right)^2}{2}\right]$$

$$\Psi = \pi^{-1/4} \sum a_n \exp\left[-\frac{\left\{x - \frac{\pi}{2}(2n+1)\right\}^2}{2}\right]$$

т.к. инстантон может тунн. только в соседнюю ячейку, то

$$i\hbar \frac{\partial}{\partial t}\bar{a} = \begin{pmatrix} 0 & -\frac{\Delta E}{2} & 0 & 0 & \cdots \\ -\frac{\Delta E}{2} & 0 & -\frac{\Delta E}{2} & 0 & \cdots \\ 0 & -\frac{\Delta E}{2} & 0 & -\frac{\Delta E}{2} & \cdots \\ \vdots & 0 & -\frac{\Delta E}{2} & 0 & \cdots \\ \vdots & & \vdots & & \ddots \end{pmatrix} \bar{a} \qquad (*)$$

$$a_k(t) = \frac{1}{\sqrt{N}} \sum_0^{N-1} a_n e^{-ink\cdot 2\pi/N}$$

$$a_n(t) = \frac{1}{\sqrt{N}} \sum_0^{N-1} a_k e^{+ink\cdot 2\pi/N}$$

$$(*):\ i\hbar \frac{\partial}{\partial t} a_n = -\frac{\Delta E}{2}(a_{n-1} + a_{n+1})$$

$$\cancel{i\hbar \frac{\partial}{\partial t} \sum a_k e^{ink\frac{2\pi}{N}} = -\frac{\Delta E}{2}\left(\sum a_k e^{ink\frac{2\pi}{N} - ik2\pi/N} + \sum a_k e^{ink + ik2\pi/N}\right)}$$

$$i\hbar \frac{\partial}{\partial t} \sum a_n e^{-ink\,2\pi/N} = -\frac{\Delta E}{2}\left(e^{-ik2\pi/N} \sum a_{n-1} e^{-i(n-1)k\cdot 2\pi/N} + e^{ik\frac{2\pi}{N}} \cdot \sum a_{n+1} e^{-i(n+1)k\frac{2\pi}{N}}\right)$$

$$\Rightarrow i\hbar \frac{\partial}{\partial t} a_k = -\frac{\Delta E}{2} e^{-ik\frac{2\pi}{N}} a_k - \frac{\Delta E}{2} e^{ik\frac{2\pi}{N}} a_k$$

$$\Rightarrow \quad i\hbar \frac{\partial}{\partial t} a_k = -\Delta E \cos\left(\frac{2\pi k}{N}\right) a_k$$

$$a_k = a_0^k \exp\left(-\Delta E \cos\left(\frac{2\pi k}{N}\right) t\right)$$

$$a_n = \frac{1}{\sqrt{N}} a_0^k \sum_0^{N-1} \exp\left(-\Delta E \cos\left(\frac{2\pi k}{N}\right) t + \frac{i n 2\pi k}{N}\right)$$

$$a_n \to \frac{1}{2\pi} \int_{-\pi}^{\pi} \exp\left(i k n - \Delta E \cos(\tilde{k})\right) d\tilde{k}$$

$$\approx \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp\left[i k n - \Delta E + \frac{\Delta E k^2}{2}\right] = \frac{\exp\left[-\Delta E + \frac{n^2}{2\Delta E}\right]\sqrt{2\pi}}{\sqrt{-\Delta E}}$$

$$\Delta E = \frac{\Delta E}{\hbar} \quad \Rightarrow \quad a_n \sim \exp\left[\frac{n^2 t}{2\Delta E}\right]$$

3) Ну, вроде как $\frac{-\Delta E}{\hbar}$ — тот самый матричный элемент. Он же амплитуда вер-ти.