№1

$$S = \sum_{k=0}^{k=N-1} p_k \Delta x_k = p_0 x_1 - p_0 x_0 + p_1 x_2 - p_1 x_1 + p_2 x_3 - p_2 x_2 =$$

$$\Delta p_{ij} = p_i - p_j$$

$$= p_2 x_f - p_0 x_2 - x_1 \Delta p_{10} - x_2 \Delta p_{21}$$

$$\langle x_f | U(t_f - t_i)(p_2 x_1 - x_2 p_1) | x_i \rangle =$$

$$-i \int D_x D_p \left(x_1 \partial x_3 - x_2 \partial x_2 \right) \exp\left[i \left(p_2 x_f - p_0 x_i + x_1 \Delta p_{10} - x_2 \Delta p_{21} \right) \right] = -i \int D_x D_p \exp(iS) = -i U(x_f, x_i)$$

↖ из условия: $\frac{\partial x_1}{\partial x_3} = 0$, $\frac{\partial x_2}{\partial x_2} = 1$

$$\langle x_f | U(t_f - t_i)(p_1 x_1 - x_2 p_1) | x_i \rangle =$$

$$-i \int D_x D_p \left(x_1 \partial x_2 - x_2 \partial x_2 \right) \exp(iS) =$$

$$= -i \int D_x D_p \frac{\partial x_2}{\partial x_2} \exp(iS) = -i U(x_f, x_i)$$

Ответ: $-i; -i$

N_2

$$\langle x_f | \hat{U}(T)\, T\left\{\frac{\left(\hat{x}(t+\tau) + - \hat{x}(t)\right)^2}{\tau^2}\right\} | x_i \rangle$$

$$= \langle x_f | \hat{U}(T)\left\{\hat{x}(t+\tau)\hat{x}(t+\tau) + \hat{x}(t)\hat{x}(t) - 2\hat{x}(t+\tau)\hat{x}(t)\right\} | x_i \rangle \equiv \left(U_{fi} \cdot \left\{ \frac{x_{free}}{\tau^2}(t+\tau) \right.\right.$$

$$\equiv U_{fi} \cdot \left\{ \frac{x^2_{free}(t+\tau)}{\tau^2} - \frac{ih}{\tau^2} G(t+\tau, t+\tau) + \frac{x^2_{free}(t)}{\tau^2} \right.$$

$$\left. - \frac{ih}{\tau^2} G(t,t) + \frac{2ih}{\tau^2} G(t+\tau, t) \pm \frac{2 x_{free}(t+\tau) x_{free}(t)}{\tau^2} \right\} =$$

$$= U_{fi} \cdot \left\{ F_{free} + \frac{ih}{m\tau^2} \left[2(t+\tau-T)t - t(t-T) - (t+\tau)(t+\tau-T) \right] \right\} = U_{fi} \cdot \left\{ F_{free} + \frac{ih}{m\tau^2} \left[2t^2 + 2\tau t - 2Tt - t^2 + tT - t^2 - t\tau + \tau T \right.\right.$$

$$\left. - Tt - \tau^2 + \tau T \right] = U_{fi} \cdot \left\{ F_{free} + \frac{ih}{m\tau^2} \left[\tau T - \tau^2 \right] \right\} \equiv$$

$$\cancel{x_f}\ F_{free} = \frac{1}{\tau^2 T^2}\Big[\big(t x_f + (T-t)x_i\big)^2 + \big((t+\tau)x_f + (T-t-\tau)x_i\big)^2 -$$

$$- \big(t x_f + (T-t)x_i\big)^{[illegible]}\big((t+\tau)x_f + (T-t)x_i\big)\Big] =$$

$$= \frac{1}{\tau^2 T^2}\Big[\big(t^2 + \tau t + \tau^2\big)x_f^2 + \big((T-2\tau)\tau + 2t(T-\tau) - 2t^2\big)x_f x_i +$$

$$+ \big((t-T)^2 + (t-T)\tau + \tau^2\big)x_i^2\Big]$$

$$\circleddash\ U_{fi}\Big\{\frac{1}{\tau^2 T^2}\Big[x_f^2\big(t^2 + t\tau + \tau^2\big) + x_f x_i\big((T-2\tau)\tau + 2t(T-\tau) - 2t^2\big) + x_i^2 \cdot$$

$$\cdot \big((t-T)^2 + (t-T)\tau + \tau^2\big)\cancel{x_i^2}\Big] + \frac{i\hbar}{m\tau^2 T}\big[T\tau - \tau^2\big]\Big\}$$

$$U_{fi} = \langle x_f | \hat{U}(T) | x_i \rangle$$

N3

$$\int Dq\,Dp \prod_{k=1}^{N-1} \frac{p_k^2}{2m} \exp(iS) =$$

$$S = q_N p_{N-1} + \sum_{k=1}^{N-1} q_k (p_{k-1} - p_k) - q_0 p_0 - \frac{\varepsilon}{2m} \sum p_k^2$$

$$\varepsilon = \Delta\tau$$

Проинтегрируем по q_k : получим $N-1$ дельта-функц.

$$= \int \frac{dp_0}{2\pi} \prod_{k=1}^{N-1} \frac{dp_k}{2\pi} \cdot \prod_{k=1}^{N-1} q_k\, p_k' \exp\left[iS\right] = \left\{ \text{воспользуемся результатами из ДЗ №2 Владимира Михайловича} \right\}$$

$$= \frac{1}{2\pi} \exp\left[\frac{im}{2\tau}(q_f - q_i)^2\right] \cdot \int dp \frac{p^2}{2m} \exp\left[-\frac{i\tau}{2m}\left(p + \frac{m}{\tau}(q_f - q_i)\right)^2\right] \cdot$$

$$= \frac{1}{2\pi} \exp\left[\frac{im}{2\tau}(q_f - q_i)^2\right] \frac{1}{2m} \cdot \frac{\sqrt{\pi}}{2} \cdot \frac{1}{\sqrt{\frac{-i\tau^3}{8m^3}}} =$$

$$= \frac{1}{2\pi} \sqrt{\frac{i\pi m}{2\tau^3}} \exp\left[\frac{im}{2\tau}(q_f - q_i)^2\right] = \frac{1}{\sqrt{\frac{im}{8\pi\tau^3}}} \exp\left[\frac{im}{2\tau}(q_f - q_i)^2\right]$$

по размерности восстанавливаем

$$\frac{\langle p^2 \rangle}{2m} = \frac{-i\hbar}{2\tau} \Rightarrow \frac{\langle p^2 \rangle}{m} = -\frac{i\hbar m}{\tau} \leftarrow \text{вроде верно}$$