$$-\frac{1}{2}\frac{\partial^2}{\partial x_0^2}\tilde{P}(s|x_0) + s x_0 \tilde{P}(s|x_0) = 0 \qquad \tilde{P}(s|0)=1, \quad \tilde{P}(s|\infty)\to 0$$

$$\frac{\partial^2}{\partial x_0^2}\tilde{P}(s|x_0) - 2 s x_0 \tilde{P}(s|x_0) = 0$$

$$x_0 \to \frac{\tilde{x}_0}{\sqrt[3]{2s}} \Rightarrow \left(\frac{\partial^2}{\partial \tilde{x}_0^2} - \tilde{x}_0\right)\tilde{P}(s|x_0) = 0$$

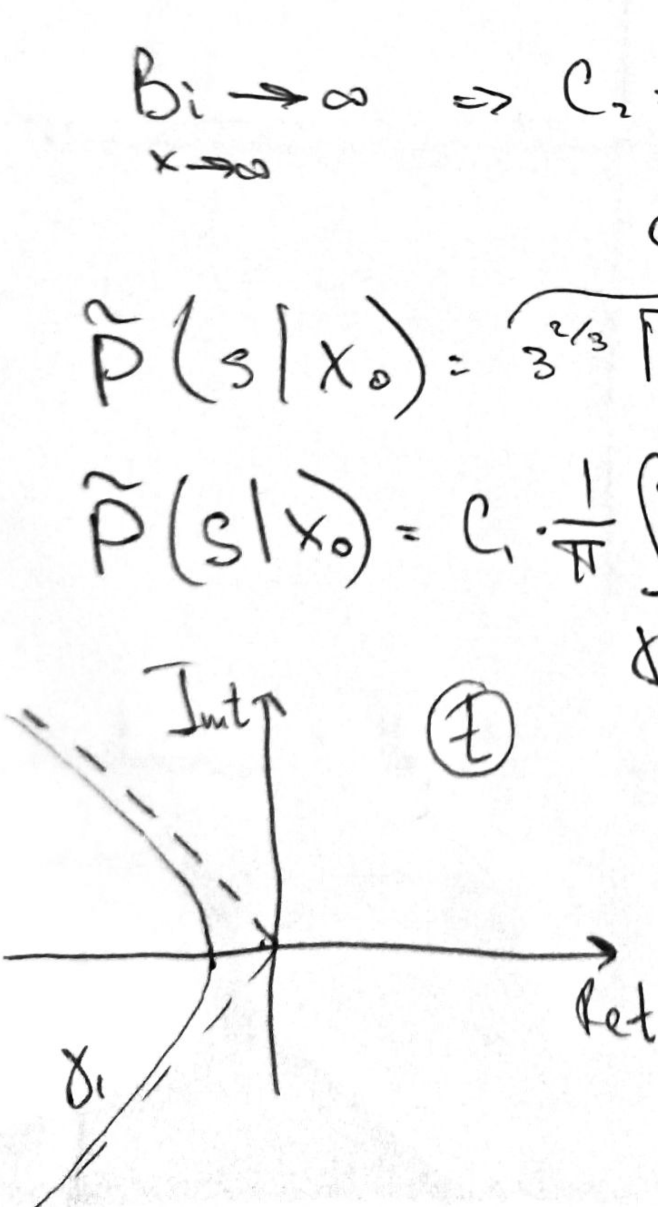
$$\tilde{P}(s|\tilde{x}_0) = Ai(\tilde{x}_0)\cdot C_1 + Bi(\tilde{x}_0)\cdot C_2$$

$$\tilde{x}_0 = x_0\sqrt[3]{2s} \qquad Ai(\infty) = 0;$$

$$Bi \xrightarrow[x\to\infty]{} \infty \Rightarrow C_2 = 0 \qquad Ai(0) = \frac{1}{3^{2/3}\Gamma(2/3)}$$

$$\tilde{P}(s|x_0) = \overbrace{3^{2/3}\Gamma(2/3)}^{C_1}\, Ai\left(x_0\sqrt[3]{2s}\right)$$

$$\tilde{P}(s|x_0) = C_1\cdot\frac{1}{\pi}\int_{\gamma_1} dt\, \exp\left((2s)^{1/3} x_0 t - \frac{t^3}{3}\right) =$$

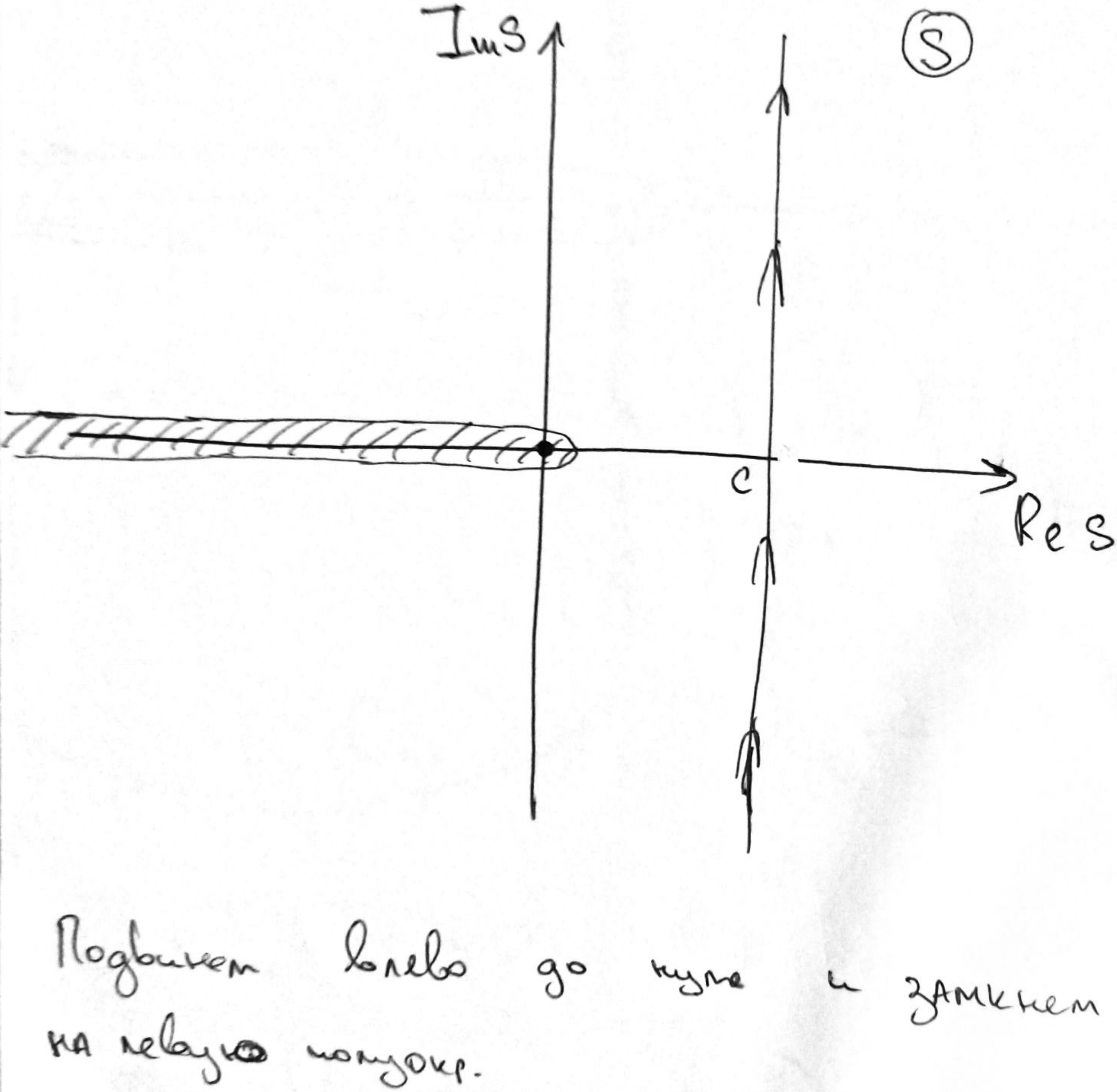


$$= \frac{C_1}{\pi}\int_{\gamma_1'} dp\, \frac{1}{(2s)^{1/3}} \exp\left(x_0 p - \frac{p^3}{6s}\right)$$

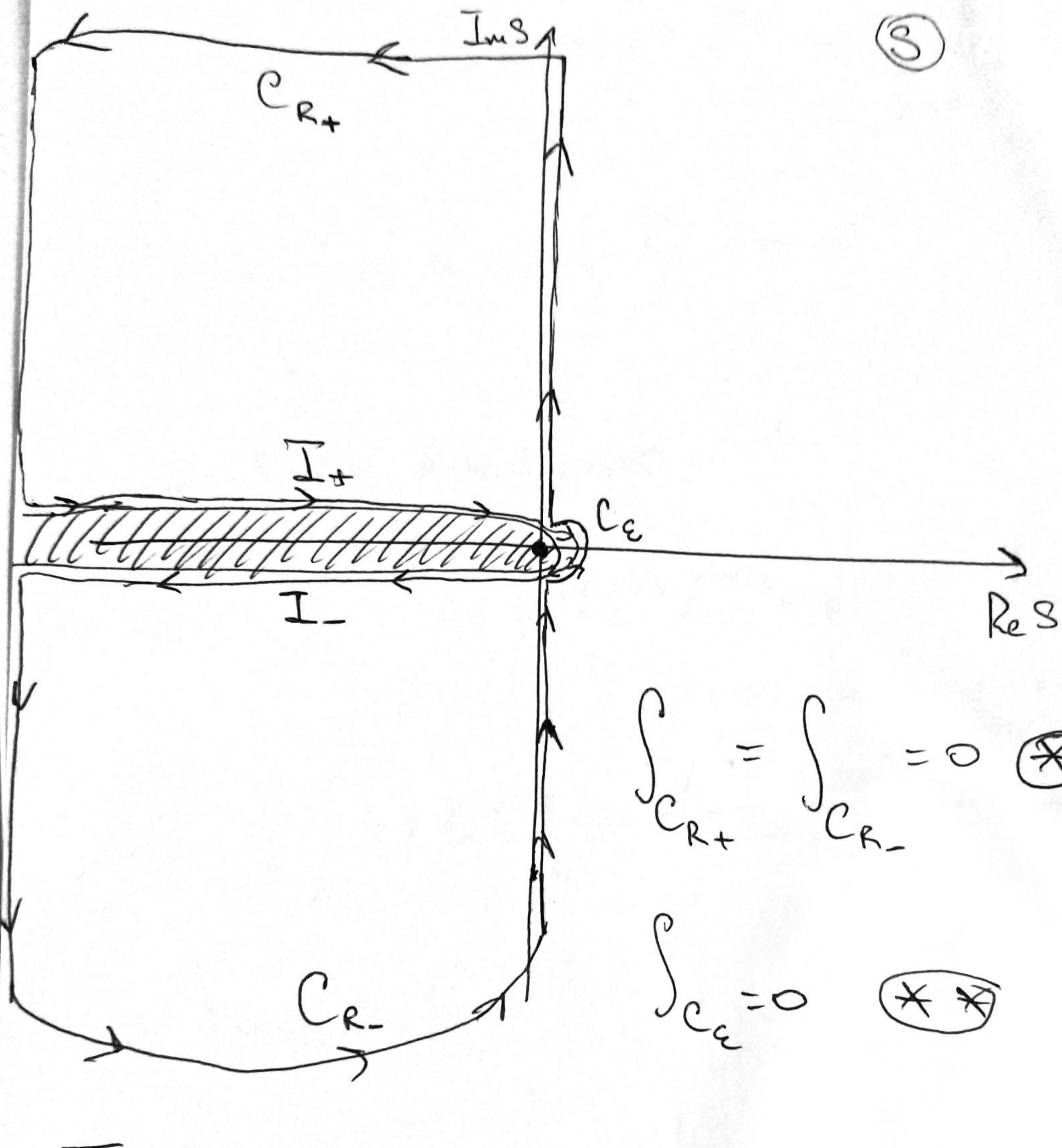
$$P(T|x_0) = \frac{C}{2\pi^2 i\, 2^{1/3}} \int_{\gamma_1^p} dp \exp(x_0 p) \underbrace{\int_{c-i\infty}^{c+i\infty} ds\, s^{-1/3} \exp\left(sT - \frac{p^3}{6s}\right)}_{I}$$

$$\beta = \frac{p^3}{6}$$

I: $s=0$ точка ветвл.



Подвинем влево до нуля и замкнем контур на левую полуокр.



$$\int_{C_{R_+}} = \int_{C_{R_-}} = 0 \quad (*)$$

$$\int_{C_\varepsilon} = 0 \quad (**)$$

$$I + I_+ + I_- = 0$$

$$I = -\int_{-\infty}^{0} f_+ - \int_{0}^{-\infty} f_- = \int_{0}^{-\infty} (f_+ - f_-)$$

$(*)$: $\displaystyle\int_{C_{R+}} = \int_{\pi/2}^{\pi} d\varphi \cdot Re^{i\varphi} \cdot R^{-1/3} \cdot e^{-i\varphi/3} \cdot \exp\Big[\underbrace{RTe^{i\varphi}}_{Re<0} - \frac{p^3}{6s}\Big]$

$$\Rightarrow \int_{C_{R+}} \sim \int d\varphi\, R^{\frac{2}{3}} \exp(-R) \to 0$$

$\displaystyle\int_{C_{R-}} = 0$ — аналогично

$(**)$: $\displaystyle\int_{C_\varepsilon} = \int_{\pi/2}^{-\pi/2} d\varphi\, \varepsilon e^{i\varphi} \varepsilon^{-1/3} e^{-i\varphi/3} \exp\Big(\varepsilon T e^{i\varphi} - \frac{p^3}{6\varepsilon} e^{-i\varphi}\Big)$

Re < 0, т.к. интегрируем по γ_1 дальше

$$\int_{C_\varepsilon} \sim \int_{\pi/2}^{-\pi/2} d\varphi\, \varepsilon^{2/3} \cdot \exp\Big(\underbrace{-\frac{p^3}{6\varepsilon} e^{-i\varphi}}_{Re<0}\Big) \to 0$$

$$I = \int_0^{-\infty} \exp\Big(sT - \frac{p^3}{6s}\Big) \cdot \frac{ds}{s^{1/3}} \cdot \Big[1 - e^{-2\pi i/3}\Big] =$$

$$= \Big(\frac{3}{2} - i\frac{\sqrt{3}}{2}\Big) \cdot \int_0^{-\infty} \exp\Big(sT - \frac{p^3}{6s}\Big) \frac{1}{s^{1/3}} ds$$

$$= \Big(-\frac{3}{2} + i\frac{\sqrt{3}}{2}\Big) \cdot e^{-i\pi/3} \int_0^{\infty} \exp\Big(-sT + \frac{p^3}{6s}\Big) \frac{1}{s^{1/3}} ds \ominus$$

$$\textcircled{=}\left(-\frac{3}{2}+i\frac{\sqrt{3}}{2}\right)\cdot\left(\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)\cdot\int_0^\infty s^{-1/3}\,ds\,\exp\left(-sT+\frac{p^3}{6s}\right)=$$

$$= i\sqrt{3}\int_0^\infty s^{-(-1/3)-1}\exp\left(-T\cdot s+\frac{p^3}{6}\frac{1}{s}\right)\textcircled{=}$$

$$\int_0^\infty x^{-\nu-1}e^{-\alpha x-\beta/x}\,dx = 2\left(\frac{\alpha}{\beta}\right)^{\nu/2}K_\nu\left(2\sqrt{\alpha\beta}\right)$$

$$\textcircled{=}\; i2\sqrt{3}\left(\frac{\alpha}{\beta}\right)^{\nu/2}K_\nu\left(2\sqrt{\alpha\beta}\right);\quad \alpha=T\quad \beta=-\frac{p^3}{6}$$

$$\nu=-2/3$$

$$\cancel{I = i2\sqrt{3}\left(\frac{\alpha}{\beta}\right)}$$

$$I = i2\sqrt{3}\left(\frac{-6T}{p^3}\right)^{-1/3}K_{-2/3}\left(2\sqrt{-\frac{p^3T}{6}}\right)$$

$$\left.\begin{aligned} K_{+\alpha}&=\frac{\pi}{2}\frac{I_{-\alpha}(x)-I_\alpha(x)}{\sin\alpha\pi}\\ K_{-\alpha}&=\frac{\pi}{2}\frac{I_\alpha(x)-I_{-\alpha}(x)}{-\sin\alpha\pi}\end{aligned}\right\}\; K_\alpha=K_{-\alpha}$$

$$I = i2\sqrt{3}\left(-\frac{6T}{p^3}\right)^{1/3}K_{+2/3}\left(2\sqrt{-\frac{p^3T}{6}}\right)$$

$$P(T|X_0)=\int_{\gamma_p}dp\oint\exp(x_0p)\cdot\frac{\sqrt{3}}{\pi^2}\cdot(3T)^{1/3}\cdot\left(\frac{1}{-p^3}\right)^{1/3}K_{2/3}\left(2\sqrt{\frac{-Tp^3}{6}}\right)$$

Не знаю што с этим делать