N1

$$K(\varphi_f,\varphi_i,t) = \langle \varphi_f | \sum |\psi_l\rangle\langle\psi_l| e^{iHt/\hbar} |\varphi_i\rangle =$$

$$= \sum \Psi_l(\varphi_f)\, e^{iEt/\hbar}\, \Psi_l^*(\varphi_i)$$

$$H = \frac{\hbar^2}{2mR^2}\frac{\partial^2}{\partial\varphi^2} \qquad L_z = -i\frac{\partial}{\partial\varphi} \qquad [H, L_z] = 0 \Rightarrow \Psi_l = e^{il\varphi}/\sqrt{2\pi}$$

$$E_l = \frac{\hbar^2 l^2}{2mR^2}$$

$$K(\varphi_f,\varphi_i,t) = \frac{1}{2\pi}\sum \exp[il(\varphi_f-\varphi_i)]\exp\left(i\hbar l^2 t/2mR^2\right)$$

2) $$K'(\varphi_f,\varphi_i,t) = \langle\varphi_f| e^{iH\Delta t}|\varphi_i\rangle = \int\limits_{\varphi(\frac{\Delta t}{2})=\varphi_i}^{\varphi(\frac{\Delta t}{2})=\varphi_f} D\varphi \exp\left[-\frac{i\hbar}{2mR^2}\int\limits_{-\frac{\Delta t}{2}}^{\frac{\Delta t}{2}}\dot\varphi^2 dt\right]$$

$$K'(\varphi_f,\varphi_i,t) = \underbrace{\int\limits_{\varphi'(\frac{\Delta t}{2})=0}^{\varphi'(\frac{\Delta t}{2})=0} D\varphi' \exp\left[-\frac{imR^2}{2\hbar}\int\limits_{-\frac{\Delta t}{2}}^{\Delta t/2}\dot\varphi'^2 dt\right]}_{=\sqrt{\frac{mR^2}{2\pi\hbar\Delta t i}}} \cdot \sum_{n=-\infty}^{\infty}\exp\left[\frac{mR^2}{2\hbar\Delta t}(\varphi_f-\varphi_i+2\pi n)^2\right] =$$

$$= \sqrt{\frac{mR^2}{2\pi i\hbar\Delta t}}\sum_{n=-\infty}^{\infty}\exp\left[-\frac{mR^2}{2\hbar\Delta t}(\varphi_f-\varphi_i+2\pi n)^2\right]$$

3) $$K'(\varphi_f,\varphi_i,t) = \frac{1}{2\pi}\int_0^{\varphi}\sum_{l=-\infty}^{\infty} e^{il(\varphi_f-\varphi_i)}\, d[\varphi_f-\varphi_i]\sqrt{\frac{mR^2}{2\pi i\hbar\Delta t}}\cdot\sum_{n=-\infty}^{\infty}\exp\left[\frac{imR^2}{2\hbar\Delta t}\varphi_f-\varphi_i+2\pi n\right]$$

$$= \frac{1}{2\pi}\cdot\sqrt{\frac{mR^2}{2\pi i\hbar\Delta t}}\sum_{l=-\infty}^{\infty}\cdot\underbrace{\sum_{n=-\infty}^{\infty}\int_0^{2\pi}}_{=\int_{-\infty}^{\infty}} d\delta\varphi\, \exp\left[i\frac{mR^2}{\hbar\Delta t}(\delta\varphi-2\pi n)^2 - il\delta\varphi\right] =$$

$\int_{-\infty}^{\infty}$ ← смещением пределов интегрирования в каждом члене ряда

$$= \frac{1}{2\pi}\sqrt{\frac{mR^2}{2\pi i\hbar\Delta t}}\sum_{l=-\infty}^{\infty}\int_{-\infty}^{\infty}dx\,\exp\left[i\frac{mR^2}{2\hbar\Delta t}x^2 - ilx\right] =$$

$$= \frac{1}{2\pi}\sum e^{il(\varphi_f-\varphi_i)}\quad\exp\left[\frac{-i\hbar l^2}{2mR^2}\Delta t\right] \Rightarrow \boxed{K' = K}$$

■

N/2

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}\left(|\psi_l\rangle + |\psi_r\rangle\right)$$

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}\left(|\psi_l\rangle - |\psi_r\rangle\right)$$

$$|\psi_l\rangle = \frac{1}{\sqrt{2}}\left(|\psi_0\rangle + |\psi_1\rangle\right)$$

$$|\psi_r\rangle = \frac{1}{\sqrt{2}}\left(|\psi_0\rangle - |\psi_1\rangle\right)$$

$$|\psi\rangle = \left(a_l|\psi_l\rangle + a_r|\psi_r\rangle\right) e^{-i\omega t/2}$$

$$i\hbar \frac{\partial}{\partial t}\begin{pmatrix} a_l \\ a_r \end{pmatrix} = H \begin{pmatrix} a_l \\ a_r \end{pmatrix}$$

$$i\hbar \frac{\partial}{\partial t}\left[\left(a_l|\psi_l\rangle + a_r|\psi_r\rangle\right) e^{-i\omega t/2}\right] = H \cdot \left[\left(a_l|\psi_l\rangle + a_r|\psi_r\rangle\right) e^{-\frac{i\omega t}{2}}\right]$$

$$-\frac{i\hbar i\omega}{2} \cdot |\psi\rangle + i\hbar \frac{\partial a_l}{\partial t}|\psi_l\rangle e^{-\frac{i\omega t}{2}} + i\hbar \frac{\partial a_r}{\partial t}|\psi_r\rangle e^{-\frac{i\omega t}{2}} = H \cdot \left[\frac{a_l}{\sqrt{2}}\left(|\psi_0\rangle + |\psi_1\rangle\right) + \right.$$

$$\left. + \frac{a_r}{\sqrt{2}}\left(|\psi_0\rangle - |\psi_1\rangle\right)\right] e^{-i\omega t/2}$$

$$\cancel{e^{\frac{i\omega t}{2}}} \frac{\hbar\omega}{2}\cancel{|\psi\rangle} \quad i\hbar \frac{\partial}{\partial t}\left(a_l|\psi_l\rangle + a_r|\psi_r\rangle\right) =$$

$$H|\psi_0\rangle \cdot \frac{1}{\sqrt{2}}\left(a_l + a_r\right) + H|\psi_1\rangle \frac{1}{\sqrt{2}}\left(a_l - a_r\right)$$

$$\frac{\hbar\omega}{2}\left(a_l|\psi_l\rangle + a_r|\psi_r\rangle\right) + i\hbar \frac{\partial}{\partial t}\left(a_l|\psi_l\rangle + a_r|\psi_r\rangle\right) = \frac{1}{\sqrt{2}}\left(E_0 - \frac{\Delta E}{2}\right)\left(a_l + a_r\right)|\psi_0\rangle$$

$$+ \frac{1}{\sqrt{2}}\left(E_0 + \frac{\Delta E}{2}\right)\left(a_l - a_r\right)|\psi_1\rangle$$

$$i\hbar\frac{\partial}{\partial t}\left(a_l|\psi_l\rangle + a_r|\psi_r\rangle\right) = -\frac{\hbar\omega}{2}\left(a_l|\psi_l\rangle + a_r|\psi_r\rangle\right) + \frac{1}{2}\left(E_0 - \frac{\Delta E}{2}\right)(a_l + a_r)\cdot$$

$$\cdot\left(|\psi_l\rangle + |\psi_r\rangle\right) + \frac{1}{2}\left(E_0 + \frac{\Delta E}{2}\right)(a_l - a_r)\left(|\psi_l\rangle - |\psi_r\rangle\right)$$

$$i\hbar\frac{\partial}{\partial t}a_l = -\frac{\hbar\omega}{2}a_l + \frac{1}{2}\left(E_0 - \frac{\Delta E}{2}\right)(a_l + a_r) + \frac{1}{2}\left(E_0 + \frac{\Delta E}{2}\right)(a_l - a_r)$$

$$i\hbar\frac{\partial}{\partial t}a_l = -\cancel{\frac{\hbar\omega}{2}}a_l + \cancel{\frac{E_0}{2}}a_l + \cancel{\frac{E_0}{2}}a_r - \cancel{\frac{E_0}{2}}a_r - \frac{\Delta E}{2}a_r$$

аналогично:

$$i\hbar\frac{\partial}{\partial t}a_r = -\frac{\Delta E}{2}a_l \quad\Rightarrow\quad i\hbar\frac{\partial}{\partial t}\begin{pmatrix}a_l\\ a_r\end{pmatrix} = \begin{pmatrix}0 & -\frac{\Delta E}{2}\\ -\frac{\Delta E}{2} & 0\end{pmatrix}\begin{pmatrix}a_l\\ a_r\end{pmatrix}$$

N3

1) $\Psi = \sum_n a_n \Psi_n \qquad \Psi_n = \pi^{-1/4} \exp\left[-\frac{\left(x - \frac{\pi}{2}(2n+1)\right)^2}{2}\right]$

$$\Psi = \pi^{-1/4} \sum_n a_n \exp\left[\frac{-\{x - \frac{\pi}{2}(2n+1)\}^2}{2}\right]$$

Инстантон может туннелировать только в соседнюю яму $\Rightarrow$

$$\Rightarrow i\hbar \frac{\partial}{\partial t} \bar{a} = \begin{pmatrix} 0 & -\mu & 0 & \dots & & & \\ -\mu & 0 & -\mu & 0 & \dots & & \\ & -\mu & 0 & -\mu & 0 & \dots & \\ 0 & & -\mu & 0 & -\mu & 0 & \dots \\ & 0 & & -\mu & 0 & -\mu & 0 \dots \\ & & & & -\mu & 0 & \mu \; 0 \dots \end{pmatrix} \bar{a} \Rightarrow$$

$$\Rightarrow \frac{\partial}{\partial t} a_n \cdot i\hbar = -\mu\left[a_{n+1} + a_{n-1}\right]$$

2) $a_k = \frac{1}{\sqrt{N}} \sum a_n \exp[-ink]$

$$a_n = \frac{1}{\sqrt{N}} \sum a_k \exp[ink]$$

$$i\hbar \frac{\partial}{\partial t} a_k = -2\mu \cos(k)\, a_k$$

$\cancel{a_k \sim \exp\left(-2\mu \cos(k)\, t\right)}$

$\cancel{a_n \to \frac{1}{2\pi} \int_{-\pi}^{\pi} \exp\left(ikn - 2\mu \cos(k)\right) dk\ ?}$

$$a_k \sim \exp\left[i \frac{2\mu}{\hbar} \cos(k)\, t\right] \Rightarrow a_n \to \frac{1}{2\pi} \int_{-\pi}^{\pi} \exp\left(ikn + \frac{i2\mu}{\hbar} \cos(k)\, t\right)$$

$$a_n = \frac{1}{2\pi}\int_{-\infty}^{\infty} \exp\left[ikn + \frac{i2\mu}{\hbar} t\left[1-\frac{k^2}{2}\right]\right] dk =$$

$$= \sqrt{\frac{\hbar}{4\pi i \mu t}} \exp\left[\frac{i\hbar n^2}{4\mu t} + \frac{2i\mu t}{\hbar}\right] \sim \frac{1}{\sqrt{t}} \exp\left[\frac{i\hbar n^2}{4\mu t} + \frac{2i\mu t}{\hbar}\right]$$

Закон дисперсии $\omega = -\frac{2\mu}{\hbar}\cos k$

$$\ddot{x} = V'(x) \Rightarrow \ddot{x} = \dot{x}\frac{d\dot{x}}{dx} = \frac{a^2}{2}\cdot 2\frac{1}{a}\cdot \sin\left(\frac{2x}{a}\right)\cdot\frac{1}{2} \Rightarrow$$

$$\Rightarrow \dot{x}\frac{d\dot{x}}{x} = \frac{a}{2}\sin\frac{2x}{a} \Rightarrow \frac{\dot{x}^2}{2} - \frac{\dot{x}_0^2}{2} = \int^{x}\frac{a}{2}\sin\frac{2x'}{a}dx'$$

$$\frac{\dot{x}^2}{2} = \frac{\dot{x}_0^2}{2} - \frac{a^2}{4}\cos\left[\frac{2x}{a}\right]$$

$$\dot{x} = \sqrt{\dot{x}_0^2 - \frac{a^2}{2}\cos\frac{2x}{a}} \qquad x = \pi/2 : \dot{x} = 0 \Rightarrow$$

$$\Rightarrow \dot{x}_0^2 = \frac{a^2}{2} \Rightarrow \dot{x} = \frac{a}{\sqrt{2}}\sqrt{1-\left(1-2\sin^2\left(\frac{x}{a}\right)\right)} =$$

$$= a\sqrt{\sin^2\frac{x}{a}} = a\left|\sin\frac{x}{a}\right|$$

$$\frac{dx}{dt} = a\left|\sin\frac{x}{a}\right| \Rightarrow \int^{x}\frac{d(x'/a)}{\left|\sin\frac{x'}{a}\right|} = t - t_0$$

$$t - t_0 = \mathrm{sign}\left(\sin\frac{x}{a}\right)\cdot \mathrm{Log}\left[\tan\frac{x}{2a}\right]$$

$$\sin\frac{x}{a} > 0 : \quad x = 2a\cdot \arctan\left[\exp(t-t_0)\right]$$

$$\dot{x} = a\left|2\sin\frac{x}{2a}\cos\frac{x}{2a}\right| = 2a\cdot\frac{e^{t-t_0}}{\sqrt{1+e^{2(t-t_0)}}}\cdot\frac{1}{\sqrt{1+e^{2(t-t_0)}}}$$

$$\dot{x} = 2a\cdot\frac{e^{t-t_0}}{1+e^{2(t-t_0)}}$$

$\sin\frac{x}{a}<0$: $\quad \dot{x} = 2a\cdot\dfrac{e^{-(t-t_0)}}{1+e^{-2(t-t_0)}}$

$$\lim_{t\to\infty}\dot{x} = 2a\cdot e^{-(t-t_0)}$$

$$\Rightarrow \dot{x}_{t\to\pm\infty} = 2a\cdot\exp\left[-|t|\right]$$

$$\lim_{t\to-\infty}\dot{x} = 2a\cdot e^{(t-t_0)}$$

$$f_1 = -A_0\dot{x}_{t\to\pm\infty} = -2aA_0\exp\left[-|t|\right]$$

$$f_2 = \mp 2aA_0\exp\left[-|t|\right]$$

$$S_1 = \int\sqrt{2V(x)}\,dx = a^2\int_{-\frac{\pi}{2}}^{\pi/2}d\varphi\cos\varphi = 2a^2$$

$$A_0 = \frac{1}{\sqrt{S_1}} = \frac{1}{\sqrt{2a^2}}$$

$$W = 8a^2A_0^2$$

$$\det O' = -\left(\frac{2aA_0}{W}\right)^2\cdot\Big\{e^{-\beta}\underbrace{\langle f_2|f_2\rangle}_{\sim e^{\beta}} - e^{\beta}\underbrace{\langle f_1|f_1\rangle}_{\sim 1}\Big\} =$$

$$= \cancel{4}\,e^{\beta}\left(\frac{2a\frac{1}{\sqrt{2a^2}}}{8a^2\frac{1}{2a^2}}\right)^2 = e^{\beta}\cdot\left(\frac{\sqrt{2}}{4}\right)^2 = e^{\beta}/8$$

$$U = e^{-S_1\cancel{/\hbar}}F_1 \qquad F_1 = \frac{\beta}{A_0}\sqrt{\frac{1}{2\pi\cdot 2\pi\det O'}} = \cancel{\frac{\beta e^{-\beta/2}}{\sqrt{8\pi^2\cdot a}}}\;\cancel{+}$$

$$\cancel{+\,\frac{\beta e^{-\beta/2}}{8\pi a}} = \beta\sqrt{\frac{2a^2\,8}{2\pi\cdot 2\pi\cdot\cancel{8}}}\;e^{-\beta/2} = \frac{\cancel{4}\,2\beta a}{\pi}e^{-\beta/2}$$

$$F_1 = \beta \frac{e^{-\beta/2}}{\sqrt{\pi}} \sqrt{K_1} \quad K_1 = \frac{4a^2}{\pi}$$

$$3) \langle \psi_n | U | \psi_1 \rangle = e^{-\beta\omega/2} \sum_{n=0}^{\infty} \frac{(\beta\mu)^{2n+1}}{(2n+1)!} =$$

$$= e^{-\beta\omega/2} \sinh(\beta \cdot \mu)$$

$F_1 = \beta \frac{e^{-\beta/2}}{\sqrt{\pi}} \sqrt{K_1} \quad K_1 = \frac{4a^2}{\pi}$

3) $\langle \psi_n | U | \psi_1 \rangle = e^{-\beta \omega/2} \sum_{n=0}^{\infty} \frac{(\beta \mu)^{2n+1}}{(2n+1)!} =$

$= e^{-\beta \omega/2} \sinh(\beta \cdot \mu)$