N3.1.

(i) $\frac{d\rho}{dt} = -\rho \cdot P \Rightarrow \frac{d\rho}{dt} = -\rho^2 \beta \Rightarrow$

$\Rightarrow \tilde{P}(T) = \left(1 - \int_0^T \tilde{P}(t)\,dt\right) \cdot \rho(T)$

$\Rightarrow -\tilde{P} = \frac{\tilde{P}' \cdot \rho - \rho' \tilde{P}}{\rho^2} \Rightarrow \cancel{\tilde{P}' = \tilde{P} \cdot / \rho}$

$\Rightarrow -\tilde{P}\rho^2 + \rho'\tilde{P} = \tilde{P}'\rho \Rightarrow \tilde{P}' = \frac{-\tilde{P}\rho^2 + \rho'\tilde{P}}{\rho}$

$\frac{\tilde{P}'}{\tilde{P}} = \frac{\rho' - \rho^2}{\rho} = \frac{(1+\rho_0\beta T)\left[-\frac{(\rho_0\beta)^2}{(1+\rho_0\beta T)^2} - \frac{(\rho_0\beta)^2}{(1+\rho_0\beta T)^2}\right]}{\rho_0\beta} =$

$= \frac{-2\rho_0\beta}{1+\rho_0\beta T} \Rightarrow \ln\tilde{P} = -2\ln(1+\rho_0\beta T) + \ln C$

$\tilde{P} = \frac{\tilde{C}}{(1+\rho_0\beta T)^2}$ тк $\tilde{C}^{-1} = \int \frac{dT}{(1+\rho_0\beta T)^2} = \frac{1}{\rho_0\beta} \Rightarrow$

$\Rightarrow \boxed{\tilde{P} = \frac{\rho_0\beta}{(1+\rho_0\beta T)^2}}$

(ii) Медиана: $\int_0^{T_{1/2}} \tilde{P}\,dt = \frac{1}{2} \Rightarrow \frac{\rho_0\beta T_{1/2}}{1+\beta\rho_0 T_{1/2}} = \frac{1}{2} \Rightarrow T_{1/2} = \frac{1}{\rho_0\beta}$