N7.1

$$\partial_t P(n,t) = -(\lambda+\mu)P(n,t) + \mu P(n+1,t) + \lambda P(n-1,t)$$

$$P_k(t) = \frac{1}{\sqrt{N}} \sum_n P(n,t)\, e^{-ink} \qquad k = -\pi, -\pi + \frac{2\pi}{N}, \ldots$$

$$\ldots 0, \frac{2\pi}{N}, \ldots \pi - \frac{2\pi}{N}$$

$P_k(t)$ - ?

$$N \to \infty : \sum_k - \frac{N}{2\pi} \int_{-\pi}^{\pi} dk \qquad \lambda t \gg 1 \qquad \mu t \gg 1$$

$$\frac{1}{\sqrt{N}} \sum \frac{\partial P_n}{\partial t} e^{-ink} = \frac{1}{\sqrt{N}} \Big[-(\lambda+\mu) \Big\{ \sum P_n e^{-ink} \Big\}$$

$$+ \mu e^{ik} \Big\{ \sum P_{n+1} e^{-ik(n+1)} \Big\} + \lambda e^{-ik} \Big\{ \sum P_{n-1} e^{-ik(n-1)} \Big\} \Big]$$

$$\frac{\partial P_k}{\partial t} = P_k \cdot \Big[\mu\big(\exp[ik] - 1\big) + \lambda\big(\exp[-ik] - 1\big) \Big]$$

$$P_k = P_0 \exp[\gamma t] \qquad \gamma = \lambda\big(\exp[-ik] - 1\big) + \mu \cdot$$
$$\cdot \big(\exp[ik] - 1\big)$$

$$P_k = \frac{1}{\sqrt{N}} \sum P(n,t) \exp[-ink] \quad ; \quad N \to \infty, \sum_k \to \frac{N}{2\pi} \int_{-\pi}^{\pi}$$

$$P_n = \frac{1}{\sqrt{N}} \sum_k P_k \exp(ink)$$

$$\gamma = \left[[\mu + \lambda]\cos k + i(\mu - \lambda)\sin k - \lambda + \mu \right]$$

$$\operatorname*{argmax}_{k} \left[\mathrm{Re}(\gamma) \right] = 0$$

$$P_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} \exp\left[ikn + \gamma t\right] \approx$$

$$\approx \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp\left[ikn + i(\mu - \lambda)k - \lambda + \mu - (\mu+\lambda)\left(1 - \frac{k^2}{2}\right)\right.$$

$$= \frac{\exp\left[-\frac{(n + (\mu-\lambda)t)^2}{2(\mu+\lambda)t}\right]}{\sqrt{2\pi t(\mu+\lambda)}}$$

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ii) $x = \frac{n}{N} \Rightarrow \cancel{P(n+1)}$

$$P\left(\frac{n+1}{N}, t\right) = P(x,t) + \frac{\partial P}{\partial x}\frac{1}{N} + \frac{\partial^2 P}{\partial x^2}\frac{1}{2N^2} + \dots$$

$$P\left(\frac{n-1}{N}, t\right) = P(x,t) - \frac{\partial P}{\partial x}\frac{1}{N} + \frac{\partial^2 P}{\partial x^2}\frac{1}{2N^2}$$

$$\Rightarrow \partial_t P = \frac{\mu+\lambda}{2N^2}\frac{\partial^2 P}{\partial x^2} + \frac{\mu-\lambda}{N}\partial_x P$$

$$P_k = \frac{1}{N} \exp\left[i\frac{(\mu-\lambda)}{N} t - \frac{\mu+\lambda}{2N^2} t k^2 \right]$$

Аналогично i) :

$$P = \int_{-\infty}^{\infty} \frac{1}{2\pi N} \exp\left[-\frac{(\mu+\lambda) t k^2}{2N^2} + i\left(\frac{\mu-\lambda}{N} t + x\right) k \right]$$

$$P \approx \frac{\exp\left[-\frac{[(\mu-\lambda)t + Nx]^2}{2t(\mu+\lambda)} \right]}{\sqrt{2\pi t(\mu+\lambda)}}$$