N1.1 $p(x) = A e^{-\lambda x}, \quad x \geq 0$

$\lambda > 0$

$$\int_0^\infty p(x)\,dx = 1 \Rightarrow A^{-1} = \int_0^\infty e^{-\lambda x}\,dx = \frac{1}{\lambda} \Rightarrow \underline{A = \lambda}$$

$$A = \lambda \Rightarrow p(x) = \lambda e^{-\lambda x}$$

$$\mathbb{E}(X) = \int_0^\infty \lambda e^{-\lambda x} \cdot x\,dx = \frac{\lambda}{\lambda^2} \int_0^\infty \xi e^{-\xi}\,d\xi = \frac{1}{\lambda}$$

$$\mathbb{E}(X^2) = \int_0^\infty \lambda e^{-\lambda x} x^2\,dx = \frac{\lambda}{\lambda^3} \int_0^\infty \xi^2 e^{-\xi}\,d\xi = 2/\lambda^2$$

$$D(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \frac{1}{\lambda^2}$$

$$G(k) = \int_{\cancel{-\infty}\,0}^\infty \lambda e^{ikx} e^{-\lambda x}\,dx = \lambda \int_{\cancel{-\infty}\,0}^\infty e^{x(-\lambda + ik)}\,dx$$

$$= \frac{\lambda}{\lambda - ik} = \left(1 - \frac{ik}{\lambda}\right)^{-1}$$

$$\left.\frac{dG}{dk}\right|_{k=0} = i \cdot \mathbb{E}(X)$$

$$\left.\frac{d^nG}{dk^n}\right|_{k=0} = i^n\, \mathbb{E}(x^n) \Rightarrow \mathbb{E}(x^n) = \frac{dG}{i^n dk}$$

$$\frac{d^nG}{dk^n}\Big|_{k=0} = \left(\frac{\lambda}{\lambda - ik}\right)^{(n)}\Big|_{k=0} \quad \frac{i^n\, n!}{\lambda^n} \cdot \left(1 - \frac{ik}{\lambda}\right)^{-n-1}\Big|_{k=0} =$$

$$= \frac{n!}{\lambda^n} \cdot i^n \quad \Rightarrow \mathbb{E}(X^n) = \frac{n!}{\lambda^n}$$