N11.1

$$P = \int_0^T (1-\alpha)\, x \, dx$$

$$\dot{x}(t) = k\alpha x \qquad x(0) = x_0 > 0$$

$$J(x,t) = \min_{\alpha} \left(\int (\alpha - 1) x \, dt + \frac{\partial J}{\partial t} + \frac{\partial J}{\partial x} \alpha \cancel{k} x \right.$$

$$\alpha^*(x,t) = \arg\min_{\alpha} \left((\alpha - 1) x \quad \frac{\partial}{\partial x} J \, k x \right.$$

$$-\frac{\partial J}{\partial t} = \min\left((\alpha - 1)x + \frac{\partial J}{\partial x} \alpha k x\right) = \begin{cases} \frac{\partial J}{\partial x} k x & \cancel{x} \cancel{\mathcal{L}} \cancel{x} > \frac{\partial J}{\partial x} \alpha k x \\ -x & -x < \frac{\partial J}{\partial x} \alpha k x \end{cases}$$

$$\partial_t J = \begin{cases} x & \partial_x J > -\frac{1}{k} \quad (\mathrm{I}) \\ -\partial_x J \, \cancel{k} x & \partial_x J \leq -\frac{1}{k} \quad (\mathrm{II}) \end{cases} \qquad \begin{array}{l} \to t > T - \frac{1}{k} \\ t < T - \frac{1}{k} \end{array}$$

$$J(x,T) = 0$$

$\cancel{\partial J}$ (I): $J = x(t - T)$

(II): $J = 0$

$$\alpha^*(x,t) = \begin{cases} \cancel{2} \alpha 0 & (\mathrm{I}) \\ \cancel{a} -x & (\mathrm{II}) \end{cases}$$

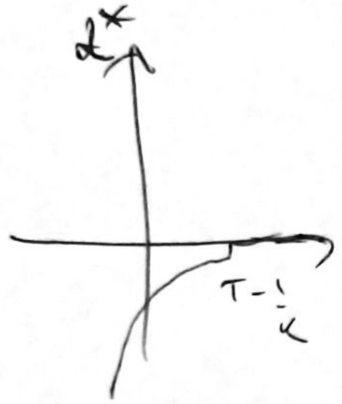
$t > T - \frac{1}{k}$: $\dot{x} = -kx^2 \Rightarrow \frac{dx}{x^2} = -kt + C \Rightarrow -\frac{1}{x} = -kt + C$

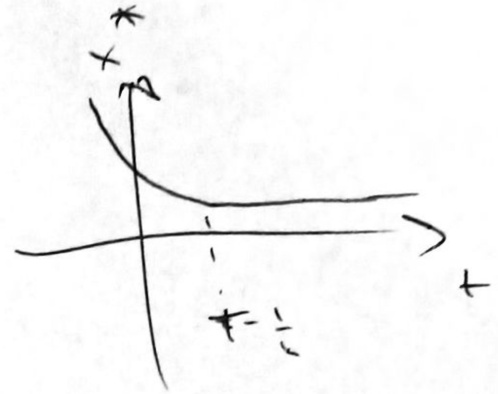
$$x = \frac{x_0}{1 + x_0 k t}$$

$t < T - \frac{1}{k}$: $x = x_0$

$t > T - \frac{1}{k}$: $x = \frac{x_0}{1 + x_0 k(T - \frac{1}{k})}$

$t < T - \frac{1}{k}$: $x = \frac{x_0}{1 + x_0 \cancel{\ldots} t}$





$$P = \int_0^t (1 - \alpha) x \, dt = \int_0^{T - \frac{1}{k}} (1 - x) x \, dt +$$

$$\int_{T - \frac{1}{k}}^{T} \frac{x_0 \, dt}{1 + x_0 k (T - \frac{1}{k})} = \int_0^{T - \frac{1}{k}} \frac{x_0}{1 + x_0 k t} dt - \int_0^{T - \frac{1}{k}} \frac{x_0^2}{(1 + x_0 k t)^2} dt +$$

$$+ \int_{T - \frac{1}{k}}^{t} \frac{x_0 \, dt}{1 + x_0 k (T - \frac{1}{k})} = \cancel{\frac{x_0}{k x_0} \log(1 + k x_0 t) \Big|_0^{T - \frac{1}{k}}}$$

$$= \frac{\ln(1 - x_0 + kTx_0)}{k} + \frac{(kt - 1) x_0^2}{k(1 - x_0 + kTx)} + \frac{1}{k}\left[\frac{x_0}{1 - x_0 kT - x_0}\right]$$