N1

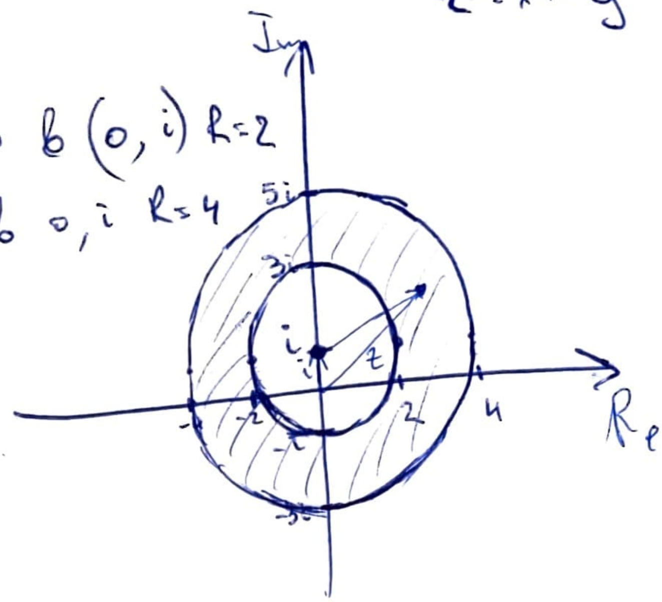
1) $2 \le |z - i| \le 4$ $\qquad z = x + iy$

границы:

$|z - i| = 2$ — окр в $(0, i)$ $R = 2$

$|z - i| = 4$ — окр в $0, i$ $R = 4$

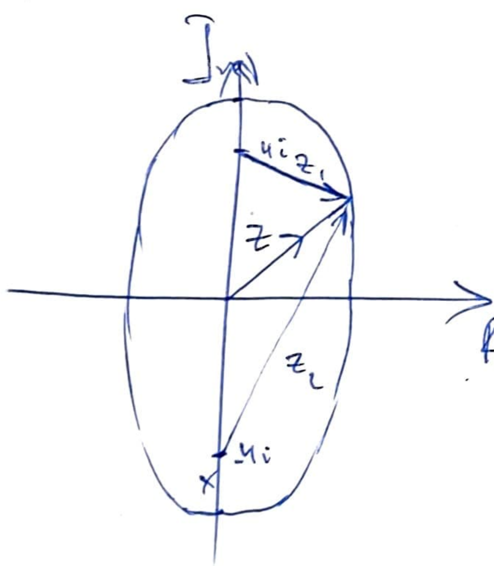
— кольцо




$A = A_2 - A_1 \qquad A_2 = \pi R_2^2 \qquad A_1 = \pi R_1^2$

$R_2 = 4 \quad R_1 = 2 \Rightarrow A = \pi(4^2 - 2^2) = 12\pi$

2) $|z - 4i| + |z + 4i| = 10$




$|z_1| + |z_2| = 10$ — сумма векторов

$z_1 = z - 4i$ $\qquad z_1$ и $z_2 = 10$

$z_2 = z + 4i$ $\qquad \Rightarrow$ эллипс центр $(0, 0)$

большая ось: $8 + x + x = 10$.

$2x = 2 \quad x = 1 \Rightarrow$ б. полуось $= \underline{5}$

$z = x + iy$: $\quad |x + iy - 4i| + |x + iy + 4i| = 10$

$x^2 + (y-4)^2 + x^2 + (y+4)^2 + 2\sqrt{x^2 + (y-4)^2}\sqrt{x^2 + (y+4)^2} = 100$

$2x^2 + 2y^2 + 2\sqrt{x^2 + (y-4)^2}\sqrt{x^2 + (y+4)^2} = 68$

$\sqrt{x^2 + (y-4)^2}\sqrt{x^2 + (y+4)^2} = 34 - x^2 - y^2$

$x^4 + x^2(y+4)^2 + x^2(y-4)^2 + (y-4)^2(y+4)^2 = 34^2 + x^4 + y^4 + 2x^2y^2 - 68(x^2+y^2)$

$x^4 + x^2(y^2+8y+16) + x^2(y^2-8y+16) + (y^2-16)^2 = 34^2 + x^4 + y^4 + 2x^2y^2 - 68(x^2+y^2)$

$x^4 - x^4 + 2x^2y^2 - 2x^2y^2 + 8x^2y - 8x^2y + 32x^2 + y^4 - 32y^2 + 16^2 - y^4 = 34^2 - 68x^2 - 68y^2$

$100x^2 + 36y^2 = (34-16)(34+16) \Rightarrow x^2 + \frac{36y^2}{100} = 9 \Rightarrow$

$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{5}\right)^2 = 1$ – эллипс, центр $(0,0)$

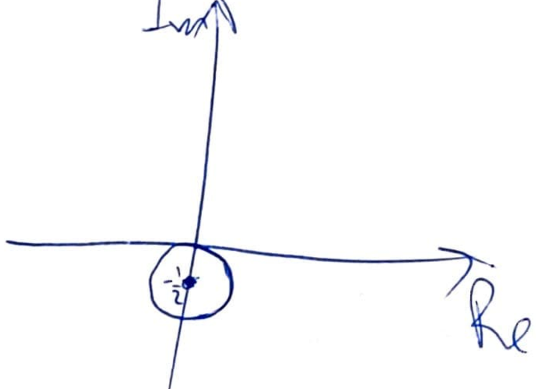
б. полуось – 5

3) $\operatorname{Im}\frac{1}{z} = 1$

$\operatorname{Im}\frac{z^*}{zz^*} = 1 \quad \operatorname{Im}\frac{x-iy}{x^2+y^2} = 1 \Rightarrow -y = x^2+y^2$

$x^2 + \left(y+\frac{1}{2}\right)^2 = \frac{1}{4}$ – ур-е окр   центр $(0,0)$

радиус $\frac{1}{2}$



N2

1) $1+2\varepsilon+3\varepsilon^2+\dots+n\varepsilon^{n-1} = \dfrac{n}{\varepsilon-1}$

$$S_n = \frac{b_1(q^n-1)}{q-1} = \frac{\varepsilon(\varepsilon^n-1)}{\varepsilon-1} = \varepsilon+\varepsilon^2+\dots+\varepsilon^n$$

$$\frac{dS_n}{d\varepsilon} = 1+2\varepsilon+3\varepsilon^2+\dots+n\varepsilon^{n-1}$$

$$\frac{dS_n}{d\varepsilon} = \frac{(\varepsilon^n-1)}{\varepsilon-1} + \frac{(n\varepsilon^n)}{\varepsilon-1} + \frac{\varepsilon(1-\varepsilon^n)}{(\varepsilon-1)^2} =$$

$$= \frac{(\varepsilon^n-1)(\varepsilon-1) + (n\varepsilon^n)(\varepsilon-1) + \varepsilon(1-\varepsilon^n)}{(\varepsilon-1)^2} =$$

$$= \frac{\cancel{\varepsilon^{n+1}} - \varepsilon^n - \cancel{\varepsilon} + 1 + n\varepsilon^{n+1} - n\varepsilon^n + \cancel{\varepsilon} - \cancel{\varepsilon^{n+1}}}{(\varepsilon-1)^2} =$$

$$= \frac{n\varepsilon^{n+1} - \varepsilon^n(n+1) + 1}{(\varepsilon-1)^2} = \frac{\varepsilon^n(n\varepsilon-n-1)+1}{(\varepsilon-1)^2} =$$

$$= \frac{(n(\varepsilon-1)-1)\varepsilon^n+1}{(\varepsilon-1)^2}.$$

2) $\varepsilon^n = 1 \Rightarrow \dfrac{n(\varepsilon-1)}{(\varepsilon-1)^2} + \dfrac{-1+1}{(\varepsilon-1)^2} = \dfrac{n}{(\varepsilon-1)}$

N3

1) $\operatorname{Im} z = 1 \qquad z \to \omega(z) = z^3 + 3z - i$

$z = x + iy \quad y = 1 \Rightarrow \quad \omega(z) = (x+i)^3 + 3(x+i) - i$

$= x^3 + 3x^2 i + \cancel{3xi^2} + \cancel{i^3} + \cancel{3x} + \cancel{3i} - \cancel{i} =$ $+i$

$= x^3 + 3ix^2 + i$

$\omega = u + iv \Rightarrow u + iv = x^3 + i(3x^2+1)$

$\begin{cases} u = x^3 \\ v = 3x^2+1 \end{cases} \Rightarrow \operatorname{Im}\omega = 1 + 3[\operatorname{Re}\omega]^{2/3}$

2) $|z - i| = 1 \qquad z \to \omega(z) = \frac{1}{z-2i}$

$z = x + iy$

$\cancel{(x+iy-i)^2 = 1}$ $\quad x^2 + (y-1)^2 = 1$

$\cancel{x^2+}$ $\quad \omega = \frac{1}{x+iy-2i} = \frac{x - i(y-2)}{(x+i(y-2))(x-i(y-2))} =$

$= \frac{x - i(y-2)}{x^2 + (y-2)^2} \qquad \operatorname{Re}(\omega) = \frac{x}{x^2+(y-2)^2} = \frac{\frac{1}{2}x}{2-y}$

$\operatorname{Im}(\omega) = \frac{2-y}{x^2+(y-2)^2} = \frac{1}{2} \cancel{x y} + 0 \cdot \text{Re}$

N4

1) $z = x + iy$

$\omega = x^2 + y^2$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \Leftrightarrow \frac{\partial \omega}{\partial x} + i\frac{\partial \omega}{\partial y} = 0$$

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

$$\frac{\partial \omega}{\partial x} + i\frac{\partial \omega}{\partial y} = 2x + i2y = 2z \neq 0 \Rightarrow \text{не аналит.}$$

2) $\omega = x^2 - y^2 + 2ixy$

$$\frac{\partial \omega}{\partial x} + i\frac{\partial \omega}{\partial y} = 2x + 2iy + i2y(-1) + 2i^2x =$$

$= 0$ — аналит.

3) $\omega = \frac{1}{x+iy}$

$$\frac{\partial \omega}{\partial x} + i\frac{\partial \omega}{\partial y} = \frac{-1}{(x+iy)^2} + i\frac{-i}{(x+iy)^2} = 0$$