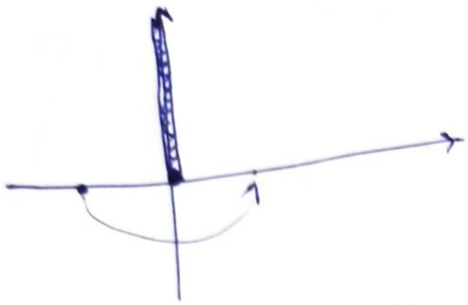
N1

$\varphi(z) = \sqrt[3]{z}$

a) $\varphi(-1) = e^{i\pi/3}$ $\qquad z \in [0; i\infty]$



$$\frac{\varphi(1)}{\varphi(-1)} = \left(\frac{e^{i\varphi_1}}{e^{i\varphi_{-1}}}\right)^{1/3}$$

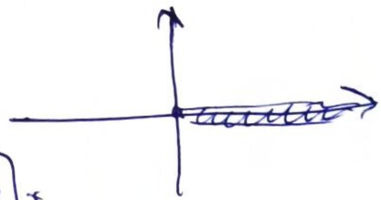
$$\Rightarrow \varphi(1) = e^{i\pi/3} \cdot \left(\frac{e^{i\cdot 0}}{e^{i\pi}}\right)^{1/3} = e^{i\pi 2/3}$$

$$\varphi(i+0) = \varphi(-1) \cdot e^{i[\varphi_{i+0} - \varphi_{-1}]\frac{1}{3}} = e^{i\pi/3 + i/3\left[\frac{\pi}{2} + \pi\right]} = e^{\frac{5i\pi}{6}}$$

$$\varphi(i-0) = \varphi(-1) \cdot e^{i[\varphi_{i-0} - \varphi_{-1}]\cdot\frac{1}{3}} = e^{i\pi/3 + \frac{i}{3}\left[-\frac{3\pi}{2} + \pi\right]} = e^{+\frac{i\pi}{6}}$$

b) $\varphi(z) = \ln z$ $\qquad z \in [0; +\infty]$ $\qquad \varphi(1 - i0) = 0$

$\cancel{\varphi(1+i\cdot 0) = \varphi(1-i0)\ln\left[e^{i[\varphi_{1+i0} - \varphi_{1-i0}]}\right]}$



$$\varphi(1+i\cdot 0) = \ln\left[e^{i[\varphi_{1+i0} - \varphi_{1-i0}]}\right] + \underset{0}{\varphi[1-i\cdot 0]} +$$

$$+ i\cdot(-2\pi) = -2\pi i$$

Анал.

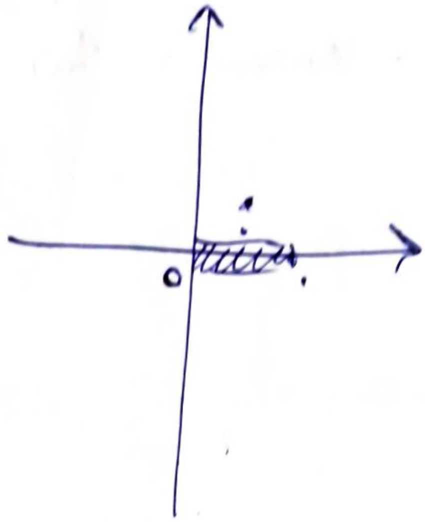
$$\varphi(i) = -\frac{3\pi}{2} i$$

$$\varphi(-i) = -\frac{\pi}{2} i$$

N4

$\varphi(z) = z^{\mu} (1-z)^{1-\mu}$

$\varphi(\frac{1}{2} + i0) = \frac{1}{2}$ $\quad z \in [0, 1]$



$\varphi(z) = \frac{1}{2} e^{i\pi(\mu-1)} \cdot \left| \frac{z^{\mu}(1-z)^{1-\mu}}{\frac{1}{2}} \right| =$

$= 2^{\mu} e^{i\pi(\mu-1)}$

$\varphi(-1) = e^{i\pi\mu} \cdot \frac{1}{2} \left| \frac{2^{1-\mu}(-1)^{\mu}}{\frac{1}{2}} \right| = 2^{1-\mu} e^{i\pi\mu}$

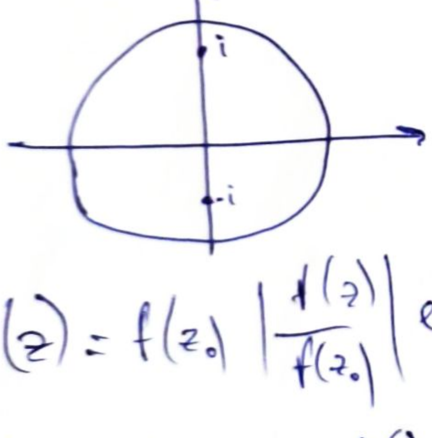
$\lim\limits_{z\to\infty} \frac{\varphi(z)}{z} = \lim\limits_{z\to\infty} \frac{1}{2} \left| \frac{z^{\mu}(1-z)^{1-\mu}}{1/2} \right| e^{i\pi(\mu-1)} \cdot \frac{1}{z} =$

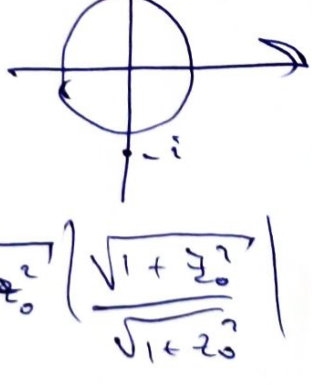
$= \lim\limits_{z\to\infty} \left| \frac{z^{\mu-1}}{(1-z)^{\mu-1}} \right| \cdot e^{i\pi(\mu-1)} = e^{i\pi(\mu-1)}$

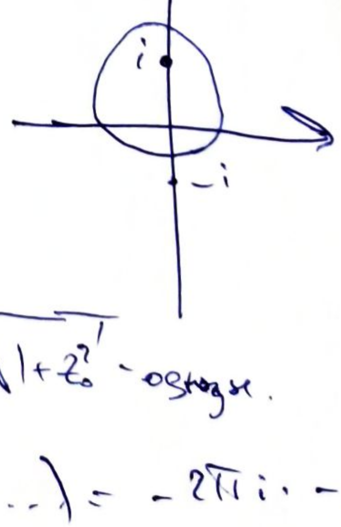
N3

$f(z) \to \sqrt{1+z^2}$ $\quad z > 0$ –ben

D_1: $\quad D_2$: $\quad D_3$:







1) $f(z) = f(z_0) \left| \frac{f(z)}{f(z_0)} \right| e^{2\pi i} = \sqrt{1+z_0^2} \left| \frac{\sqrt{1+z_0^2}}{\sqrt{1+z_0^2}} \right| = \sqrt{1+z_0^2}$ – однозн.

$\Rightarrow I = -2\pi i \cdot \operatorname{res}\limits_{z=\infty} f(z) = -2\pi i \cdot \operatorname{res}\, z\left(1 + \frac{1}{2z^2} + \ldots\right) = -2\pi i \cdot -\frac{1}{2} = \pi i$

2) Аналогич. однозн, $I = 2\pi i \sum \operatorname{res} = 0$

3) $f(z_0) \cdot e^{\pi i \cdot 1} = -f(z_0)$ – многозн.

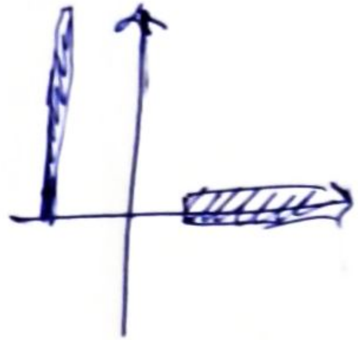
$a: f(-1) = f(+0)\, e^{i\pi} \cdot \left|\frac{f(-1)}{f(+0)}\right| = -\sqrt{2}$

$b: f(-1) =$ Anhnor $= \sqrt{2}$

N5

$$\varphi(z) = \ln(1-z^2) \qquad \varphi(0) = -2\pi i$$

$$\varphi(-2) = \ln\left[\left|\frac{1-z^2}{1-0^2}\right|\right] + \varphi(0) + i\cdot(-\pi) =$$



$$= \ln 3 - 2\pi i - i\pi = \ln 3 - 3\pi i$$

$$\varphi(-i) = \ln\left(\left|\frac{1-i^2}{1-0^2}\right|\right) + \varphi(0) + i\cdot(0) =$$

$$= \ln 2 - 2\pi i$$

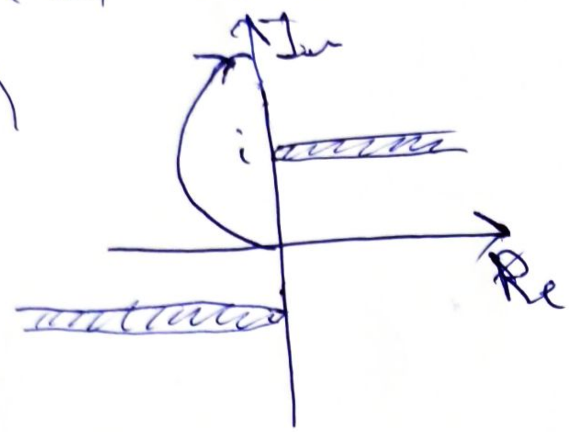
$$\varphi\left(\frac{-1+i\sqrt{3}}{2}\right) = \ln\left|1-\frac{(1-2\sqrt{3}i-3)}{4}\right| + \varphi(0) + i\cdot\left(-\frac{\pi}{6}\right) =$$

$$= \ln\sqrt{3} - \pi i\cdot\frac{11}{6}$$

N6

$\varphi(z) = \sqrt[3]{1+z^2}$

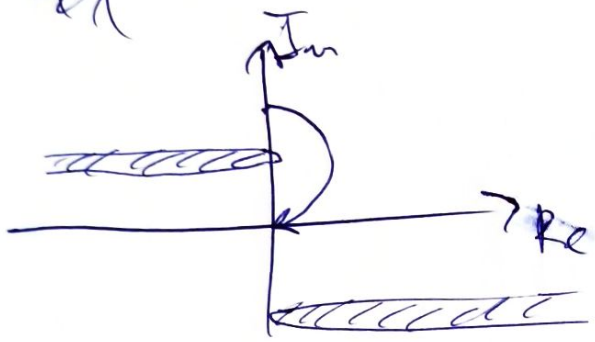
1)



$\varphi(3i) = \sqrt[3]{\left|\frac{-8}{1}\right|} \cdot e^{i\pi/3} \cdot \varphi(0) =$

$= 2 \cdot e^{-i\pi/3}$

2) $\cancel{\varphi(}$

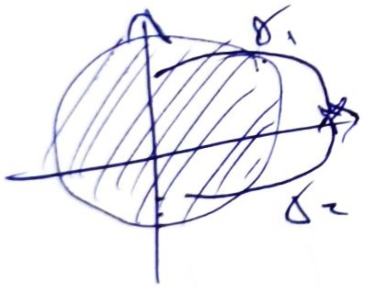


$\varphi(3i) = \sqrt[3]{\left|\frac{-8}{1}\right|} \cdot e^{i\pi/3} \cdot \varphi(0) =$

$= 2 \cdot e^{i\pi/3}$

N10

$f(z) = \sum \frac{1}{(n+1)(n+2)} z^n \qquad 1 > |z|$



$f(z) = \sum_0^\infty \frac{1}{(n+1)(n+2)} z^n = \sum_0^\infty \frac{z^n}{n+1} -$

$\sum_0^\infty \frac{z^n}{n+2} = \frac{1}{z} \cdot \sum_0^\infty \frac{z^{n+1}}{n+1} - \frac{1}{z^2} \sum_0^\infty \frac{z^{n+2}}{n+2}$

$= \frac{1}{z}\left(\cancel{\#} \sum \frac{z^{n+1}}{n+1} - \frac{1}{z} \sum \frac{z^{n+1}}{n+1} + 1\right) = \frac{1}{z^2} \ln(1-z) - \frac{1}{z} \ln(1-z)$

$+ \frac{1}{z}$

$\gamma_1: \quad \ln(1-z) = g \qquad g(z) = g(0) + \ln\left|\frac{-1}{1}\right| + i(-\pi) =$

$= \ln 1 - \pi i = -\pi i \qquad f(z) = -\frac{\pi i}{4} + \frac{\pi i}{2} + \frac{1}{2} = \frac{\pi i}{4} + \frac{1}{2}$

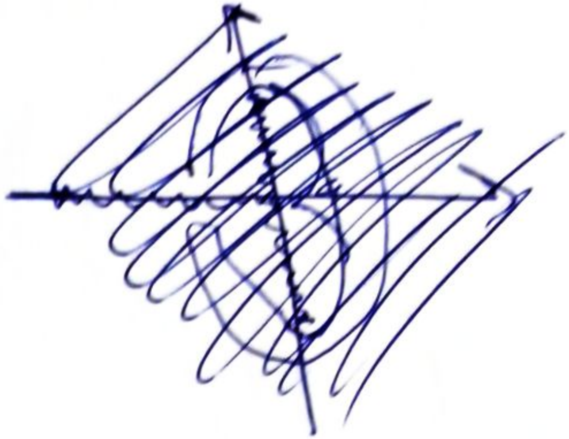
γ_2: $g(z) = g(0) + \ln\left|\frac{-1}{1}\right| + (i\pi) =$

$= \pi i \Rightarrow f(z) = \frac{-\pi i}{4} + \frac{1}{2}$

N7

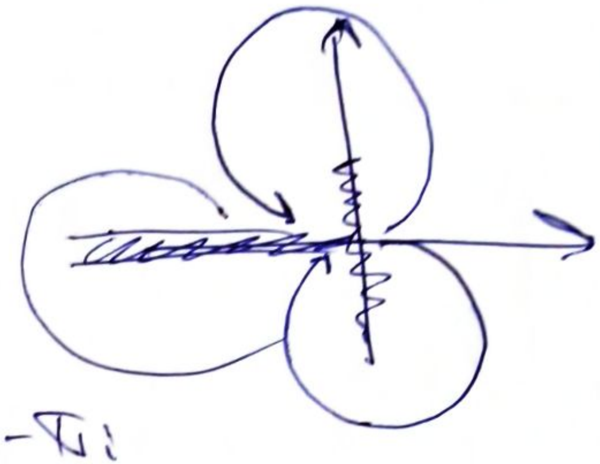
$f(x) = \ln\left((x^2+1)^{1/2}\right) =$

$= \ln\left([(x+i)(x-i)]^{1/2}\right)$



1) $\lim\limits_{\varepsilon\to 0} F(\varepsilon) = \lim\limits_{\varepsilon\to 0} \ln\sqrt{1+\varepsilon^2} = 0$

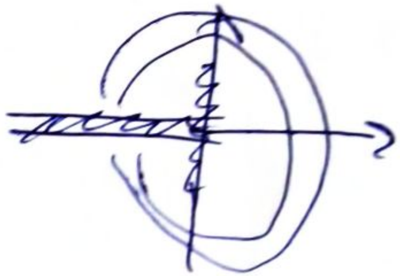
2) $\lim\limits_{\varepsilon\to} F(\varepsilon e^{3\pi i/4}) = i\pi$



3) $\lim\limits_{\varepsilon\to 0} F(\varepsilon e^{-3\pi i/4}) = -\pi i$

$F(z) = \ln\sqrt{1+z^2} = f(x)$ при об. по контуру

$\Delta\arg = 0 \to f(z)$ однозн. в A.



N 8

1) $f(z) = \ln(z)$

$f(z) = -\ln\left(\frac{1}{z}\right)$

$f(\hat{z}) = f(\hat{z}_0) + \ln|1| - 2\pi i = +2\pi i + f(\hat{z}_0)$ - многозн ~~...~~ ($z=\infty$ т. ветв)

2) $f(z) = \ln\frac{z-1}{z+1} \to \ln\frac{1/z - 1}{1/z + 1} = \frac{1-z}{1+z}$

$f(\hat{z}) = f(\hat{z}_0) + \ln 1 + 0 = f(\hat{z}_0)$ - однозн не т. ветв.

3) $f(z) = \ln(z^2 - 1) \to \ln\left(\left(\frac{1}{z} - 1\right)\left(\frac{1}{z} + 1\right)\right) =$

$= \ln\left(\frac{(1-z)(1+z)}{z^2}\right)$ $\quad f(\hat{z}) = f(\hat{z}_0) + \ln\left|\frac{z^2 - 1}{1/z_0 - 1}\right| + (-4\pi i)$

$= -4\pi i + f(\hat{z}_0)$ - неодн

4) $f(z) = \sqrt{z^2 - 1} \to \sqrt{\frac{(1-z)(1+z)}{z^2}}$

$f(\hat{z}) = e^{2\pi i} f(\hat{z}_0) = f(\hat{z}_0)$ - однозн.

N8.

$\Delta \arg f = a\Delta\arg z + b\Delta\arg(z-1) = 2\pi a$

$f(z) = z^a (z-1)^b$

$\dfrac{f(z+i0)}{f(z-i0)} = 1\cdot e^{i\cdot 2\pi a}$

$z=0$ — т. ветвления $\forall a \notin \mathbb{Z}$

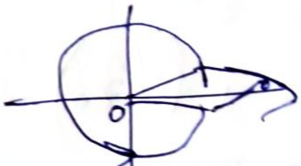
$\dfrac{f(z+i0)}{f(z-i0)} = \left|\dfrac{f(z+i0)}{f(z-i0)}\right|\cdot e^{i\Delta\arg f}$ = аналог. $e^{-i2\pi b}$.

$z=1$ — т. ветвл. $\forall b \notin \mathbb{Z}$

$z=\infty$: $f(z) = z^a(z-1)^b = \left(\frac{1}{z}\right)^{-a}\cdot z^b\left(1-\frac{1}{z}\right)^b = \left(\frac{1}{z}\right)^{-a-b}\left(1-\frac{1}{z}\right)^b$

$\tilde z = \frac{1}{z}$ $\quad f(\tilde z) = z^{-(a+b)}(1-z)^b$ аналог

$\dfrac{f(\tilde z+i0)}{f(\tilde z-i0)} = \overbrace{\left|\dfrac{f(\tilde z+i0)}{f(\tilde z-i0)}\right|}^{=1}\cdot e^{-2\pi(a+b)}$



— $z=\infty$ — т. ветвл при $a+b \notin \mathbb{Z}$

$$\Rightarrow \begin{cases} N(1,1)=0 \\ N(1,\frac{1}{2})=2 \\ N(\frac{1}{2},\frac{1}{3})=3 \\ N(\frac{2}{3},\frac{1}{3})=2 \end{cases}$$

N11

$z_n(t)$ $\{n=0,1,2)$

$z^3-3z^2+t=0$

$z_n(t\gg1)\approx t^{1/3}e^{\pi i+2\pi i n/3}$

$z^2(z-3)=-t$ $t=0:$ $z^2=0$ $z_{1,2}=0$

$z-3=0$ $z_3=3$

$z^3-2z^2=z^2-t$ $=(z-\sqrt{t})(z+\sqrt{t})$

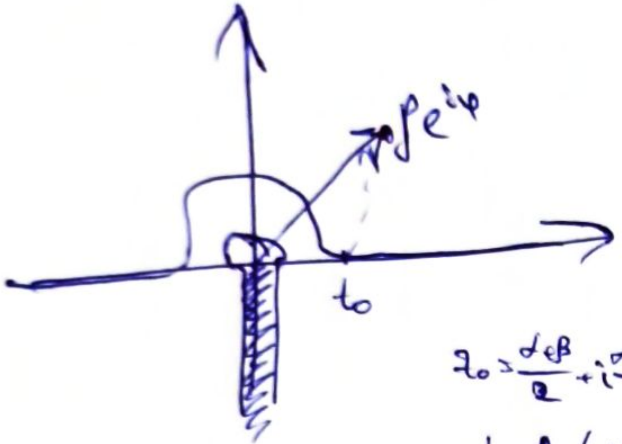
$t=0,4$

$z^2-z-2=0$

$z_{1,3}=2$

$z_2=-1$

~~№4~~ №11.2



по ф. Кардано или с помощью шампанского получим корни

$z^3 - z^2 + \lambda = 0 \qquad \lambda \approx 0$

$\alpha \approx \frac{i\sqrt{7}}{3} \qquad \beta \approx \frac{i\sqrt{7}}{3}$

$z_0 = \frac{\alpha+\beta}{2} + i\frac{\alpha-\beta}{2}\sqrt{3} \qquad z_0 \approx -\sqrt{\frac{t}{3}}$

$$f(t_0+\rho e^{i\varphi}) = \left|\frac{f(t_0+\rho e^{i})}{f(t_0+0)}\right| f(t_0+0)\, e$$

$$\arg f = \frac{1}{2}\arg t = \varphi/2$$

$$|f(t_0+0)| = \left|-\sqrt{\frac{t_0+0}{3}}\right| = -\sqrt{\frac{t_0+0}{3}} = -f(t_0+0)$$

$$|f(t_0+\rho e^{i\varphi})| = \left|-\sqrt{\frac{t_0+\rho e^{i\varphi}}{3}}\right| = \sqrt{\frac{\rho}{3}}$$

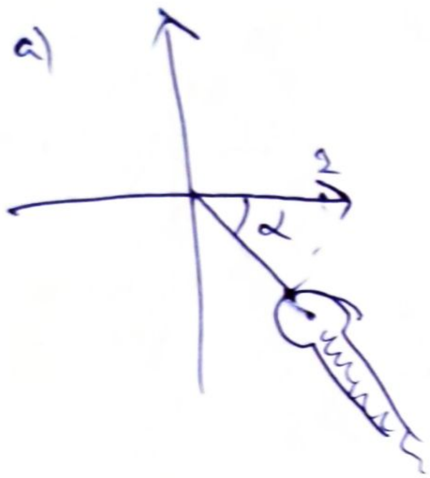
$$f = -\sqrt{\frac{\rho}{3}}\exp[i\varphi/2]$$

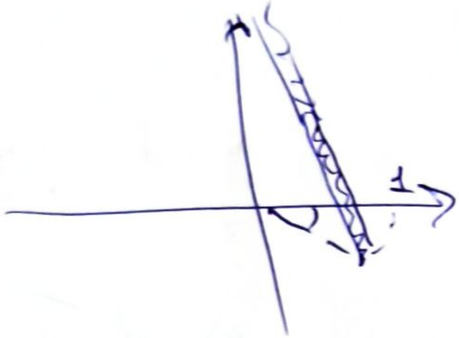
№2

$\varphi_1(z) = \sqrt{z - e^{-i\alpha}}$ $\qquad \varphi(0) = i e^{-i\alpha/2}$

$\varphi_2 = \ln(z - e^{-i\alpha})$ $\qquad \varphi_2(0) = -i\pi - i\alpha$

$g = z - e^{-i\alpha}$

a) 

b) 

a) $\varphi_1(e^{i\alpha}) = i \cdot e^{-\alpha \cdot i/2} \cdot e^{i/2\, \Delta \arg(g)} \sqrt{\frac{e^{i\alpha} - e^{-i\alpha}}{-e^{-\alpha i}}} =$

$= \sqrt{2\sin\alpha}\; e^{i/2\, \Delta\arg g} = \cancel{i e^{i\alpha/2}} \sqrt{2\sin\alpha}\; e^{i/2 \cdot \frac{\pi}{2}}$

$\varphi_1(i) = \left|\frac{g(i)}{g(0)}\right|^{1/2} \cdot \varphi_1(0) \cdot e^{i/2\, \Delta\arg(g)} =$

$= \sqrt{2 - 2\cos\frac{\pi}{2} + \alpha} \cdot e^{i\pi/2} \cdot e^{-i\alpha/2} \cdot e^{\left(-\frac{\pi}{4} + \frac{\alpha}{2}\right)\frac{i}{2}} = \sqrt{2\cos\left(\frac{\pi}{4} - \frac{\alpha}{2}\right)} \cdot \exp\left(\frac{3i\pi}{8} - \frac{i\alpha}{4}\right)$

$\varphi_2(e^{i\alpha}) = \ln[|g(e^{i\alpha})|] - \ln[|g(0)|] + \varphi_2(0) + i\Delta\arg(g(z)) =$

$= \ln[2\sin(\alpha)] - i\pi - i\alpha - \frac{i\pi}{2} + i\alpha = -\frac{3i\pi}{2} + \ln(2\sin\alpha)$

$\varphi_2(i) = \ln(|g(i)|) - \ln\{|g(0)|\} + \varphi(0) + i\Delta\arg(g) =$

$= \ln\left[2\cos\left(\frac{\pi}{4} - \frac{\alpha}{2}\right)\right] + (-i\pi - i\alpha) + i\left(\frac{\alpha}{2} - \frac{\pi}{4}\right) = -\frac{5\pi i}{4} - \frac{i\alpha}{2} +$

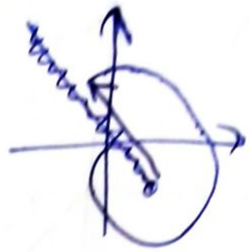
$+ \ln\left[2\cos\left(\frac{\pi}{4} - \frac{\alpha}{2}\right)\right]$

b)

$\varphi_1(e^{i\alpha}) = \left|\frac{g(e^{i\alpha})}{g(0)}\right|^{1/2} \varphi_1(0) \cdot e^{i/2\, \Delta\arg(g)} :$

$$= \sqrt{2\sin\alpha} \cdot i\, e^{-i/2\left(\alpha - \alpha + \frac{3\pi}{2}\right)} = \exp\left(\frac{5i\pi}{4}\right)\cdot\sqrt{2\sin\alpha}$$

$$\varphi_1(i) = \left|\frac{g(i)}{g(0)}\right|^{1/2} \varphi_1(0)\, e^{i/2\, \Delta\arg(g)} = \left|\frac{i - e^{-i\alpha}}{-e^{\alpha i}}\right| \cdot i\, e^{i\alpha/2} \cdot e^{i/2\, \Delta\arg(g)}$$



$x^2 = 2 - 2\cos\left(\frac{\pi}{2} - \alpha\right)$
$= 4\cos^2\left(\frac{\pi}{4} - \frac{\alpha}{2}\right)$

$$\Delta\arg_{\gamma} g = 2\pi - \frac{\pi - \left(\frac{\pi}{2} + \alpha\right)}{2} = \frac{7\pi}{4} + \frac{\alpha}{4}$$

$$\varphi_1(i) = \sqrt{2\cos\left(\frac{\pi}{4} - \frac{\alpha}{2}\right)} \cdot i\, e^{-i\alpha/2} \cdot e^{\frac{7i\pi}{8} + \frac{\alpha}{4}} =$$

$$= \sqrt{2\cos\left(\frac{\pi}{4} - \frac{\alpha}{2}\right)} \cdot \exp\left[-\frac{5i\pi}{8} - \frac{i\alpha}{4}\right]$$

$$\varphi_2(e^{i\alpha}) = \ln\left(|g(e^{i\alpha})|\right) - \ln\left(|g(0)|\right) + \varphi(0) + i\,\Delta\arg(g)$$

$$= \ln\left[2\sin(i\alpha)\right] + (-i\pi - i\alpha) + i\left(\frac{3\pi}{2} + \alpha\right) = \frac{i\pi}{2} + \ln\left[2\sin i\alpha\right]$$

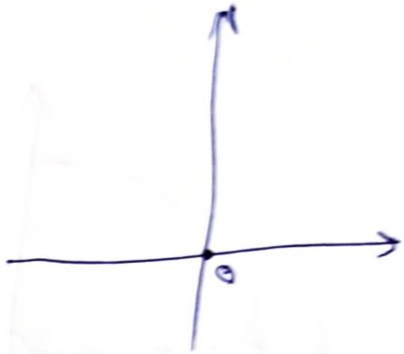
$$\varphi_2(i) = \ln\left(|g(i)|\right) - \ln\left(|g(0)|\right) + \varphi(0) + i\,\Delta\arg(g)$$

$$= \ln\left(2\cos\left(\frac{\pi}{4} - \frac{\alpha}{2}\right)\right) + (-i\pi - i\alpha) + i\left(\frac{7\pi}{4} + \frac{\alpha}{2}\right) = \ln\left(2\cos\left(\frac{\pi}{4} - \frac{\alpha}{2}\right)\right) +$$

$$+ \frac{3i\pi}{4} - \frac{i\alpha}{2}$$

№12 $\dfrac{e^{iz}}{\cos z-1} = \varphi(z)$

$$\sum_{n=-\infty}^{\infty} c_n z^n \quad \exists \in (2\pi k,\ 2\pi(k+1))$$



$$C_{-3} = \frac{1}{2\pi i}\int z^2 \varphi(z)\,dz$$

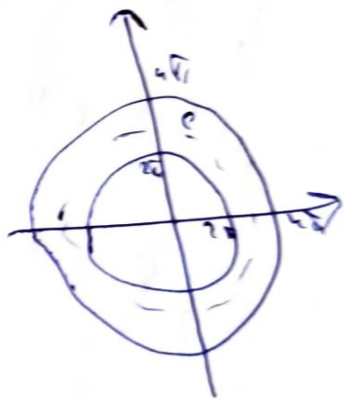
$k=0$:

$$\int_C \frac{z^2 e^{iz}}{\cos z-1} = 2\pi i \operatorname*{res}_{z=0}\left(\varphi(z)\, z^2\right)$$

$$\lim_{z\to 0} \frac{d}{dz}\left(\frac{z^4 e^{iz}}{\cos z-1}\right) = \lim \frac{e^{iz} z^3\left(-iz+z\sin z+(4+iz)\cos z-4\right)}{(\cos z-1)^2}$$

$=0$

$k=1$:



$$I=\int \frac{z^2 e^{iz}}{\cos z-1} = 2\pi i \sum \operatorname{res}\left(\varphi(z)\, z^2\right)$$

$\operatorname{res}_0 = 0$

$$\operatorname{res}_{2\pi} = \operatorname{res}\frac{(\tilde z+2\pi)^2 e^{i\tilde z}}{\cos\tilde z-1} = \operatorname{res}\frac{(\tilde z^2+4\pi\tilde z+4\pi^2)(1+i\tilde z)}{-\tilde z^2/2} = -8\pi-8\pi^2 i$$

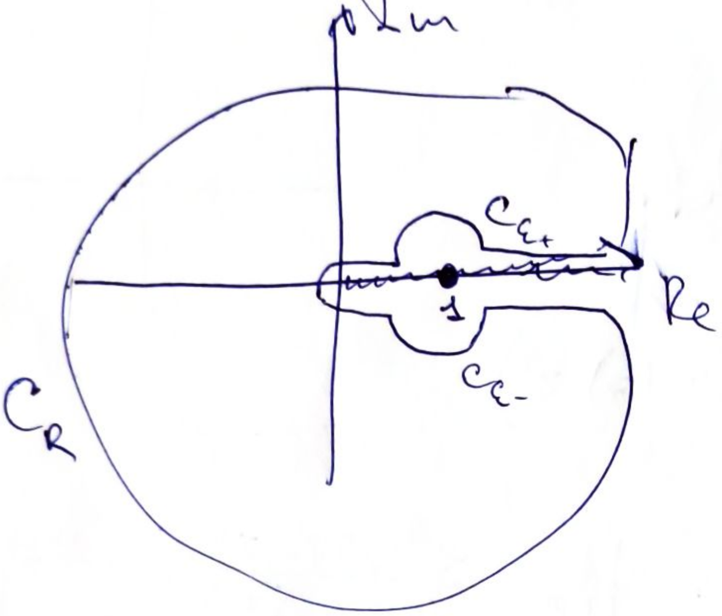
$$\operatorname{res}_{-2\pi} = \operatorname{res}\frac{(z-2\pi)^2 e^{i\tilde z}}{\cos\tilde z-1} = \operatorname{res}\frac{(\tilde z^2-4\pi\tilde z+4\pi^2)(1+i\tilde z)}{-\tilde z^2/2} = 8\pi-8\pi^2 i$$

$$\Rightarrow I = \sum \operatorname{res}\cdot 2\pi i = -16\pi^2 i$$

№3

$$I = PV \int_0^\infty \frac{x^{a-1}}{1-x^b}\,dx \qquad b > a > 0$$

a)



$$I = \int_0^\infty \frac{z^{\frac{a}{b}-1}}{b(1-z)} \qquad \oint = I + \int_{C_{\varepsilon+}} + \int_{C_{\varepsilon-}} + \cancel{\int_R}^{\,0} - e^{2\pi i \frac{a}{b}} I = 0$$

$$\int_{C_{\varepsilon+}} = \int_\pi^0 = \varepsilon i e^{i\varphi}\left(-\frac{1}{\varepsilon}e^{-i\varphi}\right)d\varphi = \pi i$$

$$\int_{C_{\varepsilon-}} = \int_0^{-\pi} \varepsilon i e^{i\varphi}\left(-\frac{1}{\varepsilon}e^{2\pi i(a/b-1)} \cdot e^{-i\varphi}\right)d\varphi = i\pi e^{2\pi i(a/b-1)}$$

$$I = -\frac{\pi i}{b}\,\frac{\left(1+e^{2\pi i\left(\frac{a}{b}-1\right)}\right)}{1-e^{2\pi i(a/b-1)}} = +\frac{\pi}{b}\,\mathrm{ctg}\left[\pi\left(\frac{a}{b}-1\right)\right] = \frac{\pi}{b}\,\mathrm{ctg}\left(\frac{\pi a}{b}\right)$$

b) $$P.V. \int_0^\infty \frac{x\,dx}{x^2+a^2 \sin bx} = \frac{1}{2} P.V. \int_{-\infty}^{\infty} \frac{x\,dx}{(x^2+a^2)\sin bx}$$

$$\oint = P.V. \int_{-a}^{a} \frac{x\,dx}{(x^2+a^2)\sin bx} + \sum \int_{C_\varepsilon} + \cancel{\int_{C_R}}^{\,0} = 2\pi i \operatorname{res}(ia)$$

$$\int_{C_\varepsilon} \frac{z\,dz}{(z^2+a^2)\sin bz} = \int_\pi^0 \frac{i(z_r+\varepsilon e^{i\varphi})\,\varepsilon e^{i\varphi}d\varphi}{((z_r+\varepsilon e^{i\varphi})^2+a^2)\sin(b(z_r+\varepsilon e^{i\varphi}))} = \int_\pi^0 \frac{i z_r\,d\varphi}{(a_r^2+a^2)b} \cdot \quad \cdot (-1)^n$$

$\Rightarrow \sum \int_{C_{\varepsilon,+}} = 0$ $\qquad$ P.V. $= 2\pi i \cdot$ ~~[illegible]~~ $\operatorname{res}(ia) = -\frac{i}{\sinh(ab)}$

~~P.V. = 2πi Σ res(...)~~ $\qquad$ Antwort: $\frac{\pi}{2\sinh(ab)}$