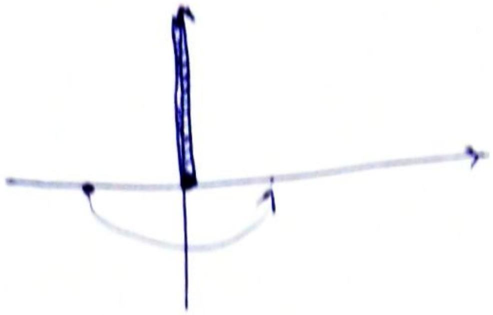
N1

$\varphi(z) = \sqrt[3]{z}$

a) $\varphi(-1) = e^{i\pi/3}$ $\quad z \in [0; i\infty]$



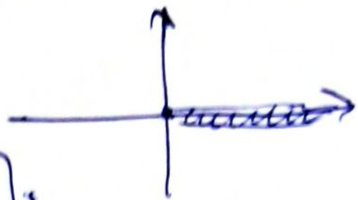
$\frac{\varphi(1)}{\varphi(-1)} = \left(\frac{e^{i\varphi_1}}{e^{i\varphi_{-1}}}\right)^{1/3}$

$\Rightarrow \varphi(1) = e^{i\pi/3} \cdot \left(\frac{e^{i\cdot 0}}{e^{-i\pi}}\right)^{1/3} = e^{i\pi 2/3}$

$\varphi(i+0) = \varphi(-1) \cdot e^{i[\varphi_{i+0} - \varphi_{-1}]\frac{1}{3}} = e^{i\pi/3 + i/3[\frac{\pi}{2} + \pi]} = e^{\frac{5i\pi}{6}}$

$\varphi(i-0) = \varphi(-1) \cdot e^{i[\varphi_{i-0} - \varphi_{-1}]\frac{1}{3}} = e^{i\pi/3 + \frac{i}{3}[-\frac{3\pi}{2} + \pi]} = e^{+\frac{i\pi}{6}}$

b) $\varphi(z) = \ln z$ $\quad z \in [0; +\infty]$ $\quad \varphi(1-i0) = 0$



$\varphi(1+i\cdot 0) = \ln\left[e^{i[\varphi_{1+i0} - \varphi_{1-i0}]}\right] + \varphi[1-i\cdot 0] +$

$\quad\quad\quad\quad\quad\quad\quad\quad 0 \quad\quad\quad\quad\quad\quad 0$

$+ i \cdot (-2\pi) = -2\pi i$

Аналог.

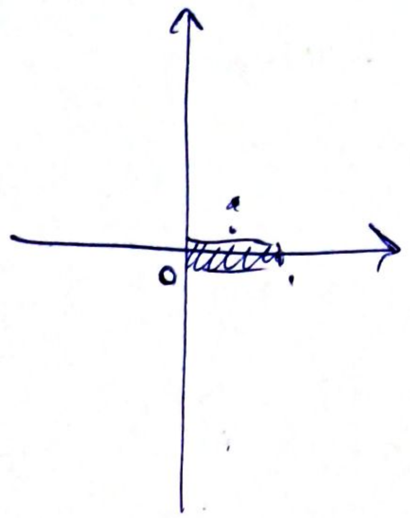
$\varphi(i) = -\frac{3\pi}{2} i$

$\varphi(-i) = -\frac{\pi}{2} i$

N4

$$\varphi(z) = z^{\mu} (1-z)^{1-\mu}$$

$$\varphi\left(\frac{1}{2} + i0\right) = \frac{1}{2} \qquad z \in [0, 1]$$



$$\varphi(2) = \frac{1}{2} e^{i\pi(\mu-1)} \cdot \left| \frac{2^{\mu}(1-2)^{1-\mu}}{\frac{1}{2}} \right| =$$

$$= 2^{\mu} e^{i\pi(\mu-1)}$$

$$\varphi(-1) = e^{i\pi\mu} \cdot \frac{1}{2} \left| \frac{2^{1-\mu}(-1)^{\mu}}{\frac{1}{2}} \right| = 2^{1-\mu} e^{i\pi\mu}$$

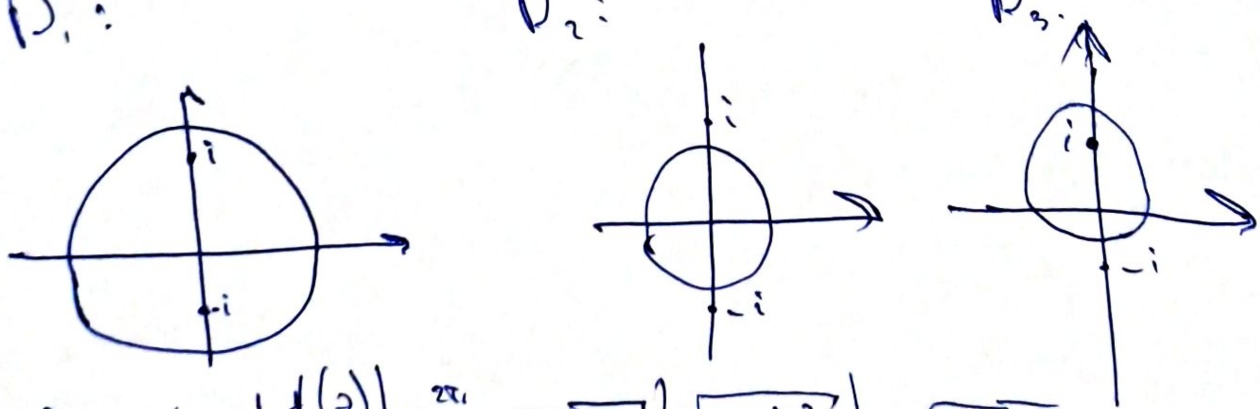
$$\lim_{z\to\infty} \frac{\varphi(z)}{z} = \lim_{z\to\infty} \frac{1}{2} \left| \frac{z^{\mu}(1-z)^{1-\mu}}{1/2} \right| e^{i\pi(\mu-1)} \cdot \frac{1}{z} =$$

$$= \lim_{z\to\infty} \left| \frac{z^{\mu-1}}{(1-z)^{\mu-1}} \right| \cdot e^{i\pi(\mu-1)} = e^{i\pi(\mu-1)}$$

N3

$f(z) \to \sqrt{1+z^2}$ $\quad z > 0$ - веще

D_1: $\qquad D_2$: $\qquad D_3$:



1) $f(z) = f(z_0) \left| \frac{f(z)}{f(z_0)} \right| e^{2\pi i} = \sqrt{1+z_0^2} \left| \frac{\sqrt{1+z_0^2}}{\sqrt{1+z_0^2}} \right| = \sqrt{1+z_0^2}$ - однозн.

$I = -2\pi i \cdot \operatorname{res}_{z=\infty} f(z) = -2\pi i \cdot \operatorname{res} z\left(1 + \frac{1}{2z^2} + \ldots\right) = -2\pi i \cdot -\frac{1}{2} =$

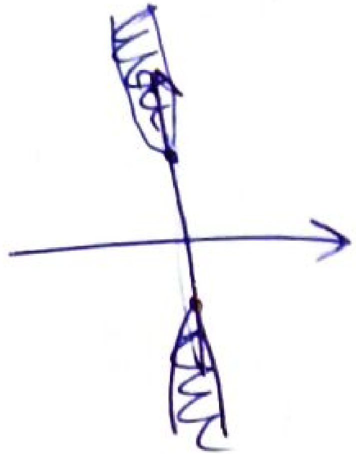
2) Аналогич. однозн, $I = 2\pi i \sum \operatorname{res} = 0$

3) $f(z_0) \cdot e^{\pi i} \cdot 1 = -f(z_0)$ - многозн.

$a: f(-1) = f_{+0}\, e^{i\pi} \cdot \left|\frac{f(-1)}{f(+0)}\right| = -\sqrt{2}$

$b: f(-1) =$ AnAnor $= \sqrt{2}$

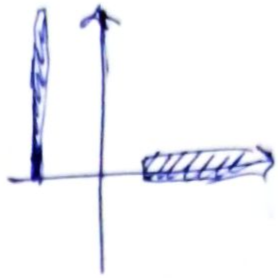
a:



N5

$\varphi(z) = \ln(1-z^2)$ $\varphi(0) = -2\pi i$;

$\varphi(-2) = \ln\left[\left|\frac{1-z^2}{1-0^2}\right|\right] + \varphi(0) + i\cdot(-\pi) =$



$= \ln 3 - 2\pi i - i\pi = \ln 3 - 3\pi i$

$\varphi(-i) = \ln\left(\left|\frac{1-i^2}{1-0^2}\right|\right) + \varphi(0) + i\cdot(0) =$

$= \ln 2 - 2\pi i$

$\varphi\left(\frac{-1+i\sqrt{3}}{2}\right) = \ln\left|1-\frac{(1-2\sqrt{3}i-3)}{4}\right| + \varphi(0) + i\cdot\left(-\frac{\pi}{6}\right) =$

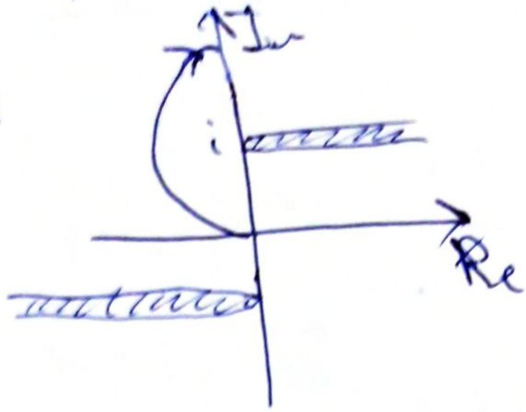
$= \ln\sqrt{3} - \pi i\cdot\frac{11}{6}$

N11.2

N6

$\varphi(z) = \sqrt[3]{1+z^2}$

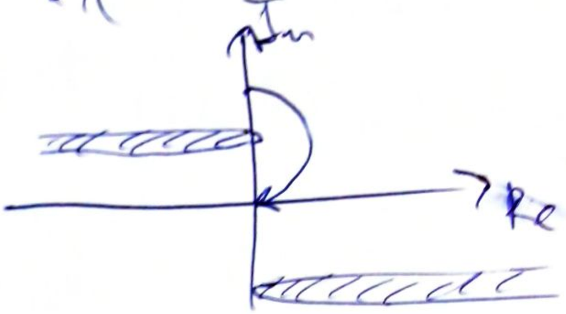
1)



$\varphi(3i) = \sqrt[3]{\left|\frac{-8}{1}\right|}\cdot e^{i\pi/3}\cdot\varphi(0) =$

$= 2\cdot e^{-i\pi/3}$

2) ~~φ(~~

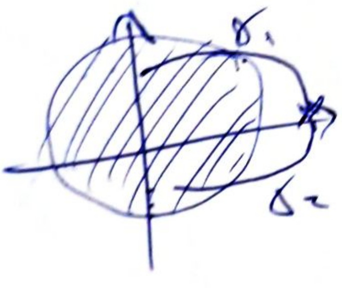


$\varphi(3i) = \sqrt[3]{\left|\frac{-8}{1}\right|}\cdot e^{i\pi/3}\cdot\varphi(0) =$

$= 2\cdot e^{i\pi/3}$

N10

$f(z) = \sum \frac{1}{(n+1)(n+2)} z^n \qquad 1 > |z|$



$f(z) = \sum_0^\infty \frac{1}{(n+1)(n+2)} z^n = \sum_0^\infty \frac{z^n}{n+1} -$

$\sum_0^\infty \frac{z^n}{n+2} = \frac{1}{z}\cdot\sum_0^\infty \frac{z^{n+1}}{n+1} - \frac{1}{z^2}\sum_0^\infty \frac{z^{n+2}}{n+2}$

$= \frac{1}{z}\left(\sum \frac{z^{n+1}}{n+1} - \frac{1}{z}\sum\frac{z^{n+1}}{n+1} + 1\right) = \frac{1}{z^2}\ln(1-z) - \frac{1}{z}\ln(1-z)$

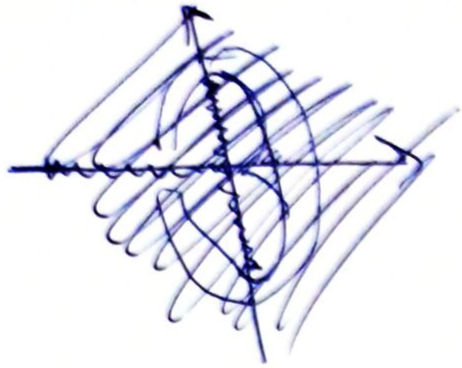
$+ \frac{1}{z}$

$\gamma_1: \quad \ln(1-z) = g \qquad g(z) = g(0) + \ln\left|\frac{-1}{1}\right| + i(-\pi) =$

$= \ln 1 - \pi i = -\pi i \qquad f(z) = -\frac{\pi i}{4} + \frac{\pi i}{2} + \frac{1}{2} = \frac{\pi i}{4} + \frac{1}{2}$

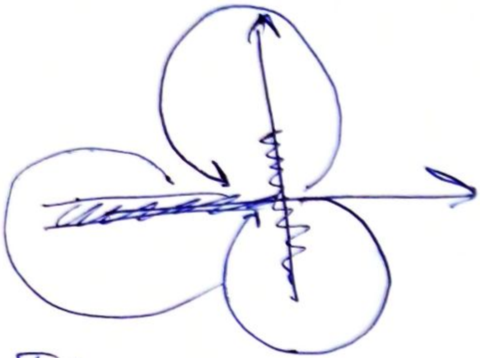
N7

$$f(x) = \ln\left((x^2+1)^{1/2}\right) = \ln\left([(x+i)(x-i)]^{1/2}\right)$$



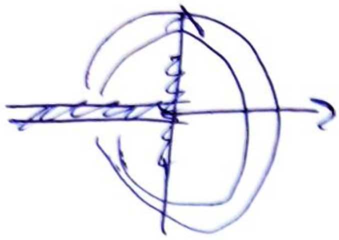
1) $\lim\limits_{\varepsilon\to 0} F(\varepsilon) = \lim\limits_{\varepsilon\to 0} \ln\sqrt{1+\varepsilon^2} = 0$

2) $\lim\limits_{\varepsilon\to} F(\varepsilon e^{5\pi i/4}) = i\pi$



3) $\lim\limits_{\varepsilon\to 0} F(\varepsilon e^{-3\pi i/4}) = -\pi i$

$F(z) = \ln\sqrt{1+z^2} = f(x)$ при gl. по контуру



$\Delta \arg = 0 \Rightarrow F(z)$ одн. в A.

N 8

1) $f(z) = \ln(z)$

$f(z) = -\ln\left(\frac{1}{z}\right)$

$f(\hat z) = f(\hat z_0) + \ln|1| - 2\pi i = -2\pi i + f(\hat z_0)$ - многозн. ~~[illegible]~~ ($z=\infty$ т. ветв)

2) $f(z) = \ln\frac{z-1}{z+1} \to \ln\frac{1/\hat z - 1}{1/\hat z + 1} = \frac{1-\hat z}{1+\hat z}$

$f(\hat z) = f(\hat z_0) + \ln 1 + 0 = f(\hat z_0)$ - однозн.

не т. ветв.

3) $f(z) = \ln(z^2-1) \to \ln\left(\left(\frac{1}{\hat z} - 1\right)\left(\frac{1}{\hat z}+1\right)\right) =$

$= \ln\left(\frac{(1-\hat z)(1+\hat z)}{\hat z^2}\right)$ $f(\hat z) = f(\hat z_0) + \ln\left|\frac{1/\hat z^2 - 1}{1/\hat z_0^2 - 1}\right| + (-4\pi i)$

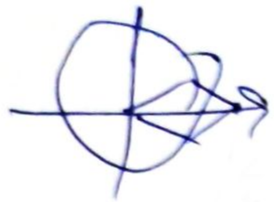
$= -4\pi i + f(\hat z_0)$ - неодн

4) $f(z) = \sqrt{z^2-1} \to \sqrt{\frac{(1-\hat z)(1+\hat z)}{\hat z^2}}$

$f(\hat z) = e^{2\pi i} f(\hat z_0) = f(\hat z_0)$ - однозн.

№8. $\Delta \arg f = a\Delta \arg z + b\Delta \arg(z-1) = 2\pi a$

$$f(z) = z^a (z-1)^b$$



$$\frac{f(z+i0)}{f(z-i0)} = 1 \cdot e^{i \cdot 2\pi a}$$

$z=0$ – т. ветвления $\forall a \notin \mathbb{Z}$

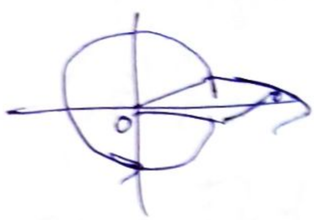
$$\frac{f(z+i0)}{f(z-i0)} = \left|\frac{f(z+i0)}{f(z-i0)}\right| \cdot e^{i\Delta \arg f} = \text{аналог. } e^{-i2\pi b}$$

$z=1$ – т. ветвл. $\forall b \notin \mathbb{Z}$

$$z=\infty: \; f(z) = z^a (z-1)^b = \left(\frac{1}{z}\right)^{-a} \cdot z^b \left(1-\frac{1}{z}\right)^b = \left(\frac{1}{z}\right)^{-a-b} \left(1-\frac{1}{z}\right)^b$$

$$\tilde{z} = \frac{1}{z} \qquad f(\tilde{z}) = z^{-(a+b)} (1-z)^b \quad \text{аналог}$$

$$\frac{f(\tilde{z}+i0)}{f(\tilde{z}-i0)} = \underbrace{\left|\frac{f(\tilde{z}+i0)}{f(\tilde{z}-i0)}\right|}_{=1} \cdot e^{-2\pi i(a+b)}$$



– $z=\infty$ – т. ветвл. при $a+b \notin \mathbb{Z}$

$$\Rightarrow \begin{cases} N(1,1) = 0 \\ N(1, 1/2) = 2 \\ N(1/2, 1/3) = 3 \\ N(2/3, 1/3) = 2 \end{cases}$$

N11

$z_n(t)$ $\quad \{n=0,1,2)$

$z^3 - 3z^2 + t = 0$

$z_n(t \gg 1) \approx t^{1/3} e^{\pi i + 2\pi i n/3}$

$z^2(z-3) = -t$ $\qquad t=0:$ $\quad z^2=0 \quad z_{1,2}=0$

$\qquad z-3=0 \quad z_3=3$

$z^3 - 2z^2 = z^2 - 1$

$z^2 - z - 2 = 0$ $\qquad t = 0,4$

$z_{1,3} = 2$

$z_2 = -1$

$$f(t_0+\rho e^{i\varphi}) = \left|\frac{f(t_0+\rho e^{i})}{f(t_0-0)}\right| f(t_0+0)\, e$$

$$\arg f = \frac{1}{2}\arg t = \varphi/2$$

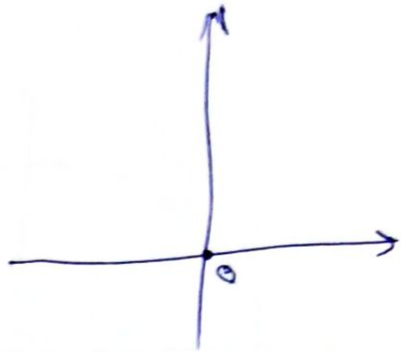
$$|f(t_0+0)| = 1-\sqrt{\frac{t_0+0}{3}} = -\sqrt{\frac{t_0+0}{3}} = -f(t_0+0)$$

$$f(t_0+\rho e^{i\varphi}) = 1-\sqrt{\frac{t_0+\rho e^{i\varphi}}{3}} = \sqrt{\frac{\rho}{3}}$$

$$f = -\sqrt{\frac{\rho}{3}}\exp\left[i\varphi/2\right]$$

$$\frac{N12\ e^{iz}}{\cos z - 1} = \varphi(z)$$

$$\sum_{n=-\infty}^{\infty} c_n z^n \quad z \in (2\pi k,\ 2\pi(k+1))$$



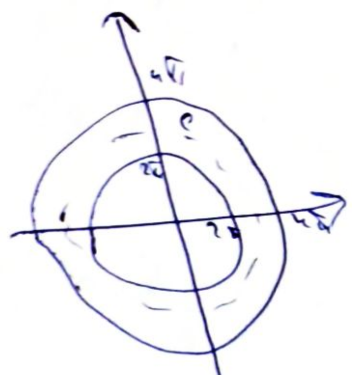
$$C_{-3} = \frac{1}{2\pi i} \int z^2 \varphi(z)\, dz$$

$k=0$:

$$\int_C \frac{z^2 e^{iz}}{\cos z - 1} = 2\pi i \operatorname*{res}_{z=0} \left(\varphi(z)\, z^2\right)$$

$$\lim_{z\to 0} \frac{d}{dz}\left(\frac{z^4 e^{iz}}{\cos z - 1}\right) = \lim \frac{e^{iz} z^3(-iz + z\sin z + (4+iz)\cos z - 4)}{(\cos z - 1)^2}$$

$= 0$



$k=1$:

$$I = \int \frac{z^2 e^{iz}}{\cos z - 1} = 2\pi i \sum \operatorname{res}\left(\varphi(z)\, z^2\right)$$

$\operatorname{res}_0 = 0$

$$\operatorname{res}_{2\pi} = \operatorname{res} \frac{(\tilde z + 2\pi)^2 e^{i\tilde z}}{\cos \tilde z - 1} = \operatorname{res} \frac{(\tilde z^2 + 4\pi \tilde z + 4\pi^2)(1 + i\tilde z)}{-\tilde z^2/2} = -8\pi - 8\pi^2 i$$

$$\operatorname{res}_{-2\pi} = \frac{\operatorname{res}(z-2\pi)^2 e^{i\tilde z}}{\cos \tilde z - 1} = \operatorname{res} \frac{(\tilde z^2 - 4\pi \tilde z + 4\pi^2)(1 + i\tilde z)}{-\tilde z^2/2} = 8\pi - 8\pi^2 i$$

$$\Rightarrow I = \sum \operatorname{res} \cdot 2\pi i = -16\pi^2 i$$

$\Rightarrow \sum \int_{C_{\varepsilon_+}} = 0$ $\qquad$ P.V. $= 2\pi i$ ~~...~~ $\operatorname{res}(ia) = -\frac{i}{\sinh(a b)}$

Answer: $\frac{\pi}{2\sinh(ab)}$