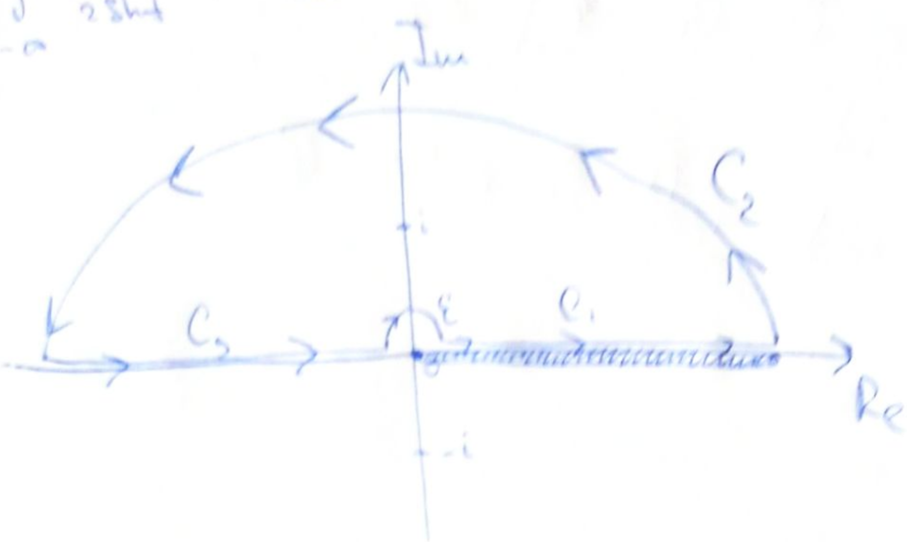
NC

$$I = \int_0^\infty \frac{\ln x\,dx}{x^2+1} \qquad \begin{cases} x = e^t \\ dx = e^t dt \end{cases} \Rightarrow I = \int \frac{dt\cdot t e^t}{e^{2t}+1} =$$

$$= \int_{-\infty}^{\infty} \frac{2t}{2\,\mathrm{sh}\,t}\,dt = \underline{0}$$



$$\oint = \int_\varepsilon + I_{1+2+3} = 2\pi i\cdot \mathrm{res}\,(z=i)$$

$$\int_\varepsilon = \lim_{\varepsilon\to 0}\int_\pi^0 \underbrace{\frac{\ln(\varepsilon e^{i\varphi})\cdot i\varepsilon e^{i\varphi}\,d\varphi}{\varepsilon^2 e^{2i\varphi}+1}}_{J} = \int_\pi^0 \lim_{\varepsilon\to 0} J = 0$$

$$2\pi i\,\mathrm{res}(z=i) = 2\pi i\cdot\frac{\ln i}{2i} = \pi\ln i = \pi i\cdot\Delta\arg z = \frac{i\pi^2}{2}$$

$$I_3 = \int_\infty^0 \frac{\ln(-z)\,dz}{z^2+1} = \int\left[\cancel{\ln\left|\frac{-z}{-z+0i}\right|}^{\,0} + \ln(z) + i\Delta\arg z\right]\frac{dz}{z^2+1} =$$

$$= I_1 + \int_0^\infty \frac{\pi i\,dx}{1+x^2}$$

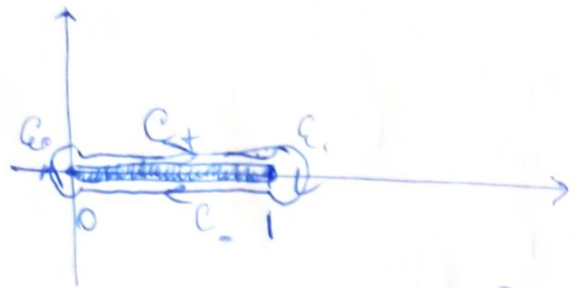
$$\oint = I_1 + I_3 = 2I_1 + \int_0^\infty \frac{\pi i\,dx}{x^2+1} = \frac{1}{2}\int_{-\infty}^{\infty}\frac{\pi i\,dx}{1+x^2} + 2I_1 =$$

$$= 2\pi i\cdot\frac{\pi i}{2}\cdot\mathrm{res}\,(z=i) + 2I_1 = 2I_1 + \frac{\pi^2 i}{2} = \frac{\pi^2 i}{2} \Rightarrow$$

$$\Rightarrow 2I_1 = 0 \Rightarrow \underline{I_1 = 0}$$

N7

$$I(\alpha) = \int_0^1 \frac{\overbrace{x^\alpha (1-x)^{2-\alpha}}^{f}}{x+1}\,dx$$



$$\oint = \int_{C+} + \int_{C-} + \overset{\to 0}{\int_{\varepsilon_0}} + \overset{\to 0}{\int_{\varepsilon_1}}$$

$$f(x-i0) = \frac{f(x-i0)}{f(x+i0)} \cdot f(x+i0) \cdot e^{i\Delta\arg f}$$

$$\Delta\arg f = \cancel{\alpha+2-\alpha}\,\Delta\arg z + (2-\alpha)\Delta\arg(1-z) =$$

$$= 2\pi\alpha + (2-\alpha)\cdot 0 = 2\pi\alpha$$

$$f(x-i0) = f(x+i0)\cdot e^{2\pi i\alpha} \Rightarrow \int_{C-} = -e^{2\pi i\alpha}\cdot\int_{C+} = -e^{2\pi i\alpha}\cdot I$$

$$\Rightarrow \oint = I - e^{2\pi i\alpha}\cdot I = 2\pi i \cdot \sum_{\text{outside}} \operatorname{res}$$

$$\operatorname{res}(z=-1) = (-1)^\alpha \cdot 2^{2-\alpha} = \left|\frac{-1}{1+i0}\right|^\alpha \cdot (1+i0)^\alpha \cdot \overset{\cdot 2^{2-\alpha}}{\underbrace{e^{i\Delta\arg z}}_{=e^{i\pi\alpha}}}\cdot$$

$$\cdot 2^{2-\alpha}$$

$$\operatorname*{res}_{z=\infty} \frac{z^\alpha(1-z)^{2-\alpha}}{z+1} \overset{res}{=} z\,\frac{\left(\frac{1}{z}-1\right)^{2-\alpha}}{1+\frac{1}{z}} \overset{res}{\approx} z\left(1-\frac{1}{z}+\frac{1}{z^2}\right)\left((-1)^{2-\alpha} + (-1)^{-\alpha}(-2+\alpha)\cdot\frac{1}{z}\right.$$

$$\left.+\ \frac{1}{2}(-1)^{-\alpha}(-2+\alpha)\cdot(-1+\alpha)\cdot\frac{1}{z^2}\right)$$

порядок $\frac{1}{z}$: $\frac{1}{z}\left((-1)^{-\alpha}(-\alpha+2) + \frac{1}{2}(-1)^{-\alpha}(\alpha-2)(\alpha-1) +\right.$

$$\left. + (-1)^{2-\alpha}\right) =$$

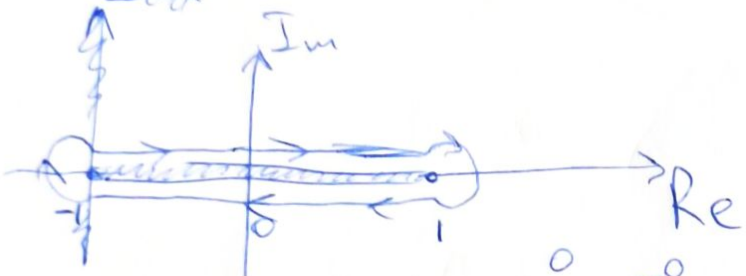
$$= \frac{1}{z(-1)^\alpha}\left(\frac{\alpha^2}{2} - \frac{5\alpha}{2}+4\right) \Rightarrow \operatorname{res}_{z=\infty} = -\frac{1}{e^{i\pi\alpha}}\left(\frac{\alpha^2}{2}-\frac{5\alpha}{2}+4\right)$$

$$\Rightarrow I = 2\pi i\left(-\frac{e^{-i\pi\alpha}}{2}\left(\alpha^2-5\alpha+8\right)+e^{i\pi\alpha}\cdot 2^{2-\alpha}\right)\cdot\left(1-e^{2\pi i\alpha}\right)^{-1} =$$

$$= \frac{\pi\cdot 2i}{e^{i\pi\alpha}-e^{-i\pi\alpha}}\left(\frac{\alpha^2}{2}-\frac{5\alpha}{2}+4-2^{2-\alpha}\right) = \frac{\pi}{\sin\pi\alpha}\cdot\left(\frac{\alpha^2}{2}-\frac{5\alpha}{2}+4-2^{2-\alpha}\right)$$

N8

$$I(a) = \int_{-1}^{1}\frac{(1-x)^a(1+x)^{1-a}}{x^2+1}\,dx = \int_{-1}^{1}\frac{f\,dx}{x^2+1}$$



$$\oint = \int_{C_+}+\int_{C_-}+\underbrace{\cancel{\int_{\varepsilon_{-1}}}}_{\to 0}+\underbrace{\cancel{\int_{\varepsilon_1}}}_{\to 0}$$

$$f(x-i0) = \left|\frac{f(x-i0)}{f(x+i0)}\right| f(x+i0)\cdot e^{i\Delta\arg f}$$

$$\Delta\arg f = a\cdot\Delta\arg(1-z)+(1-a)\,\Delta\arg(1+z) = 2\pi-2\pi a$$

$$\Rightarrow f(x-i0) = f(x+i0)\cdot e^{-2\pi i a} \Rightarrow \int_{C_-} = -e^{-2\pi i a}\int_{C_+} = -e^{-2\pi i a}I$$

$$\Rightarrow \oint = I\left(1-e^{-2\pi i a}\right) = 2\pi i\sum_{\text{outside}}\operatorname{res}$$

$$\operatorname{res}_{z=\pm i} = \frac{(1\mp i)^a(1\pm i)^{1-a}}{\pm 2i}$$

$$f(i) = \left|(1-x)^a(1+x)^{1-a}\right| e^{i\Delta\arg f}$$

$$\Delta\arg f = (1-a)\,\Delta\arg(1+z)+a\,\Delta\arg(1-z) =$$

$$= \frac{\pi}{4}-\frac{\pi a}{2}$$

$$f(-i) = \left|\frac{f(-i)}{f(i\cdot 0)}\right| f_{i0}\, e^{i\Delta \arg f}$$

$$\Delta \arg f = a\Delta\arg(1-z) + (1-a)\Delta\arg(1+z) = \frac{7\pi}{4} - \frac{3\pi a}{2}$$

$$\cancel{\operatorname{res} f(z)} \quad \operatorname{res}(z=i) = -\frac{i}{\sqrt{2}} \exp\left[i\left(\frac{\pi}{4} - \frac{\pi a}{2}\right)\right]$$

$$\operatorname{res}(z=-i) = \frac{i}{\sqrt{2}} \exp\left[i\left(\frac{7\pi}{4} - \frac{3\pi a}{2}\right)\right]$$

$$\operatorname{res}(z=\infty)$$

$$f(z) = \left|\frac{f(z)}{f(z+i0)}\right| \cdot f(z_0 + i0) \cdot e^{i\Delta\arg f}$$

$$\Delta \arg f = a\,\Delta\arg(1-z) + (1-a)\Delta\arg(1+z) = -\pi a$$

$$f(z) = |f(z)|\, e^{-i\pi a} = (x-1)^a (x+1)^{1-a} e^{-i\pi a}$$

$$\operatorname{res}(z=\infty) = \operatorname{res}_{z=\infty} \frac{(z-1)^a (z+1)^{1-a}}{z^2+1} e^{-i\pi a} \overset{\operatorname{res}}{=} \frac{z^a z^{1-a}\left(1-\frac{1}{z}\right)^a\left(1+\frac{1}{z}\right)^{1-a}}{z^2\left(1+\frac{1}{z^2}\right)} e^{-i\pi a}$$

$$\overset{\operatorname{res}}{\sim} e^{-i\pi a}\cdot\frac{1}{z} = e^{-i\pi a}$$

$$\oint = 2\pi i \sum_{out} \operatorname{res} = 2\pi i\left[\frac{i}{\sqrt{2}}\left[\exp\left(i\left(\frac{7\pi}{4} - \frac{3\pi a}{2}\right)\right) - \exp\left[i\left(\frac{\pi}{4} - \frac{\pi a}{2}\right)\right]\right] - e^{-i\pi a}\right) = 2\pi i\left(-\frac{i}{\sqrt{2}}\left(\exp\left[i\left(\frac{\pi}{4} - \frac{\pi a}{2}\right)\right] - \exp\left[-i\left(\frac{\pi}{4} - \frac{\pi a}{2}\right)\right]\right) - e^{-\pi a i}\right)$$

$$= 2\pi i\, e^{-i\pi a}\left(-\frac{i}{\sqrt{2}}\, 2i\sin\left(\frac{\pi}{4} + \frac{\pi a}{2}\right) - 1\right) = 2\pi i\, e^{-i\pi a}\cdot$$

$$\left(\sqrt{2}\left(\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi a}{2}\right) + \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi a}{2}\right)\right) - 1\right) =$$

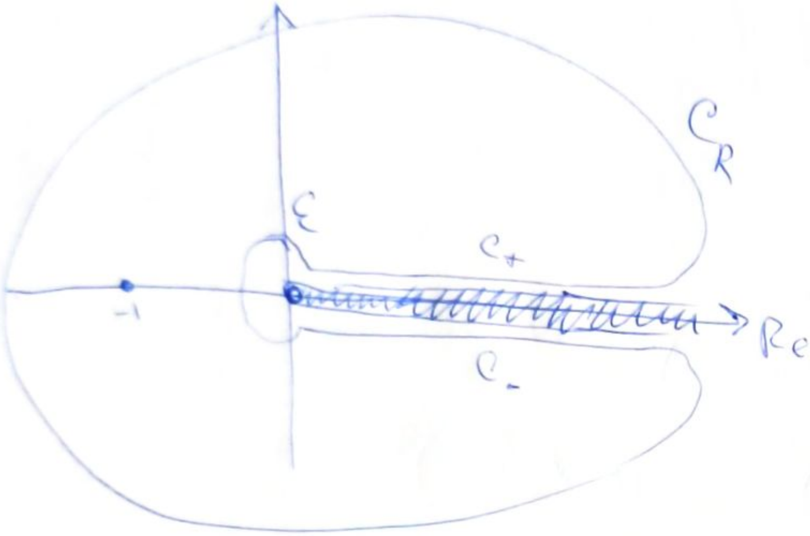
$$= 2\pi i\, e^{-i\pi a}\left(-1 + \cos\left(\frac{\pi a}{2}\right) + \sin\left(\frac{\pi a}{2}\right)\right) = \partial$$

$$I = \frac{\partial}{1 - e^{-2\pi i a}} = \frac{2\pi i}{e^{i\pi a} - e^{-i\pi a}}\left(-1 + \cos\frac{\pi a}{2} + \sin\frac{\pi a}{2}\right) =$$

$$= \frac{\pi}{\sin(\pi a)}\left(-1 + \cos\frac{\pi a}{2} + \sin\frac{\pi a}{2}\right)$$

N 9

$$I = \int_0^\infty \frac{\ln(x)}{\sqrt[3]{x}(x+1)^2}dx = \int_0^\infty f\,dz$$



$$\oint = \int_{C_+} + \int_{C_-} + \cancel{\int_{C_R}}^{\,0} + \int_\varepsilon \qquad \int_\varepsilon = \lim_{\varepsilon\to 0}\int_\pi^0 \frac{\ln(\varepsilon e^{i\varphi})\cdot i\varepsilon e^{i\varphi}}{\sqrt[3]{\varepsilon e^{i\varphi}}(\varepsilon e^{i\varphi}+1)^2}d\varphi =$$

$$= 0$$

$$f_1 = \frac{1}{\sqrt[3]{z}} \qquad f_2 = \ln z \qquad f_1(z-i0) = \left|\frac{f_1(z_0)}{f_1(z_0+i0)}\right| f_1(z_0+i0)\cdot$$

$$\cdot e^{i\Delta \arg f_1} = f_1(z_0 \cdot i0)\cdot e^{-\frac{2\pi i}{3}}$$

$$f_2(z_0-i0) = f_2(z_0+i0) + 2\pi i$$

$$C_- = \int_{+\infty}^0 \frac{(f_2(z_0+i0)+2\pi i)\,g_1(z_0+i0)e^{-\frac{2\pi i}{3}}}{(x+1)^2}d\varphi = -e^{-\frac{2\pi i}{3}}\cdot I - e^{-\frac{2\pi i}{3}}\cdot\overbrace{\int_0^\infty \frac{2\pi i}{\sqrt[3]{x}(x+1)^2}}^{I_0}$$

$$\text{res}(z=-1) = \lim_{z\to-1}\frac{d}{dz}\left((z+1)^2 f\right) = \lim_{z\to -1}\left(\frac{1}{z\sqrt[3]{z}} - \frac{\ln z}{3z\sqrt[3]{z}}\right) =$$

$$= \frac{-1}{\sqrt[3]{-1}}\left(1 - \frac{\ln(-1)}{3}\right)$$

$$f_1(-1) = \left|\frac{f_1(-1)}{f_1(1)}\right| f_1(1)\cdot e^{i\Delta \arg f_1} = e^{-i\pi/3}$$

$$f_2 = i\cdot\Delta\arg z = \pi i$$

$$\oint = 2\pi i \frac{-1}{e^{i\pi/3}}\left(1-\frac{i\pi}{3}\right) = \frac{-2\pi^2 e^{-i\pi/3}}{3} - 2\pi i\, e^{-i\pi/3}$$

$$\left(I + 2\pi i\cdot I_0\right)\left(-e^{2\pi i/3}\right) \cdot I = -2\pi e^{-i\pi/3}\left(\frac{\pi}{3}+i\right)$$

$$I\left(e^{\frac{i\pi}{3}} - e^{-\frac{i\pi}{3}}\right) - 2\pi i\, e^{-i\pi/3} I_0 = -2\pi i\left(1-\frac{\pi i}{3}\right)$$

$$\begin{cases} I\frac{\sqrt{3}}{2} - \pi\cos\left(-\frac{\pi}{3}\right)\cdot I_0 = -\pi \\ -\pi\sin\left(-\frac{\pi}{3}\right) I_0 = \frac{\pi^2}{3} \end{cases} \Rightarrow \begin{cases} I_0 = \frac{2\pi}{3\sqrt{3}} \\ \underline{\underline{I = \frac{2\pi^2}{3} - \frac{2\sqrt{3}}{3}\pi}} \end{cases}$$

N 10

$$I = \int_0^1 \ln\left(\frac{1-x}{x}\right)\frac{dx}{x^2+1}$$

$$dx = -\frac{1}{t^2}dt \qquad \cancel{I = \int_1^\infty \ln\, 1-x} \qquad \int_1^\infty \ln(t-1)\frac{dt}{t^2\left(1+\frac{1}{t^2}\right)} =$$

$$x = \frac{1}{t}$$

$$= \int_1^\infty \frac{\ln(t-1)}{t^2+1} = \int_0^\infty \frac{\ln \xi}{\xi^2+2\xi+2}\,d\xi$$

$$I_0 = \int_0^\infty \frac{d\xi\, \ln^2\xi}{\xi^2+2\xi+2}$$

$$\oint = \int_{C_+} + \int_{C_-} + \oint_\varepsilon^{\to 0} + \oint_R^{\to 0}$$

$$\int_{C_-} = \int_\infty^0 \frac{\ln^2\xi + 4\pi i\ln\xi - 4\pi^2}{\xi^2+2\xi+2} = -\int_0^\infty \frac{\ln^2\xi}{\xi^2+2\xi+2} - 4\pi i\int_0^\infty \frac{\ln\xi}{\xi^2+2\xi+2}$$

$$+ 4\pi^2\int_0^\infty \frac{d\xi}{\xi^2+2\xi+2} \Rightarrow \oint = -4\pi i I + 4\pi^2\int_0^\infty \frac{d\xi}{\xi^2+2\xi+2}$$

$f(z) = \frac{\ln^2 z}{z^2+2z+2}$ $\qquad \oint = 2\pi i \sum res$

$res(z=-1+i) = \left(\ln\sqrt{2} + \frac{3\pi i}{4}\right)^2 \cdot \frac{1}{2i}$

$res(z=-1-i) = -\frac{\left(\ln\sqrt{2} + \frac{5\pi i}{4}\right)^2}{2i}$

$\oint = 2\pi i \cdot \frac{1}{2i}\left(\left(\ln\sqrt{2} + \frac{3\pi i}{4}\right)^2 - \left(\ln\sqrt{2} + \frac{5\pi i}{4}\right)^2\right) =$

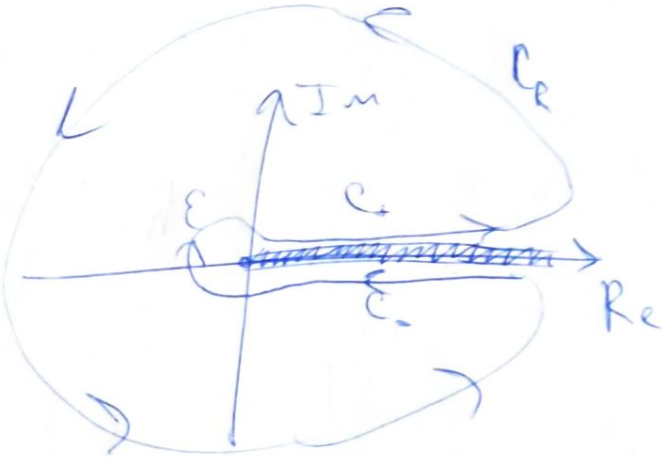
$= \pi^3 - \pi^2 i \ln\sqrt{2} \Rightarrow -4\pi I \cdot i + 4\pi \int_0^\infty \frac{d\xi}{\xi^2+2\xi+2} = \pi^3 - \pi^2 i \ln\sqrt{2}$

$\Rightarrow 4\pi I = \pi^2 \ln\sqrt{2} \Rightarrow I = \frac{\pi}{8}\ln 2$

N11

$I = \int_0^\infty \frac{\ln x}{x^2+2ax\cos\lambda+a^2}dx$

$I_0 = \int_0^\infty \frac{\ln^2 x\,dx}{x^2+2ax\cos\lambda+a^2}$



$\oint = \int_{C_+} + \int_{C_-} + \int_{\varepsilon} + \int_{C_R}$, $\int_{\varepsilon} \to 0$, $\int_{C_R} \to 0$

$\int_{C_-} = \int_\infty^0 \frac{\ln^2(x) + 4\pi i \ln(x) - 4\pi^2}{x^2+2ax\cos\lambda+a^2} = -\int_0^\infty \frac{\ln^2 x}{x^2+2ax\cos\lambda+a^2} -$

$-4\pi i \int_0^\infty \frac{\ln x}{x^2+2ax\cos\lambda+a^2} + 4\pi^2 \int_0^\infty \frac{dx}{x^2+2ax\cos\lambda+a^2}$

$\oint = -4\pi i I + 4\pi^2 \int_0^\infty \frac{dx}{x^2+2ax\cos\lambda+a^2}$

$\oint = 2\pi i \sum res f(z)$

$$\operatorname{res}(z=ae^{i\lambda}) = \frac{\ln^2(-ae^{i\lambda})}{-2ai\sin\lambda} = i\frac{(\ln(-a)+i\lambda)^2}{2a\sin\lambda}$$

$$\operatorname{res}(z=-ae^{-\lambda i}) = \frac{\ln^2(-ae^{-i\lambda})}{2ai\sin\lambda} = \frac{-i}{2a\sin\lambda}(\ln(-a)-i\lambda)^2$$

$$\oint = 2\pi i\cdot\frac{i}{2a\sin\lambda}\left((\ln(-a)+i\lambda)^2-(\ln(-a)-i\lambda)^2\right) =$$

$$= -\frac{\pi}{a\sin\lambda}\left(4i\lambda\ln(-a)-2\lambda^2\right) = -\frac{2\pi}{a\sin\lambda}\left(2i\lambda\ln a-2\pi\lambda-\lambda^2\right)$$

$$-4\pi i - I + 4\pi^2\int_0^\infty\frac{dx}{x^2+2ax\cos\lambda+a^2} = -\frac{2\pi}{a\sin\lambda}\left(2i\lambda\ln(a)-2\pi\lambda-\lambda^2\right)$$

$$\Rightarrow I = \frac{\lambda\ln a}{a\sin\lambda}$$

N 12.

$$I = \int_0^\infty\frac{x^{a-1}}{1+x^b}\,dx$$

$$I = \int_0^\infty\frac{t^{\frac{a-1}{b}}\cdot\frac{1}{b}\cdot t^{\frac{1}{b}-1}}{1+t}\,dt = \frac{1}{b}\int_0^\infty\frac{t^{a/b-1}}{1+t}\,dt$$

$$I_1 = \int_0^\infty\frac{t^{a/b-1}}{1+t}\,dt = bI$$

$$f = t^{\frac{a}{b}-1} = t^{\varphi-1}$$

$$f(t_0-i0) = f(t_0+i0)\cdot e^{i\Delta\arg f}$$

$$\Delta\arg f = (\varphi-1)\Delta\arg f = 2\pi(\varphi-1)$$

$$f(t_0-i0) = f(t_0+i0)\,e^{2\pi i\cdot\varphi}$$

$$\Rightarrow \int_{C_-} = -e^{2\pi i\varphi}\int_{C_+} \qquad \oint = 2\pi i\operatorname{res}(z=-1)$$

$\mathrm{res}(z=-1) = f(-1) \Rightarrow f(-1) = 1^{\varphi-1} \cdot 1 \cdot e^{i\Delta \arg f}$

$\Delta \arg f = \pi(\varphi-1)$

$\mathrm{res}(z=-1) = e^{i\pi(\varphi-1)}$

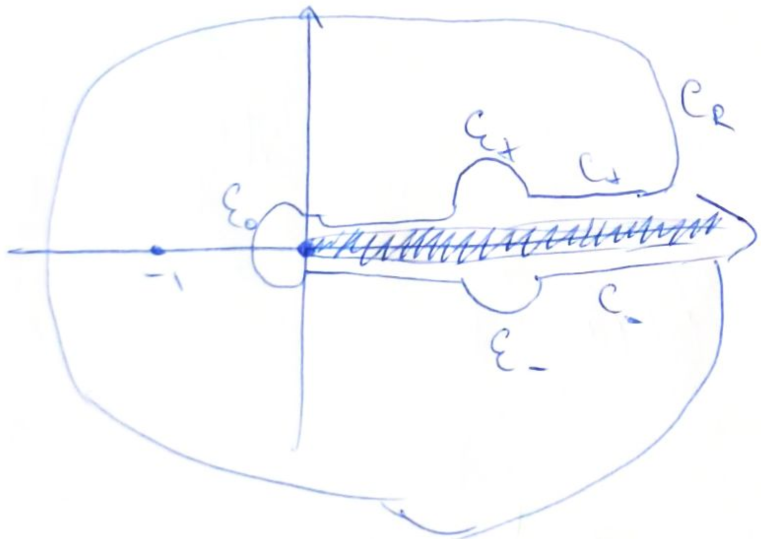
$\oint = I_1(1-e^{2\pi i\varphi}) = 2\pi i e^{i\pi(\varphi-1)}$

$I_1 = \dfrac{2\pi i e^{i\pi(\varphi-1)}}{1-e^{2\pi i\varphi}} = \dfrac{2\pi i\, e^{-i\pi}}{e^{-\pi i\varphi} - e^{i\pi\varphi}} = \dfrac{\pi}{\sin\pi\varphi} \Rightarrow$

$I = \dfrac{\pi}{b\sin(\pi\frac{a}{b})}$

N13

$I = PV\int_0^\infty \dfrac{\sqrt{x}\,dx}{x^2-1}$



$f_1 = \dfrac{\sqrt{z}}{z^2-1}$ $\qquad \oint = \int_{C_+} + \int_{C_-} + \int_{\varepsilon_+} + \int_{\varepsilon_-} + \int_{C_R}^{\to 0} + \int_{\varepsilon_0}^{\to 0}$

$f_2 = \sqrt{z}$ $\qquad f_2(z_0 - i0) = 1^{1/2} \cdot f_2(z_0 + i0) \cdot e^{i\Delta \arg f_2}$

$\Delta \arg f_2 = \frac{1}{2}\Delta \arg z = \pi$

$g(z_0 - i0) = -g(z_0 + i0)$

$$\int_{C_-} = -\int_{\infty}^{0} \frac{f_2(z_0+i0)}{z^2-1} = \int_{0}^{\infty} \frac{f_2(z_0+i0)}{z^2-1}\,dz = \int_{C_+}$$

$$\int_{\mathcal{E}_+} = \lim_{\varepsilon\to 0} \int_{\pi}^{0} \frac{\sqrt{1+\varepsilon e^{i\varphi}}\cdot \varepsilon i e^{i\varphi}\,d\varphi}{\varepsilon^2 e^{2i\varphi}+2\varepsilon e^{i\varphi}} = \lim_{\varepsilon\to 0}\left[\frac{\sqrt{1+\varepsilon e^{i\varphi}}\; i}{\varepsilon e^{i\varphi}+2}\right] d\varphi =$$

$$= \int_{\pi}^{0} \frac{i}{2}\,d\varphi = -\frac{i\pi}{2}$$

$$\int_{\mathcal{E}_-} = \lim_{\varepsilon\to 0} \int_{0}^{-\pi} \frac{-\sqrt{1+\varepsilon e^{i\varphi}}\,\varepsilon i e^{i\varphi}\,d\varphi}{\varepsilon^2 e^{2i\varphi}+2\varepsilon e^{i\varphi}} = \lim_{\varepsilon\to 0} \int_{0}^{-\pi} \frac{-\sqrt{1+\varepsilon e^{i\varphi}}\cdot i\,d\varphi}{\varepsilon e^{i\varphi}+2}$$

$$= \int_{0}^{-\pi} \lim_{\varepsilon\to 0} \frac{-\sqrt{1+\varepsilon e^{i\varphi}}\, i\,d\varphi}{\varepsilon e^{i\varphi}+2} = \int_{0}^{-\pi} \frac{-i\,d\varphi}{2} = \frac{i\pi}{2}$$

$$\oint = 2\pi i \operatorname{res} f(z=-1) = 2\pi i \frac{\cdot}{2\sqrt{z}}\Big|_{z=-1} = -\pi i f_2(-1)$$

$$f_2(-1) = 1^{1/2}\cdot f_2(1+i0)\cdot e^{i\Delta\arg f_2}$$

$$\Delta\arg f_2 = \frac{1}{2}\Delta\arg z = \frac{\pi}{2} \Rightarrow f_2(-1) = i$$

$$\oint = \pi \to I = \frac{\oint}{2} = \frac{\pi}{2}$$

N3

$$I_0(z) = \int_0^\infty t^z e^{-t}\,dt \qquad I_1(z) = \frac{1}{z+1}\int_0^\infty t^{z+1} e^{-t}\,dt$$

$$I_2(z) = \frac{1}{z+1}\cdot\frac{1}{z+2}\cdot\int_0^\infty t^{z+2} e^{-t}\,dt \qquad \operatorname{Re}(z) > -3$$

$$I_3(z) = \frac{1}{(z+1)(z+2)(z+3)}\int_0^\infty t^{z+3} e^{-t}\,dt \qquad \operatorname{Re}(z) > -4$$

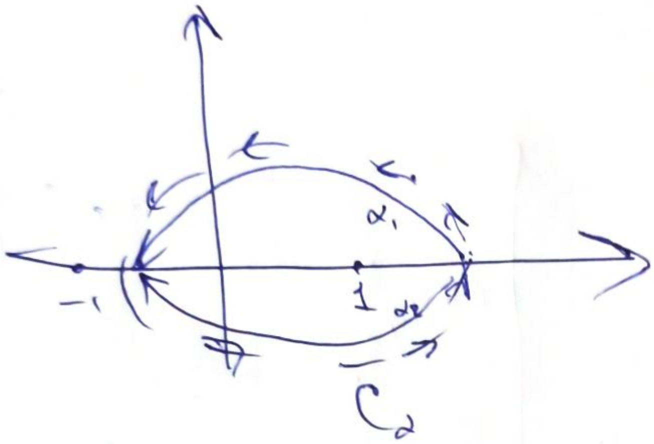
$\operatorname{res}_{z=-3} f(z)$ $\quad\quad$ $\int_0^\infty e^{-t}\,dt = 1$

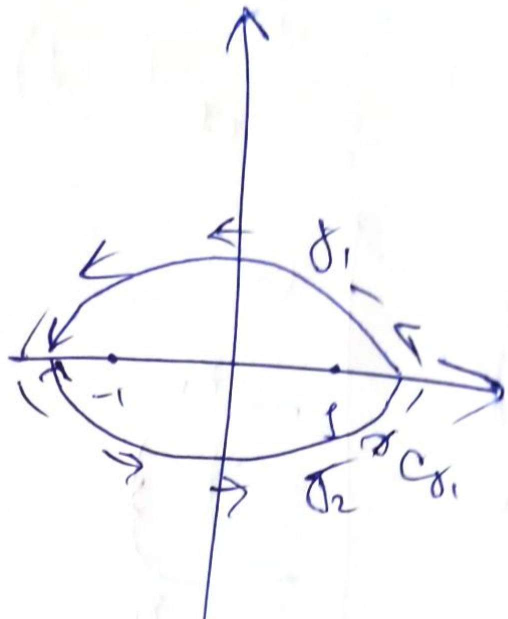
$$\underset{z \to -3}{f(z)} = \frac{1}{(z+3)(z+2)(z+1)} = \frac{1}{2} \cdot \frac{1}{z+3} \Rightarrow$$

$$\Rightarrow \operatorname{res}_{z=-3} f = \frac{1}{2}$$

N 8

i) $I(x) = \int_{-\infty}^{x} \frac{e^{i\pi x}}{x^2-1} dx \qquad x > 1$





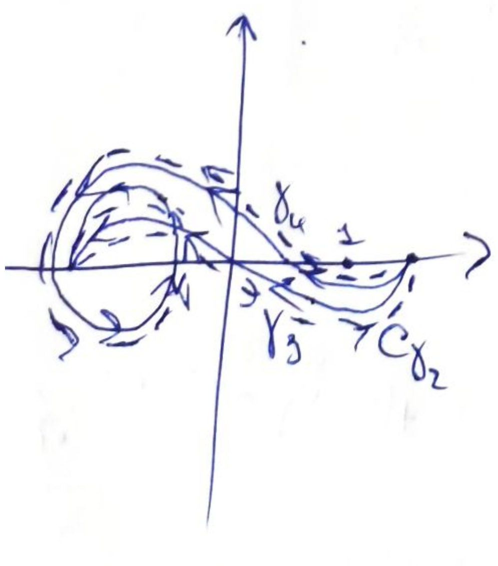
$I_{\alpha_1}(x) - I_{\alpha_2}(x) = \int_{\alpha_1} - \int_{\alpha_2} = \oint_{C_\alpha} = 2\pi i \cdot res(z=1) =$

$= 2\pi i \cdot \frac{e^{i\pi z}}{2z}\Big|_{z=1} = \underline{\underline{-i\pi}}$

$I_{\gamma_1}(x) - I_{\gamma_2}(x) = \int_{\gamma_1} - \int_{\gamma_2} = \oint_{C_{\gamma_1}} = 2\pi i \left[res(z=1) + res(z=-1)\right]$

$= -i\pi + 2\pi i \cdot \frac{e^{i\pi z}}{2z}\Big|_{z=-1} = -i\pi + i\pi = 0$

ii)



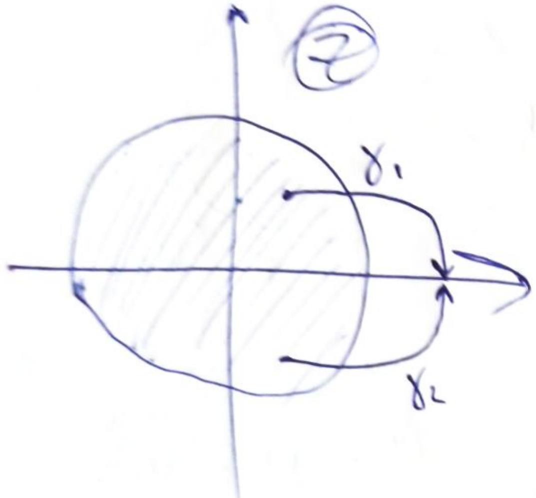
$I_{\gamma_3}(x) - I_{\gamma_2}(x) = \int_{\gamma_3} - \int_{\gamma_2} =$

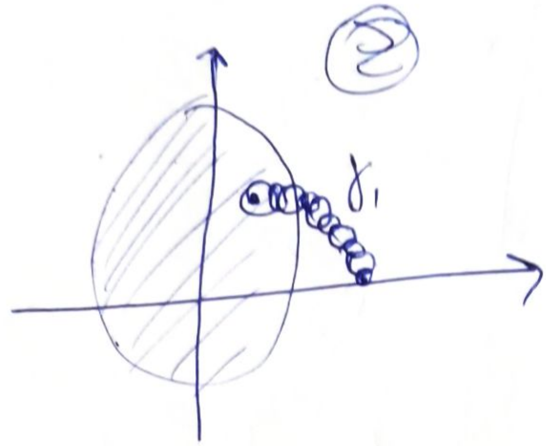
$= \oint_{C_{\gamma_2}} = 2\pi i \cdot res(z=-1) = \underline{\pi i}$

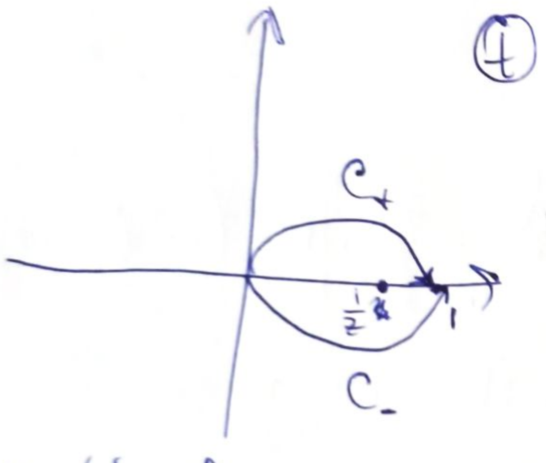
$$I_{\gamma_4}(x) - I_{\gamma_3}(x) = \int_{\gamma_4} - \int_{\gamma_3} = 2\pi i \cdot res(z = -1) = \underline{\underline{\pi i}}$$

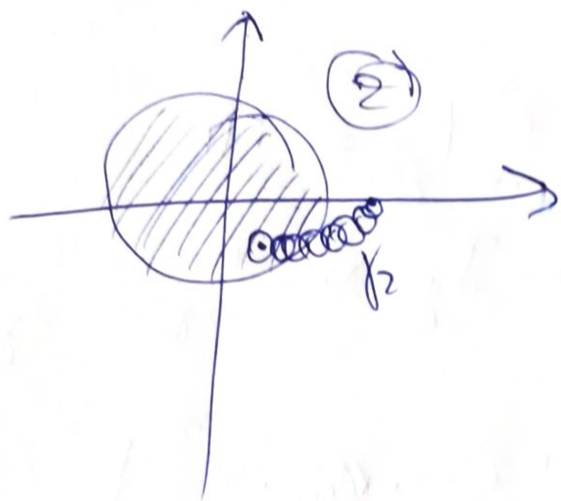
N2

$$F_\alpha(z) = \int_0^1 \frac{t^\alpha}{1-zt}(1-t)^\alpha \, dt$$









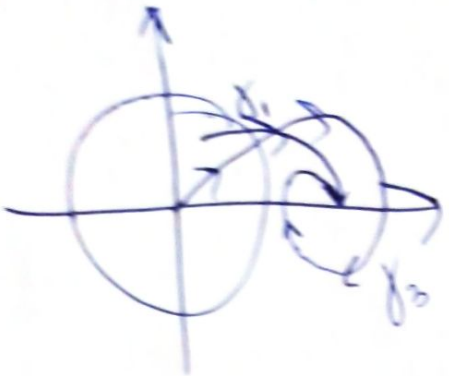
$$F_\alpha(x_1) = \int_{C_+}$$

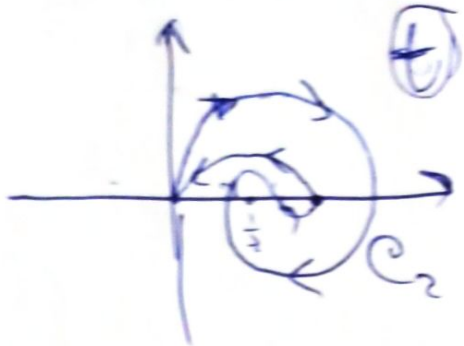
$$\Rightarrow F_\alpha(x_1) - F_\alpha(x_2) = \int_{C_+} - \int_{C_-} =$$

$$F_\alpha(x_2) = \int_{C_-}$$

$$= 2\pi i \cdot res\left(t = \frac{1}{z}\right) =$$

$$= 2\pi i \cdot res\left[\frac{t^\alpha(1-t)^\alpha}{1-zt}\right] = \frac{t^\alpha(1-t)^\alpha}{-z}\Big|_{t=\frac{1}{z}} = \underline{\underline{+(z-1)^\alpha z^{-1-2\alpha} \cdot 2\pi i}}$$





$$\oint F(\gamma_1) - F(\gamma_3) = \oint_{C_2} = 2\pi i \cdot \operatorname{res}\left(t = \tfrac{1}{2}\right) =$$

$$= 2\pi i \cdot (z-1)^{\alpha} \cdot z^{-1-2\alpha}, \text{ но } t\left\{\tfrac{1}{z} + i0\right\} \longleftrightarrow t = \tfrac{1}{z} - i0$$

пусть $g = (z-1)^{\alpha} z^{-2\alpha - 1}$

$$g = t^{\alpha}(1-t)^{\alpha} \Rightarrow g\left(\tfrac{1}{z} - i0\right) = \left|\frac{g\left(\frac{1}{z} - i0\right)}{g\left(\frac{1}{z} + i0\right)}\right| \cdot g\left(\tfrac{1}{z} + i0\right) \cdot e^{i\Delta \arg g}$$

$$\Delta \arg g = \alpha \Delta \arg t + \alpha \Delta \arg (1-t) = -\alpha + 2\pi$$

$$g\left(\tfrac{1}{z} - i \cdot 0\right) = \left(\tfrac{1}{z}\right)^{\alpha}\left(1 - \tfrac{1}{z}\right)^{\alpha} \cdot e^{-2\pi i \alpha}$$

$$\Rightarrow F(\gamma_1) - F(\gamma_3) = -2\pi i \cdot e^{-2\pi i \alpha}(z-1)^{\alpha} z^{-1-2\alpha}$$

№4

$$f(z) = \int_1^z \left(\frac{1}{\omega} + \frac{\alpha}{\omega^3}\right)\cos\omega \, d\omega = I$$

$$f(z) = \int_1^z \frac{1}{\omega}\cos\omega \, d\omega + \frac{\alpha}{2} \cdot -\frac{\cos\omega}{\omega^2}\Big|_1^z + \frac{\alpha}{2}\int_1^z -\sin\omega \frac{1}{\omega^2} d\omega =$$

$$= I - \frac{\alpha}{2}\left(\frac{\cos z}{z^2} - \cos 1 - \frac{\sin\omega}{\omega}\Big|_1^z + \int_1^z \frac{1}{\omega}\cos\omega \, d\omega\right) =$$

$$= I - \frac{\alpha}{2}\cdot\frac{I}{1} - \frac{\alpha}{2}\underbrace{\left(\frac{\cos z}{z^2} - \frac{\sin(z)}{z} + \sin(1) - \cos(1)\right)}_{\text{однозначн.}}$$

$$\Rightarrow I - \frac{\alpha}{2}I - \frac{\alpha}{2}\cdot g(z) = f(z) \Rightarrow$$

при $\frac{\alpha}{2} = 1$ $f(z)$ однозн $\Rightarrow \underline{\alpha = 2}$

N5

$$z u'' + (\gamma - 1 - iz)u' + iu = 0$$

$z \to x$ $\quad u_1 = \gamma - 1 - ix$:

$$\frac{d^2}{dx^2}(\gamma-1-ix) + (\gamma-1-ix)\frac{d}{dx}(\gamma-1-ix) + i(\gamma-1-ix) = 0$$

$(\gamma-1-ix)(-i) + i(\gamma-1-ix) = 0 \Rightarrow \underline{u_1 \text{ подошло}}$

$$u_2 = \int_i^{-\infty} e^{xt}\frac{(t-i)^{\gamma-1}}{t^2}dt :$$

$$x\frac{d^2}{dx^2}\int_i^{-\infty}\exp xt\cdot\frac{(t-i)^{\gamma-1}}{t^2}dt + \overbrace{(\gamma-i-ix)\frac{d}{dx}\int_i^{-\infty}\exp(xt)\frac{(t-i)^{\gamma-1}}{t^2}dt}^{I_1} +$$

$$+ i\int_i^{-\infty}\exp[xt]\frac{(t-i)^{\gamma-1}}{t^2}dt = x\int_t^{-\infty} e^{xt}(t-i)^{\gamma-1}dt + (\gamma-1-ix)\cdot$$

$$\cdot\int_i^{-\infty} e^{xt}\frac{(t-i)^{\gamma-1}}{t}dt + \underbrace{i\int_i^{-\infty} e^{xt}\frac{(t-i)^{\gamma-1}}{t^2}dt}_{I_0} = x\cdot\frac{1}{x}\left(e^{xt}(t-i)^{\gamma-i}\right|_i^{-\infty} -$$

$$- \int_i^{-\infty}(\gamma-1)(t-i)^{\gamma-2}\exp(xt)dt + i\int\frac{xe^{xt}(t-i)^{\gamma-1} + (\gamma-1)(t-i)^{\gamma-2}e^{xt}}{t}dt +$$

$$\cancel{\ldots} + I_1 = \cancel{\ldots}$$

$$= -(\gamma-1)\int_i^{-\infty}(t-1)^{\gamma-2}\exp(xt)\,dt + (\gamma-1-ix)\int_i^{-\infty}\frac{e^{xt}(t-i)^{\gamma-1}}{t}\,dt$$

$$+ix\int_i^{-\infty}\frac{e^{xt}}{t}(t-i)^{\gamma-1}\,dt + i\int_i^{-\infty}\frac{(\gamma-1)(t-1)^{\gamma-2}}{t}e^{xt}\,dt =$$

$$= (\gamma-1)\left(-\int_i^{-\infty}e^{xt}\frac{(t-i)^{\gamma-1}}{t}\,dt + \int_i^{-\infty}e^{xt}\frac{(t-i)^{\gamma-1}}{t}\,dt\right) = 0$$

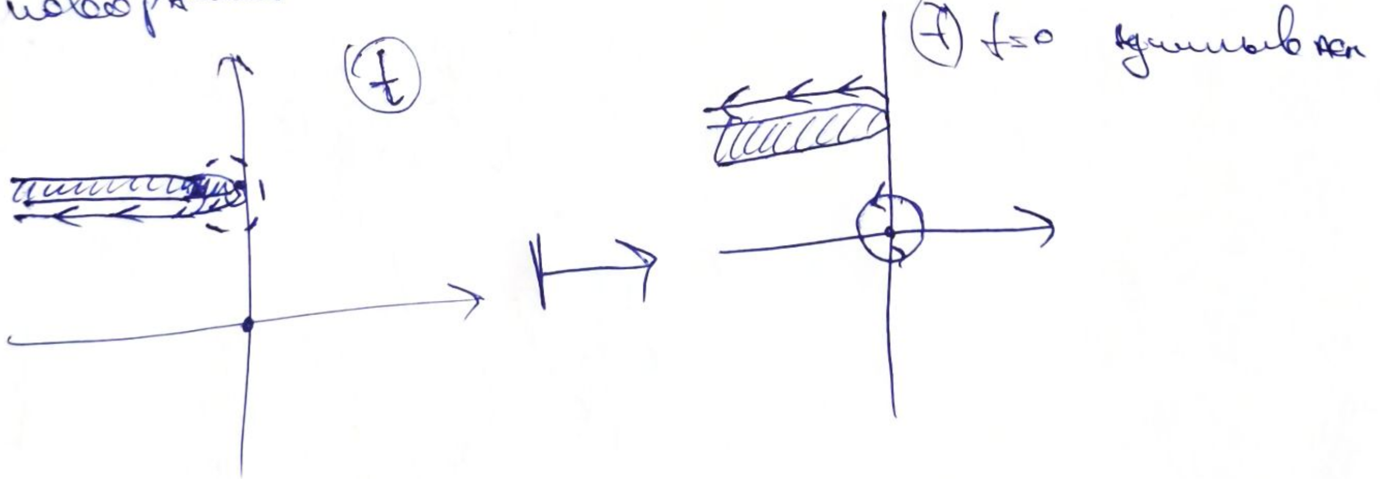
$\Rightarrow$ u_2 подошло

$g(t) = (t-i)^{\gamma-1}$ $\quad g(i+\infty) > 0$

$x \to x\,e^{-2i\pi}$

~~экс~~ экспонента должна быть везде верной

$\Rightarrow$ вместе с x нужно поворачивать t. Контур поворачивание произвольно. повор x ~~на~~



$\Rightarrow$ $u_2(x\,e^{-2\pi i}) = u_2' * - \oint_C$

~~где~~ $g(i+t_0+i0) = \left|\dfrac{g(i+t_0+i0)}{g(i+t_0-i0)}\right| \cdot g(i+t_0-i0)\cdot e^{i\Delta\arg g}$

$\Delta\arg g = (\gamma-1)2\pi \Rightarrow g(i+t_0+i0) = g(i+t_0-i0)\,e^{2\pi i\gamma}$

$$\oint_C = 2\pi i \operatorname{res}(t=0) = 2\pi i \cdot \left(\lim \frac{d}{dt}\left[e^{xt}(t-i)^{\gamma-1}\right]\right) =$$

$$= \lim_{t\to 0}\left(x\, e^{xt}(t-i)^{\gamma-1} + (\gamma-1)(t-i)^{\gamma-2} e^{xt}\right) =$$

$$= (\gamma-1-ix)(-i)^{\gamma-2} = -u_1 e^{-i\pi\gamma\cdot\frac{1}{2}}$$

$$\Rightarrow u_2 = e^{2\pi i\gamma}\cdot u_2 - 2\pi i e^{-i\pi\gamma/2}\cdot u_1$$