N1

$$\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right)^{2021}=\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)^{2021}=\cos\frac{2021\pi}{4}+i\sin\frac{2021\pi}{4}$$

$$=\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4}=-\frac{\sqrt{2}}{2}-i\frac{\sqrt{2}}{2}$$

N2

$$\left[\frac{e^z-\sin z-1}{z^2(\cos z-1)^2}\right]_{res,0}=\left[\frac{z+\frac{z^2}{2}+\frac{z^3}{6}-z+\frac{z^3}{6}+\frac{z^4}{4!}+\frac{z^5}{5!}-\frac{z^5}{5!}+1-1+\dots}{z^2\cdot z^4\left(1/2!+\frac{z^2}{4!}+\dots\right)^2}\right]_{res,0}\approx$$

$$\approx\left[\frac{4}{z^6}\cdot\left(\frac{z^2}{2}+\frac{z^3}{3}+\frac{z^4}{4!}\right)\left(1+2\cdot\frac{2z^2}{4!}\right)\right]_{res,0}\approx\left[\frac{16}{4!\cdot 3z}\right]_{res,0}=$$

$$=\frac{16}{4!\cdot 3}=\frac{2}{9}$$

N3

$$\frac{z^3}{\sin^8(2z)}\approx\frac{z^3}{\left(2z-\frac{4z^3}{6}\right)^8}\approx\frac{z^3}{2^8z^8\left(1-\frac{2z^2}{3}\right)^8}\approx$$

$$\approx\frac{1}{2^8z^5}\left(1+\frac{16z^2}{3}\right)\approx\frac{1}{256z^5}+\frac{1}{48z^3}+\dots$$

N4

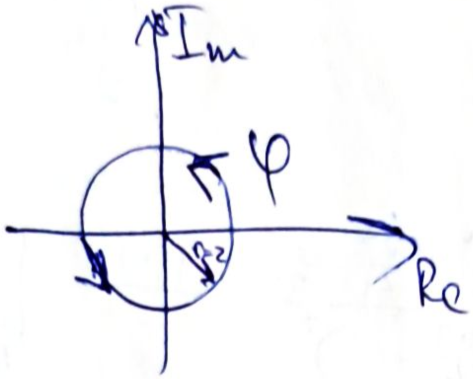
$$\frac{e^{\frac{1}{z}}e^z}{(z-1)^2}=\frac{\left(1+z+\frac{z^2}{2}+\dots\right)\left(1+\frac{1}{z}+\frac{1}{2z^2}+\frac{1}{6z^3}+\dots\right)}{(z-1)^2}$$

$z=0$ – сущ. особая точка $z=1$ – полюс второго порядка.

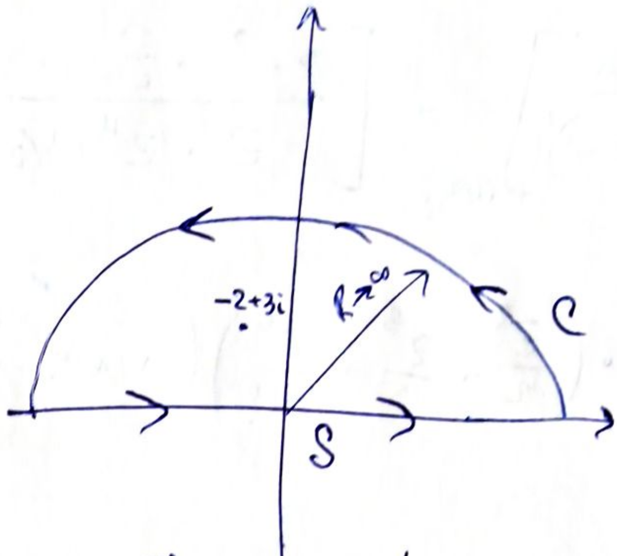
N5

$$z=e^{i\varphi}\cdot r$$

$$\oint\limits_{|z|=2} (|z|+1)^3\,\bar{z}\,dz = \int_0^{2\pi} i\,(2+1)^3\cdot 2\cdot e^{-i\varphi}\cdot e^{i\varphi}\cdot 2\,d\varphi = 216\pi i$$



N6



$$I=\int_{-\infty}^{\infty}\frac{\sin 2x}{(x^2+4x+13)^2}\,dx$$

$$I=\operatorname{Im}\int_{-\infty}^{\infty}\frac{\exp(2ix)\,dx}{(x+2+3i)^2(x+2-3i)^2}=\operatorname{Im}\int_{-\infty}^{\infty}\frac{\exp(2iz)\,dz}{(z+2+3i)^2(z+2-3i)^2}$$

$$\oint=\int_C+\int_S=2\pi i\cdot\operatorname*{Res}_{z=-2+3i}\left(\frac{\exp(2iz)\,\cancel{dz}}{(z+2+3i)^2(z+2-3i)^2}\right)$$

$\int_C=0$ (Л. Жордана)

$$\operatorname{Res}=\lim_{z\to-2+3i}\frac{d}{dz}\left[\frac{\exp(2iz)}{(z+2+3i)^2}\right]=\lim_{z\to-2+3i}\left[\frac{2i\exp(2iz)}{(z+2+3i)^2}+\frac{-2\exp(2iz)}{(z+2+3i)^3}\right]$$

$$=\lim_{z\to-2+3i}\frac{2ie^{2iz}(z+2+4i)}{(z+2+3i)^3}=\frac{2i\,e^{-4i-6}(7i)}{(6i)^3}=\frac{-7i}{108}\exp[-6-4i]$$

$$I = \operatorname{Im} 2\pi i \frac{-7i}{108} \exp[-6-4i] = \frac{-7}{54}\pi \exp[-6] \sin[4]$$

N8

$$\oint\limits_{|z|=6} \frac{\sin\left(\frac{1}{z}\right)\cos\left(\frac{1}{z-2}\right)}{z-3}$$

Поменяем напр обхода, так, чтобы

$$\oint = -2\pi i \sum Res_{снаружи}.$$

$$\sum Res_{снаружи} = Res_{\infty}$$

$$Res_{\infty} = \left(\frac{\sin\left(\frac{1}{z}\right)\cos\left(\frac{1}{z-2}\right)}{(z-3)}\right)_{res,\infty} = \left\{\left[\frac{1}{z} - \frac{1}{6z^3} + \dots\right] \cdot \right.$$

$$\left. \cdot \left[1 - \frac{1}{2z^2} + \dots\right] \cdot \frac{1}{z}\left(1 + \frac{3}{z} + \dots\right)\right\}_{res,0} =$$

$$= 0, \text{ т.к. } \frac{0}{z} + \dots$$

$$I = \oint -2\pi i \cdot Res_{\infty} = 0$$

N7

$$\int_0^{\infty} \frac{3\sin 2x - 2\sin 3x}{x^3} dx = \frac{1}{2}\int_{-\infty}^{\infty} \frac{3\sin 2x - 2\sin 3x}{x^3} dx =$$

$$= \mathrm{Im}\left[\int_{-\infty}^{\infty} \frac{\frac{3}{2}\exp 2ix - \exp 3ix}{x^3}\,dx\right] = \mathrm{Im}\left[\int_{\varepsilon}^{\infty} \frac{\frac{3}{2}\exp 2ix}{x^3}\,dx\right.$$

$$\left. - \int_{\varepsilon}^{\infty} \frac{\exp 3ix}{x^3}\,dx + \int_{-\infty}^{-\varepsilon} \frac{\frac{3}{2}\exp 2ix}{x^3}\,dx - \int_{-\infty}^{\varepsilon} \frac{\exp 3ix}{x^3}\,dx\right]$$

$$\oint = \overset{=0}{\int_C} + \int_S + \int_{\varepsilon'} = 0$$

$$\int_S = -\int_{\varepsilon'} \Rightarrow \int_S = \frac{2\pi i}{2}\,\mathrm{Res}\left[\frac{\frac{3}{2}\exp 2iz - \exp 3iz}{z^3}\right]_{z=0}$$

$$= \frac{1}{2}\cdot\pi i\cdot\lim_{z\to 0}\frac{d}{dz}\left[\frac{3}{2}\exp 2iz - \exp 3iz\right] =$$

$$= \frac{\pi i}{2}\cdot\left[-\frac{4\cdot 3}{2} + 9\right] = \frac{3\pi i}{2} \Rightarrow I = \mathrm{Im}\,\frac{3\pi i}{2} = \frac{3\pi}{2}$$