N6

1) $|f| = e^{r^2\cos 2\varphi}$ $\quad z = re^{i\varphi}$

$|f| = R \quad \operatorname{Arg} f = \Phi$

$$\begin{cases} \dfrac{\partial R}{\partial r} = \dfrac{R}{r}\dfrac{\partial \Phi}{\partial \varphi} \\ R\dfrac{\partial \Phi}{\partial r} = -\dfrac{\partial R}{r\partial \varphi} \end{cases} \Rightarrow \begin{cases} 2r\cos 2\varphi\, R = \dfrac{R}{r}\dfrac{\partial \Phi}{\partial \varphi} \\ +\dfrac{2r^2\sin 2\varphi}{r} R = R\dfrac{\partial \Phi}{\partial r} \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} \dfrac{\partial \Phi}{\partial \varphi} = 2r^2\cos 2\varphi \\ \dfrac{\partial \Phi}{\partial r} = +2r\sin 2\varphi \end{cases} \Rightarrow \begin{cases} \Phi = r^2\sin 2\varphi + C(r) \\ \Phi = r^2\sin 2\varphi + C(\varphi) \end{cases}$$

$\Rightarrow \Phi = r^2\sin 2\varphi \qquad f = R\cdot \exp(i\Phi)$

$f = e^{r^2(\cos 2\varphi + i\sin 2\varphi)} = e^{r^2 e^{2i\varphi}} = e^{z^2}$

2) ~~$\operatorname{Arg} f = xy = \Phi$~~

~~$\begin{cases} \dfrac{\partial R}{\partial r} = \dfrac{R}{r}\dfrac{\partial \Phi}{\partial \varphi} \\ R\dfrac{\partial \Phi}{\partial r} = \end{cases} \Rightarrow$~~

2) $\cancel{}$ $\operatorname{Arg} f = xy$

$$\frac{\partial R}{\partial x} = R\frac{\partial \phi}{\partial y} \Rightarrow \begin{cases} R \cdot x = \dfrac{\partial R}{\partial x} \\ -Ry = \dfrac{\partial R}{\partial y} \end{cases} \Rightarrow$$

$$\frac{\partial R}{\partial y} = -R\frac{\partial \phi}{\partial x}$$

$$\ln R = \frac{x^2}{2} + C(y)$$

$$\ln R = -\frac{y^2}{2} + C(\cancel{x}) \Rightarrow R = \exp\left(\frac{x^2}{2} - \frac{y^2}{2}\right)$$

$$f = R\exp(i\phi) \Rightarrow f = \exp\left[\frac{x^2}{2} + ixy - \frac{y^2}{2}\right] = e^{\frac{z^2}{2}}$$

$\cancel{}$ $x + iy = z \qquad z^2 = x^2 - y^2 + 2ixy$

N6

1) $u = \varphi(x^2-y^2)$ $\qquad f = u + iv$

~~$\Delta f = 0$~~ $\quad \Delta u = 0$

$u = \varphi(x^2-y^2) = \varphi(t)$, $\quad x^2-y^2 = t$

$$\varphi_x(t) = \frac{\partial\varphi}{\partial t}\frac{\partial t}{\partial x} = 2x\frac{\partial\varphi}{\partial t}$$

$$\varphi_{xx} = 2\frac{\partial\varphi}{\partial t} + \frac{\partial^2\varphi}{\partial t^2}\cdot 2^2x^2$$

$$\varphi_y = -2y\frac{\partial\varphi}{\partial t}$$

$$\varphi_{yy} = -2\frac{\partial\varphi}{\partial t} - 2^2y^2\frac{\partial^2\varphi}{\partial t^2}$$

$$\frac{\partial^2\varphi}{\partial t^2}(x^2-y^2) = 0 \qquad t\varphi''(t) = 0 \qquad \varphi = C_1 + C_2 t$$

$u = C_1 + C_2(x^2-y^2)$ $\qquad C_1 = a \qquad C_2 = b$

$$\begin{cases} \dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y} \Rightarrow 2xC_1 = \dfrac{\partial v}{\partial y} \\[2ex] \dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x} \Rightarrow -2yC_2 = -\dfrac{\partial v}{\partial x} \end{cases} \Rightarrow \begin{matrix} v = 2C_2xy + C(x) \\ v = +2C_2xy + C(y) \end{matrix}$$

$v = 2xy(b)$

$f(z) = a + b(x^2-y^2) + i2xy(b) = a + bz^2$

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2) $u = \varphi\left(\frac{y}{x}\right)$ $\qquad \frac{y}{x} = t$

$f(x) - ?$

$$\varphi_x = \frac{\partial\varphi}{\partial t}\cdot\left(\frac{-y}{x^2}\right)$$

$$\varphi_{xx} = \frac{\partial\varphi}{\partial t}\cdot\frac{+2y}{x^3} + \frac{\partial^2\varphi}{\partial^2 t^2}\frac{y^2}{x^4}$$

$$\varphi_y = \frac{\partial\varphi}{\partial t}\frac{1}{x}$$

$$\varphi_{yy} = \frac{\partial^2\varphi}{\partial t^2}\frac{1}{x^2}$$

$$\Delta\varphi = 0 \Rightarrow \frac{1}{x^2}\frac{\partial^2\varphi}{\partial^2 t^2}\left(1+t^2\right) + \frac{1}{x^2}\frac{\partial\varphi}{\partial t}(2t) = 0$$

$$\varphi''\left(1+t^2\right) + 2\varphi' t = 0$$

$$\varphi' = g$$

$$\frac{dg}{dt}\left(1+t^2\right) + 2gt = 0 \Rightarrow \frac{dg}{g} = \frac{-2t}{1+t^2}dt$$

$$\ln g = -\ln\left(t^2+1\right) + C$$

$$g = \frac{C_1}{t^2+1} \qquad u = \int g\, dt =$$

$$= C_1\cdot\operatorname{arctan} t + C_2$$

$$\begin{cases} \dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y} \\ \dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x} \end{cases} \Rightarrow \quad -\frac{yC_1}{x^2+y^2} = \frac{\partial v}{\partial y}$$

$$\frac{xC_1}{x^2+y^2} = -\frac{\partial v}{\partial x}$$

$$V = -C_1 \ln(x^2+y^2) + C_2(x)$$

$$V = -C_1 \ln(x^2+y^2) + C(y)$$

$$V = -C_1 \frac{1}{2} \ln(x^2+y^2)$$

$$f = u + iv = C_1\left(\arctan\left(\frac{y}{x}\right) - \frac{1}{2} i \ln(x^2+y^2)\right) + C_2$$

$$= C_1 \cdot \frac{1}{2} i \left(\ln \frac{z^2}{x^2+y^2} + \ln(x^2+y^2)\right) + C_2 = C_1 \frac{1}{2} i \ln z^2 + C_2$$

N7

$z = e^{i\varphi}$

1) $\int_C z\,dz = \int_0^{2\pi} e^{i\varphi} \cdot i e^{i\varphi}\,d\varphi = i\int_0^{2\pi} e^{2i\varphi}\,d\varphi = 0$

2) $\int_C z^* dz = \int_0^{2\pi} e^{-i\varphi} \cdot i e^{i\varphi}\,d\varphi = 2\pi i$

N8

1) $\int_C \frac{y\,dx - x\,dy}{x^2+y^2} \overset{\substack{x=\cos\varphi \\ y=\sin\varphi}}{=} \int_0^{2\pi} \frac{-1}{1}\,d\varphi = -2\pi$

2) $\oint_C (L\,dx + M\,dy) = \iint_D \left(\frac{\partial M}{\partial x} - \frac{\partial L}{\partial y}\right) dx\,dy \Rightarrow$

$\Rightarrow \oint_C \frac{y\,dx - x\,dy}{x^2+y^2} = \cancel{\iint\left(\frac{2xy}{(x^2+y^2)^2} + \frac{2xy}{x^2+y^2}\right) dx\,dy}$

$= \iint_C \left(\frac{-1 \quad -1}{x^2+y^2} + \frac{2x^2}{(x^2+y^2)^2} + \frac{2y^2}{(x^2+y^2)^2}\right) dx\,dy =$

$= 0$

N9

$\frac{1}{1-z^k} = 1 + z^k + z^{2k} + \ldots$

$\oint z^n dz = 0$ $\oint z^{-1} dz = 2\pi i$

~~[illegible]~~ $n \neq -1$ $c: |z| < 1$

$P(n) = \frac{1}{2\pi i} \oint_c dz\, z^{-1-n} \prod_1^\infty \frac{1}{1-z^k}$

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~~P(t)dt~~

$$\prod_1 \frac{1}{1-z^k} = (1+z+z^2+\dots)(1+z^2+\dots)(1+z^3+\dots)$$

т. е. ненулев. будут только чл. z^{-1} т. е. $p(n) = \int \frac{1}{2\pi i} z^{-1-n} (1+z+2z^2+a_3z^3+\dots$

$+a_nz^n) = \int b_1 z^{-1-n} + \int b_2 z^{-1-n+1} + \int b_3 z^{-1-n+2} + \dots$

~~ненулевой только тот~~ $= \int b_k z^{-1}$

$$p(1) = \int \frac{1}{2\pi i} \cdot z^{-2} \cdot z\,dz = 1$$

$$p(4) = \int \frac{5}{2\pi i} \cdot z^{-5} z^4 dz = 5$$

N11 (N10 см. ниже)

$$f(z) = \frac{1+2z^2}{z^3+z^5} = \frac{1}{z^3}\,\frac{1+2z^2}{1+z^2} = \frac{1}{z^3}(1+2z^2)\cdot$$

$$\cdot(1-z^2+z^4-\dots)$$

$$\frac{1}{z^3} + \frac{0}{z^2} + \frac{1}{z} + \dots$$

$a_{-3} = 1$

$a_{-2} = 0$

$a_{-1} = 1$

N10

$$a)\ y_\pi = \int_C dz\, y'(z) = \int_C \frac{1}{2z} = \int_0^\pi \frac{1}{2} i\,d\varphi = \frac{\pi i}{2}$$

b) $y = \int_0^{\pi} \frac{i}{2} d\varphi = -\frac{\pi i}{2}$

N12

$$f(z) = \frac{1}{z(e^z - 1)} = \frac{1}{z\left(z + \frac{z^2}{2} + \frac{z^3}{6} + \dots\right)}$$

$$f(z) = \frac{1}{z^2\left(1 + \frac{z}{2}\right)} = \frac{1}{z^2} \cdot \left(1 - \frac{z}{2} + \frac{z^2}{4} + \dots\right)$$

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вычет $= -\frac{1}{2}$

N13

$$f(z) = \frac{1}{(z-1)z} = \frac{-1}{z}\left(1 + z + z^2 + z^3 + \dots\right)$$

$$f(z) = \frac{+1}{z^2\left(1 - \frac{1}{z}\right)} = \frac{+1}{z^2}\left(1 + \frac{1}{z} + \frac{1}{z^2} + \dots\right)$$

N14

$$\frac{z}{z^2+1} = \frac{z}{(z-i)(z+i)} = \cancel{\frac{-i}{z-i} \cdot z\left(1 - \frac{z}{i} + \frac{z^2}{i^2} - \frac{z^3}{i^3}\right)}$$

$$= \frac{z}{(z-i)((z-i)+2i)} = \frac{z}{2i(z-i)\left(1 + \frac{z-i}{2i}\right)} = \frac{1}{z-i} \cdot \frac{z}{2i} \cdot$$

$$\cdot \left(1 - \frac{z-i}{2i} - \frac{(z-i)^2}{2^2 i^2} - \dots\right) = \frac{1}{2}\frac{1}{z-i} - \frac{i}{4}\sum_0^{\infty}\left(\frac{i}{2}\right)^n (z-i)^n$$

$|z - i| < 2$