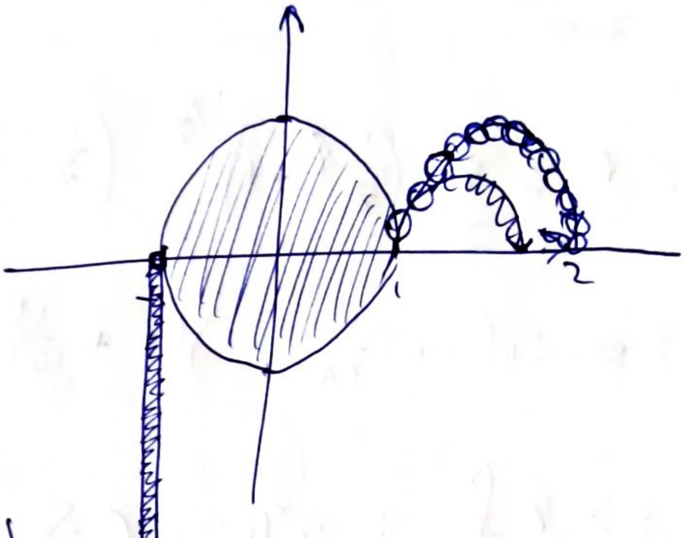
N8

$f(z) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} z^{n+1}}{n} = -z \sum \frac{(-1)^n z^n}{n} = z \cdot \ln(z+1)$

Сходимая при $|z| < 1$ т.к R до ближайшей т. сингулярности 1

$1 - e^{it} \; \{ t \in [0; 2\pi]$



$\frac{\ln 2}{2\pi i}$

$\tilde{f}(z) = \ln(1+z)$

$g(z) = 1+z$

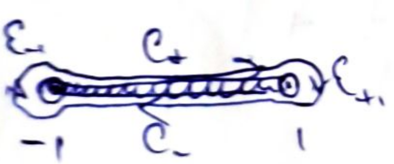
$\tilde{f}(z) = \tilde{f}(0) + \ln(|g(z)|) - \ln|g(0)|) + i\Delta\arg(g)$

$\tilde{f}(0) = 0$

$\tilde{f}(2) = \ln(|3|) + i\Delta\arg(g) \Rightarrow f(2) = 2\ln(3)$

N7

$\int_{-1} \sqrt{\frac{x+1}{1-x}}\, dx$ $\qquad g(z) = (z+1)^{1/2} (1-z)^{-1/2}$



$\oint = \int_{C_+} + \int_{C_-} + \cancelto{0}{\int_{\varepsilon_{-1}}} + \cancelto{0}{\int_{\varepsilon_1}}$

$g(x-i0) = \cancelto{1}{\left| \frac{g(x-i0)}{g(x+i0)} \right|} g(x+i0)\, e^{i\Delta\arg g}$

$\Delta\arg(g) = \frac{1}{2}\Delta\arg(z+1) \mp \frac{1}{2}\Delta\arg(1-z) = \frac{1}{2}\cdot 2\pi \mp 2\pi\cdot\frac{1}{2} =$

$= 0$

$$g(x-i0) = g(x+i0)\, e^{\pi i k 0}$$

$$\int_{C_-} = \int_{C_+} \Rightarrow \oint = 2\int_{C_+} = 2\pi \sum_{\text{outside}} \operatorname{res}$$

$$\underset{z=\infty}{\operatorname{res}} f(z) = \overset{\text{res}}{\cancel{z}}(1+z)^{1/2}(\cancel{z}1-z)^{-\frac{1}{2}} = \cancel{\operatorname{res}(1+\tfrac{1}{2}}$$

$$= \operatorname{res} \cancel{z} \left(\frac{1}{z}\cancel{z}+1\right)^{1/2}\left(\frac{1}{z}-1\right)^{-\frac{1}{2}} = \operatorname{res} \frac{1}{\sqrt{-1}}\left(\frac{1}{z}+1\right)^{1/2}\left(-\frac{1}{z}+1\right)^{-\frac{1}{2}}$$

$$= \operatorname{res} -i\left(1+\frac{1}{2z}+\ldots\right)\left(1+\frac{1}{2z}+\ldots\right) = -i$$

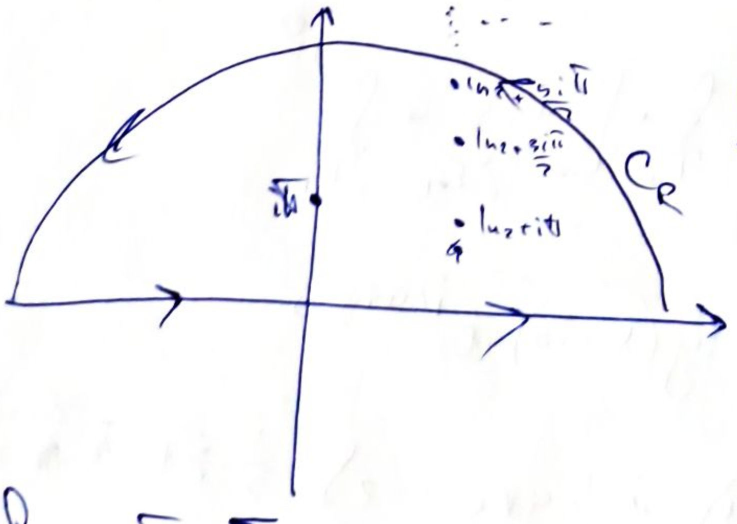
$$\Rightarrow 2\int_{C_+} = 2\pi i\cdot(-i) \Rightarrow \int_{C_+} = \pi$$

P.S. пределы ~~от~~ -1; 1 (спросил у К.С., там опечатка)

№9

$$\int_0^\infty \frac{1}{(x+2)(\ln^2 x+\pi^2)}dx = \int_{-\infty}^{\infty} \frac{e^t\, dt}{(e^t+2)(t^2+\pi^2)} = I$$

$t_0 = \pm i\pi$; $e^{t_0}+2 = 0 \Rightarrow t_0 = \ln 2 + i\pi(1+2n)$ $n\in\mathbb{Z}$



$$\oint = \cancel{I}\int + \cancel{\int_{C_R}}^{\to 0} =$$

$$= I$$

$$\oint = 2\pi i \sum \operatorname{res}$$

$$\text{res}(i\pi) = \frac{1}{2\pi i}\cdot(-1) = -\frac{1}{2\pi i}$$

$$\text{res}(\ln 2 + i\pi + 2i\pi n) = \frac{\text{[illegible]} - 4}{4\left(\pi^2 + (\ln 2 + \pi i + 2\pi i n)^2\right)} =$$

$$= \frac{-\text{[illegible]}\,1}{(\ln 2 + 2\pi i n)^2 + 2\pi i(\ln 2 + 2\pi i n) + 0} =$$

$$= \frac{-1}{(\ln 2 + 2\pi i n)(\ln 2 + 2\pi i n + \pi i)} = \frac{1}{(2\pi i)^2\left(n + 1 + \frac{\ln 2}{2\pi i}\right)\left(n + 1 + \frac{\ln 2}{2\pi i}\right.}$$

$$\sum_{n=0}^{\infty} \frac{1}{(n+1+x)(n+x)} = \frac{1}{x} - \frac{1}{x+1} + \frac{1}{x+1} - \frac{1}{x+2} + \frac{1}{x+2} + \dots$$

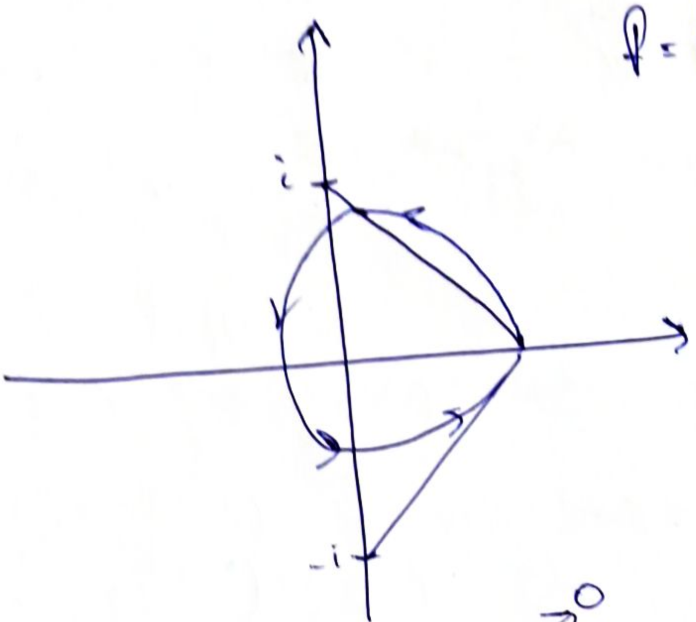
$$= \frac{1}{x} \Rightarrow \oint = \left(-\frac{1}{2\pi i} + \frac{8\pi i}{\ln 2(2\pi i)^2}\right) 2\pi i = \underline{\underline{\ln(2)^{-1}\ \text{[illegible]} - 1}}$$

N2

$g = \sqrt{\log(z^2+1)} = \sqrt{\log((z+i)(z-i))}$

$f = \log((z+i)(z-i))$

$\hat{f}(z) = (z+i)(z-i)$



$$f(z_0 - i\cdot 0) = \cancelto{0}{\ln\left(\left|\frac{\hat{f}(z_0 - i0)}{\hat{f}(z_0 + i\cdot 0)}\right|\right)} + f(z_0 + i0) + \cancelto{0}{i\,\Delta\arg \hat{f}}$$

$\Delta\arg \hat{f} = 0 + 0 = 0$

$f(z_0 - i0) = f(z_0 + i0) \Rightarrow z = 0$ не т. ветвл. для $f(z)$

$\cancel{g(f)} = g(z) = \sqrt{f(z)}$ $f(z=0) = 0$

$$g(z_0 - i0) = \left|\frac{g(z_0 - i0)}{g(z_0 + i0)}\right| g(z_0 + i0)\, e^{i\Delta\arg g}$$

$\Delta\arg g = \frac{1}{2}\cdot 2\pi = \pi$

$g(z_0 - i0) = g(z_0 + i0)\, e^{i\pi} \Rightarrow z = 0$ — т. ветвл. для $g(z)$

$g(z)$ принимает знач. $\left.\left|\sqrt{\ln(z^2+1)}\right| \exp(i\pi n)\right|_{z=0} = 0$

N1

$\log(\sqrt{z^4+1})$ а точки ветвления : $\sqrt{z^4+1} = 0$

$z^4+1=0 \Rightarrow z^4=-1 \Rightarrow z_n = e^{\pi i/4+\frac{\pi n}{2}}$, $n=[1,2,3,4]$; $z=\infty$ тоже т. в

Ответ: $z=e^{\pi i/4}$, $z=e^{3\pi i/4}$, $z=e^{5\pi i/4}$, $z=e^{7\pi i/4}$, $z=\infty$

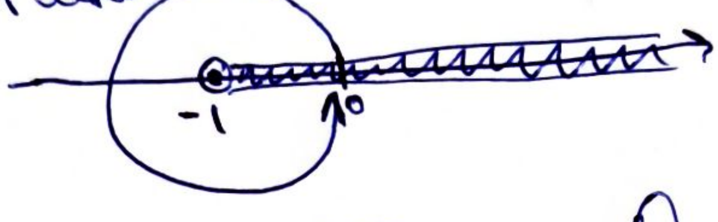
N~~2~~3

$f = z^2\sqrt{z+1}$ $z=-1, \infty$ - т. ветвл. $\left(\frac{1}{\xi}+1\right)^{\frac{1}{2}}\left(\frac{1}{\xi}\right)^2 = \frac{1}{\xi^{\frac{5}{2}}}\cdot(1+\xi)^{\frac{1}{2}}$

∞ - т. ветвл.

-1 аналогично

$g = \sqrt{z+1}$



$\Delta \arg g = \frac{1}{2}\cdot 2\pi = \pi$

Ответ: $-1, \infty$ - т. ветв
$\Delta\arg = \pi$