Lec 8. HW1

$$J(\mu,\Sigma)=\left(\max(0,1-\mu^T\phi(x)y)\right)^2+\phi(x)^T\Sigma\phi(x)+\gamma\left\{\log\frac{\det(\tilde{\Sigma})}{\det(\Sigma)}+\mathrm{tr}(\tilde{\Sigma}^{-1}\Sigma)+(\mu-\tilde{\mu})^T\tilde{\Sigma}^{-1}(\mu-\tilde{\mu})-d\right\}$$

に対し、$\hat{\mu}=\underset{\mu}{\mathrm{argmin}}\,J(\mu,\Sigma)$ を求める。

$$\frac{\partial}{\partial\mu}J(\mu,\Sigma)=\underbrace{\frac{\partial}{\partial\mu}\left[\left(\max(0,1-\overset{1\times b}{\mu^T}\overset{b\times1}{\phi(x)}\overset{1\times1}{y})\right)^2\right]}_{①}+2\gamma\tilde{\Sigma}^{-1}(\mu-\tilde{\mu})$$

① に関して。

$1-\mu^T\phi(x)y>0$ のとき。

$$① \to \frac{\partial}{\partial\mu}(1-\mu^T\phi(x)y)^2=\frac{\partial}{\partial\mu}\|1-\mu^T\phi(x)y\|^2 \quad \because (1-\mu^T\phi(x)y)\in\mathbb{R}$$

$$=-2\phi(x)y(1-\mu^T\phi(x)y)$$

$$=-2\phi(x)y(1-y\phi(x)^T\mu)$$ より。

$$\frac{\partial}{\partial\mu}J=0$$

$$\to -\phi(x)y+\phi(x)\underbrace{y^2}_{=1}\phi(x)^T\hat{\mu}+\gamma\tilde{\Sigma}^{-1}(\hat{\mu}-\tilde{\mu})=0$$

$$\to (\phi(x)\phi(x)^T+\gamma\tilde{\Sigma}^{-1})\hat{\mu}=\phi(x)y+\gamma\tilde{\Sigma}^{-1}\tilde{\mu}$$

$$\to \hat{\mu}=(\gamma\tilde{\Sigma}^{-1}+\phi(x)\phi(x)^T)^{-1}(\gamma\tilde{\Sigma}^{-1}\tilde{\mu}+\phi(x)y)$$

$B=\phi(x)$

$D=\frac{y^2}{\gamma}I$

$C=\phi(x)^T$

Woodburyの恒等式

$$=\left(\tilde{\Sigma}^{-1}+\frac{1}{\gamma}\phi(x)\phi(x)^T\right)^{-1}\left(\tilde{\Sigma}^{-1}\tilde{\mu}+\frac{y}{\gamma}\phi(x)\right)$$

$$=\left\{\tilde{\Sigma}-\tilde{\Sigma}\phi(x)[\gamma+\phi(x)^T\tilde{\Sigma}\phi(x)]^{-1}\phi(x)^T\tilde{\Sigma}\right\}\left(\tilde{\Sigma}^{-1}\tilde{\mu}+\frac{y}{\gamma}\phi(x)\right)$$

$$=\left(\tilde{\Sigma}-\frac{\tilde{\Sigma}\phi(x)\phi(x)^T\tilde{\Sigma}}{\phi(x)^T\tilde{\Sigma}\phi(x)+\gamma}\right)\left(\tilde{\Sigma}^{-1}\tilde{\mu}+\frac{y}{\gamma}\phi(x)\right)$$

$$=\tilde{\mu}+\frac{y}{\gamma}\tilde{\Sigma}\phi(x)-\frac{\tilde{\Sigma}\phi(x)\phi(x)^T\tilde{\mu}+\tilde{\Sigma}\phi(x)\boxed{\phi(x)^T\tilde{\Sigma}\phi(x)}\frac{y}{\gamma}}{\phi(x)^T\tilde{\Sigma}\phi(x)+\gamma}$$

$$=\tilde{\mu}-\frac{\tilde{\Sigma}\phi(x)\boxed{\phi(x)^T\tilde{\mu}}-y\tilde{\Sigma}\phi(x)}{\phi(x)^T\tilde{\Sigma}\phi(x)+\gamma}$$ スカラー

$$=\tilde{\mu}+\frac{y\tilde{\Sigma}\phi(x)-\tilde{\mu}\phi(x)\tilde{\Sigma}\phi(x)}{\phi(x)^T\tilde{\Sigma}\phi(x)+\gamma}$$

$$=\tilde{\mu}+\frac{y(1-\mu^T\phi(x)y)}{\phi(x)^T\tilde{\Sigma}\phi(x)+\gamma}\tilde{\Sigma}\phi(x)$$

(続く。)

… (A)