Lecq. HW. 2

$D_E^2(p_{test}, q_\pi) = 2\mathbb{E}\|x' - x\| - \mathbb{E}\|x' - \tilde{x}'\| - \mathbb{E}\|x - \tilde{x}\|$ — ⊛

풀이. $x, \tilde{x} \sim q_\pi(x) = \pi p_{train}(x|y=+1) + (1-\pi) p_{train}(x|y=-1)$

$x', \tilde{x}' \sim p_{test}$.

· $\mathbb{E}\|x' - x\| = \int_{-\infty}^{\infty} \|x' - x\| q_\pi(x)\, dx$

$= \pi \int_{-\infty}^{\infty} \|x' - x\| p_{train}(x|y=+1)\, dx + (1-\pi) \int_{-\infty}^{\infty} \|x' - x\| p_{train}(x|y=-1)\, dx$

$= \pi b_{+1} + (1-\pi) b_{-1}$ — (1)

· $\mathbb{E}\|x' - \tilde{x}'\| = const$ — (2)

· $\mathbb{E}\|x - \tilde{x}\| = \pi \mathbb{E}_{x, \tilde{x} \sim p_{train}(x|+1)} \|x - \tilde{x}\| + (1-\pi) \mathbb{E}_{x, \tilde{x} \sim p_{train}(x|-1)} \|x - \tilde{x}\|$

$= \pi \{ \pi \mathbb{E}_{x \sim p_{train}(x|+1), \tilde{x} \sim p_{train}(x|+1)} \|x - \tilde{x}\| + (1-\pi) \mathbb{E}_{x \sim p_{train}(x|-1), \tilde{x} \sim p_{train}(\tilde{x}|+1)} \|x - \tilde{x}\| \}$

$+ (1-\pi) \{ \pi \mathbb{E}_{x \sim p_{train}(x|+1), \tilde{x} \sim p_{train}(\tilde{x}|-1)} \|x - \tilde{x}\|$

$+ (1-\pi) \mathbb{E}_{x \sim p_{train}(x|-1), \tilde{x} \sim p_{train}(\tilde{x}|-1)} \|x - \tilde{x}\| \}$

$= \pi^2 A_{+1,+1} + \pi(1-\pi) A_{-1,+1}$

$+ \pi(1-\pi) A_{+1,-1} + (1-\pi)^2 A_{-1,-1}$

$= \pi^2 A_{+1,+1} + 2\pi A_{+1,-1} - 2\pi^2 A_{+1,-1} + \pi^2 A_{-1,-1} - 2\pi A_{-1,-1} + A_{-1,-1}$ — (3)

(1) ~ (3) 대입

⊛ $\longrightarrow 2\pi b_{+1} - 2\pi b_{-1} - (\pi^2 A_{+1,+1} + 2\pi A_{+1,-1} - 2\pi^2 A_{+1,-1} + \pi^2 A_{-1,-1} - 2\pi A_{-1,-1} + A_{-1,-1})$

$+ const$

$= (2A_{+1,-1} - A_{+1,+1} - A_{-1,-1})\pi^2$

$- 2(A_{+1,-1} - A_{-1,-1} - b_{+1} - b_{-1})\pi + const$
