lec11. HW2.

$$S^{(w)} = \sum_{y=1}^{c} \sum_{i:y_i=y} \overset{b\times 1}{(x_i-\mu_y)}(x_i-\mu_y)^T \quad \cdots ①$$

$= \frac{1}{2}\sum_{i,i'=1}^{n} Q^{(w)}_{i,i'}(x_i - x_{i'})(x_i - x_{i'})^T$ を示す。 但し、$Q^{(w)}_{i,i'} = \begin{cases} 1/n_y & (y_i = y_{i'} = y) \\ 0 & (y_i \neq y_{i'}) \end{cases}$

$$① \rightarrow \sum_{y=1}^{c}\sum_{i:y_i=y}(x_i x_i^T - x_i\mu_y^T - \mu_y x_i^T + \mu_y\mu_y^T)$$

$$= \sum_{y=1}^{c}\sum_{i:y_i=y} x_i x_i^T - \sum_{y=1}^{c} n_y\mu_y\mu_y^T - \sum_{y=1}^{c} n_y\mu_y\mu_y^T + \sum_{y=1}^{c} n_y\mu_y\mu_y^T$$

$$= \sum_{y=1}^{c}\sum_{i:y_i=y} x_i x_i^T - \sum_{y=1}^{c} n_y\mu_y\mu_y^T$$

$$= \sum_{i=1}^{n} x_i x_i^T - \sum_{y=1}^{c} n_y\mu_y\mu_y^T \quad —— \quad ②$$

$$= \underline{\sum_{i,i'=1}^{n} Q_{i,i'}^{(w)} x_i x_i^T} - \sum_{y=1}^{c}\frac{1}{n_y}\Big(\sum_{i:y_i=y} x_i\Big)\Big(\sum_{i':y_{i'}=y} x_{i'}\Big)^T$$

$$= \underline{\frac{1}{2}\sum_{i,i'=1}^{n} Q_{i,i'}^{(w)}(x_i x_i^T + x_{i'}x_{i'}^T)} - \frac{1}{2}\sum_{i,i'=1}^{n} Q_{i,i'} x_i x_{i'}^T - \frac{1}{2}\sum_{i,i'=1}^{n} Q_{i,i'} x_{i'} x_i^T$$

$$= \frac{1}{2}\sum_{i,i'=1}^{n} Q_{i,i'}^{(w)}(x_i - x_{i'})(x_i - x_{i'})^T$$

また、

$$C = \sum_{i=1}^{n} x_i x_i^T$$

$$= \sum_{i,i'=1}^{n} \frac{1}{n} x_i x_i^T$$

$$= \frac{1}{2}\sum_{i,i'=1}^{n}\frac{1}{n}(x_i x_i^T + x_{i'}x_{i'}^T)$$

$$= \frac{1}{2}\sum_{i,i'=1}^{n}\frac{1}{n}(x_i x_i^T + x_{i'}x_{i'}^T) \overbrace{- \frac{1}{2}\sum_{i,i'=1}^{n}\frac{1}{n}(x_i x_{i'}^T + x_{i'}x_i^T)}^{0}$$

$$= \frac{1}{2}\sum_{i,i'=1}^{n}\frac{1}{n}(x_i - x_{i'})(x_i - x_{i'})^T$$

ここで、 $S^{(b)} = C - S^{(w)}$ より。

$$S^{(b)} = \frac{1}{2}\sum_{i,i'=1}^{n}\underbrace{\Big(\frac{1}{n} - Q_{i,i'}^{(w)}\Big)}_{= Q_{i,i'}^{(b)}}(x_i - x_{i'})(x_i - x_{i'})^T$$

但し、

$$Q_{i,i'}^{(b)} = \begin{cases} 1/n - 1/n_y & (y_i = y_{i'} = y) \\ 1/n & (y_i \neq y_{i'}) \end{cases}$$