

Lec 10 HW. 1

$$\sum_{i,i'=1}^n W_{i,i'} \|Tx_i - Tx_{i'}\|^2 = 2 \text{tr}(TXLX^T T^T)$$

$$\sum_{i,i'=1}^n W_{i,i'} \|Tx_i - Tx_{i'}\|^2 = \sum_{i,i'=1}^n W_{i,i'} (x_i^T T^T - x_{i'}^T T^T) (Tx_i - Tx_{i'})$$

$$= \sum_{i,i'=1}^n W_{i,i'} (x_i^T T^T Tx_i - 2 x_{i'}^T T^T Tx_i + x_{i'}^T T^T Tx_{i'})$$

$$= \sum_{i,i'=1}^n W_{i,i'} (x_i^T T^T Tx_i + x_{i'}^T T^T Tx_{i'}) - 2 \sum_{i,i'=1}^n W_{i,i'} x_{i'}^T T^T Tx_i$$

$$= 2 \text{diag} \left(\sum_{i=1}^n W_{i,i} x_i^T T^T Tx_i \right) - 2 \sum_{i,i'=1}^n W_{i,i'} x_{i'}^T T^T Tx_i$$

$$= 2 \text{tr}(TXDX^T T^T) - 2 \text{tr}(TXWX^T T^T)$$

$$= 2 \text{tr}(TXLX^T T^T)$$

————— tr