Lec4 /H.W.1

$f_\theta(x) = \sum_{j=1}^{b} \theta_j \phi_j(x)$

$\min_\theta \frac{1}{2} \sum_{i=1}^{n} \hat{w}_i (f_\theta(x_i) - y_i$

$f_\theta(x_i) = \bar{x}_i \theta$

$$
\begin{aligned}
L(\theta) &= \frac{1}{2} \sum_{i=1}^{n} \hat{w}_i (\bar{x}_i \theta - y_i)^2 \\
&= \frac{1}{2} \sum_{i=1}^{n} (\hat{w}_i^{\frac{1}{2}} \bar{x}_i \theta - w_i^{\frac{1}{2}} y_i)^2 \\
&= \frac{1}{2} \| \hat{W}^{\frac{1}{2}} \bar{X} \theta - \hat{W}^{\frac{1}{2}} y \|^2 \\
&= \frac{1}{2} (\hat{W}^{\frac{1}{2}} \bar{X} \theta - \hat{W}^{\frac{1}{2}} y)^T (\hat{W}^{\frac{1}{2}} \bar{X} \theta - \hat{W}^{\frac{1}{2}} y) \\
&= \frac{1}{2} (\theta^T \bar{X}^T \hat{W}^{\frac{1}{2}} - y^T \hat{W}^{\frac{1}{2}}) (\hat{W}^{\frac{1}{2}} \bar{X} \theta - \hat{W}^{\frac{1}{2}} y) \\
&= \frac{1}{2} (\theta^T \bar{X}^T \hat{W} \bar{X} \theta - \theta^T \bar{X}^T \hat{W} y - y^T \hat{W} \bar{X} \theta + y^T \hat{W} y)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial L}{\partial \theta} &= \frac{1}{2} ((\bar{X}^T \hat{W} \bar{X} + \bar{X}^T \hat{W} \bar{X}) \theta - \bar{X}^T \hat{W} y - \bar{X}^T \hat{W} y) \\
&= \bar{X}^T \hat{W} \bar{X} \theta - \bar{X}^T \hat{W} y = 0
\end{aligned}
$$