Tensor Network Decoders for Quantum Error Correcting Codes

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Contents

Todo list

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Chapter 1

Literature Review

1.1 Decoding

- Chubb2021[9] discusses tensor networking decoding of 2D local codes. These are under phenomenological noise models for bit-flip, phase-flip and depolarizing noise. This is the results to replicate by mid October
- BSV2014[3] discusses the equivalence between maximum Likelihood decoding and tensor network contraction.
- Conservartion Laws and QEC[6] is a very good review on decoding and different methods and discusses generalizing it for LDPC Codes
- On MLD with circuit-level errors[27] gives a description for MLD given a measurement circuit.
- appendix from google's jul 2022[1] on scaling up the surface code.

1.2 Tensor Networks

- Quantum Lego[7] considers building quantum error correcting codes as tensor networks.
- TN Codes[19] introduces tensor network stabilizer codes.
- Hand-Wavy[5] is an introduction to Tensor Networks.
- review on area laws of entanglement entropy[17]
- Parallel decoding in tensor-network codes[18]

  to read later
- Biamonte's textbook on Quantum Tensor Networks[2]
- TenPy lecture notes on Tensor Networks[23]
- DMRG review using tensor networks[30]
- Computational Studies of Quantum Spin Systems, review on Monte Carlo and different algorithms[29]

1.3 Noise Models, Codes and Simulations

- Stim[21] is probably the backend we are going to be using to generate noise data.
- Chamberland2022[8] has a section on defining what circuit level noise is. Local vs Global decoders and their combination is discussed.
- Honeycomb Code[22] is benchmaked using Stim in this paper. It also discusses standard metrics in benchmarking QEC codes like logical error rates, thresholds, lambdas and teraquop qubit counts
- Niko's ug thesis on subsystem codes[4]

1.4 Stat Mech

- DLKP's[14] paper on Topological Quantum Memory is one of the first paper to discuss the statistical physics of error recovery.
- Infamous-Chubb-Flammia[10] generalises the proof for the statistical mechanical mapping from DLKP[14] for independent noise to weakly correlated noise. It also discusses the link between Maximum Likelihood Decoding and Tensor Network Contraction.
- The Nishimory Line in the RBIM[26] is a review on the Nishimori Line in the two dimensional Random Bond Ising Model.
- Disorder[28] is a review from a 2015 summer school on disorder in condensed matter systems.
- 3D color codes via stat-mech mapping[25] studies the disorder temperature phase transition diagram using 2 new models: 4-body and 6-body 3D RBIM.

Chapter 2

Statistical Mechanical Mapping of QEC Codes

2.1 The Statistical Physics of Error Recovery

Conditions and Assumptions about Fault Tolerance[13]

- Constant Error Rate
- Weakly correlated errors
- Parallel Operation
- Resuable Memory
- Fast Measurements
- Fast and accurate classical processing
- No leakage, however leakage errors do exist and we have to deal with them
- Non local quantum gates
- However if local gates are only available, a high coordination number is demanded. (a lot more nearest neighbors per qubit)

An order parameter is formulated that distinguishes two phases of a quantum memory.

- “ordered” phase: reliable storage of encoded quantum information is possible.
- “disordered” phase: errors afflict the encoded quantum information.

The Error Model used

We assume that X and Z errors are equally likely with probability p and these are uncorrelated and independent. The error channel is then represented as

$$\rho \rightarrow (1-p)^2 I\rho I + p(1-p)X\rho X + p(1-p)Z\rho Z + p^2 Y\rho Y \tag{2.1}$$

Measurement errors are also allowed to occur. The probability that a particular sydrome bit is faulty is q. Measurement errors are also uncorrelated with qubit errors in both time and space.

2.2 Statistical Mechanical Model

A classical spin model and its' statistical mechanical properties capture the error correction properties of the quantum code, in a way that the threshold of the error correcting code is the phase transition of the classical spin model.[11, 13]

Definition 1 (Stat Mech Hamiltonian: *independent noise*)**.** For a Pauli E $\in P^(\times n)$, and coupling strengths $\{J_i : \mathcal{P}_i \to \mathrm{R}\}_i$, the hamilition of a spin configuration $\vec{c}$ is given as

$$H_E(\vec{c}) = -\sum_{i,\sigma\in\mathcal{P}_i} J_i(\sigma)[\sigma, E]\prod_k [\sigma, S_k]^{c_k}$$

The sum is taken over all the sites i and all elements σ in the single site Pauli group at site i. The commutator used here is the scalar commutator, which is defined as

$$[\mathrm{A}, \mathrm{B}] \tag{2.2}$$

Chapter 3

Noise Models

The difference between phenomenological and circuit level noise is not exactly clear.
RBIM -> phenomenological and circuit sims-> generate syndrome -> decode

- Does phenomenological noise include measurement errors?
- The goal for next meeting is to implement this for surface codes under both phenomenological and circuit level noise before moving onto other codes.

3.1 Phenomenological Noise Models

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3.2 Circuit Level Noise Models

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Appendix

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