Tensor Network Decoders for Quantum Error Correcting Codes

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Chapter 1

Literature Review

1.1 Decoding

- Chubb2021[4] discusses tensor networking decoding of 2D local codes. These are under phenomenological noise models for bit-flip, phase-flip and depolarizing noise. This is the results to replicate by mid October
- BSV2014[1] discusses the equivalence between maximum Likelihood decoding and tensor network contraction.
-

1.2 Tensor Networks

- Quantum Lego[3] considers building quantum error correcting codes as tensor networks.
- TN Codes[14] introduces tensor network stabilizer codes.
- Hand-Wavy[2] is an introduction to Tensor Networks.

1.3 Noise Models and Simulations

- Stim[16] is probably the backend we are going to be using to generate noise data.
-

1.4 Stat Mech

- DLKP's[9] paper on Topological Quantum Memory is one of the first paper to discuss the statistical physics of error recovery.
- Infamous-Chubb-Flammia[5] generalises the proof for the statistical mechanical mapping from DLKP[9] for independent noise to weakly correlated noise. It also discusses the link between Maximum Likelihood Decoding and Tensor Network Contraction.
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Chapter 2

Statistical Mechanical Mapping of QEC Codes

2.1 The Statistical Physics of Error Recovery

Conditions and Assumptions about Fault Tolerance[8]

- Constant Error Rate
- Weakly correlated errors
- Parallel Operation
- Resuable Memory
- Fast Measurements
- Fast and accurate classical processing
- No leakage, however leakage errors do exist and we have to deal with them
- Non local quantum gates
- However if local gates are only available, a high coordination number is demanded. (a lot more nearest neighbors per qubit)

An order parameter is formulated that distinguishes two phases of a quantum memory.

- "ordered" phase: reliable storage of encoded quantum information is possible.
- "disordered" phase: errors afflict the encoded quantum information.

The Error Model used

We assume that X and Z errors are equally likely with probability p and these are uncorrelated and independent. The error channel is then represented as

$$\rho \to (1-p)^2 I\rho I + p(1-p)X\rho X + p(1-p)Z\rho Z + p^2 Y\rho Y \tag{2.1}$$

Measurement errors are also allowed to occur. The probability that a particular sydrome bit is faulty is q. Measurement errors are also uncorrelated with qubit errors in both time and space.

2.2 Statistical Mechanical Model

A classical spin model and its' statistical mechanical properties capture the error correction properties of the quantum code, in a way that the threshold of the error correcting code is the phase transition of the classical spin model.[6, 8]

Definition 1 (Stat Mech Hamiltonian: *independent noise*)**.** For a Pauli E $\in P^(\times n)$, and coupling strengths $\{J_i : \mathcal{P}_i \to \mathrm{R}\}_i$, the hamilition of a spin configuration $\vec{c}$ is given as

$$H_E(\vec{c}) = -\sum_{i,\sigma\in\mathcal{P}_i} J_i(\sigma)[\sigma, E]\prod_k [\sigma, S_k]^{c_k}$$

The sum is taken over all the sites i and all elements σ in the single site Pauli group at site i. The commutator used here is the scalar commutator, which is defined as

$$[\mathrm{A}, \mathrm{B}] \tag{2.2}$$

Chapter 3

Noise Models

3.1 Phenomenological Noise Models

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3.2 Circuit Level Noise Models

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Appendix

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