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Electric power demand forecasting using interval time series: A comparison between VAR and iMLP

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ABSTRACT

Electric power demand forecasts play an essential role in the electric industry, as they provide the basis for making decisions in power system planning and operation. A great variety of mathematical methods have been used for demand forecasting. The development and improvement of appropriate mathematical tools will lead to more accurate demand forecasting techniques.

In order to forecast the monthly electric power demand per hour in Spain for 2 years, this paper presents a comparison between a new forecasting approach considering vector autoregressive (VAR) forecasting models applied to interval time series (ITS) and the iMLP, the multi-layer perceptron model adapted to interval data.

In the proposed comparison, for the VAR approach two models are fitted per every hour, one composed of the centre (mid-point) and radius (half-range), and another one of the lower and upper bounds according to the interval representation assumed by the ITS in the learning set. In the case of the iMLP, only the model composed of the centre and radius is fitted. The other interval representation composed of the lower and upper bounds is obtained from the linear combination of the two.

This novel approach, obtaining two bivariate models each hour, makes possible to establish, for different periods in the day, which interval representation is more accurate. Furthermore, the comparison between two different techniques adapted to interval time series allows us to determine the efficiency of these models in forecasting electric power demand. It is important to note that the iMLP technique has been selected for the comparison, as it has shown its accuracy in forecasting daily electricity price intervals.

This work shows the ITS forecasting methods as a potential tool that will lead to a reduction in risk when making power system planning and operational decisions.

1. Introduction

Accurate models for electric power demand and price forecasting are necessary for the operation and planning of power systems and its consequences are critical in energy efficiency and sustainability issues. Nowadays, the role of forecasting in deregulated energy industries is essential in key decision making, such as purchasing and generating electric power, load switching, and infrastructure development. In addition, sustainability analysis (see, for example, Linares et al. (2008) for the Spanish electricity sector) relies on accurate quantitative predictions.

Depending on the time horizon selected, demand forecasting can be classified as: short-term from 1 h to 1 week, medium-term from a week to a year and long-term for more than a year (see Hahn et al. (2009) for details and implications in decision making).

Most forecasting methods based on classic data (single valued) use statistical techniques or artificial intelligence tools. The initial studies were based on statistical models, using for instance integrated moving average models (ARIMA) (Abdel-Aal and Al-Garni, 1997; Saab et al., 2001) or models based on regression (Mohgram and Rahman, 1989; Papalexopoulos and Hesterberg, 1990). Their low accuracy in time series with non-linear characteristics prompted the application of artificial intelligence techniques, such as neural networks (Papalexopoulos et al., 1994; Chowand and Leung, 1996), hybrids methods (Srinivasan et al., 1999; Padmakumari et al., 1999) or genetic algorithms (Tzafestas and Tzafestas, 2001). Taylor and McSharry (2007) evaluate different short-term load forecasting methods: (i) ARIMA model; (ii) Periodic AR model; (iii) an extension for double seasonality of Holt–Winters exponential smoothing method; (iv) an alternative exponential smoothing method; (v) a method based on the principal component analysis (PCA) of the daily demand profiles, concluding from the results obtained that the double seasonal Holt–Winters exponential smoothing method is the best of these

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methods. Franco et al. (2006) apply an extension of the VAR models, the Vector error correction models (VECM) to forecast the electricity demand of the Venezuelan electric system for the period 2004–2024. Erdogdu (2007) analyzed electricity demand using cointegration and ARIMA modelling. Arroyo and Maté (2009) adapt the *k*-Nearest Neighbours (*k*-NN) algorithm to forecast histogram time series (HTS).

As it can be seen, a large variety of mathematical methods and ideas have been used for demand forecasting (see Hahn et al. (2009) for a recent survey). Further development and improvement of mathematical tools will surely lead to more accurate demand forecasting techniques, such as indicated by Gonzalez-Romera et al. (2006).

In the field of interval time-series (ITS) forecasting, different techniques have been developed in recent years. Arroyo et al. (2007) develop three exponential smoothing methods for ITS. Maia et al. (2006) apply autoregressive moving average models (ARMA) to ITS in a hybrid model with neural nets and Maia et al. (2008) present approaches to interval-valued time-series forecasting based on AR, ARIMA and Artificial Neural Networks (ANN) models. Cheung (2007) proposes an empirical model for daily highs and lows for three US stock indexes using the VECM. Han et al. (2008) propose an interval linear model to investigate the dynamic relationships between interval processes.

Moreover, Neural Nets applied to Interval data (INN) have been developed by several researchers (Ishibuchi et al., 1993; Simoff, 1996; Beheshti et al., 1998; Rossi and Conan-Guez, 2002; Patiño-Escarcina et al., 2004; Muñoz San Roque et al., 2007). The model proposed by Muñoz San Roque et al. (2007) is the iMLP used here for comparison.

In addition, Zhao et al. (2008) propose a statistical approach for interval forecasting of the electricity price, but the method is based on classic time series (single valued) not in interval time series. The prediction interval is obtained by forecasting the price value and its variance. The Support Vector Machine (SVM) is employed to forecast the value of the price, and a statistical model – obtained by introducing a heteroskedastic variance equation to the SVM – to forecast the price variance.

Two interval time-series forecasting models are proposed in this paper in order to show their accuracy in demand forecasting. In the work reported here the VAR technique is adapted to ITS as it is done in Maia et al. (2006) for ARMA models. The iMLP is applied in accordance with Muñoz San Roque et al. (2007), whose model presents accuracy results in electricity price forecasting, in order to observe its behaviour in electricity demand forecasting. Depending on the representation of the interval assumed by the ITS, the interval time series are split up into two time series: the time series of the lower bound and the time series of the upper bound, or the time series of the centre and the time series of the radius. Once the two time series are obtained, the application of the VAR and the iMLP methods to obtain the forecasts of the electric power demand is straightforward.

The paper is divided into five sections: Section 2 introduces the interval analysis; Section 3 introduces the vector auto-regression models; Section 4 shows the iMLP; Section 5 shows the application of the forecasting technique to the interval time series under study, compares both methods and analyzes the results. Finally, Section 6 concludes.

2. Interval analysis

2.1. Introduction

Under the assumption that observations and estimations in the real world are incomplete to represent real data exactly, Ramon Moore in 1959 proposed the Interval Analysis as a tool for automatic control of the errors in a computed result that arise from input error, from rounding errors during computation, and from truncation errors when using a numerical approximation to the mathematical problem. Hence, if precision is needed, data must be represented by intervals. Since 1960, Interval Analysis has been an active focus on research.

Moore (1966) establishes the basis for Interval Analysis and Moore and Bierbaum (1979) deal with an important set of techniques providing a mathematically rigorous and complete error analysis for computational results. They show that interval analysis provides a powerful set of tools with direct applicability to important problems in scientific computing. In addition, Chavent and Saracco (2008) have obtained basic descriptive statistics such as central tendency and dispersion measures for interval data. More recently, Moore et al. (2009) present an updated introduction to Interval Analysis.

2.2. Interval data

Interval data is a particular case of symbolic data. See Billard and Diday (2003), Billard and Diday (2006) and Diday and Noirhomme (2008) for key references in the field. The main difference between classic and symbolic data is that a classic data point takes as its value a single point in *p*-dimensional space, whereas a symbolic one takes as its value a hypercube (or hyperrectangle) in *p*-dimensional space, or it is the Cartesian product of *p* distributions in *p*-dimensional space or a mixture of both. In short, symbolic data have internal variation and structure. It is important to mention that interval data may be in many instances the result of an aggregation procedure, spatial or temporal, over information collected at a very disaggregated level.

An interval $[x]$ over the base set $(E, \leq)$ is an ordered pair $[x]=[x_L; x_U]$ where $x_L, x_U \in E$ are the endpoints or bounds of the interval such that $x_L \leq x_U$.

Table 1 shows the interval-valued variables in every month of the hourly spot electricity price and the electric energy demand in Spain in 2007.

An interval can be represented by its lower and upper bounds $[x]=[x_L,x_U]/-\infty<x_L\leq x_U<\infty$ or by its centre (mid-point) and radius (half-range) as $[x]=\langle x_C,x_R\rangle$ where $x_C=(x_L+x_U)/2$ and $x_R=(x_U-x_L)/2$. In Fig. 1 the structure of an interval is presented.

2.3. Interval time series

An interval time series (ITS) is a chronological sequence of interval-valued variables and it is denoted by $\{[x_t]\} = \{[x_{Lt},x_{Ut}]\} = \{\langle x_{Ct},x_{Rt}\rangle\}$ for t=1,2,…, n. The interval time series of the monthly electric power demand in 2000 for the hour 1, H1, is shown in Fig. 2.

2.4. Interval time-series forecasting techniques

Several interval time-series forecasting techniques have been developed in recent years (see Arroyo et al. (2010) for a survey).

Table 1
Interval-valued variables (data available at http://www.ree.es and http://www.omel.es).

Year 2007	Final electricity price (€/MWh)	Electricity demand (MWh)
January	[10, 84.79]	[18,644, 43,201]
February	[5, 77]	[20,082, 40,745]
March	[7, 61.32]	[19,907, 39,593]
…	…	…
…	…	…

Arroyo et al. (2007) develop three exponential smoothing methods for ITS: Simple Exponential Smoothing (SES), Exponential Smoothing with additive Trend (EST) and Exponential Smoothing with Seasonality (ESS). Maia et al. (2006) apply the ARMA models to interval time series by analyzing two time series: the interval mid-point series and the half-range interval series. The model proposed is the same as that used for classic data, but transforming the interval time series into two series of single values (mid-point and half-range series). Vector error correction models (VECM) are extended to interval time series by Cheung (2007), who proposes an empirical model for daily highs and lows for three US stock indexes. It shows that daily highs and lows are cointegrated and that data on openings, closings and trading volume offer incremental explanatory power for the variations in highs and lows within a VECM framework.

In the case of neural nets applied to interval data, the type of data (interval or single valued) used for the parameters of the INN (inputs, outputs, weights and biases), is the main difference between different methods.

Ishibuchi et al. (1993) propose two models. In the first one, weights, biases and output are interval valued, while inputs are single valued. However, due to the difficulty of the learning algorithm, they established a second model, taking the weights and the biases of the hidden units to be single values. Another INN model is proposed by Simoff (1996). In this model, all the parameters of the net are interval valued. The properties of the INN are analyzed, but it does not establish a learning algorithm. Beheshti et al. (1998) also assume all the parameters to be interval values and propose an algorithm in order to obtain optimal weights and biases. Rossi and Conan-Guez (2002) propose different approaches in order to allow intervals being the output and inputs of a classical MLP. Patiño-Escarcina et al. (2004) obtain a new model of neural network, including interval arithmetic in the traditional neural network structure (learning algorithm). It is applied to a one-layer perceptron classification model. The output is represented in binary form, while weights, biases and inputs are intervals. The activation function is a binary function for interval data.

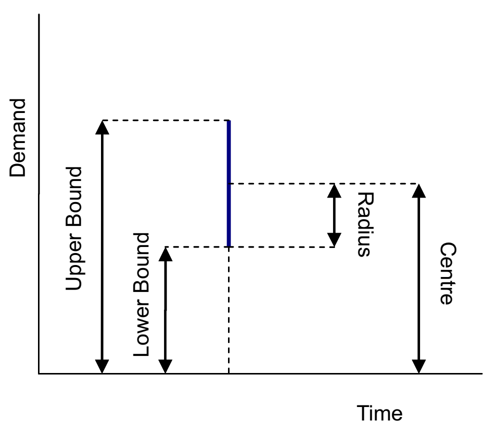


Fig. 1. Interval structure.

2.5. *Forecasting accuracy measures for interval time series*

Arroyo and Maté (2006) propose error measures based on distances for interval data, such as the Ichino-Yaguchi and the Haussdorf distances, the latter also being proposed by Chavent and Saracco (2008).

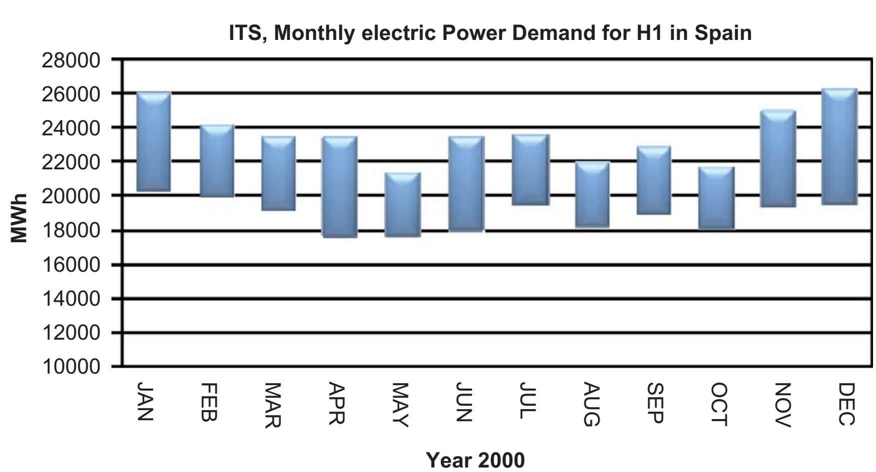
Let $\{[x_t]\}$be the observed ITS and $\{[\hat{x}_t]\}$ be the forecast of this ITS with t=1,2, …, n. The mean distance error based on Hausdorff distance is defined as

$$MDE_H = \frac{1}{n}\sum_{t=1}^{n}\left[\left|x_{Ct}-\hat{x}_{Ct}\right|+\left|x_{Rt}-\hat{x}_{Rt}\right|\right] \quad (1)$$

The mean distance error based on the Ichino-Yaguchi distance is defined as

$$MDE_{IY} = \frac{1}{n}\sum_{t=1}^{n}0.5\left[\left|x_{Lt}-\hat{x}_{Lt}\right|+\left|x_{Ut}-\hat{x}_{Ut}\right|\right] \quad (2)$$

However, these methods are not independent of the scale of the data, as they are focused either on the centre and the radius, or on the lower and upper limits. Thus, in order to be able to analyze the forecasting performance of the model for both bivariate systems, a scaled error measure is needed.



Upper Bound	26091	24219	23513	23491	21373	23445	23609	22008	22842	21768	24956	26302
Lower Bound	20328	19920	19137	17679	17582	18000	19494	18198	18909	18141	19342	19523
	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC

Fig. 2. Interval time series, ITS. Monthly electric power demand for H1 in Spain. Year 2000 (data available at http://www.ree.es).

Table 2
Benchmark naive forecasting methods.

Benchmark method	Description	Notation
1	Without trend and seasonality	x_{i-1}
2	Without seasonality but with trend in level	$x_{i-1}+(x_{i-1}-x_{i-2})$
3	With seasonality but without trend	x_{i-s}
4	With seasonality and trend on the seasonality	$x_{i-s}+(x_{i-s}-x_{i-2s})$
5	With seasonality and with trend in level	$x_{i-s}+((x_{i-1}-x_{i-(s-1)})-(x_{i-2}-x_{i-(s-2)}))$
6	With seasonality, trend on the seasonality and trend in level	$x_{i-s}+(x_{i-s}-x_{i-2s})+(x_{i-1}-x_{i-2})$

Hyndman and Koehler (2006) propose error measures that remove the effect of the scale. These measures consist of scaling the error based on the in-sample Mean Absolute Error (MAE) from the naïve forecast method (which assumes a random walk model).

A scaled error in t is defined as

$$q_t = \frac{x_t - \hat{x}_t}{(1/n-1)\sum_{i=2}^{n}|x_i - x_{i-1}|} \tag{3}$$

This error is less than one if it is the result of a better forecast than the average one-step naïve forecast computed in sample. However, it is greater than one if the forecast is worse than the average one-step naïve forecast computed in sample.

The Mean Absolute Scaled Error (MASE) proposed by Hyndman and Koehler (2006) is

$$MASE = mean\ (|q_t|) \tag{4}$$

They state that measures such as root mean squared scaled error (RMSSE) can be defined analogously. This method will be applied here.

Before defining the RMSSE, we present different benchmark forecast methods or naïve methods in Table 2 (s is the length of seasonality), because scaling the error depends on the time-series characteristics.

Our time series under study presents seasonality and trend in level, thus method number 5 will be used in order to define the RMSSE.

Hence, the squared scaled error at time t is defined as

$$s_t = \frac{(x_t - \hat{x}_t)^2}{(1/n-s)\sum_{i=s}^{n}(x_{i-s}+((x_{i-1}-x_{i-(s-1)})-(x_{i-2}-x_{i-(s-2)})))^2} \tag{5}$$

The scaled factor is the in-sample mean square scaled error. The RMSSE can be defined as

$$RMSSE = \sqrt{mean(s_t)} \tag{6}$$

3. Vector autoregressive models (VAR)

The vector autoregressive models (VAR) are a generalization of the autoregressive models in the multivariate context (Lütkepohl, 2004, 2006).

The VAR(p) model for K variables can be written as follows:

$$Y_t = C + \varphi_p(B)Y_t + \varepsilon_t \tag{7}$$

where ε_t is the $K\times 1$ vector of disturbances of the system, is assumed to be a white noise error (i.e., with a mean of zero, a constant variance and zero covariances) and is an identically and independently distributed random variable, C is the $K\times 1$ vector of constants and Y_t is a K-dimensional stationary vector time series, and

$$\varphi_p(B) = \varphi_1 B + \varphi_2 B^2 + \cdots + \varphi_p B^p \tag{8}$$

is the autoregressive $K\times K$ matrix polynomial of order p, with all its roots outside the unit circle, and B is the backshift operator:

$$B^j Y_t = Y_{t-j} \tag{9}$$

It should be noted that before the VAR model is estimated it is necessary to identify the characteristics of the variables used in the model. That is, it should be determined whether the underlying data processes are stationary or not. If the time series are found to be non-stationary, the order of integration will need to be determined, and the stationary form of the variable should be added to the VAR model. Furthermore, if there is any cointegrated relation between the variables, the relation must be included in the model by using a VECM. The VECM are an extension of the VAR models. They contain cointegrated variables, that is, variables with a long-term equilibrium relation between them.

The VECM(p) model for K variables can be written as follows:

$$\Delta Y_t = C + \pi Y_{t-1} + \boldsymbol{\Gamma}_1 \Delta Y_{t-1} + \boldsymbol{\Gamma}_2 \Delta Y_{t-2} + \cdots + \boldsymbol{\Gamma}_{p-1}\Delta Y_{t-p-1} + \varepsilon_t \tag{10}$$

where Δ refers to $\Delta Y_t = Y_t - Y_{t-1}$, C is a vector of K constants, and coefficient matrices are in the following form: $\pi = -(I_K - \varphi_1 - \cdots - \varphi_p)$ and $\boldsymbol{\Gamma}_j = -(\varphi_{j+1} + \cdots + \varphi_p)$with $j = 1, \ldots, p-1$.

In this paper two different bivariate systems are analyzed, one for the lower and the upper bounds series and other for the radius and centre series. Thus, there will be two different VAR models for the two different bivariate systems given by: (1) Lower and Upper bounds series, $(\{x_{Lt}\},\{x_{Ut}\})$; and (2) Radius and Centre series, $(\{x_{Ct}\},\{x_{Rt}\})$.

4. Interval multi-layer perceptron, iMLP

The new type of multi-layer perceptron adapted to interval data, the iMLP, is similar to the traditional one but with different transfer functions to enable it to operate with interval-valued data. Muñoz San Roque et al. (2007) propose an interval neural net (INN), specifically an interval multi-layer perceptron (iMLP), where inputs and outputs are interval-valued data, but biases and weights are single values.

In this model, the iMLP is comprised of a hidden and an output layer. The output of the units of the hidden layer is obtained from a weighted linear combination of n interval inputs and bias, while the activation function of this layer is the tanh function.

The output of the network, the forecasted value, is obtained by transforming the activations of the hidden units, using a second layer of processing units via a linear combination of the activations of the hidden layer and the bias.

This model has two applications:

- As an interval-valued function approximation.
- As an instrument to evaluate the prediction interval of a pre-adjusted crisp MLP model fed with interval-valued input data.

The scheme of the iMLP model is shown in detail in the appendix.

5. Demand forecasting

The energy demand of a country or region is one of the most important variables in determining the main characteristics of the energy industry, for instance, the range values of energy prices as well as the energy policy assumed in energy production. Thus, it is

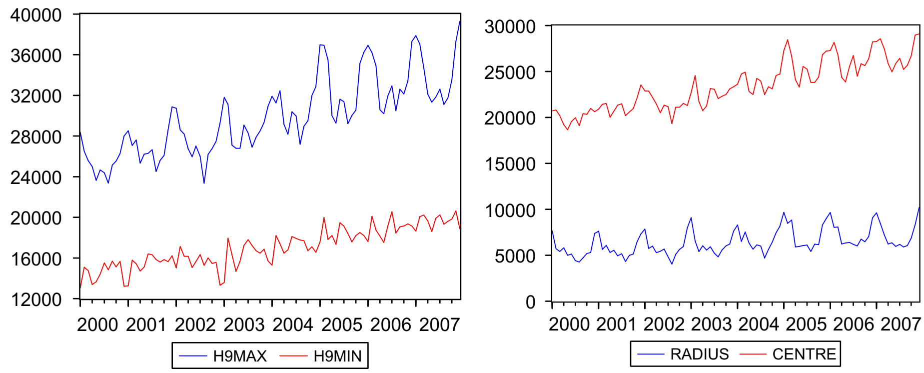


Fig. 3. Components of the interval time series for H9 (data are available at http://www.ree.es).

essential to obtain accurate forecasts of this important variable with such influential consequences.

In this section we will demonstrate the efficiency of the VAR and the iMLP models when applied to interval time series to forecast the monthly electric power demand per hour in Spain.

5.1. Data treatment

One of the sources of interval-valued data is the summarization of high-frequency sample data. First of all, we have to operate with the original time series in order to obtain the interval time series. The available data represent the electric power demand in Spain in MWh per day and per hour from January 1, 2000 to December 12, 2007. Data can be obtained at the website of the Spanish Power System Operator, Red Eléctrica de España (www.ree.es).

We calculate the maximum and minimum value of the demand per hour and month from 2000 to 2007. This produces an interval time series where each observation is formed by an interval that collects, as its lower bound, the minimum value of the electric demand and, as its upper bound, the maximum value of the electric demand for a specific hour, month and year.

Once we have the interval time series where every observation is denoted by its lower and upper limits, obtaining the interval time series for the radius and centre is straightforward by applying $x_C = (x_L + x_U)/2$ and $x_R = (x_U - x_L)/2$.

Thus, there are two single-valued time series in each interval representation. On the one hand, the lower bound series and the upper bound series. On the other hand, the radius series and the centre series. As it is shown in Fig. 3 it is important to note that the seasonality pattern is more noticeable in the radius time series than in the centre time series.

Taking into account the number of series, it is necessary to estimate 48 VAR models, one for each hour and every interval representation and 24 iMLP, as the interval representation applied to the iMLP is the one formed by the centre and the radius.

5.2. VAR estimation

A requirement of the VAR technique is the stationarity of the series. We study their integration grade. As we can see in Fig. 3, it is obvious that the series are not stationary, as they present trend and seasonality. The augmented Dickey–Fuller test at 1%, 5% and 10% levels of significance demonstrate that these series are integrated of grade 1, I(1). For this reason, it is necessary to transform the series in order to convert them to stationary form.

The first idea should be to apply first differences in order to remove the trend and a seasonal difference for the seasonality, but according to Sims (1980) it is not appropriate to differentiate the series, even if they have a unit root, since it can eliminate the relationships between the variables.

Lütkepohl (2005) and Enders (2004) propose that differentiating the series can distort interesting features of the relationships between the original variables, such as the co-movements between the data or the possible cointegration relationships.

Furthermore, the number of observations per each serie is reduced. Therefore the utilization of other methods instead of differentiating to make the series stationary is highly recommended. Hence, the Hodrick–Prescott filter has been used to identify and remove the trend (see Arias and Torres (2004)), and the Census X12 method has been used for the seasonality.

In this work the VAR model is adapted to interval time series as it is done in Maia et al. (2006) for ARMA models. Depending on the representation of the ITS assumed by the interval time series, the interval time series is split up into two time series, the time series of the lower bound and the time series of the upper bound, or the time series of the centre and the time series of the radius. Once the two time series are calculated, the application of the VAR method is straightforward in order to obtain the forecast of the electric power demand. The series used to estimate the model are the ITS from 2000 to 2005.

To select the VAR order, the Final Prediction Error Criterion (FPE) has been applied, and the Akaike's information criterion (AIC) has been used instead of the Hannan–Quinn criterion (HQ) or the Schwarz criterion (SC) (see Akaike, 1969, 1971; Hannan and Quinn, 1979; Schwarz, 1978). The VAR order for both bivariate systems is shown in Table 3.

After having determined the VAR order, we analyze if there are any cointegrated relations between the variables (see Murray (2004)). A short view of the linear regresssion for H19 once the series have been deseasonalized, give us a light evidence that there is no cointegration between the variables as the slopes presents a significant difference (see Fig. 4). Formally, the Johansen test (see Erdogdu (2007)) run with the Software Eviews indicates that there is not any cointegration between the variables, which means that it is not possible to fit a VECM to the bivariate systems.

Once the VAR model per every hour for both bivariate systems is estimated using the Eviews, the Pormanteau test and the Lagrange Multiplier test are used to analyze the residual autocorrelation (see Arias and Torres (2004)). Finally, the performance evaluation of the estimated VAR models for the interval time series under study is checked through the accuracy measure RMSSE presented before.

5.3. iMLP estimation

In this section, the monthly electric power demand per hour is modelled as a function of the monthly electric power demand per hour of past observations (see Table 4).

An iMLP with 15 neurons in the hidden layer has been trained with 5 years of data as a training set (2001–2005) and validated with 2 years (2006–2007).

The interval outputs of the iMLP are represented by the radius and the centre. Applying $x_C=(x_L+x_U)/2$ and $x_R=(x_U-x_L)/2$, the computation of the interval outputs represented by the upper and the lower bounds is straightforward.

As it is done for the VAR model, the performance evaluation of the estimated iMLP models for the interval time series under study is checked through the accuracy measure, RMSSE.

Table 3
VAR order for both bivariate systems in the estimation period (2000–2005).

Hour	Centre-radius	Upper-lower bound
1	1	3
2	1	3
3	3	3
4	1	2
5	1	1
6	1	1
7	1	1
8	1	1
9	1	1
10	1	2
11	1	1
12	1	2
13	1	1
14	1	1
15	1	1
16	1	2
17	1	2
18	1	1
19	1	1
20	1	2
21	1	1
22	2	1
23	2	1
24	1	1

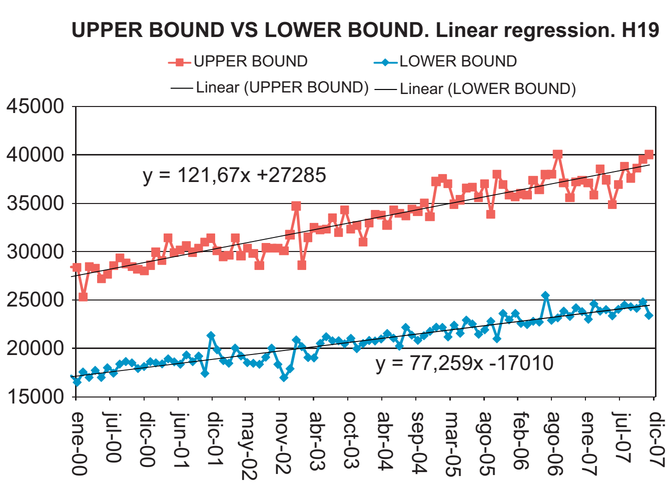


Fig. 4. Linear regression H19.

Table 4
Past observations variables.

Input	Description
$[x_{t-1}]$	Electric power demand for a specific hour 1 month ago
$[x_{t-2}]$	Electric power demand for a specific hour 2 months ago
$[x_{t-12}]$	Electric power demand for a specific hour 12 months ago

5.4. RMSSE values

The results obtained from the calculation of the RMSSE shown in Table 5 confirm the predictability of an iMLP and VAR model for ITS of electric power demand, as their values are lower than one.

5.5. Comparison

The comparison of the efficiency of both methods is shown in Fig. 5, where the values of the RMSSE for the different components of the interval representations are compared.

Looking at Fig. 5, it is clear that the VAR model is more efficient than the iMLP when forecasting the monthly electric power demand per hour as its RMSSE values are lower than the RMSSE values of the iMLP.

Nevertheless, it should be noted that the iMLP shows a great potential for forecasting electric power demand as its values of the RMSSE are lower than one. Furthermore, for further research it would be interesting to compare both methods when forecasting daily electricity prices intervals, where the iMLP model is likely to be more efficient than the VAR model, as daily electricity prices present less linearity than electric power demand.

Taking these considerations into account, in the next section the results from the VAR model will be analyzed.

5.6. Results

Before analyzing the results obtained, it is necessary to make some observations.

As it is known, the hourly electric power demand depends on various factors, such as temperature, day of the week, etc. In Fig. 6 an hourly average demand curve is shown comparing the summer period vs. the winter period for 2005 (data are available http://www.ree.es).

Looking at Fig. 6, it is clear that in the period from 18 to 22 h, the demand shows the highest variability, as its behaviour differs considerably from winter to summer due to the temperature effect. In contrast, from 22 to 18 h the demand curve for both periods is very stable as, in both seasons, in the period from 22 to 6 h the majority of companies and factories are closed and most of people are asleep. From 22 h to 18 h the variability of the demand is lower than from 18 to 22 h.

Thus, according to the variability of the demand, it is possible to classify the hours of the day in two different periods or sections (see Table 6).

According to Table 5 and the aforementioned hours classification, the lower bound has the lowest error in all periods. This is obvious, since the minimum value of the demand in an hour presents less variability and uncertainty than the others providing the maximum level of the demand, the critical value in demand forecasting.

In the case of the radius, as we can see in Fig. 7, the values of the RMSSE are higher from 18 to 22 h, which is the period of highest variability, taking the maximum value at 19 h, (0.41). On the other hand, for the rest of the hours the values of the RMSSE are lower, with the minimum at 4 h, (0.15).

For the upper bound, in contrast to the radius case, it can be seen in Fig. 8 that the higher values of the RMSSE are found from

Table 5
RMSSE values.

iMLP					VAR				
Hour	Upper	Lower	Centre	Radius	Hour	Upper	Lower	Centre	Radius
1	0.4944	0.3093	0.4540	0.3991	1	0.3884	0.2640	0.3253	0.3065
2	0.5412	0.3181	0.4625	0.4766	2	0.3777	0.2654	0.3274	0.3010
3	0.5524	0.2992	0.4667	0.4824	3	0.4023	0.2493	0.3372	0.3271
4	0.5125	0.3613	0.2757	0.2211	4	0.3834	0.2866	0.1952	0.1435
5	0.4721	0.3567	0.4272	0.4193	5	0.3880	0.3175	0.3217	0.3111
6	0.4727	0.3329	0.4030	0.4412	6	0.3185	0.2641	0.2952	0.2983
7	0.5133	0.3146	0.4246	0.4598	7	0.3393	0.2605	0.3199	0.2914
8	0.4529	0.3182	0.3745	0.4617	8	0.3268	0.2246	0.3021	0.2437
9	0.4684	0.2724	0.4002	0.4262	9	0.2889	0.2257	0.2688	0.2431
10	0.4977	0.2246	0.3710	0.4283	10	0.3329	0.1689	0.2684	0.2708
11	0.4885	0.2439	0.3318	0.4581	11	0.3437	0.1753	0.2479	0.2957
12	0.4810	0.2781	0.3291	0.4913	12	0.3238	0.2020	0.2533	0.3008
13	0.4749	0.2981	0.3467	0.5012	13	0.3276	0.2303	0.2888	0.3036
14	0.4637	0.4489	0.4108	0.5308	14	0.3370	0.2651	0.2963	0.3275
15	0.4479	0.3664	0.3709	0.4938	15	0.3680	0.2727	0.3305	0.3343
16	0.4654	0.3759	0.4120	0.4748	16	0.3734	0.2908	0.3523	0.3458
17	0.4821	0.3425	0.4170	0.4752	17	0.3883	0.2915	0.3619	0.3528
18	0.4608	0.3495	0.4002	0.4771	18	0.3809	0.2932	0.3582	0.3785
19	0.4305	0.3748	0.4071	0.4248	19	0.3554	0.2774	0.3150	0.4071
20	0.4298	0.3970	0.3963	0.4489	20	0.3074	0.2681	0.2691	0.3580
21	0.4147	0.3846	0.3589	0.5105	21	0.2677	0.2708	0.2323	0.3428
22	0.5303	0.3401	0.4185	0.5201	22	0.3741	0.2300	0.2907	0.3394
23	0.5331	0.3704	0.4844	0.4301	23	0.4244	0.2348	0.3877	0.2986
24	0.4477	0.3875	0.4362	0.4141	24	0.3751	0.2788	0.3437	0.3007

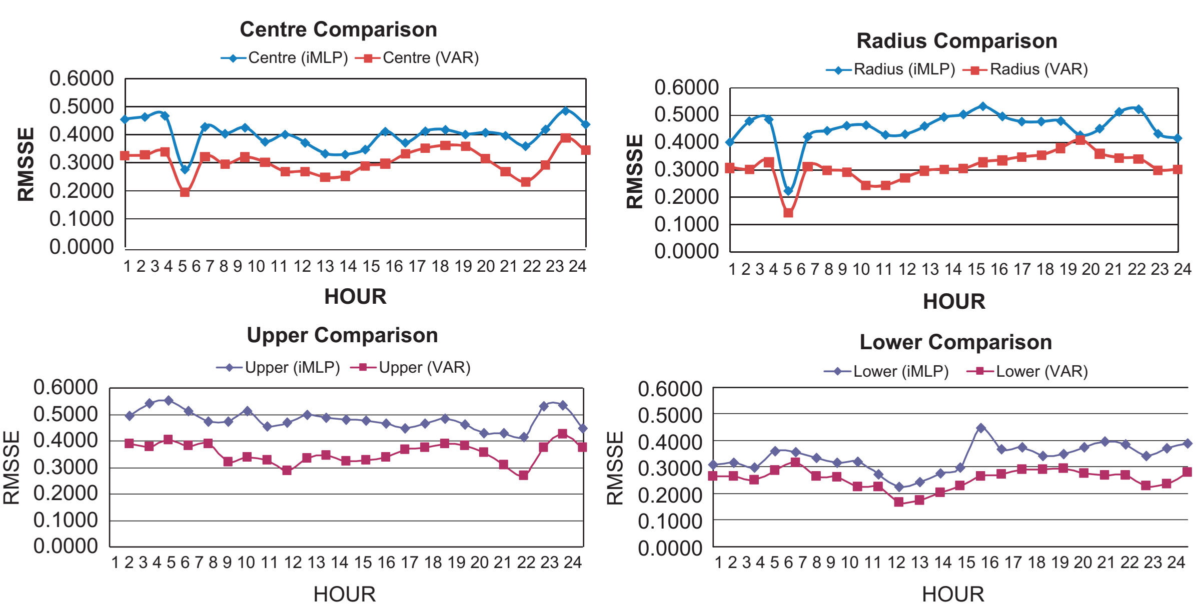


Fig. 5. RMSSE comparison between VAR and iMLP for the different variables.

22 to 18 h, while the lower values are from 18 to 22 h, with the minimum value at 21 h (0.27).

Thus, depending on the period, it is possible to establish which variable is more accurate in forecasting the electric power demand. In addition, we must take into account that through different combinations of the variables, these can be forecasted. For example, the forecast of the upper bound could be calculated by adding the forecast of the radius to the forecast of the centre. Therefore, we have these 12 possible combinations for obtaining the forecasts (see Table 7):

Table 8 shows the values of the RMSSE obtained using this method.

Now, comparing the values of the RMSSE for the different hours and periods, the method presenting the lowest RMSSE is selected (see Table 9).

Looking at Table 9 we conclude that obtaining the forecast of the lower and upper bounds, radius and centre and applying these combinations produce an improvement in the RMSSE of 1% in some hours, which it is not very significant. Thus, for these variables the best way to forecast them is by directly applying the VAR model.

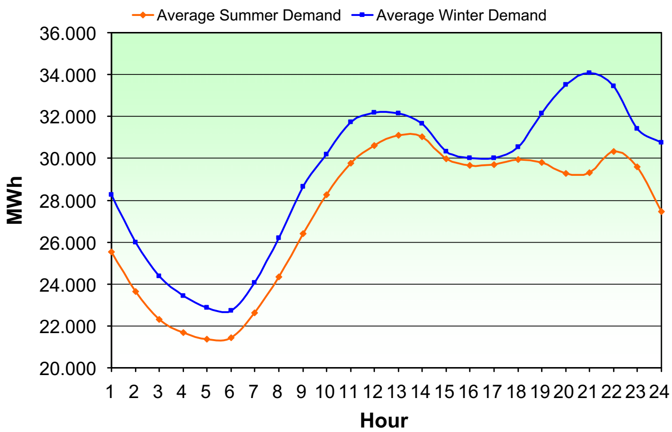


Fig. 6. Average electric power demand curve.

Table 6
Hours classification.

Period 1	From 22 to 18 h
Period 2	From 18 to 22 h

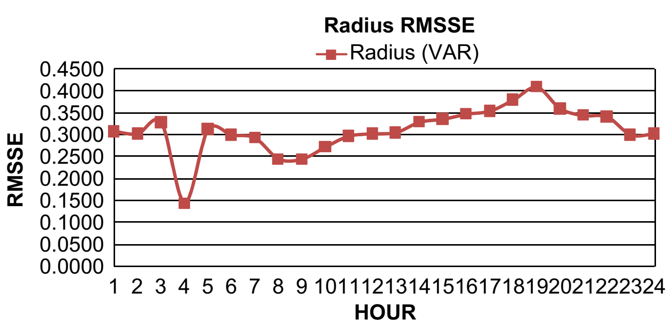


Fig. 7. Radius RMSSE.

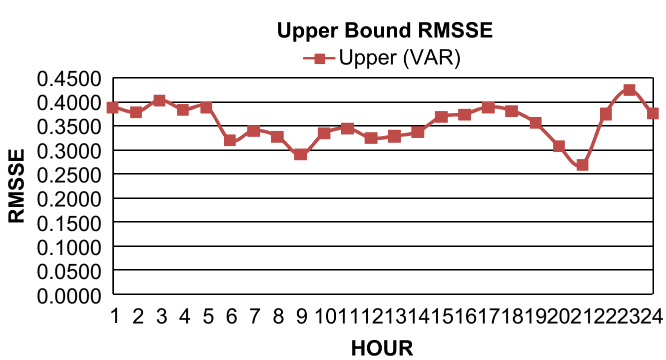


Fig. 8. Upper bound RMSSE.

Table 7
Formulas.

Centre, C	Radius, R	Upper bound, U	Lower bound, L
$U-R$	$C-L$	$L+2R$	$U-2R$
$L+R$	$U-C$	$C+R$	$C-R$
$(U+L)/2$	$(U-L)/2$	$2C-L$	$2C-U$

Now, comparing the values of the RMSSE for the different variables we establish which representation of the interval is more accurate in defining the forecasts of the electric power demand depending on the period or hour of the day.

If we look at Fig. 9 to forecast the monthly electric power demand per hour, it is quite clear that for the period of maximum variability, the interval representation composed of the lower and the upper bound shows higher accuracy than the other one composed of the radius and the centre. For the other period, the radius shows, in the majority of the hours, higher accuracy than the upper bound. So for this period, considering that these variables are more relevant than the others, since they provide the maximum value of the demand, we conclude that the interval representation composed of the radius and the centre is more appropriate. On the other hand, if we are interested in obtaining the forecast of the minimum level of demand the best interval representation for all the periods except for H4, H5, H20 and H21 is the one composed of the upper and lower bounds, {Upper Bound, Lower Bound} since the lower bound is the variable which presents the fewest RMSSE.

Therefore, in accordance with these considerations and looking at Fig. 9, we establish the best interval representation to obtain the most accurate predictions (see Tables 9 and 10, where Period 2 is indicated in bold).

6. Conclusions

Electric Power Demand forecasts are a key issue in the electricity industry, as they provide the basis for making decisions in power system planning and operation. It is therefore necessary to develop new techniques for reducing the uncertainty of the predictions. The use of symbolic data in the energy forecasting field constitutes a potential step forward, as it provides more complete and rigorous information about uncertainty than classic data does.

A novel approach to electric power demand forecasting which incorporates interval data in time series has been proposed in this article. The vector autoregressive (VAR) model for multivariate time series is adapted to interval time series (ITS) and compared with the interval multi-layer perceptron (iMLP), in order to forecast the monthly electric power demand per hour in Spain from 2006 to 2007. The calculations reveal that the VAR model provides better results than the iMLP in electric power demand forecasting.

In the case under study, given the variability of the demand, the hours of the day are classified in two different periods: (a) Period 1 from 22 to 18 h and (b) Period 2 from 18 to 22 h. Period 2 is the period of most variability, as the value of the electric power demand changes sharply from winter to summer in this period.

From the results obtained we establish the best interval representation to obtain the most accurate predictions of the maximum level of demand: Period 1 is represented with an interval based on the centre and the radius, and Period 2 is represented with an interval based on the lower and upper bounds.

On the other hand, if we are interested in obtaining the forecast of the minimum level of demand, the best interval representation for almost all the hours of both periods is the one composed of the upper and lower bounds, {Upper Bound, Lower Bound}, since the lower bound is the variable which presents the fewest root mean squared scaled error (RMSSE).

Working with different representations for the ITS, and taking into account the hourly behaviour of the demand makes possible to define different periods in the day, in order to get better forecasts depending on the ITS representation. The availability of

Table 8
New RMSSE values.

Hour	$C=(U+L)/2$	$R=(U-L)/2$	$C=L+R$	$R=U-C$	$C=U-R$	$R=C-L$	$U=L+2R$	$L=U-2R$	$U=C+R$	$L=C-R$	$U=2C-L$	$L=2C-U$
1	0.3630	0.3182	0.3586	0.4243	0.3951	0.3652	0.3789	0.3236	0.3586	0.2362	0.3780	0.3185
2	0.3198	0.3692	0.3559	0.4237	0.3602	0.3570	0.3750	0.2691	0.3584	0.2405	0.3774	0.3453
3	0.3303	0.3875	0.3539	0.4620	0.3719	0.3622	0.3916	0.2651	0.3705	0.2625	0.3944	0.3604
4	0.2075	0.1715	0.2209	0.2183	0.2195	0.1493	0.3814	0.2778	0.3509	0.2550	0.3391	0.3435
5	0.3608	0.3529	0.4118	0.5078	0.3572	0.3987	0.4282	0.2775	0.3517	0.2711	0.3702	0.3925
6	0.2881	0.3104	0.3427	0.3516	0.2827	0.2902	0.3767	0.2454	0.3336	0.2441	0.3165	0.3064
7	0.3036	0.3131	0.3576	0.3396	0.2939	0.3269	0.3912	0.2235	0.3593	0.2399	0.3644	0.3088
8	0.2970	0.2798	0.2888	0.2827	0.3466	0.3147	0.3020	0.2978	0.3073	0.2426	0.3486	0.2389
9	0.2737	0.2605	0.2927	0.2496	0.2751	0.2909	0.2966	0.2054	0.2869	0.1961	0.3021	0.1999
10	0.2695	0.2697	0.2758	0.2749	0.2679	0.2818	0.3393	0.1672	0.3342	0.1653	0.3395	0.1739
11	0.2496	0.3062	0.2601	0.3124	0.2507	0.3286	0.3458	0.1623	0.3399	0.1653	0.3583	0.1811
12	0.2502	0.3074	0.2686	0.2852	0.2483	0.3610	0.3362	0.1904	0.3330	0.1807	0.3638	0.1791
13	0.2692	0.3247	0.2845	0.2871	0.2765	0.4000	0.3289	0.2179	0.3416	0.2120	0.3920	0.2223
14	0.2833	0.3567	0.2946	0.3280	0.2915	0.4133	0.3294	0.2425	0.3413	0.2421	0.3829	0.2527
15	0.3132	0.3769	0.3131	0.3652	0.3382	0.4188	0.3433	0.2639	0.3603	0.2669	0.4062	0.2914
16	0.3465	0.3535	0.3706	0.3569	0.3507	0.3930	0.3886	0.2881	0.3759	0.2847	0.4009	0.3085
17	0.3625	0.3588	0.3897	0.3641	0.3644	0.3909	0.4060	0.2853	0.3874	0.2825	0.4069	0.2991
18	0.3427	0.3812	0.3589	0.3819	0.3550	0.4131	0.3916	0.3185	0.3846	0.3145	0.4101	0.3317
19	0.3028	0.3791	0.3352	0.3819	0.2950	0.4042	0.3967	0.3045	0.3758	0.2921	0.3793	0.3072
20	0.2579	0.3419	0.2932	0.3311	0.2580	0.3729	0.3505	0.2860	0.3286	0.2683	0.3390	0.2728
21	0.2398	0.3428	0.2736	0.3454	0.2278	0.3960	0.3077	0.2523	0.2721	0.2532	0.2902	0.2614
22	0.2928	0.3623	0.3152	0.3999	0.3057	0.3874	0.3826	0.2283	0.3582	0.2309	0.3873	0.2700
23	0.3607	0.3232	0.3959	0.3508	0.3706	0.3440	0.4399	0.2194	0.4321	0.2350	0.4631	0.3070
24	0.3324	0.3706	0.3378	0.4161	0.3706	0.4083	0.3461	0.2851	0.3511	0.2806	0.4044	0.3610

Table 9
Formulation with lowest RMSSE per hour.

Hour	Period	Upper	Lower	Centre	Radius
1		$C+R$	$C-R$	C	R
2		$C+R$	$C-R$	$(U+L)/2$	R
3		$C+R$	L	$(U+L)/2$	R
4		$2C-L$	$C-R$	C	R
5		$C+R$	$C-R$	C	R
6		$2C-L$	$C-R$	C	$C-L$
7		U	$U-2R$	$U-R$	R
8		$L+2R$	L	$L+R$	R
9	1	$C+R$	$C-R$	C	R
10		U	$C-R$	$U-R$	$(U-L)/2$
11		$C+R$	$U-2R$	C	R
12		U	$2C-U$	$(U+L)/2$	$U-C$
13		U	$C-R$	$(U+L)/2$	$U-C$
14		$L+2R$	$C-R$	$(U+L)/2$	R
15		$L+2R$	$U-2R$	$L+R$	R
16		U	$C-R$	$(U+L)/2$	R
17		$C+R$	$C-R$	C	R
18		$\boldsymbol{U}$	$\boldsymbol{L}$	$\boldsymbol{(U+L)/2}$	$\boldsymbol{R}$
19		$\boldsymbol{U}$	$\boldsymbol{L}$	$\boldsymbol{U-R}$	$\boldsymbol{(U-L)/2}$
20	**2**	$\boldsymbol{U}$	$\boldsymbol{L}$	$\boldsymbol{U-R}$	$\boldsymbol{U-C}$
21		$\boldsymbol{U}$	$\boldsymbol{U-2R}$	$\boldsymbol{U-R}$	$\boldsymbol{R}$
22		$\boldsymbol{C+R}$	$\boldsymbol{U-2R}$	$\boldsymbol{C}$	$\boldsymbol{R}$
23	1	U	$U-2R$	$(U+L)/2$	R
24		$L+2R$	L	$(U+L)/2$	R

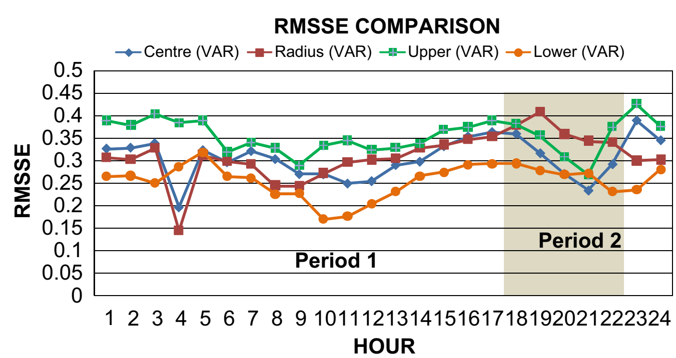


Fig. 9. RMSSE values.

Table 10
Interval representation per period.

Period	Interval representation
1	{Centre, Radius}
2	**{Upper bound, Lower bound}**

the interval rather than the single value for the electric power demand provides greater accuracy in the calculation of the electric power reserve needed to ensure energy supply and system security, which implies a reduction in operation costs which directly affect consumers. This developed methodology provides an important risk management tool in the electricity industry. Several quiet interesting studies could be developed analyzing the interval approach versus the current approach based on single values. One of them would be to evaluate the change in costs of operational decisions. Other one would be to consider the implications in energy efficiency policies.

Further research should be undertaken in order to evaluate if this method could provide accurate results for other important energy issues. The comparison between the VAR and the iMLP model, to evaluate their efficiency in forecasting daily electricity prices could provide interesting conclusions.

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We acknowledge the Project "Forecasting Models for Symbolic Data (PRESIM)", www.upcomillas.es/otri/propios/presim, funded by Universidad Pontificia Comillas. We are grateful to Antonio Muñoz San Roque for comments on a preliminary version of this work. We are also grateful to reviewers ́comments.

Appendix

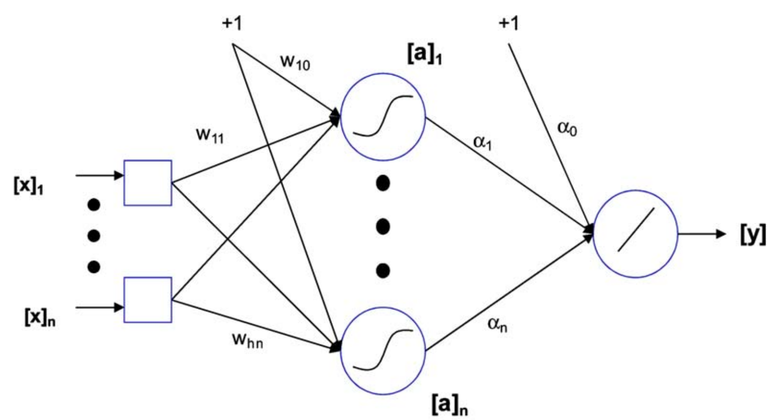
See Table A1.

Table A1

iMLP.

Authors	Model	Interval values	Single values
(Muñoz San Roque et al., 2007)	They propose a new model of Multilayer Perceptron for handling interval valued data, iMLP, by means of interval arithmetic.	Inputs and outputs	Weights and biases

Model



iMLP structure

- $[x]=\langle x_C, x_R \rangle$
- $$[s]_j = w_{j0} + \sum_{i=1}^{n} w_{ji}[x]_i = \left\langle w_{j0} + \sum_{i=1}^{n} w_{ji} x_{i,C}, \sum_{i=1}^{n} |w_{ji}| x_{i,R} \right\rangle$$
- $$[a]_j = g([s]_j) = \tanh([s]_j) = [\tanh(s_{j,C} - s_{j,R}), \tanh(s_{j,C} + s_{j,R})] = \left\langle \frac{\tanh(s_{j,C} - s_{j,R}) + \tanh(s_{j,C} + s_{j,R})}{2}, \frac{\tanh(s_{j,C} + s_{j,R}) - \tanh(s_{j,C} - s_{j,R})}{2} \right\rangle$$
- $$[y] = \sum_{j=1}^{h} \alpha_j [a]_j + \alpha_0 = \left\langle \sum_{j=1}^{h} \alpha_j a_{j,C} + \alpha_0, \sum_{j=1}^{h} |\alpha_j| a_{j,R} \right\rangle$$
- Learning algorithm, see (Muñoz San Roque et al., 2007).

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