# What is Additional Mathematics all about?

**Secondary 2 Subject Allocation Exercise** 

If you choose Pure Sciences, you have to take Additional Mathematics.

#### O-Level Additional Mathematics (Code – 4049, First Year of Examination – 2021)

Paper	Duration	Description	Marks	Weighting
1	2 h 15 min	and lengths up to 10 marks per question		50%
2	2 h 15 min	There will be 9 – 11 questions of varying marks and lengths, up to 12 marks per question. Candidates are required to answer all questions.	90	50%

### Some of the topics under Algebra:

#### A4 Polynomials and Partial Fractions

- 4.1 Multiplication and division of polynomials
- 4.2 Use of remainder and factor theorems, including factorising polynomials and solving cubic equations
- 4.3 Use of:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$
  
 $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ 

- 4.4 Partial fractions with cases where the denominator is no more complicated than:
  - (ax+b)(cx+d)
  - $(ax+b)(cx+d)^2$
  - $\bullet \quad (ax+b)(x^2+c^2)$

#### A6 Exponential and Logarithmic Functions

- 6.1 Exponential and logarithmic functions  $a^x$ ,  $e^x$ ,  $\log_a x$ ,  $\ln x$  and their graphs, including
  - laws of logarithms
  - equivalence of  $y = a^x$  and  $x = \log_a y$
  - · change of base of logarithms
- 6.2 Simplifying expressions and solving simple equations involving exponential and logarithmic functions
- 6.3 Using exponential and logarithmic functions as models

- 1.1 Six trigonometric functions for angles of any magnitude (in degrees or radians)
- 1.2 Principal values of  $\sin^{-1} x$ ,  $\cos^{-1} x$ ,  $\tan^{-1} x$
- 1.3 Exact values of the trigonometric functions for special angles  $(30^{\circ},45^{\circ},60^{\circ})$  or  $(\frac{\pi}{6},\frac{\pi}{4},\frac{\pi}{3})$
- 1.4 Amplitude, periodicity and symmetries related to sine and cosine functions
- 1.5 Graphs of  $y = a\sin(bx) + c$ ,  $y = a\sin(\frac{x}{b}) + c$ ,  $y = a\cos(bx) + c$ ,  $y = a\cos(\frac{x}{b}) + c$  and  $y = a\tan(bx)$ , where a is real, b is a positive integer and c is an integer

1.6 Use of:

- $\frac{\sin A}{\cos A} = \tan A, \quad \frac{\cos A}{\sin A} = \cot A,$  $\sin^2 A + \cos^2 A = 1,$  $\sec^2 A = 1 + \tan^2 A,$  $\csc^2 A = 1 + \cot^2 A$
- the expansions of  $sin(A \pm B)$ ,  $cos(A \pm B)$  and  $tan(A \pm B)$
- the formulae for  $\sin 2A$ ,  $\cos 2A$  and  $\tan 2A$
- the expression of  $a\cos\theta + b\sin\theta$  in the form  $R\cos(\theta \pm \alpha)$  or  $R\sin(\theta \pm \alpha)$

Calculus deals with the concept of change. It builds on algebra, geometry and trigonometry. It has widespread applications in science, economics, business, engineering and medicine and provides useful tools for analysing and modelling change and behaviour. The interpretation of these phenomena requires the study of the *functions* used in these models.

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## In Mathematics, you learn how to solve simultaneous linear equations:

Solve the following pair of simultaneous equations.

$$3x - 2y = 1$$

$$x - 2y = 2$$

By elimination, 2x = -1.

Therefore, 
$$x = -\frac{1}{2}$$
 and  $y = -\frac{5}{4}$ .

## In Additional Mathematics, you learn how to solve simultaneous quadratic equations:

Solve the following pair of simultaneous equations.

$$x^{2} - 2xy + y^{2} = 1$$
 -----equation ①
$$x - 2y = 2$$
 -----equation ②

The answers are x = 0, y = -1 or x = -4, y = -3.

Can you solve this? Try it!

A little hint: Substitute x = 2y + 2 into  $x^2 - 2xy + y^2 = 1$ .

#### In Mathematics, you learn how to solve inequalities:

Find the range of values of x such that

$$8x + 9 < 2x + 1$$
.  
 $6x < -8$ 

$$x < -\frac{4}{3}$$

## In Additional Mathematics, you learn how to solve quadratic inequalities

Find the range of values of x such that  $8x + 9 < (2x + 1)^2$ .

$$8x+9 < 4x^{2} + 4x + 1$$

$$0 < 4x^{2} + 4x + 1 - 8x - 9$$

$$4x^{2} - 4x - 8 > 0$$

$$x^{2} - x - 2 > 0$$

$$(x-2)(x+1) > 0$$

Continue on the next slide

$$(x-2)(x+1) > 0$$

## Do you think the answer is (A) or (B)?

(A) 
$$x > 2$$
 or  $x > -1$ 

**(B)** 
$$x > 2$$
 or  $x < -1$ 

#### In Mathematics, you learn how to combine algebraic fractions:

$$\frac{3}{x+1} + \frac{1}{x-4} = \frac{3(x-4)+1(x+1)}{(x+1)(x-4)}$$
$$= \frac{3x-12+x+1}{x^2-3x-4}$$
$$= \frac{4x-11}{x^2-3x-4}$$

## In Additional Mathematics, you learn how to "dismantle" an algebraic fraction:

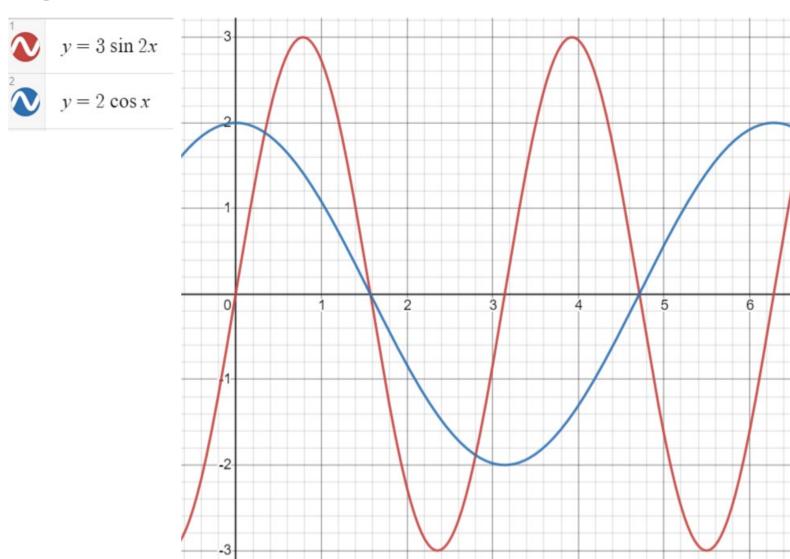
From the previous slide, we know that

$$\frac{4x-11}{x^2-3x-4} = \frac{3}{x+1} + \frac{1}{x-4}.$$

Do you know which 2 fractions is  $\frac{3x+5}{x^2+3x+2}$  equal to?

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#### **Derivatives of trigonometric functions**

If u is a function of x, x is in radians, a, b and n are constants, and n is rational, then

• 
$$\frac{d}{dx}(\sin x) = \cos x$$

• 
$$\frac{d}{dx}(\cos x) = -\sin x$$

• 
$$\frac{\mathrm{d}}{\mathrm{d}x}(\tan x) = \sec^2 x$$

• 
$$\frac{\mathrm{d}}{\mathrm{d}x}(\sin u) = \cos u \cdot \frac{\mathrm{d}u}{\mathrm{d}x}$$

• 
$$\frac{d}{dx}(\cos u) = -\sin u \cdot \frac{du}{dx}$$

• 
$$\frac{\mathrm{d}}{\mathrm{d}x}(\tan u) = \sec^2 u \cdot \frac{\mathrm{d}u}{\mathrm{d}x}$$

• 
$$\frac{d}{dx}[\sin(ax+b)] = a\cos(ax+b)$$

• 
$$\frac{\mathrm{d}}{\mathrm{d}x}[\cos(ax+b)] = -a\sin(ax+b)$$

• 
$$\frac{d}{dx}[\tan{(ax+b)}] = a \sec^2{(ax+b)}$$

• 
$$\frac{\mathrm{d}}{\mathrm{d}x}(\sin^n x) = n \sin^{n-1} x \cdot \cos x$$

• 
$$\frac{\mathrm{d}}{\mathrm{d}x}(\cos^n x) = -n\cos^{n-1} x \cdot \sin x$$

• 
$$\frac{\mathrm{d}}{\mathrm{d}x}(\tan^n x) = n \tan^{n-1} x \cdot \sec^2 x$$

Prove the identity 
$$\frac{\sec A(1+\sin A)}{4(\sin A+\sin^2 A)} = \frac{1}{2}\csc 2A$$

Solve the equation  $1 + \sin x = \sqrt{3} \cos x$  for  $0^{\circ} < x < 180^{\circ}$ 

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The 4 basic calculus rules that you need to memorise & use well:

1. Basic differentiation 
$$\frac{d}{dx}ax^n = nax^{n-1}$$

2. Chain Rule 
$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

3. Product Rule 
$$\frac{d}{dx}uv = u\frac{dv}{dx} + v\frac{du}{dx}$$
 where *u* and *v* are functions of *x*

4. Quotient Rule 
$$\frac{d}{dx} \left( \frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} \text{ where } u \text{ and } v \text{ are functions of } x$$

Differentiate  $y = \frac{2x}{\sqrt{3x-1}}$  with respect to x.

$$\frac{d}{dx} \left( \frac{2x}{\sqrt{3x-1}} \right) = \frac{\sqrt{3x-1} \frac{d}{dx} (2x) - 2x \frac{d}{dx} \sqrt{3x-1}}{\left(\sqrt{3x-1}\right)^2}$$

$$= \frac{2\sqrt{3x-1} - 2x \cdot \frac{3}{2\sqrt{3x-1}}}{\left(\sqrt{3x-1}\right)^2}$$

$$= \frac{3x-2}{(3x-1)\sqrt{3x-1}}$$

## To be able to choose Additional Mathematics:

- You must be scoring high marks consistently in Mathematics and Science Overall Combined.
- You must ALGEBRA, have a strong foundation in algebra as there are a lot of algebraic manipulation.
- Also, there are 19 chapters to be completed in 2 years. It is like 4
  years of Mathematics compressed into 2 years.



### JAE Booklet (JAE: Joint Admission Exercise)

https://www.moe.gov.sg/-/media/files/post-secondary/2022-jae-information-booklet.pdf

#### Page 102 of booklet details the cut-off points of JC

- 1 Junior Colleges & Millennia Institute
  - 1.1 Previous Year's Aggregate Scores of Junior Colleges

Table D-1 shows the L1R5 net aggregate of the lowest ranked students (course cut-off point) who were admitted to the junior college (JC) course through the previous year's (2021) Joint Admissions Exercise (JAE). These aggregate scores are meant as a reference for applicants applying to the JCs and do not constitute the admission scores for subsequent admission exercises. The L1R5 net aggregate scores for the courses are not pre-determined by MOE or schools before posting. The eventual course aggregate scores for 2022 JAE may vary from previous years, depending on students' GCE O-Level examination results and their course choices during the JAE.

Table D-1: L1R5 Aggregate Scores (with Bonus Points) of Students Admitted to JCs in the Previous Year's (2021) JAE

	JC Name	Arts		Science/IB	
S/No		Course Code	Previous (2021) JAE Net L1R5	Course Code	Previous (2021) JAE Net L1R5
1	Anderson Serangoon Junior College	39A	11	39S	11
2	Anglo-Chinese Junior College	22A	9	22S	8
3	Anglo-Chinese School (Independent)	-	-	511	5
4	Catholic Junior College	23A	13	23S	13

#### 1 Junior Colleges & Millennia Institute

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3	Anglo-Chinese School (Independent)	-	-	51I	5
4	Catholic Junior College	23A	13	23S	13
5	Dunman High School	44A	9	44S	8
6	Eunoia Junior College	38A	8	38S	7
7	Hwa Chong Institution	24A	5	24S	4
8	Jurong Pioneer Junior College	40A	16	40S	15
9	Nanyang Junior College	26A	6	26S	6
10	National Junior College	27A	8	27S	7
11	Raffles Institution	28A	5	28S	4
12	River Valley High School	45A	9	45S	8
13	St. Andrew's Junior College	30A	11	30S	10
14	St. Joseph's Institution	-	-	521	8
15	Tampines Meridian Junior College	41A	13	41S	13
16	Temasek Junior College	32A	9	32S	9
17	Victoria Junior College	33A	8	33S	7
18	Yishun Innova Junior College	42A	19	42S	20

#### Combination 1:

Pure Sciences + Additional Mathematics caters mainly to students who aspires to go to a JC.



To be eligible for consideration for admission to the various courses in Ngee Ann Polytechnic, applicants must obtain **26 points or better for the net ELR2B2 aggregate score** (i.e. English Language, 2 relevant subjects and best 2 other subjects, **including CCA Bonus Points**) and meet the minimum entry requirements as shown in the table of courses in this section. CCA cannot be used to meet the minimum entry requirements. For Aggregate Types Computation Criteria (for polytechnic courses), please refer to Table 6 under Section 5 – Aggregate Types Computation Criteria.

The "Net ELR2B2 Range for Previous (2021) JAE" in the table below shows the net ELR2B2 aggregate of the highest to lowest ranked students who were admitted to these courses in the 2021 Joint Admissions Exercise (JAE). These aggregate scores are meant as a reference for applicants applying to these courses, and do not constitute the admission scores for subsequent admission exercises. The net ELR2B2 aggregate scores for the courses are not pre-determined by MOE or polytechnics before posting. The eventual course aggregate scores for the 2022 JAE may vary from previous years, depending on students' GCE O-Level examination results and their course choices during the JAE.

Courses	Course Code	Aggregate Type	Net ELR2B2 Range for Previous (2021) JAE	Minimum Entry Requirements
APPLIED SCIENCES  Biomedical Science	N59	ELR2B2-C	4 to 8	Subject Grade  a) English Language 1-7 b) Mathematics (Elementary/Additional) 1-6 c) Any one of the following subjects: 1-6  • Biology • Biotechnology • Chemistry • Combined Science • Design & Technology • Food & Nutrition • Electronics / Fundamentals of Electronics • Physics / Engineering Science • Science (Chemistry, Biology)
Pharmaceutical Science ①	N73	ELR2B2-C	5 to 9	Science (Physics, Biology)     Science (Physics, Chemistry) / Physical Science

Polytechnic courses are very competitive.

Although the minimum entry requirements are:

(a) English Language A1 – D7

(b) Mathematics OR A1 – C6

**Additional Mathematics** 

do you think you can get into this course if you only obtained passed to EL & Maths?

What would your EL + R2 + B2 score?