



Singapore–Cambridge General Certificate of Education Ordinary Level (2023)

Mathematics (Syllabus 4052)

(To be implemented from year of examination in 2023)

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INTRODUCTION

The syllabus is intended to provide students with fundamental mathematical knowledge and skills. The content is organised into three strands, namely, *Number and Algebra*, *Geometry and Measurement*, and *Statistics and Probability*. Besides conceptual understanding and skill proficiency explicated in the content strands, important mathematical processes such as reasoning, communication and application (including the use of models) are also emphasised and assessed.

AIMS

The O-Level Mathematics syllabus aims to enable all students to:

- acquire mathematical concepts and skills for continuous learning in mathematics and to support learning in other subjects
- develop thinking, reasoning, communication, application and metacognitive skills through a mathematical approach to problem-solving
- connect ideas within mathematics and between mathematics and other subjects through applications of mathematics
- build confidence and foster interest in mathematics.

ASSESSMENT OBJECTIVES

The assessment will test candidates' abilities to:

AO1 Use and apply standard techniques

- recall and use facts, terminology and notation
- read and use information directly from tables, graphs, diagrams and texts
- carry out routine mathematical procedures

AO2 Solve problems in a variety of contexts

- interpret information to identify the relevant mathematics concept, rule or formula to use
- translate information from one form to another
- make and use connections across topics/subtopics
- formulate problems into mathematical terms
- analyse and select relevant information and apply appropriate mathematical techniques to solve problems
- interpret results in the context of a given problem

AO3 Reason and communicate mathematically

- justify mathematical statements
- provide explanation in the context of a given problem
- write mathematical arguments

Approximate weightings for the assessment objectives are as follows:

AO1	45%
AO2	40%
AO3	15%

SCHEME OF ASSESSMENT

Paper	Duration	Description	Marks	Weighting
Paper 1	2 hours 15 minutes	There will be about 26 short answer questions. Candidates are required to answer all questions.	90	50%
Paper 2	2 hours 15 minutes	There will be 9 to 10 questions of varying marks and lengths. The last question in this paper will focus specifically on applying mathematics to a real-world scenario. Candidates are required to answer all questions.	90	50%

NOTES

- Omission of essential working will result in loss of marks.
- 2. Relevant mathematical formulae will be provided for candidates.
- 3. Candidates should also have geometrical instruments with them for both papers.
- 4. Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question. In questions which explicitly require an answer to be shown to be correct to a specific accuracy, the answer must be first shown to a higher degree of accuracy.
- 5. SI units will be used in questions involving mass and measures.
 - Both the 12-hour and 24-hour clock may be used for quoting times of the day. In the 24-hour clock, for example, 3.15 a.m. will be denoted by 03 15; 3.15 p.m. by 15 15.
- 6. Candidates are expected to be familiar with the solidus notation for the expression of compound units, e.g. 5 cm/s for 5 centimetres per second, 13.6 g/cm³ for 13.6 grams per cubic centimetre.
- 7. Unless the question requires the answer in terms of π , the calculator value for π or π = 3.142 should be used.
- 8. Spaces will be provided in each question paper for working and answers.

PROBLEMS IN REAL-WORLD CONTEXTS

Notwithstanding the presentation of the topics in 3 separate strands in the syllabus document, it is envisaged that some examination questions (including the extended problem involving real-world contexts at the end of Paper 2) may integrate ideas from more than one topic.

Problems in real-world contexts may be based on contexts:

- In everyday life (including travel/excursion plans, transport schedules, sports and games, recipes, floor plans, navigation etc.)
- Involving personal and household finance (including simple and compound interest, taxation, instalments, utilities bills, money exchange, etc.)

These problems may also require:

- Interpreting and analysing data from tables and graphs, including distance-time and speed-time graphs;
- Interpreting the solution in the context of the problem.

USE OF CALCULATORS

An approved calculator may be used in **both** Paper 1 and Paper 2.

SUBJECT CONTENT

No.	Topic/Sub-topics	Content
NUMI	BER AND ALGEBRA	
N1	Numbers and their operations	 primes and prime factorisation finding highest common factor (HCF) and lowest common multiple (LCM), squares, cubes, square roots and cube roots by prime factorisation negative numbers, integers, rational numbers, real numbers, and their four operations calculations with calculator representation and ordering of numbers on the number line use of the symbols <, >, ≤, ≥ approximation and estimation (including rounding off numbers to a required number of decimal places or significant figures and estimating the results of computation) use of standard form A × 10ⁿ, where n is an integer, and 1 ≤ A < 10 positive, negative, zero and fractional indices laws of indices
N2	Ratio and proportion	 ratios involving rational numbers writing a ratio in its simplest form map scales (distance and area) direct and inverse proportion
N3	Percentage	 expressing one quantity as a percentage of another comparing two quantities by percentage percentages greater than 100% increasing/decreasing a quantity by a given percentage reverse percentages
N4	Rate and speed	 average rate and average speed conversion of units (e.g. km/h to m/s)

No.	Topic/Sub-topics	Content
N5	Algebraic expressions and formulae	• using letters to represent numbers • interpreting notations: $-ab \text{ as } a \times b$ $-\frac{a}{b} \text{ as } a + b \text{ or } a \times \frac{1}{b}$ $-a^2 \text{ as } a \times a, \ a^3 \text{ as } a \times a \times a, \ a^2b \text{ as } a \times a \times b, \dots$ $-3y \text{ as } y + y + y \text{ or } 3 \times y$ $-3(x + y) \text{ as } 3 \times (x + y)$ $-\frac{3 + y}{5} \text{ as } (3 + y) \div 5 \text{ or } \frac{1}{5} \times (3 + y)$ • evaluation of algebraic expressions and formulae • translation of simple real-world situations into algebraic expressions • recognising and representing patterns/relationships by finding an algebraic expression for the <i>n</i> th term • addition and subtraction of linear expressions • simplification of linear expressions such as: $-2(3x - 5) + 4x$ $\frac{2x}{3} - \frac{3(x - 5)}{2}$ • use brackets and extract common factors • factorisation of linear expressions of the form $ax + bx + kay + kby$ • expansion of the product of algebraic expressions • changing the subject of a formula • finding the value of an unknown quantity in a given formula • use of: $-(a + b)^2 = a^2 + 2ab + b^2$ $-(a - b)^2 = a^2 - 2ab + b^2$ $-(a - b)^2 = a^2 - 2ab + b^2$ $-(a - b)^2 = a^2 - 2ab + b^2$ $-(a - b)^2 = a^2 - 2ab + b^2$ • multiplication and division of simple algebraic fractions such as: $(\frac{3a}{4b^2})(\frac{5ab}{3})$ $\frac{3a}{4b^2}(\frac{9a^2}{3})$ • addition and subtraction of algebraic fractions with linear or quadratic denominator such as: $\frac{1}{x^2 - y} + \frac{2}{x - 3}$ $\frac{1}{x^2 - 9} + \frac{2}{(x - 3)^2}$

N6 Funct graph	tions and ns	 Cartesian coordinates in two dimensions graph of a set of ordered pairs as a representation of a relationship between two variables linear functions (y = ax + b) and quadratic functions (y = ax² + bx + c) graphs of linear functions the gradient of a linear graph as the ratio of the vertical change to the
		 the gradient of a linear graph as the ratio of the vertical change to the
		 horizontal change (positive and negative gradients) graphs of quadratic functions and their properties: positive or negative coefficient of x² maximum and minimum points symmetry sketching the graphs of quadratic functions given in the form: y = (x - p)² + q y = (x - p)² + q y = (x - a)(x - b) y = (x - a)(x - b) graphs of power functions of the form y = ax², where n = -2, -1, 0, 1, 2, 3, and simple sums of not more than three of these graphs of exponential functions y = ka², where a is a positive integer
	tions and nalities	 estimation of the gradient of a curve by drawing a tangent solving linear equations in one variable solving simple fractional equations that can be reduced to linear equations such as: \[\frac{x}{3} + \frac{x-2}{4} = 3 \] \[\frac{3}{x-2} = 6 \] solving simultaneous linear equations in two variables by substitution and elimination methods graphical method solving quadratic equations in one unknown by factorisation use of formula completing the square for \$y = x^2 + px + q\$ graphical method solving fractional equations that can be reduced to quadratic equations such as: \frac{6}{x+4} = x+3\$ \frac{1}{x-2} + \frac{2}{x-3} = 5\$ formulating equations to solve problems solving linear inequalities in one variable, and representing the solution on

No.	Topic/Sub-topics	Content
N8	Set language and notation	 use of set language and the following notation: Union of A and B
N9	Matrices METRY AND MEASUR	 display of information in the form of a matrix of any order interpreting the data in a given matrix product of a scalar quantity and a matrix problems involving the calculation of the sum and product (where appropriate) of two matrices
G1	Angles, triangles and polygons	 right, acute, obtuse and reflex angles vertically opposite angles, angles on a straight line and angles at a point angles formed by two parallel lines and a transversal: corresponding angles, alternate angles, interior angles properties of triangles, special quadrilaterals and regular polygons (pentagon, hexagon, octagon and decagon), including symmetry properties classifying special quadrilaterals on the basis of their properties angle sum of interior and exterior angles of any convex polygon construction of simple geometrical figures from given data using compasses, ruler, set squares and protractors, where appropriate

No.	Topic/Sub-topics	Content
G2	Congruence and similarity Properties of	 congruent figures and similar figures properties of similar triangles and polygons: corresponding angles are equal corresponding sides are proportional enlargement and reduction of a plane figure scale drawings properties and construction of perpendicular bisectors of line segments and angle bisectors determining whether two triangles are congruent similar ratio of areas of similar plane figures ratio of volumes of similar solids solving simple problems involving similarity and congruence symmetry properties of circles:
G3	circles	 symmetry properties of circles: equal chords are equidistant from the centre the perpendicular bisector of a chord passes through the centre tangents from an external point are equal in length the line joining an external point to the centre of the circle bisects the angle between the tangents angle properties of circles: angle in a semicircle is a right angle angle between tangent and radius of a circle is a right angle angle at the centre is twice the angle at the circumference angles in the same segment are equal angles in opposite segments are supplementary
G4	Pythagoras' theorem and trigonometry	 use of Pythagoras' theorem determining whether a triangle is right-angled given the lengths of three sides use of trigonometric ratios (sine, cosine and tangent) of acute angles to calculate unknown sides and angles in right-angled triangles extending sine and cosine to obtuse angles use of the formula ½ ab sin C for the area of a triangle use of sine rule and cosine rule for any triangle problems in two and three dimensions including those involving angles of elevation and depression and bearings

No.	Topic/Sub-topics	Content
G5	Mensuration	 area of parallelogram and trapezium problems involving perimeter and area of composite plane figures volume and surface area of cube, cuboid, prism, cylinder, pyramid, cone and sphere conversion between cm² and m², and between cm³ and m³ problems involving volume and surface area of composite solids arc length, sector area and area of a segment of a circle use of radian measure of angle (including conversion between radians and degrees)
G6	Coordinate geometry	 finding the gradient of a straight line given the coordinates of two points on it finding the length of a line segment given the coordinates of its end points interpreting and finding the equation of a straight line graph in the form y = mx + c geometric problems involving the use of coordinates
G7	Vectors in two dimensions	 use of notations: \$\begin{align*} x \ y \end{align*}\$, \$\overline{AB}\$, \$\overline{a}\$, \$\overline{AB}\$ and \$ \overline{a} \$ representing a vector as a directed line segment translation by a vector position vectors magnitude of a vector \$\begin{align*} x \ y \end{align*}\$ as \$\sqrt{x^2 + y^2}\$ use of sum and difference of two vectors to express given vectors in terms of two coplanar vectors multiplication of a vector by a scalar geometric problems involving the use of vectors

No.	Topic/Sub-topics	Content
STAT	STATISTICS AND PROBABILITY	
S1	Data handling and analysis	 simple concepts in collecting, classifying and tabulating data analysis and interpretation of: tables bar graphs pictograms line graphs pie charts dot diagrams histograms with equal class intervals stem-and-leaf diagrams cumulative frequency diagrams box-and-whisker plots purposes and uses, advantages and disadvantages of the different forms of statistical representations drawing simple inference from statistical diagrams explaining why a given statistical diagram leads to misinterpretation of data mean, mode and median as measures of central tendency for a set of data purposes and use of mean, mode and median calculation of the mean for grouped data quartiles and percentiles range, interquartile range and standard deviation as measures of spread for a set of data calculation of the standard deviation for a set of data (grouped and ungrouped) using the mean and standard deviation to compare two sets of data
S2	Probability	 probability as a measure of chance probability of single events (including listing all the possible outcomes in a simple chance situation to calculate the probability) probability of simple combined events (including using possibility diagrams and tree diagrams, where appropriate) addition and multiplication of probabilities (mutually exclusive events and independent events)

MATHEMATICAL FORMULAE

Compound interest

Total amount =
$$P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone
$$=\frac{1}{3}\pi r^2 h$$

Volume of a sphere
$$=\frac{4}{3}\pi r^3$$

Area of triangle
$$ABC = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$Mean = \frac{\sum fx}{\sum f}$$

Standard deviation =
$$\sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

MATHEMATICAL NOTATION

The list which follows summarises the notation used in Cambridge's Mathematics examinations. Although primarily directed towards A-Level, the list also applies, where relevant, to examinations at all other levels.

1. Set Notation	
€	is an element of
∉	is not an element of
$\{x_1, x_2, \ldots\}$	the set with elements $x_1, x_2,$
{x:}	the set of all x such that
n(A)	the number of elements in set A
Ø	the empty set
8	universal set
A'	the complement of the set A
\mathbb{Z}	the set of integers, {0, ±1, ±2, ±3,}
\mathbb{Z}^+	the set of positive integers, {1, 2, 3,}
\mathbb{Q}	the set of rational numbers
\mathbb{Q}^+	the set of positive rational numbers, $\{x \in \mathbb{Q}: x > 0\}$
\mathbb{Q}_0^+	the set of positive rational numbers and zero, $\{x \in \mathbb{Q}: x \ge 0\}$
\mathbb{R}	the set of real numbers
\mathbb{R}^{+}	the set of positive real numbers, $\{x \in \mathbb{R}: x > 0\}$
\mathbb{R}^{+}_{0}	the set of positive real numbers and zero, $\{x \in \mathbb{R}: x \ge 0\}$
\mathbb{R}^n	the real <i>n</i> -tuples
\mathbb{C}	the set of complex numbers
⊆	is a subset of
C	is a proper subset of
⊈	is not a subset of
⊄	is not a proper subset of
U	union
\cap	intersection
[a, b]	the closed interval $\{x \in \mathbb{R}: a \leqslant x \leqslant b\}$
[a, b)	the interval $\{x \in \mathbb{R}: a \leq x \leq b\}$
(a, b]	the interval $\{x \in \mathbb{R}: a < x \leq b\}$
(a, b)	the open interval $\{x \in \mathbb{R}: a < x < b\}$

2. Miscellaneous Symbols

= is equal to

≠ is not equal to

≡ is identical to or is congruent to

≈ is approximately equal to

 ∞ is proportional to

< is less than

≼; ≯ is less than or equal to; is not greater than

> is greater than

≽; < is greater than or equal to; is not less than

 ∞ infinity

3. Operations

a + b a plus b

a-b a minus b

 $a \times b$, ab, a.b a multiplied by b

 $a \div b$, $\frac{a}{b}$, a/b a divided by b

a: b the ratio of a to b

 $\sum_{i=1}^{n} a_i \qquad \qquad a_1 + a_2 + \dots + a_n$

 \sqrt{a} the positive square root of the real number a

the modulus of the real number a

n! n factorial for $n \in \mathbb{Z}^+ \cup \{0\}$, (0! = 1)

 $\frac{n(n-1)...(n-r+1)}{r!}, \text{ for } n \in \mathbb{Q}, r \in \mathbb{Z}^+ \cup \{0\}$

4. Functions

f the function f

f(x) the value of the function f at x

f: $A \rightarrow B$ f is a function under which each element of set A has an image in set B

f: $x \mapsto y$ the function f maps the element x to the element y

f⁻¹ the inverse of the function f

 $g \circ f$, gf the composite function of f and g which is defined by

 $(g \circ f)(x) \text{ or } gf(x) = g(f(x))$

 $\lim_{x\to a} f(x)$ the limit of f(x) as x tends to a

 Δx ; δx an increment of x

 $\frac{dy}{dx}$ the derivative of y with respect to x

 $\frac{d^n y}{dx^n}$ the *n*th derivative of y with respect to x

 $f'(x), f''(x), ..., f^{(n)}(x)$ the first, second, ... nth derivatives of f(x) with respect to x

 $\int y \, dx$ indefinite integral of y with respect to x

 $\int_{a}^{b} y \, dx$ the definite integral of y with respect to x for values of x between a and b

 \dot{x} , \ddot{x} , the first, second, ...derivatives of x with respect to time

5. Exponential and Logarithmic Functions

e base of natural logarithms e^x , exp x exponential function of x $log_a x$ logarithm to the base a of x

 $\ln x$ natural logarithm of x $\log x$ logarithm of x to base 10

6. Circular Functions and Relations

sin, cos, tan, cosec, sec, cot } the circular functions

 \sin^{-1} , \cos^{-1} , \tan^{-1} \csc^{-1} , \sec^{-1} , \cot^{-1} } the inverse circular functions

7. Complex Numbers

i the square root of -1

z a complex number, z = x + iy

= $r(\cos\theta + i\sin\theta), r \in \mathbb{R}_0^+$

= $re^{i\theta}$, $r \in \mathbb{R}_0^+$

Re z the real part of z, Re (x + iy) = x

Im z the imaginary part of z, Im (x + iy) = y

|z| the modulus of z, $|x+iy| = \sqrt{x^2 + y^2}$, $|r(\cos\theta + i\sin\theta)| = r$

arg z the argument of z, $arg(r(\cos \theta + i \sin \theta)) = \theta$, $-\pi < \theta \le \pi$

 z^* the complex conjugate of z, $(x + iy)^* = x - iy$

8. Matrices

M a matrix M

 \mathbf{M}^{-1} the inverse of the square matrix \mathbf{M}

 \mathbf{M}^{T} the transpose of the matrix \mathbf{M}

det **M** the determinant of the square matrix **M**

9. Vectors

a the vector a

The vector represented in magnitude and direction by the directed line segment AB

â a unit vector in the direction of the vector **a**

i, j, k unit vectors in the directions of the Cartesian coordinate axes

a the magnitude of a

 $|\overrightarrow{AB}|$ the magnitude of $|\overrightarrow{AB}|$

a.b the scalar product of a and b

 $\mathbf{a} \times \mathbf{b}$ the vector product of \mathbf{a} and \mathbf{b}

10. Probability and Statistics

A, B, C, etc. events

 $A \cup B$ union of events A and B

 $A \cap B$ intersection of the events A and B

P(A) probability of the event A

A' complement of the event A, the event 'not A' P($A \mid B$) probability of the event A given the event B

X, Y, R, etc. random variables

x, y, r, etc. value of the random variables X, Y, R, etc.

 x_1, x_2, \dots observations

 $f_1, f_2,...$ frequencies with which the observations, x_1, x_2 ...occur

p(x) the value of the probability function P(X = x) of the discrete random variable X

 $p_1, p_2...$ probabilities of the values $x_1, x_2, ...$ of the discrete random variable X

f(x), g(x)... the value of the probability density function of the continuous random variable X F(x), G(x)... the value of the (cumulative) distribution function P($X \le x$) of the random variable X

E(X) expectation of the random variable X

E[g(X)] expectation of g(X)

Var(X) variance of the random variable X

B(n, p) binominal distribution, parameters n and p

Po(μ) Poisson distribution, mean μ

 $N(\mu, \sigma^2)$ normal distribution, mean μ and variance σ^2

 μ population mean σ^2 population variance

 σ population standard deviation

x sample mean

unbiased estimate of population variance from a sample,

 s^2

$$s^2 = \frac{1}{n-1} \sum_{x} \left(x - \overline{x} \right)^2$$

φ probability density function of the standardised normal variable with distribution N (0, 1)

Φ corresponding cumulative distribution function

ρ linear product-moment correlation coefficient for a population

r linear product-moment correlation coefficient for a sample