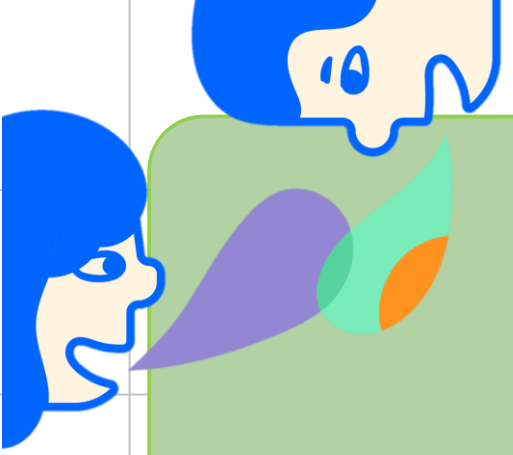


Reasoning & Communication in Mathematical Problem-Solving

24 March 2023

By Mdm Leong Fong Fong

Inspiring Excellence
PALM VIEW PRIMARY

Why Reasoning and Communication is important in the Primary Mathematics Classroom?

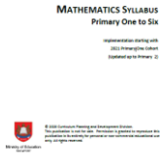
2021 Primary Mathematics Syllabus

Primary Mathematics Curriculum

Primary education is a stage where students acquire important basic numeracy as well as develop logical reasoning and problem-solving skills that are required in many disciplines. It lays the foundation for the learning of mathematics for all students, equipping them with a tool for everyday life and the knowledge and skills for learning mathematics at the next level. It is also a stage where students' confidence and interest in the subject are built and their attitudes towards the discipline are shaped.

For these reasons, the Primary Mathematics Syllabus aims to enable all students to:

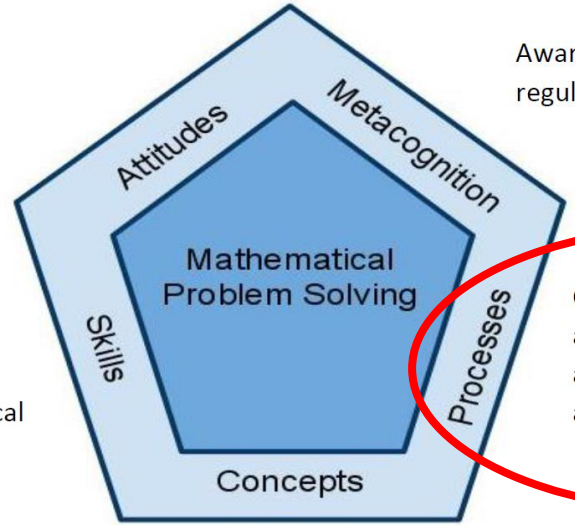
- acquire mathematical concepts and skills for everyday use and continuous learning in mathematics;
- develop thinking, reasoning, communication, application and metacognitive skills through a mathematical approach to problem solving; and
- build confidence and foster interest in mathematics.



Mathematics Curriculum Framework

Belief, appreciation, confidence, motivation, interest and perseverance

Awareness, monitoring and regulation of thought processes



Proficiency in carrying out operations and algorithms, visualising space, handling data and using mathematical tools

Competencies in abstracting and reasoning, representing and communicating, applying and modelling

Understanding of the properties and relationships, operations and algorithms

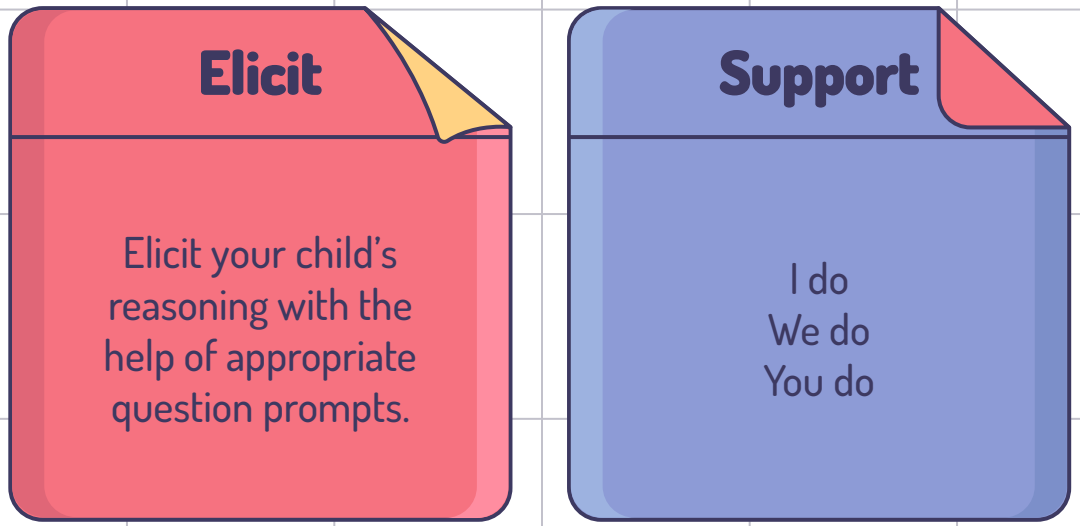
Mathematical Reasoning

refers to the ability to think, understand and form opinions or judgements that are based on facts.

Communication

refers to the ability to use mathematical language to express mathematical ideas and arguments precisely, concisely and logically.

How do we infuse Mathematical Reasoning and Communication in Problem-solving?



Problem-Solving Process

Stages		Question Prompts
Read	Topic	What is known? What is unknown? What do you need to find?
Identify Keywords	Type of Question	Are there any keywords that leads to type of question/ concept?
Get A Plan	Heuristics	Have you seen it before? What strategy would you use? Is there other ways to solve this question?
Have It Done	M.E.W.A.S	Is each step correct? Can you prove that each step is correct? What makes you say that?
Triple Check	N.T.U.C	Is the solution reasonable? Can you show that the solution is correct?

I DO

Checklist

RIGHT

READ

- What is known?
- What is unknown?
- What do you need to find?

IDENTIFY KEYWORDS

- Are there any keywords that leads to the type of question or concept?

GET A PLAN

- Have you seen it before?
- What strategy would you use?
- Is there other ways to solve this question?

HAVE IT DONE

- Is each step correct?
- Can you prove that each step is correct?
- What makes you say that?

TRIPLE CHECK

- Is the solution reasonable?
- Can you show that the solution is correct?

Repeated Identity

Unknown Beginning

Age Word Problems

Unchanged Difference

Before and After

Unchanged Total

True / False /
Not Possible To Tell

Spatial Visualisation

True / False / Not Possible To Tell

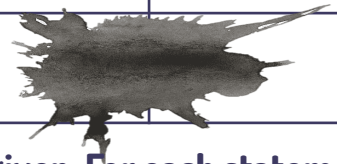
True / False / Not Possible To Tell

True	The statement agrees with the information given all the time.
False	The statement contradicts the information given.
Not Possible To Tell	There is not enough information on this. OR The statement can sometimes be true and sometimes be false.

Q1) **PSLE 2020 Paper 1 Booklet B**

The table shows the number of storybooks read by each pupil in a group. Part of the table is covered by an ink blot. There were 45 pupils who read at least 2 storybooks.

Number of storybooks	0	1	2	3	4
Number of pupils	7	8	20		



Each of the statements is either true, false or not possible to tell from the information given. For each statement, put a tick (✓) to indicate your answer.

Read → Topic → P4 Tables

Identify Keywords → Type of Question → True / False / Not Possible To Tell

Get A Plan → Heuristics → Logical Reasoning

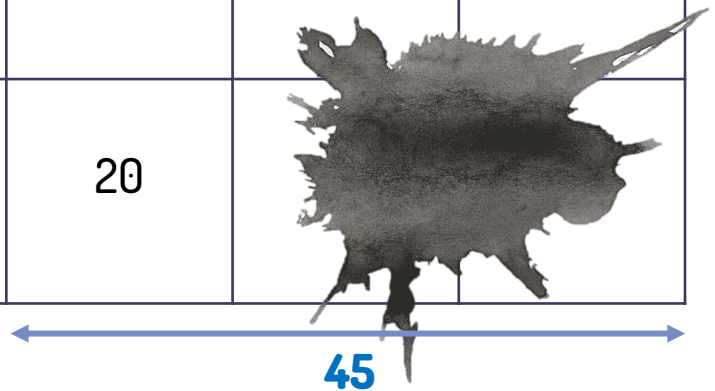
Q1)

The table shows the number of storybooks read by each pupil in a group. Part of the table is covered by an ink blot. There were 45 pupils who read at least 2 storybooks.

Number of pupils who read 2, 3 or 4 storybooks.

2, 3 or 4 storybooks

Number of storybooks	0	1	2	3	4
Number of pupils	7	8	20		



45

READ

- What is known?
- What is unknown?
- What do you need to find?

IDENTIFY KEYWORDS

- Are there any keywords that leads to the type of question or concept?

I DO

Q1)

Number of storybooks	0	1	2	3	4
Number of pupils	7	8	20		

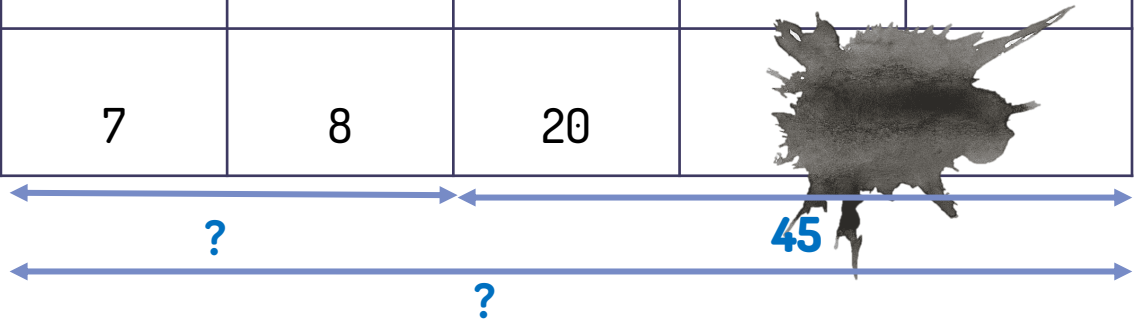
Each of the statements is either true, false or not possible to tell from the information given. For each statement, put a tick (✓) to indicate your answer.

Statement	True	False	Not Possible To Tell
7 pupils did not read any storybooks.	✓		

I DO

Q1)

Number of storybooks	0	1	2	3	4
Number of pupils	7	8	20		



Each of the statements is either true, false or not possible to tell from the information given. For each statement, put a tick (✓) to indicate your answer.

Statement	True	False	Not Possible To Tell
There were 80 pupils in the group.			

total

GET A PLAN

- Have you seen it before?
- What strategy would you use?
- Is there other ways to solve this question?

HAVE IT DONE

- Is each step correct?
- Can you prove that each step is correct?
- What makes you say that?

I DO

Q1)

Number of storybooks	0	1	2	3	4
Number of pupils	7	8	20		

Number of pupils who read 0 and 1 storybooks → 7 + 8

= 15

Total number of pupils → 15 + 45

= 60

I DO

Q1)

Number of storybooks	0	1	2	3	4
Number of pupils	7	8	20		

Statement	True	False	Not Possible To Tell
There were 80 pupils in the group.		✓	

total

I DO

Q1)

Number of storybooks	0	1	2	3	4
Number of pupils	7	8	20		

Statement	True	False	Not Possible To Tell
The number of pupils who read 3 storybooks was equal to the number of pupils who read 4 storybooks.			

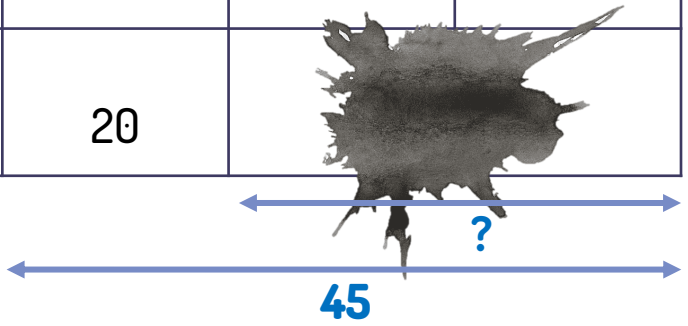
same

What makes you say that?

I DO

Q1)

Number of storybooks	0	1	2	3	4
Number of pupils	7	8	20		

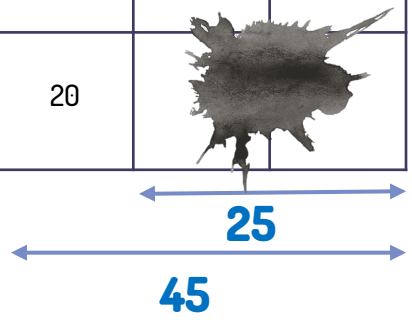


Number of pupils who read 3 and 4 storybooks → 45 – 20
= 25

I DO

Q1)

Number of storybooks	0	1	2	3	4
Number of pupils	7	8	20		



HAVE IT DONE

- Is each step correct?
- Can you prove that each step is correct?
- What makes you say that?

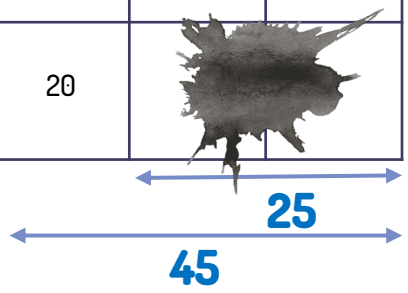
Statement	True	False	Not possible to tell
The number of pupils who read 3 storybooks was equal to the number of pupils who read 4 storybooks.			

There are 25 pupils who read 3 and 4 books.

Is it possible for the two categories to have the same number of students?

Q1)

Number of storybooks	0	1	2	3	4
Number of pupils	7	8	20		



HAVE IT DONE

- Is each step correct?
- Can you prove that each step is correct?
- What makes you say that?

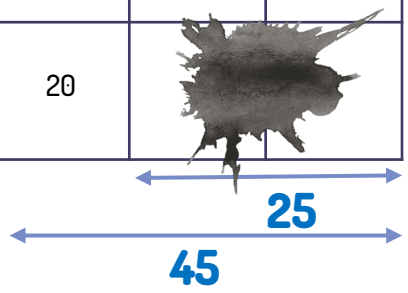
Statement	True	False	Not possible to tell
The number of pupils who read 3 storybooks was equal to the number of pupils who read 4 storybooks.		✔	

If the number of pupils who read 3 and 4 storybooks are the same, this would mean there are $25 \div 2 = 12.5$ students.

The number of pupils must be a whole number, therefore, the statement is false.

Q1)

Number of storybooks	0	1	2	3	4
Number of pupils	7	8	20		



HAVE IT DONE

- Is each step correct?
- Can you prove that each step is correct?
- What makes you say that?

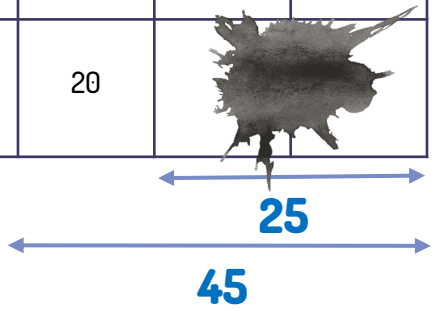
Statement	True	False	Not possible to tell
The number of pupils who read 3 storybooks was equal to the number of pupils who read 4 storybooks.		✓	

25 is an odd number and not divisible by 2.

The number of pupils must be a whole number, therefore, the statement is false.

Q1)

Number of storybooks	0	1	2	3	4
Number of pupils	7	8	20		



Statement	True	False	Not possible to tell
The number of pupils who read 3 storybooks was equal to the number of pupils who read 4 storybooks.		✓	

TRIPLE CHECK

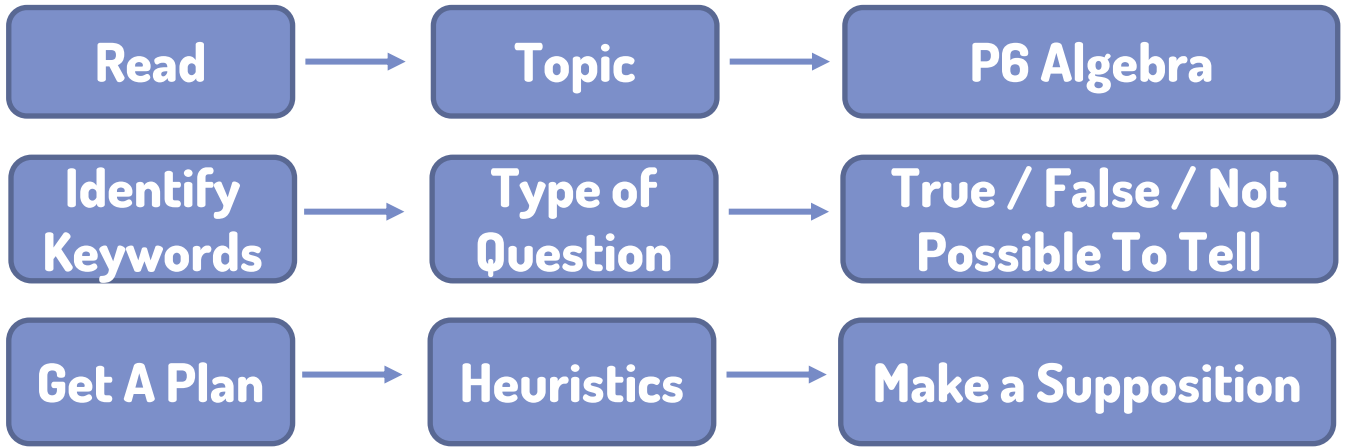
- Is the solution reasonable?
- Can you show that the solution is correct?

Q2) **PSLE 2018 Paper 2**

For a recycling project, Ali collected 17 bottles, Bala collected 2m bottles and Carl collected (2 + m) bottles.

Each of the statements is either true, false or not possible to tell from the information given. For each statement, put a tick (✓) to indicate your answer.

Statement	True	False	Not possible to tell
Ali collected the most number of bottles.			



Q2)

For a recycling project, Ali collected 17 bottles, Bala collected 2m bottles and Carl collected (2 + m) bottles.

Each of the statement is either true, false or not possible to tell from the information given. For each statement, put a tick (✓) to indicate your answer.

Statement	True	False	Not possible to tell
Ali collected the most number of bottles.			
Bala collected more bottles than Carl.			
The 3 boys collected 3m + 19 bottles altogether.			

Q2)

For a recycling project, Ali collected 17 bottles, Bala collected 2m bottles and Carl collected $(2 + m)$ bottles.

Given:

Number of bottles collected by Ali → 17

Number of bottles collected by Bala –> 2m

Number of bottles collected by Carl → $(2 + m)$

Unknown:

The value of m.

READ

- What is known?
- What is unknown?
- What do you need to find?

Q2)

For a recycling project, Ali collected 17 bottles, Bala collected $2m$ bottles and Carl collected $(2 + m)$ bottles.

Statement	True	False	Not possible to tell
Ali collected the most number of bottles.	✔		✔

If m = 5,

Bala collected → 2×5

$= 10$

Carl collected → $2 + 5$

$= 7$

READ

- What is known?
- What is unknown?
- What do you need to find?

IDENTIFY KEYWORDS

- Are there any keywords that leads to the type of question or concept?

GET A PLAN

- Have you seen it before?
- What strategy would you use?
- Is there other ways to solve this question?

Make A Supposition

Q2)

For a recycling project, Ali collected 17 bottles, Bala collected 2m bottles and Carl collected (2 + m) bottles.

Statement	True	False	Not possible to tell
Ali collected the most number of bottles.	✓		✓

If m = 5,
Bala collected → 2×5
= 10
Carl collected → $2 + 5$
= 7

If m = 10,
Bala collected → 2×10
= 20
Carl collected → $2 + 10$
=12

Make A Supposition

HAVE IT DONE

- Is each step correct?
- Can you prove that each step is correct?
- What makes you say that?

As the value of m was <u>not</u> provided in the question, we cannot conclude whether Ali has collected the most number of bottles.

Q2)

For a recycling project, Ali collected 17 bottles, Bala collected 2m bottles and Carl collected (2 + m) bottles.

Statement	True	False	Not possible to tell
Bala collected more bottles than Carl.	✔		

$2 \times m$ is more than $2 + m$.

If m = 8,

Bala collected → 2×8

= 16

Carl collected → $2 + 8$

= 10

If m = 2,

Bala collected → 2×2

= 4

Carl collected → $2 + 2$

= 4

READ

- What is known?
- What is unknown?
- What do you need to find?

IDENTIFY KEYWORDS

- Are there any keywords that leads to the type of question or concept?

Q2)

For a recycling project, Ali collected 17 bottles, Bala collected $2m$ bottles and Carl collected $(2 + m)$ bottles.

Statement	True	False	Not possible to tell
Bala collected more bottles than Carl.	✔		

$2 \times m$ is more than $2 + m$.

If m = 8,

Bala collected → 2×8
= 16

Carl collected → $2 + 8$
= 10

If m = 2,

Bala collected → 2×2
= 4

Carl collected → $2 + 2$
= 4

GET A PLAN

- Have you seen it before?
- What strategy would you use?
- Is there other ways to solve this question?

HAVE IT DONE

- Is each step correct?
- Can you prove that each step is correct?
- What makes you say that?

Q2)

For a recycling project, Ali collected 17 bottles, Bala collected $2m$ bottles and Carl collected $(2 + m)$ bottles.

Statement	True	False	Not possible to tell
Bala collected more bottles than Carl.	✔		✔

TRUE

If m = 8,

Bala collected → 2×8

$= 16$

Carl collected → $2 + 8$

$= 10$

FALSE

If m = 2,

Bala collected → 2×2

$= 4$

Carl collected → $2 + 2$

$= 4$

FALSE

If m = 1,

Bala collected → 2×1

$= 2$

Carl collected → $2 + 1$

$= 3$

Q2)

For a recycling project, Ali collected 17 bottles, Bala collected 2m bottles and Carl collected (2 + m) bottles.

Statement	True	False	Not possible to tell
The 3 boys collected 3m + 19 bottles altogether.	✔		

total

Total number of bottles collected

→ 17 + 2m + (2 + m)

= 2m + m + 17 + 2

= 3m + 19

READ

- What is known?
- What is unknown?
- What do you need to find?

IDENTIFY KEYWORDS

- Are there any keywords that leads to the type of question or concept?

GET A PLAN

- Have you seen it before?
- What strategy would you use?
- Is there other ways to solve this question?

Q2)

For a recycling project, Ali collected 17 bottles, Bala collected 2m bottles and Carl collected (2 + m) bottles.

Statement	True	False	Not possible to tell
The 3 boys collected 3m + 19 bottles altogether.	✔		

total

Total number of bottles collected

→ 17 + 2m + (2 + m)

= 2m + m + 17 + 2

= 3m + 19

HAVE IT DONE

- Is each step correct?
- Can you prove that each step is correct?
- What makes you say that?

TRIPLE CHECK

- Is the solution reasonable?
- Can you show that the solution is correct?

Q3) **PSLE 2022 Paper 2**

The pie charts show the number of each type of fish in two fish tanks, A and B. The total number of fish in Tank A is twice the total number of fish in Tank B.

Each of the statements is either true, false or not possible to tell from the information given. For each statement, put a tick (✓) to indicate your answer.

Read → Topic → P6 Pie Chart

Identify Keywords → Type of Question → True / False / Not Possible To Tell

Get A Plan → Heuristics → Make a Supposition

Each of the statement is either true, false or not possible to tell from the information given. For each statement, put a tick (✓) to indicate your answer.

Statement	True	False	Not possible to tell
There are 55 swordtails in Tank A.			
$\frac{1}{3}$ of the fish in Tank B are rainbow fish.			
There are more rainbow fish in Tank A than in Tank B.			

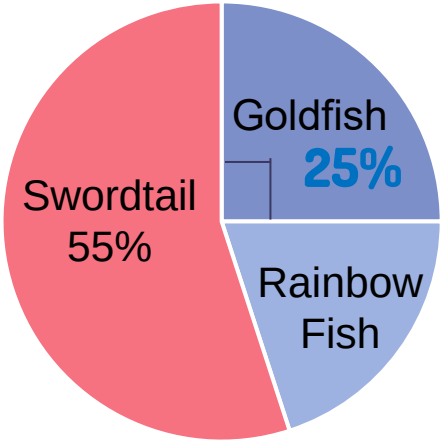
Q3)

The pie charts show the number of each type of fish in two fish tanks, A and B. The total number of fish in Tank A is twice the total number of fish in Tank B.

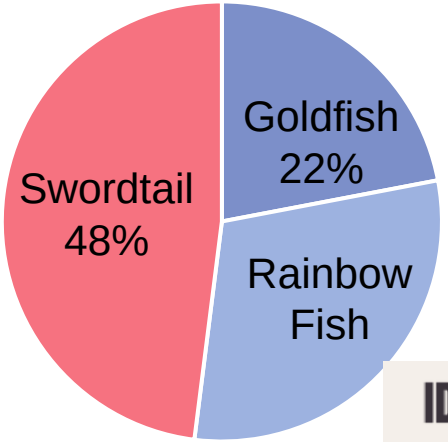
2 units

1 unit

Tank A



Tank B



READ

- What is known?
- What is unknown?
- What do you need to find?

IDENTIFY KEYWORDS

- Are there any keywords that leads to the type of question or concept?

Each of the statement is either true, false or not possible to tell from the information given. For each statement, put a tick (✓) to indicate your answer.

Statement	True	False	Not possible to tell
There are 55 swordtails in Tank A.			✓

The total number of fish in Tank A is unknown.

There is not enough information to support the statement above.

GET A PLAN

- Have you seen it before?
- What strategy would you use?
- Is there other ways to solve this question?

Each of the statement is either true, false or not possible to tell from the information given. For each statement, put a tick (✓) to indicate your answer.

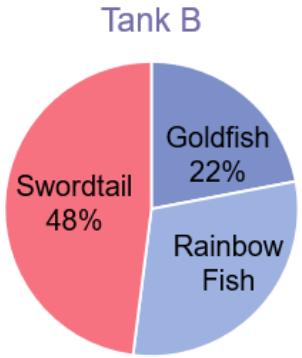
Statement	True	False	Not possible to tell
$\frac{1}{3}$ of the fish in Tank B are rainbow fish.		✓	

Percentage of rainbow fish in Tank B

→ 100% − 48% − 22%

= 30%

$\frac{1}{3} \times 100\% = 33\frac{1}{3}\%$



IDENTIFY KEYWORDS

- Are there any keywords that leads to the type of question or concept?

GET A PLAN

- Have you seen it before?
- What strategy would you use?
- Is there other ways to solve this question?

The pie charts show the number of each type of fish in two fish tanks, A and B. The total number of fish in Tank A is twice the total number of fish in Tank B.

2 units

1 unit

Each of the statement is either true, false or not possible to tell from the information given. For each statement, put a tick (✓) to indicate your answer.

Statement	True	False	Not possible to tell
There are more rainbow fish in Tank A than in Tank B.			

more

less

IDENTIFY KEYWORDS

- Are there any keywords that leads to the type of question or concept?

GET A PLAN

- Have you seen it before?
- What strategy would you use?
- Is there other ways to solve this question?

Each of the statement is either true, false or not possible to tell from the information given. For each statement, put a tick (✓) to indicate your answer.

Statement	True	False	Not possible to tell
There are more rainbow fish in Tank A than in Tank B.	✓		

more less

Percentage of rainbow fish in Tank A
→ 100% – 55% – 25%
= 20%

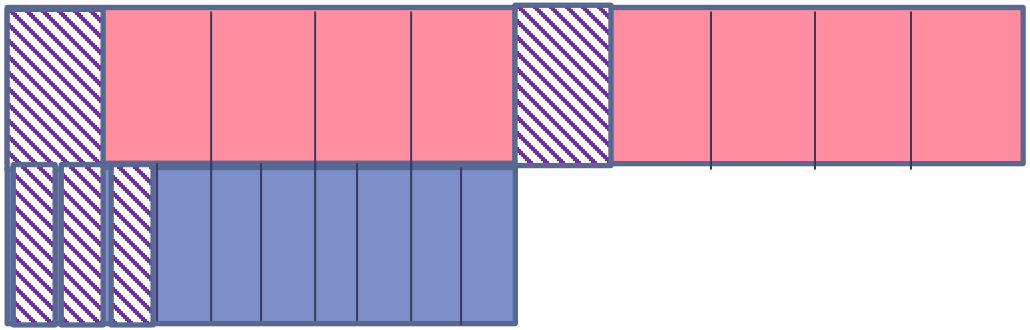
Assuming total number of fish in Tank A is 200, Number of rainbow fish in Tank A
→ $\frac{20}{100} \times 200 = 40$

Percentage of rainbow fish in Tank B
→ 100% – 48% – 22%
= 30%

Assuming total number of fish in Take B is 100, Number of rainbow fish in Tank B
→ $\frac{30}{100} \times 100 = 30$

Tank A

Tank B



Number of rainbow fish in Tank A

$\rightarrow \frac{20}{100} \times 2$ units

$= \frac{1}{5} \times 2$ units

$= \frac{2}{5}$

Number of rainbow fish in Tank B

$\rightarrow \frac{30}{100} \times 1$ unit

$= \frac{3}{10} \times 1$ unit

$= \frac{3}{10}$

$\frac{2}{5}$ is greater than $\frac{3}{10}$.

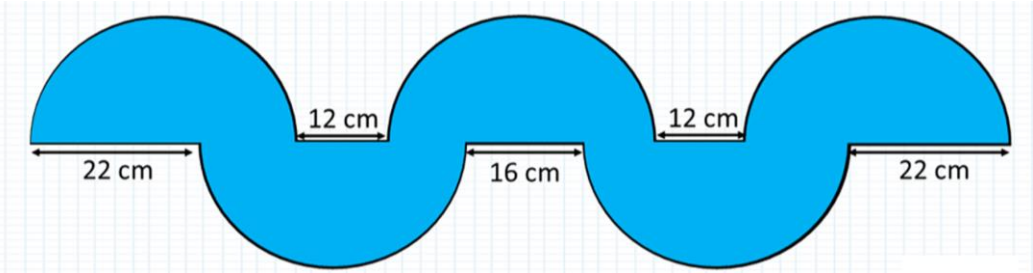
TRIPLE CHECK

- Is the solution reasonable?
- Can you show that the solution is correct?

Spatial Visualisation

Q4) **PSLE 2019 Paper 2**

The figure is formed by 5 identical semicircles.



What is the diameter of each semicircle?

Read → Topic → P6 Circles

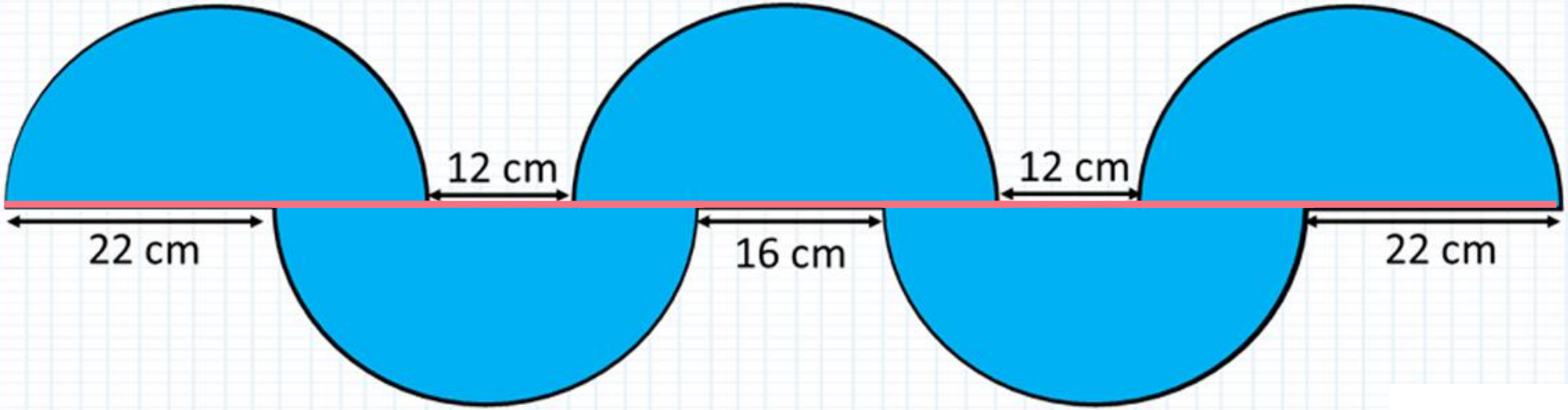
Identify Keywords → Type of Question → Spatial Visualisation

Get A Plan → Heuristics → Cut and Paste

Q4)

same

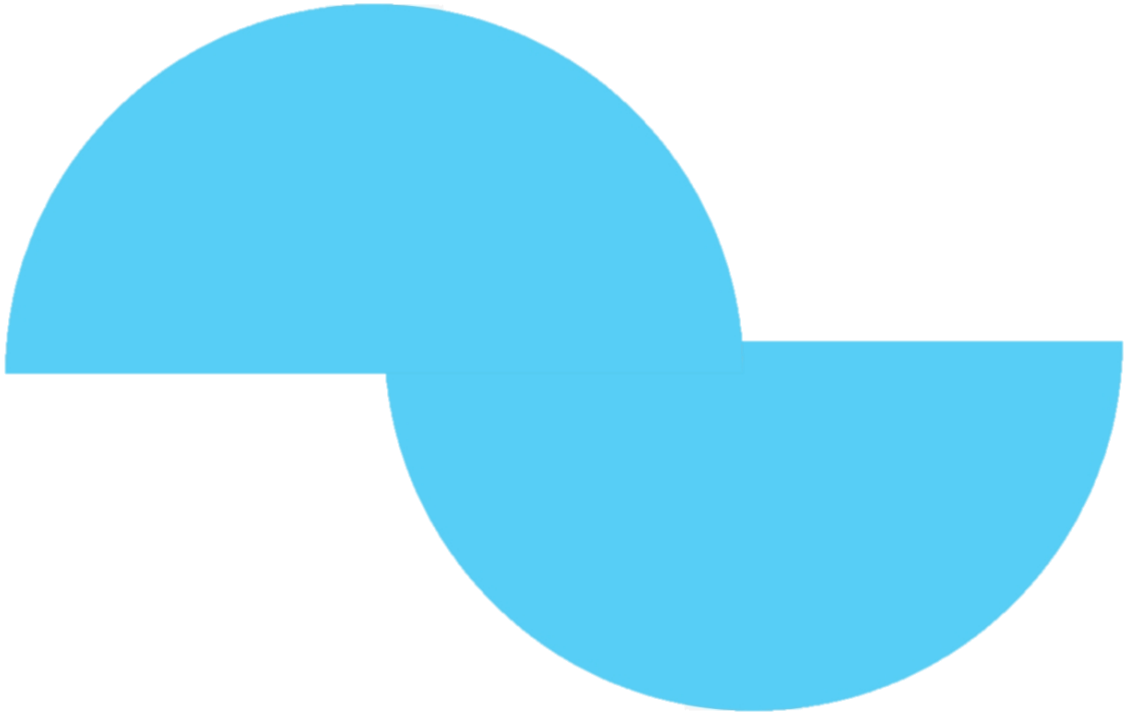
The figure is formed by 5 identical semicircles.



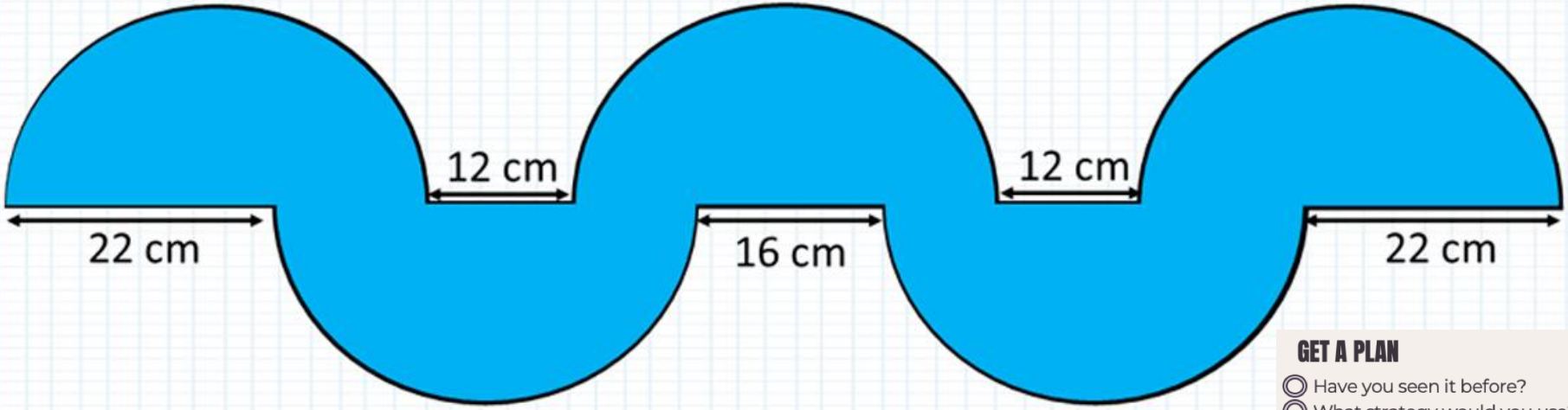
What is the diameter of each semicircle?

What shapes do we have?

How to move the shapes?

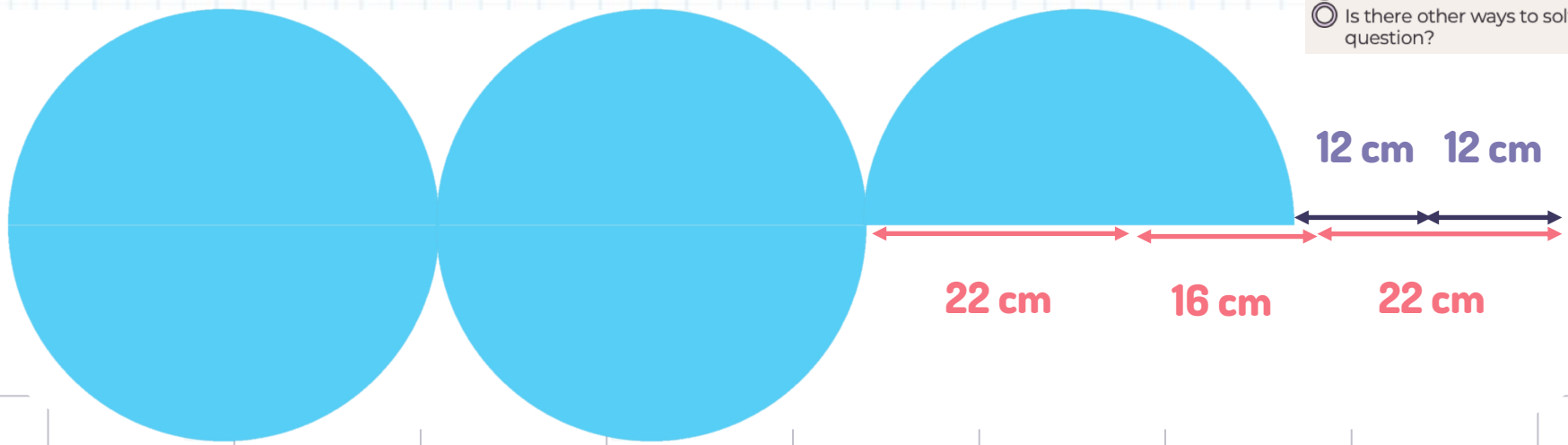


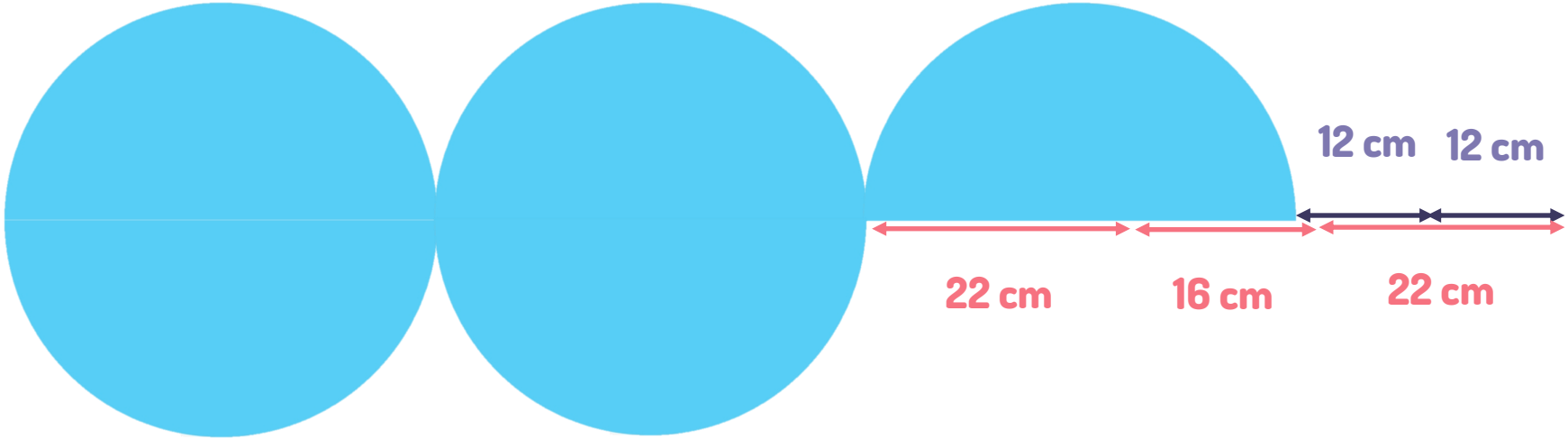
From side to side



GET A PLAN

- Have you seen it before?
- What strategy would you use?
- Is there other ways to solve this question?





Diameter of 1 semicircle

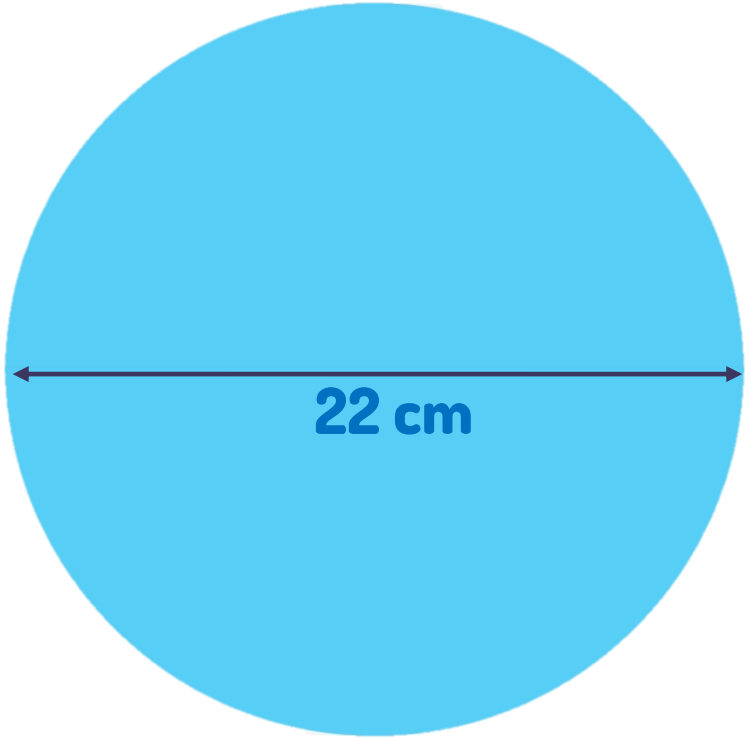
$\rightarrow (22 + 16 + 22) - (12 + 12)$

$= 60 - 24$

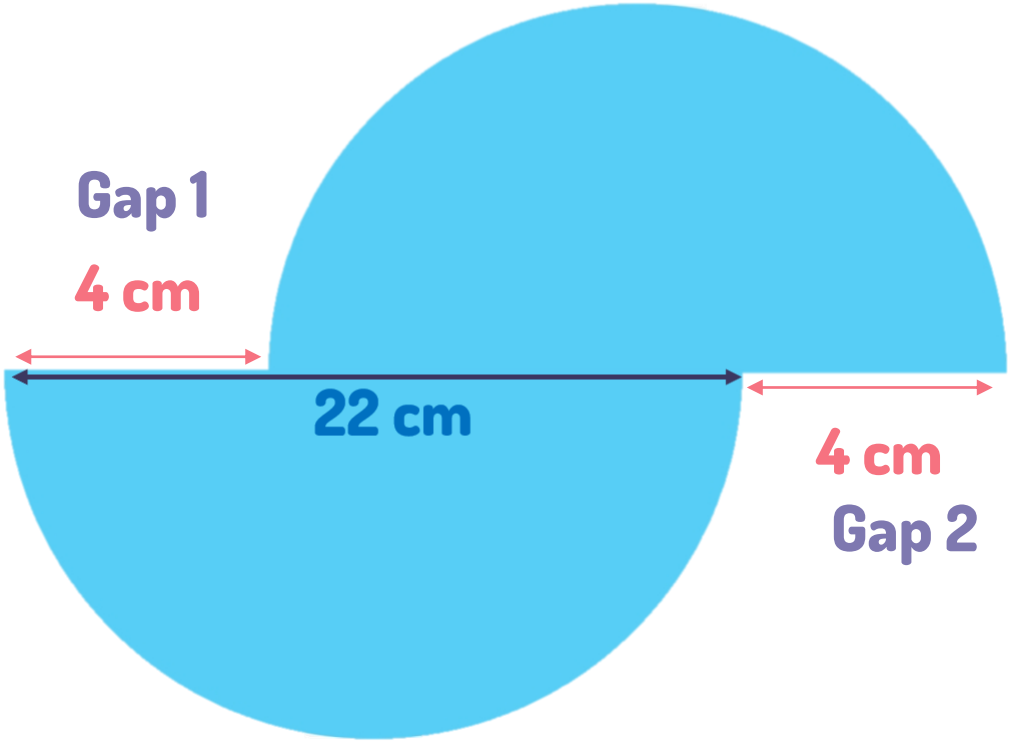
$= 36$

Ans: 36 cm

Main Idea



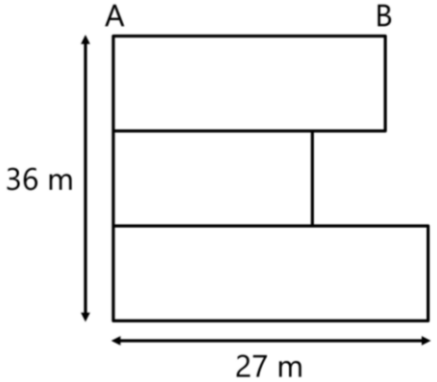
Main Idea



Gap 1 = Gap 2

Q4) **PSLE 2022 Paper 2**

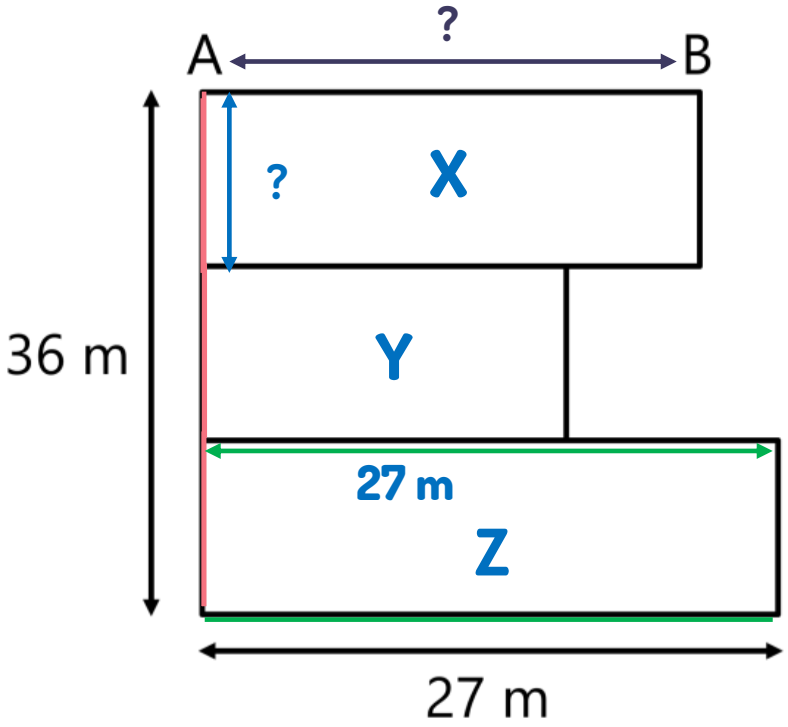
A plot of land of area 876 m^2 is divided into three rectangular fields of equal width. The fields are fenced using 177 m of fencing.



Read	Topic	P4 Perimeter
Identify Keywords	Type of Question	Spatial Visualisation
Get A Plan	Heuristics	Overlapping

Q5)

A plot of land of area 876 m^2 is divided into three rectangular fields of equal width. The fields are fenced using 177 m of fencing.



total length of the fencing

3 widths → 36 m

1 width → 36 ÷ 3

= 12

(a) Find the length of AB.

2 lengths of rectangular X

→ 177 – 36 – 36 – 27 – 27

= 51

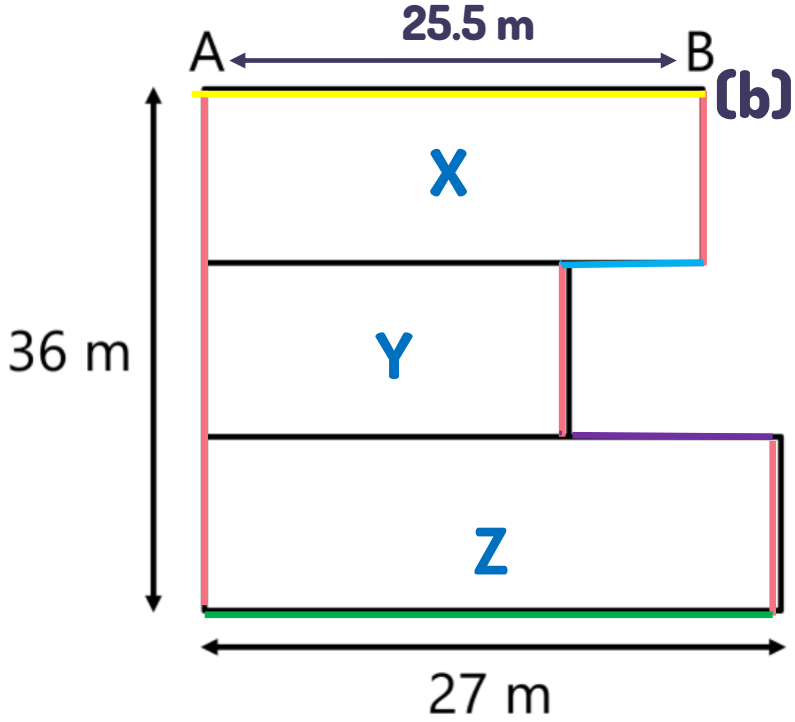
Length of AB → 51 ÷ 2

= 25.5 Ans: (a) 25.5 m

Q5)

A plot of land of area 876 m^2 is divided into three rectangular fields of equal width. The fields are fenced using 177 m of fencing.

total length of the fencing



(b) Find the perimeter of the plot of land.

Area of Z → 27×12

$= 324$

Area of X → 25.5×12

$= 306$

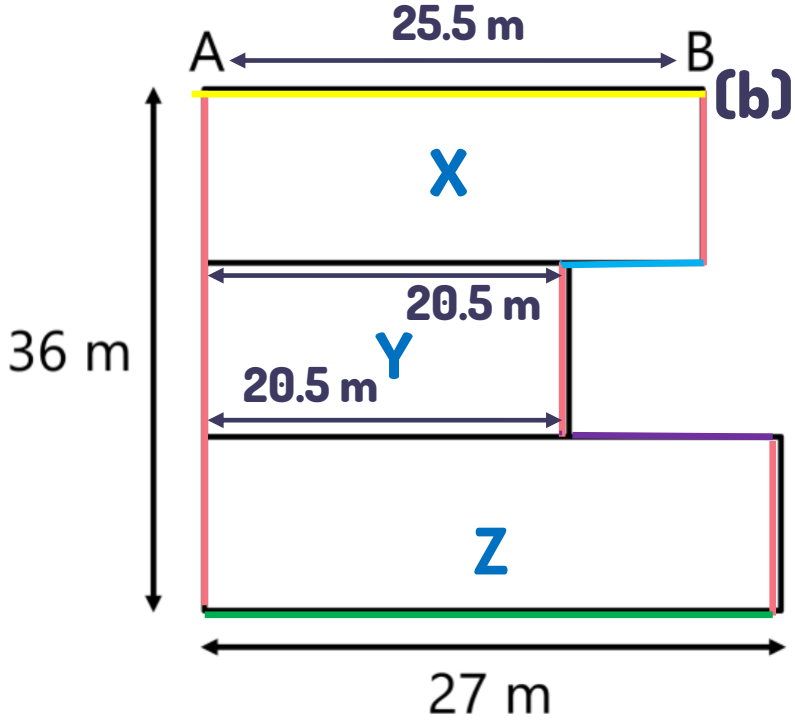
Area of Y → $876 - 324 - 306$

$= 246$

Q5)

A plot of land of area 876 m^2 is divided into three rectangular fields of equal width. The fields are fenced using 177 m of fencing.

total length of the fencing



(b) Find the perimeter of the plot of land.

Length of Y → 246 ÷ 12
= 20.5

Perimeter of the plot of land
→ 177 − 20.5 − 20.5
= 136

Ans: (b) 136 m

We Do

Parallel Question

Tips

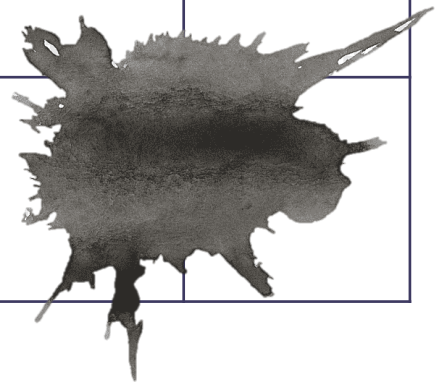
We Do
What Questions Can You Answer?

Tips

Q1)

The table below shows the number of storybooks read by each pupil in a group. Part of the table is covered by an ink blot. There were 45 pupils who read at least 2 storybooks.

Number of storybooks	0	1	2	3	4
Number of pupils	7	8	20		



READ

- What is known?
- What is unknown?
- What do you need to find?

What questions can I answer?

“I’m going to give you some time to think about which questions can I generate from these given information before I ask you to respond.”

IDENTIFY KEYWORDS

- Are there any keywords that leads to the type of question or concept?

GET A PLAN

- Have you seen it before?
- What strategy would you use?
- Is there other ways to solve this question?

HAVE IT DONE

- Is each step correct?
- Can you prove that each step is correct?
- What makes you say that?

Can you explain to me the steps you intend to use to solve the problem? Why?

Do you see any relationships that might help you to solve the problem?

What are the mathematical concepts in this problem?

GET A PLAN

- Have you seen it before?
- What strategy would you use?
- Is there other ways to solve this question?

HAVE IT DONE

- Is each step correct?
- Can you prove that each step is correct?
- What makes you say that?

What are the missing information we can find out with the given information?

Can I ask a ratio or fractions question for the given information?

Which topic can the question be of?

How do you know your question / solution is reasonable? How can you convince me that your answer makes sense?

Explain why _____ would have been an unreasonable question?

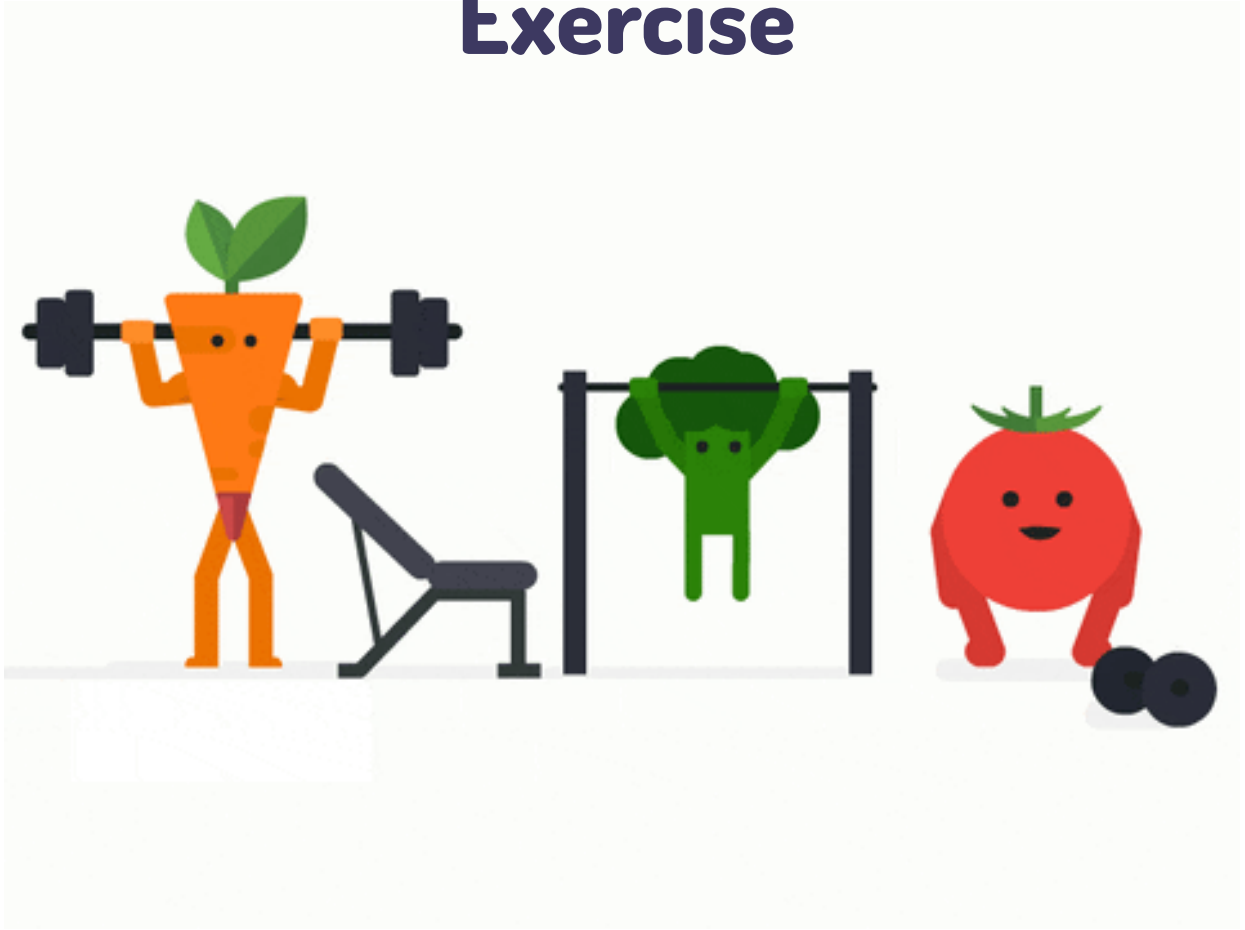
TRIPLE CHECK

- ◎ Is the solution reasonable?
- ◎ Can you show that the solution is correct?

Seeing the JOY in Math

BUDGET

Exercise



Shopping

Thank You