



Singapore Examinations and Assessment Board



## **Singapore–Cambridge General Certificate of Education Advanced Level Higher 2 (2027)**

# **Mathematics (Syllabus 9758)**

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## PREAMBLE

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Mathematics is a basic and important discipline that contributes to the developments and understandings of sciences and other disciplines. It is used by scientists, engineers, business analysts and psychologists, etc. to model, understand and solve problems in their respective fields. A good foundation in mathematics and the ability to reason mathematically are therefore essential for students to be successful in their pursuit of various disciplines.

H2 Mathematics is designed to prepare students for a range of university courses, including mathematics, sciences, engineering and related courses, where a good foundation in mathematics is required. It develops mathematical thinking and reasoning skills that are essential for further learning of mathematics. Through applications of mathematics, students also develop an appreciation of mathematics and its connections to other disciplines and to the real world.

## SYLLABUS AIMS

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The aims of H2 Mathematics are to enable students to:

- (a) acquire mathematical concepts and skills to prepare for their tertiary studies in mathematics, sciences, engineering and other related disciplines
- (b) develop thinking, reasoning, communication and modelling skills through a mathematical approach to problem-solving
- (c) connect ideas within mathematics and apply mathematics in the contexts of sciences, engineering and other related disciplines
- (d) experience and appreciate the nature and beauty of mathematics and its value in life and other disciplines.

## ASSESSMENT OBJECTIVES (AO)

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The assessment will test candidates' abilities to:

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|-----|---|
| AO1 | <b>Use mathematical techniques and procedures</b>   |
|     | <ul style="list-style-type: none"> <li>• Recall facts, formulae and notation and use them directly.</li> <li>• Read and use information from tables, graphs, diagrams and texts.</li> <li>• Carry out straightforward mathematical procedures.</li> </ul>   |
| AO2 | <b>Formulate and solve problems including those in real-world contexts</b>  |
|     | <ul style="list-style-type: none"> <li>• Select relevant mathematical concept or strategy to apply.</li> <li>• Formulate problems into mathematical expressions or models.</li> <li>• Integrate mathematical concepts to solve mathematical problems.</li> <li>• Translate between equivalent forms of mathematical expressions or statements.</li> <li>• Interpret results in the context of a given problem.</li> </ul> |
| AO3 | <b>Reason and communicate mathematically</b>  |
|     | <ul style="list-style-type: none"> <li>• Explain the choice of mathematical models or strategies.</li> <li>• Make deductions, inferences and generalisations.</li> <li>• Formulate conjectures and justify mathematical statements.</li> <li>• Construct mathematical arguments and proofs.</li> </ul>  |

Approximate weightings for the assessment objectives are as follows:

AO1	30%
AO2	60%
AO3	10%

## USE OF A GRAPHING CALCULATOR (GC)

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The use of an approved GC without computer algebra system will be expected. The examination papers will be set with the assumption that candidates will have access to GC. As a general rule, unsupported answers obtained from GC are allowed unless the question states otherwise. Where unsupported answers from GC are not allowed, candidates must present the mathematical steps using mathematical notations and not calculator commands. For questions where graphs are used to find a solution, candidates should sketch these graphs as part of their answers. Incorrect answers without working will receive no marks. However, if there is written evidence of using GC correctly, method marks may be awarded.

Candidates should be aware that there are limitations inherent in GC. For example, answers obtained by tracing along a graph to find roots of an equation may not produce the required accuracy.

## LIST OF FORMULAE AND RESULTS

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Candidates will be provided in the examination with a list of formulae and results.

## INTEGRATION AND APPLICATION

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Notwithstanding the presentation of the topics in the syllabus document, it is envisaged that some examination questions may integrate ideas from more than one topic, and that topics may be tested in the contexts of problem solving and application of mathematics.

Possible list of H2 Mathematics applications and contexts:

Applications and contexts	Some possible topics involved
Kinematics and dynamics (e.g. free fall, projectile motion, collisions)	Functions; Calculus; Vectors
Optimisation problems (e.g. maximising strength, minimising surface area)	Inequalities; System of linear equations; Calculus
Electrical circuits	Complex numbers; Calculus
Population growth, radioactive decay, heating and cooling problems	Differential equations
Financial maths (e.g. banking, insurance)	Sequences and series; Probability; Sampling distributions
Standardised testing	Normal distribution; Probability

Applications and contexts	Some possible topics involved
Market research (e.g. consumer preferences, product claims)	Sampling distributions; Hypothesis testing; Correlation and regression
Clinical research (e.g. correlation studies)	Sampling distributions; Hypothesis testing; Correlation and regression

The list illustrates some types of contexts in which the mathematics learnt in the syllabus may be applied, and is by no means exhaustive. While problems may be set based on these contexts, no assumptions will be made about the knowledge of these contexts. All information will be self-contained within the problem.

## SCHEME OF EXAMINATION PAPERS

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For the examination in H2 Mathematics, there will be two 3-hour papers, each carrying 50% of the total mark, and each marked out of 100, as follows:

### PAPER 1 (3 hours)

A paper consisting of 10 to 12 questions of different lengths and marks based on the Pure Mathematics section of the syllabus.

There will be one question on application of Mathematics in real-world contexts, including those from sciences and engineering. This question will carry at least 12 marks and may require concepts and skills from more than one topic.

Candidates will be expected to answer **all** questions.

### PAPER 2 (3 hours)

A paper consisting of two sections, Sections A and B.

**Section A** (Pure Mathematics – 40 marks) will consist of 4 to 5 questions of different lengths and marks based on the Pure Mathematics section of the syllabus.

**Section B** (Probability and Statistics – 60 marks) will consist of 6 to 8 questions of different lengths and marks based on the Probability and Statistics section of the syllabus.

There will be one question in Section B on application of Mathematics in real-world contexts, including those from sciences and engineering. This question will carry at least 12 marks and may require concepts and skills from more than one topic.

Candidates will be expected to answer **all** questions.

## CONTENT OUTLINE

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Knowledge of the content of the O-Level Mathematics syllabus is assumed. The assumed knowledge for O-Level Additional Mathematics is appended after this section.

	Topic/Sub-topics	Content
<b>SECTION A: PURE MATHEMATICS</b>		
<b>1</b>	<b>Functions and graphs</b>	
1.1	Functions	<p>Include:</p> <ul style="list-style-type: none"> <li>concepts of function, domain and range</li> <li>inverse functions and composite functions</li> <li>conditions for the existence of inverse functions and composite functions</li> <li>domain restriction to obtain an inverse function</li> <li>relationship between graphs of a one-to-one function and its inverse</li> </ul> <p>Exclude the use of the relation <math>(fg)^{-1} = g^{-1}f^{-1}</math>, and restriction of domain to obtain a composite function.</p>
1.2	Graphs and transformations	<p>Include:</p> <ul style="list-style-type: none"> <li>use of a graphing calculator or a graphing software to graph a given function</li> <li>important characteristics of graphs such as symmetry, intersections with the axes, turning points and asymptotes of the following:</li> </ul> $y^2 = ax ; x^2 = by$ $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 ; \frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$ $y = \frac{ax+b}{cx+d}$ $y = \frac{ax^2+bx+c}{dx+e}$ <ul style="list-style-type: none"> <li>equations of asymptotes, axes of symmetry, and restrictions on the possible values of <math>x</math> and/or <math>y</math></li> <li>effect of transformations on the graph of <math>y = f(x)</math> as represented by <math>y = af(x)</math>, <math>y = f(x) + a</math>, <math>y = f(x+a)</math> and <math>y = f(ax)</math> and combinations of these transformations</li> <li>relating the graphs of <math>y =  f(x) </math>, <math>y = f( x )</math>, and <math>y = \frac{1}{f(x)}</math> to the graph of <math>y = f(x)</math></li> <li>simple parametric equations and their graphs</li> </ul>

	<b>Topic/Sub-topics</b>	<b>Content</b>
1.3	Equations and inequalities	<p>Include:</p> <ul style="list-style-type: none"> <li>formulating an equation, a system of linear equations, or inequalities from a problem situation</li> <li>solving an equation exactly or approximately using a graphing calculator or a graphing software</li> <li>solving a system of linear equations using a graphing calculator or a graphing software</li> <li>solving inequalities of the form <math>\frac{f(x)}{g(x)} &gt; 0</math> where <math>f(x)</math> and <math>g(x)</math> are linear expressions or quadratic expressions</li> <li>concept of <math> x </math>, and use of relations  <math> x - a  &lt; b \Leftrightarrow a - b &lt; x &lt; a + b</math> and  <math> x - a  &gt; b \Leftrightarrow x &lt; a - b</math> or <math>x &gt; a + b</math></li> <li>solving inequalities by graphical methods</li> </ul>
<b>2</b>	<b>Sequences and series</b>	
2.1	Sequences and series	<p>Include:</p> <ul style="list-style-type: none"> <li>concepts of sequence and series for finite and infinite cases</li> <li>sequence as function <math>y = f(n)</math> where <math>n</math> is a positive integer</li> <li>relationship between <math>u_n</math> (the <math>n</math>th term) and <math>S_n</math> (the sum to <math>n</math> terms)</li> <li>sequence given by a formula for the <math>n</math>th term</li> <li>sequence generated by the relation <math>u_{n+1} = f(u_n)</math>, including the use of a graphing calculator or a computer to generate the sequence</li> <li>sum and difference of two series</li> <li>convergence of a series and the sum to infinity</li> <li>formula for the <math>n</math>th term and the sum of a finite arithmetic series</li> <li>formula for the <math>n</math>th term and the sum of a finite geometric series</li> <li>condition for convergence of an infinite geometric series</li> <li>formula for the sum to infinity of a convergent geometric series</li> </ul>
<b>3</b>	<b>Vectors</b>	
3.1	Basic properties of vectors in two and three dimensions	<p>Include:</p> <ul style="list-style-type: none"> <li>addition and subtraction of vectors, multiplication of a vector by a scalar, and their geometrical interpretations</li> <li>position vectors, displacement vectors and direction vectors</li> <li>magnitude of a vector</li> <li>unit vectors</li> <li>distance between two points</li> <li>collinearity</li> </ul>

	<b>Topic/Sub-topics</b>	<b>Content</b>
		<ul style="list-style-type: none"> <li>use of the ratio theorem in geometrical applications</li> </ul>
3.2	Scalar and vector products in vectors	<p>Include:</p> <ul style="list-style-type: none"> <li>concepts of scalar product and vector product of vectors and their properties</li> <li>angle between two vectors</li> <li>geometrical meanings of <math> \mathbf{a} \cdot \hat{\mathbf{n}} </math> and <math> \mathbf{a} \times \hat{\mathbf{n}} </math>, where <math>\hat{\mathbf{n}}</math> is a unit vector</li> </ul> <p>Exclude triple products <math>\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}</math> and <math>\mathbf{a} \times \mathbf{b} \times \mathbf{c}</math></p>
3.3	Three-dimensional vector geometry	<p>Include:</p> <ul style="list-style-type: none"> <li>vector and cartesian equations of lines and planes</li> <li>foot of the perpendicular and distance from a point to a line or to a plane</li> <li>angle between two lines, between a line and a plane, or between two planes</li> <li>relationships between <ul style="list-style-type: none"> <li>(i) two lines (coplanar or skew)</li> <li>(ii) a line and a plane</li> <li>(iii) two planes</li> </ul> </li> </ul> <p>Exclude:</p> <ul style="list-style-type: none"> <li>shortest distance between two skew lines</li> <li>common perpendicular to two skew lines</li> </ul>
<b>4</b>	<b>Introduction to Complex numbers</b>	
4.1	Complex numbers expressed in cartesian form and Argand diagrams	<p>Include:</p> <ul style="list-style-type: none"> <li>extension of the number system from real numbers to complex numbers</li> <li>complex roots of quadratic equations</li> <li>modulus, argument and conjugate of a complex number</li> <li>four operations of complex numbers</li> <li>equality of complex numbers</li> <li>conjugate roots of a polynomial equation with real coefficients</li> <li>representation of complex numbers in the Argand diagram</li> <li>geometrical effects of conjugation, negation, addition, subtraction, and multiplication by <math>i</math></li> </ul> <p>Exclude complex numbers expressed in polar (or modulus-argument) form and exponential form.</p>

	<b>Topic/Sub-topics</b>	<b>Content</b>
<b>5</b>	<b>Calculus</b>	
5.1	Differentiation	<p>Include:</p> <ul style="list-style-type: none"> <li>• graphical interpretation of           <ul style="list-style-type: none"> <li>(i) <math>f'(x) &gt; 0</math>, <math>f'(x) = 0</math> and <math>f'(x) &lt; 0</math></li> <li>(ii) <math>f''(x) &gt; 0</math> and <math>f''(x) &lt; 0</math></li> </ul> </li> <li>• relating the graph of <math>y = f'(x)</math> to the graph of <math>y = f(x)</math></li> <li>• differentiation of simple functions defined implicitly or parametrically</li> <li>• determining the nature of the stationary points (local maximum and minimum points and points of inflection) analytically, in simple cases, using the first derivative test or the second derivative test</li> <li>• locating maximum and minimum points using a graphing calculator or a <i>graphing software</i></li> <li>• finding the approximate value of a derivative at a given point using a graphing calculator or a <i>graphing software</i></li> <li>• problems involving tangents and normals to curves, including cases where the curve is defined implicitly or parametrically</li> <li>• local maxima and minima problems</li> <li>• connected rates of change problems</li> </ul> <p>Exclude non-stationary points of inflection and finding second derivative of functions defined parametrically.</p>
5.2	Maclaurin series	<p>Include:</p> <ul style="list-style-type: none"> <li>• standard series expansion of <math>(1+x)^n</math> for any rational <math>n</math>, <math>e^x</math>, <math>\sin x</math>, <math>\cos x</math> and <math>\ln(1+x)</math></li> <li>• derivation of the first few terms of the Maclaurin series by           <ul style="list-style-type: none"> <li>- repeated differentiation, e.g. <math>\sec x</math></li> <li>- repeated implicit differentiation, e.g. <math>y^3 + y^2 + y = x^2 - 2x</math></li> <li>- using standard series, e.g. <math>e^x \cos 2x</math>, <math>\ln\left(\frac{1+x}{1-x}\right)</math></li> </ul> </li> <li>• range of values of <math>x</math> for which a standard series converges</li> <li>• concept of Maclaurin's series as an approximation of a function</li> <li>• small angle approximations: <math>\sin x \approx x</math>, <math>\cos x \approx 1 - \frac{1}{2}x^2</math>, <math>\tan x \approx x</math></li> </ul> <p>Exclude problems involving derivation of the general term of a series.</p>

	<b>Topic/Sub-topics</b>	<b>Content</b>
5.3	Integration techniques	<p>Include:</p> <ul style="list-style-type: none"> <li>integration of <math>f'(x)[f(x)]^n</math> (including <math>n = -1</math>), <math>f'(x)e^{f(x)}</math>, <math>\sin^2 x, \cos^2 x, \tan^2 x</math>, <math>\frac{1}{a^2 + x^2}, \frac{1}{\sqrt{a^2 - x^2}}, \frac{1}{a^2 - x^2}</math> and <math>\frac{1}{x^2 - a^2}</math></li> <li>integration by a given substitution</li> <li>integration by parts</li> </ul> <p>Exclude reduction formulae.</p>
5.4	Definite integrals	<p>Include:</p> <ul style="list-style-type: none"> <li>concept of definite integral as a limit of sum</li> <li>definite integral as the area under a curve</li> <li>evaluation of definite integrals</li> <li>area of a region bounded by a curve and lines parallel to the coordinate axes, between a curve and a line, or between two curves</li> <li>area below the <math>x</math>-axis</li> <li>volume of revolution about the <math>x</math>- or <math>y</math>-axis</li> <li>finding the approximate value of a definite integral using a graphing calculator or a graphing software</li> </ul> <p>Exclude area and volume of revolution about the <math>x</math>-axis or <math>y</math>-axis where curve is defined parametrically.</p>
5.5	Differential equations	<p>Include:</p> <ul style="list-style-type: none"> <li>solving for the general solutions and particular solutions of differential equations of the form <math>\frac{dy}{dx} = f(x)g(y)</math>, including reducing a given differential equation to this form by means of a given substitution</li> <li>formulating a differential equation from a problem situation</li> <li>interpreting a differential equation and its solution in terms of a problem situation</li> </ul>

	Topic/Sub-topics	Content
<b>SECTION B: PROBABILITY AND STATISTICS</b>		
<b>6</b>	<b>Probability and Statistics</b>	
6.1	Probability	<p>Include:</p> <ul style="list-style-type: none"> <li>addition and multiplication principles for counting</li> <li>concepts of permutation (<math>{}^n P_r</math>) and combination (<math>{}^n C_r</math>)</li> <li>arrangements of objects in a line or in a circle, including cases involving repetition and restriction</li> <li>addition and multiplication of probabilities</li> <li>mutually exclusive events and independent events</li> <li>use of tables of outcomes, Venn diagrams, tree diagrams, and permutations and combinations techniques to calculate probabilities</li> <li>calculation of conditional probabilities in simple cases</li> <li>use of:  <math>P(A') = 1 - P(A)</math>  <math>P(A \cup B) = P(A) + P(B) - P(A \cap B)</math>  <math>P(A B) = \frac{P(A \cap B)}{P(B)}</math></li> </ul>
6.2	Discrete random variables	<p>Include:</p> <ul style="list-style-type: none"> <li>concept of discrete random variables, probability distributions, expectations and variances</li> <li>concept of binomial distribution <math>B(n, p)</math> as an example of a discrete probability distribution and use of <math>B(n, p)</math> as a probability model, including conditions under which the binomial distribution is a suitable model</li> <li>use of mean and variance of binomial distribution (without proof)</li> </ul> <p>Exclude finding cumulative distribution function of a discrete random variable.</p>

	Topic/Sub-topics	Content
6.3	Normal distribution	<p>Include:</p> <ul style="list-style-type: none"> <li>• concept of continuous random variables*</li> <li>• concept of a normal distribution as an example of a continuous probability model and its mean and variance; use of <math>N(\mu, \sigma^2)</math> as a probability model</li> <li>• standard normal distribution</li> <li>• finding the value of <math>P(X &lt; x_1)</math> or a related probability, given the values of <math>x_1, \mu, \sigma</math></li> <li>• symmetry of the normal curve and its properties</li> <li>• finding a relationship between <math>x_1, \mu, \sigma</math> given the value of <math>P(X &lt; x_1)</math> or a related probability</li> <li>• solving problems involving the use of <math>E(aX + b)</math> and <math>\text{Var}(aX + b)</math></li> <li>• solving problems involving the use of <math>E(aX + bY)</math> and <math>\text{Var}(aX + bY)</math>, where <math>X</math> and <math>Y</math> are independent</li> </ul> <p>Exclude normal approximation to binomial distribution.</p>
6.4	Sampling	<p>Include:</p> <ul style="list-style-type: none"> <li>• concepts of population and simple random sample</li> <li>• concept of the sample mean <math>\bar{X}</math> as a random variable with <math>E(\bar{X}) = \mu</math> and <math>\text{Var}(\bar{X}) = \frac{\sigma^2}{n}</math></li> <li>• distribution of sample mean from a normal population</li> <li>• use of the Central Limit Theorem to treat sample mean as having normal distribution when the sample size is sufficiently large (e.g. <math>n \geq 30</math>)</li> <li>• use of unbiased estimates of the population mean and variance from a sample, including cases where the data are given in summarised form <math>\sum x</math> and <math>\sum x^2</math>, or <math>\sum(x-a)</math> and <math>\sum(x-a)^2</math></li> </ul>

\* For teaching and learning only.

Topic/Sub-topics	Content
6.5 Hypothesis testing	<p>Include:</p> <ul style="list-style-type: none"> <li>• concepts of null hypothesis (<math>H_0</math>) and alternative hypotheses (<math>H_1</math>), test statistic, critical region, critical value, level of significance, and <math>p</math>-value</li> <li>• formulation of hypotheses and testing for a population mean based on <ul style="list-style-type: none"> <li>– a sample from a normal population of known variance</li> <li>– a large sample from any population</li> </ul> </li> <li>• 1-tail and 2-tail tests</li> <li>• interpretation of the results of a hypothesis test in the context of the problem</li> </ul> <p>Exclude the use of the term ‘Type I error’, concept of Type II error and testing the difference between two population means.</p>
6.6 Correlation and linear regression	<p>Include:</p> <ul style="list-style-type: none"> <li>• use of scatter diagram to judge if there is a plausible linear relationship between the two variables</li> <li>• correlation coefficient as a measure of the fit of a linear model to the scatter diagram</li> <li>• interpreting the product moment correlation coefficient (in particular, values close to <math>-1</math>, <math>0</math> and <math>1</math>)</li> <li>• concepts of linear regression and method of least squares to find the equation of the regression line</li> <li>• concepts of interpolation and extrapolation</li> <li>• use of the appropriate regression line to make prediction or estimate a value in practical situations, including explaining how well the situation is modelled by the linear regression model</li> <li>• use of a square, reciprocal or logarithmic transformation to achieve linearity</li> </ul> <p>Exclude:</p> <ul style="list-style-type: none"> <li>• problems involving derivation of formulae</li> <li>• relationship <math>r^2 = b_1 b_2</math>, where <math>b_1</math> and <math>b_2</math> are regression coefficients</li> <li>• hypothesis tests</li> </ul>

## ASSUMED KNOWLEDGE

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Content from O-Level Additional Mathematics	
ALGEBRA	
A1	<p>Quadratic functions; Equations and inequalities</p> <ul style="list-style-type: none"> <li>Finding the maximum or minimum value of a quadratic function using the method of completing the square</li> <li>Conditions for a quadratic equation to have:           <ul style="list-style-type: none"> <li>(i) two real roots</li> <li>(ii) two equal roots</li> <li>(iii) no real roots</li> </ul> </li> <li>Conditions for <math>ax^2 + bx + c</math> to be always positive (or always negative)</li> <li>Solving simultaneous equations in two variables by substitution, with one of the equations being a linear equation</li> </ul>
A2	<p>Surds</p> <ul style="list-style-type: none"> <li>Four operations on surds, including rationalising the denominator</li> <li>Solving equations involving surds.</li> </ul>
A3	<p>Polynomials and partial fractions</p> <ul style="list-style-type: none"> <li>Multiplication and division of polynomials</li> <li>Use of remainder and factor theorems</li> <li>Partial fractions with cases where the denominator is not more complicated than:           <ul style="list-style-type: none"> <li><math>(ax + b)(cx + d)</math></li> <li><math>(ax + b)(cx + d)^2</math></li> <li><math>(ax + b)(x^2 + c^2)</math></li> </ul> </li> </ul>
A4	<p>Exponential and logarithmic functions</p> <ul style="list-style-type: none"> <li>Exponential and logarithmic functions <math>a^x</math>, <math>e^x</math>, <math>\log_a x</math>, <math>\ln x</math> and their graphs, including:           <ul style="list-style-type: none"> <li>laws of logarithms</li> <li>equivalence of <math>y = a^x</math> and <math>x = \log_a y</math></li> <li>change of base of logarithms</li> </ul> </li> <li>Simplifying expressions and solving simple equations involving exponential and logarithmic functions</li> </ul>
GEOMETRY AND TRIGONOMETRY	
B5	<p>Coordinate geometry in two dimensions</p> <ul style="list-style-type: none"> <li>Coordinate geometry of the circle with the equation in the form <math>(x - a)^2 + (y - b)^2 = r^2</math> or <math>x^2 + y^2 + 2gx + 2fy + c = 0</math></li> </ul>
B6	<p>Trigonometric functions, identities and equations</p> <ul style="list-style-type: none"> <li>Six trigonometric functions, and principal values of the inverses of sine, cosine and tangent</li> <li>Trigonometric equations and identities (see List of Formulae)</li> <li>Expression of <math>a \cos \theta + b \sin \theta</math> in the forms <math>R \sin(\theta \pm \alpha)</math> and <math>R \cos(\theta \pm \alpha)</math></li> </ul>

**Content from O-Level Additional Mathematics****CALCULUS**

C7	<p>Differentiation and integration</p> <ul style="list-style-type: none"> <li>• Derivative of <math>f(x)</math> as the gradient of the tangent to the graph of <math>y = f(x)</math> at a point</li> <li>• Derivative as rate of change</li> <li>• Derivatives of <math>x^n</math> for any rational <math>n</math>, <math>\sin x</math>, <math>\cos x</math>, <math>\tan x</math>, <math>e^x</math> and <math>\ln x</math>, together with constant multiples, sums and differences</li> <li>• Use of Chain Rule</li> <li>• Derivatives of products and quotients of functions</li> <li>• Increasing and decreasing functions</li> <li>• Stationary points (maximum and minimum turning points and points of inflexion)</li> <li>• Use of second derivative test to discriminate between maxima and minima</li> <li>• Connected rates of change</li> <li>• Maxima and minima problems</li> <li>• Integration as the reverse of differentiation</li> <li>• Integration of <math>x^n</math> for any rational <math>n</math>, <math>e^x</math>, <math>\sin x</math>, <math>\cos x</math>, <math>\sec^2 x</math> and their constant multiples, sums and differences</li> <li>• Integration of <math>(ax + b)^n</math> for any rational <math>n</math>, <math>\sin(ax + b)</math>, <math>\cos(ax + b)</math> and <math>e^{ax + b}</math></li> </ul>
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## MATHEMATICAL NOTATION

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The list which follows summarises the notation used in Singapore–Cambridge Mathematics examinations. Although primarily directed towards A-Level, the list also applies, where relevant, to examinations at all other levels.

### 1. Set Notation

$\in$	is an element of
$\notin$	is not an element of
$\{x_1, x_2, \dots\}$	the set with elements $x_1, x_2, \dots$
$\{x: \dots\}$	the set of all $x$ such that
$n(A)$	the number of elements in set $A$
$\emptyset$	the empty set
$\mathcal{E}$	universal set
$A'$	the complement of the set $A$
$\mathbb{Z}$	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, \dots\}$
$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, \dots\}$
$\mathbb{Q}$	the set of rational numbers
$\mathbb{Q}^+$	the set of positive rational numbers, $\{x \in \mathbb{Q}: x > 0\}$
$\mathbb{Q}_0^+$	the set of positive rational numbers and zero, $\{x \in \mathbb{Q}: x \geq 0\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^+$	the set of positive real numbers, $\{x \in \mathbb{R}: x > 0\}$
$\mathbb{R}_0^+$	the set of positive real numbers and zero, $\{x \in \mathbb{R}: x \geq 0\}$
$\mathbb{R}^n$	the real $n$ -tuples
$\mathbb{C}$	the set of complex numbers
$\subseteq$	is a subset of
$\subset$	is a proper subset of
$\not\subseteq$	is not a subset of
$\not\subset$	is not a proper subset of
$\cup$	union
$\cap$	intersection
$[a, b]$	the closed interval $\{x \in \mathbb{R}: a \leq x \leq b\}$
$[a, b)$	the interval $\{x \in \mathbb{R}: a \leq x < b\}$
$(a, b]$	the interval $\{x \in \mathbb{R}: a < x \leq b\}$
$(a, b)$	the open interval $\{x \in \mathbb{R}: a < x < b\}$

**2. Miscellaneous Symbols**

=	is equal to
≠	is not equal to
≡	is identical to or is congruent to
≈	is approximately equal to
∞	is proportional to
<	is less than
≤; ≥	is less than or equal to; is not greater than
>	is greater than
≥; ≤	is greater than or equal to; is not less than
∞	infinity

**3. Operations**

$\sum_{i=1}^n a_i$	$a_1 + a_2 + \dots + a_n$
$\sqrt[n]{a}$	the nth root of the real number $a$
$ a $	the modulus of the real number $a$
$n!$	$n$ factorial for $n \in \mathbb{Z}^+ \cup \{0\}$ , ( $0! = 1$ )
$\binom{n}{r}$	the binomial coefficient $\frac{n!}{r!(n-r)!}$ , for $n, r \in \mathbb{Z}^+ \cup \{0\}$ , $0 \leq r \leq n$
$\frac{n(n-1)\dots(n-r+1)}{r!}$	, for $n \in \mathbb{Q}$ , $r \in \mathbb{Z}^+ \cup \{0\}$

**4. Functions**

$f$	the function $f$
$f(x)$	the value of the function $f$ at $x$
$f: A \rightarrow B$	$f$ is a function under which each element of set $A$ has an image in set $B$
$f: x \mapsto y$	the function $f$ maps the element $x$ to the element $y$
$f^{-1}$	the inverse of the function $f$
$gf$	the composite function of $f$ and $g$ which is defined by $gf(x) = g(f(x))$
$\lim_{x \rightarrow a} f(x)$	the limit of $f(x)$ as $x$ tends to $a$
$\Delta x; \delta x$	an increment of $x$
$\frac{dy}{dx}$	the derivative of $y$ with respect to $x$
$\frac{d^n y}{dx^n}$	the $n$ th derivative of $y$ with respect to $x$
$f'(x), f''(x), \dots, f^{(n)}(x)$	the first, second, ... $n$ th derivatives of $f(x)$ with respect to $x$
$\int y dx$	indefinite integral of $y$ with respect to $x$
$\int_a^b y dx$	the definite integral of $y$ with respect to $x$ for values of $x$ between $a$ and $b$
$\dot{x}, \ddot{x}, \dots$	the first, second, ...derivatives of $x$ with respect to time
$\frac{\partial z}{\partial x}$	the partial derivative of $z$ with respect to $x$
$\frac{\partial^2 z}{\partial y \partial x}$	the partial derivative of $z$ with respect to $x$ then with respect to $y$
$f_x$	the partial derivative of $f$ with respect to $x$
$f_{xy}$	the partial derivative of $f$ with respect to $x$ then with respect to $y$

**5. Exponential and Logarithmic Functions**

$e$	base of natural logarithms
$e^x, \exp x$	exponential function of $x$
$\log_a x$	logarithm to the base $a$ of $x$
$\ln x$	natural logarithm of $x$
$\lg x$	logarithm of $x$ to base 10

**6. Circular Functions and Relations**

$\sin, \cos, \tan,$ $\text{cosec}, \sec, \cot$	$\left. \begin{array}{l} \text{the circular functions} \\ \text{the inverse circular functions} \end{array} \right\}$
$\sin^{-1}, \cos^{-1}, \tan^{-1}$ $\text{cosec}^{-1}, \sec^{-1}, \cot^{-1}$	

**7. Complex Numbers**

$i$	the square root of $-1$
$z$	a complex number, $z = x + iy$ $= r(\cos \theta + i \sin \theta), r \in \mathbb{R}^+$ $= re^{i\theta}, r \in \mathbb{R}^+$
$\operatorname{Re} z$	the real part of $z$ , $\operatorname{Re}(x + iy) = x$
$\operatorname{Im} z$	the imaginary part of $z$ , $\operatorname{Im}(x + iy) = y$
$ z $	the modulus of $z$ , $ x + iy  = \sqrt{x^2 + y^2}$ , $ r(\cos \theta + i \sin \theta)  = r$
$\arg z$	the argument of $z$ , $\arg(r(\cos \theta + i \sin \theta)) = \theta, -\pi < \theta \leq \pi$
$z^*$	the complex conjugate of $z$ , $(x + iy)^* = x - iy$

**8. Matrices**

$\mathbf{M}$	a matrix $\mathbf{M}$
$\mathbf{M}^{-1}$	the inverse of the square matrix $\mathbf{M}$
$\mathbf{M}^T$	the transpose of the matrix $\mathbf{M}$
$\det \mathbf{M}$	the determinant of the square matrix $\mathbf{M}$

## 9. Vectors

$\begin{pmatrix} x \\ y \end{pmatrix}$	a column vector in $xy$ -plane
$\begin{pmatrix} x \\ y \\ z \end{pmatrix}$	a column vector in $xyz$ -space
<b>a</b>	the vector <b>a</b>
$\overrightarrow{AB}$	the vector represented in magnitude and direction by the directed line segment $AB$
$\hat{\mathbf{a}}$	a unit vector in the direction of the vector <b>a</b>
<b>i, j, k</b>	unit vectors in the directions of the cartesian coordinate axes
$  \mathbf{a}  $	the magnitude of <b>a</b>
$  \overrightarrow{AB}  $	the magnitude of $\overrightarrow{AB}$
$\mathbf{a} \cdot \mathbf{b}$	the scalar product of <b>a</b> and <b>b</b>
$\mathbf{a} \times \mathbf{b}$	the vector product of <b>a</b> and <b>b</b>

**10. Probability and Statistics**

$A, B, C$ , etc.	events
$A \cup B$	union of events $A$ and $B$
$A \cap B$	intersection of the events $A$ and $B$
$P(A)$	probability of the event $A$
$A'$	complement of the event $A$ , the event ‘not $A$ ’
$P(A   B)$	probability of the event $A$ given the event $B$
$X, Y, R$ , etc.	random variables
$x, y, r$ , etc.	value of the random variables $X, Y, R$ , etc.
$x_1, x_2, \dots$	observations
$f_1, f_2, \dots$	frequencies with which the observations, $x_1, x_2, \dots$ occur
$p(x)$	the value of the probability function $P(X = x)$ of the discrete random variable $X$
$p_1, p_2, \dots$	probabilities of the values $x_1, x_2, \dots$ of the discrete random variable $X$
$f(x), g(x) \dots$	the value of the probability density function of the continuous random variable $X$
$F(x), G(x) \dots$	the value of the (cumulative) distribution function $P(X \leq x)$ of the random variable $X$
$E(X)$	expectation of the random variable $X$
$E[g(X)]$	expectation of $g(X)$
$\text{Var}(X)$	variance of the random variable $X$
$B(n, p)$	binomial distribution, parameters $n$ and $p$
$\text{Po}(\mu)$	Poisson distribution, mean $\mu$
$\text{Geo}(p)$	Geometric distribution, mean $\frac{1}{p}$
$N(\mu, \sigma^2)$	normal distribution, mean $\mu$ and variance $\sigma^2$
$\mu$	population mean
$\sigma^2$	population variance
$\sigma$	population standard deviation
$\bar{x}$	sample mean
$s^2$	unbiased estimate of population variance from a sample
$r$	linear product-moment correlation coefficient for a sample