

# Calculus Formulae

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## 1 Some Integrals

### 1.1 Integrating Hyperbolic Functions

$$\int \sinh(ax)dx = \frac{1}{a}\cosh ax + C$$

$$\int \sinh^2(ax)dx = \frac{1}{4a}\sinh 2ax - \frac{x}{2} + C$$

$$\int \sinh^n(ax)dx = \frac{1}{na}(\sinh^{n-1} 2ax)(\cosh ax) - \frac{n-1}{n} \int \sinh^{n-2}(ax)dx$$

$$\int x \sinh(ax)dx = \frac{1}{a}x \cosh ax - \frac{1}{a^2}\sinh ax + C$$

$$\int \cosh(ax)dx = \frac{1}{a}\sinh ax + C$$

$$\int \cosh^2(ax)dx = \frac{1}{4a}\sinh 2ax - \frac{x}{2} + C$$

$$\int \cosh^n(ax)dx = \frac{1}{na}(\cosh^{n-1} 2ax)(\sinh ax) + \frac{n-1}{n} \int \cosh^{n-2}(ax)dx$$

$$\int x \cosh(ax) dx = \frac{1}{a} x \sinh ax - \frac{1}{a^2} \cosh ax + C$$

$$\int \tanh(ax) dx = \frac{1}{a} \ln(\cosh ax) + C$$

$$\int \coth(ax) dx = \frac{1}{a} \ln(\sinh ax) + C$$

## 1.2 Integrating to Hyperbolic Functions

$$\int \frac{1}{\sqrt{a^2 + x^2}} dx = \operatorname{arcsinh}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \operatorname{arccosh}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{a^2 - x^2} dx = \frac{1}{a} \operatorname{arctanh}\left(\frac{x}{a}\right) + C \quad (x^2 < a^2)$$

$$\int \frac{1}{a^2 - x^2} dx = \frac{1}{a} \operatorname{arccoth}\left(\frac{x}{a}\right) + C \quad (x^2 > a^2)$$

$$\int \frac{1}{x\sqrt{a^2 - x^2}} dx = -\frac{1}{a} \operatorname{arcsech}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{x\sqrt{a^2 + x^2}} dx = -\frac{1}{a} \operatorname{arcsech}\left|\frac{x}{a}\right| + C$$

### 1.3 Integrating Trigs

$$\int \sin^2(ax) dx = \frac{x}{2} - \frac{1}{4a} \sin 2ax + C$$

$$\int x \sin(ax) dx = \frac{\sin ax}{a^2} - \frac{x \cos ax}{a} + C$$

$$\int \cos^2(ax) dx = \frac{x}{2} + \frac{1}{4a} \sin 2ax + C$$

$$\int x \cos(ax) dx = \frac{\cos ax}{a^2} - \frac{x \sin ax}{a} + C$$

## 2 Some Vector Calculus

### Tensors

$$\varepsilon_{ijk} \varepsilon_{klm} = \delta_{il} \delta_{jm} - \delta_{im} \delta_{jl}$$

$$\varepsilon_{ijk} \varepsilon_{ilm} = \delta_{jl} \delta_{km} - \delta_{jm} \delta_{kl}$$

### Grad, Div, Curl

#### Directional Derivative

$$\begin{aligned} \frac{\partial \phi}{\partial s} &= \frac{\partial \phi}{\partial n} (\hat{n} \cdot \hat{s}) \\ &= \hat{s} \cdot \nabla \phi \end{aligned}$$

## Laplacian

$$\begin{aligned}\nabla^2\phi &= \operatorname{div}(\nabla\phi) \\ &= \frac{\partial^2\phi}{\partial x_i^2}\end{aligned}$$