Continuous Time Stochastic Processes

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1 Preliminaries

A **right continuous** stochastic process.

There are three types of right continuous processes

- **Normal**
- **Absorption**
- **Explosion**

The **jump times** are random variables

The **holding times** are random variables defined as

A **jump process**

Compute probabilities using **countable union**

A **counting process** is a stochastic process $\{N_t\}_{t\geq 0}$ satisfying

- $N_0 = 0$
- $\forall t \geq 0, N_t \in \mathbb{N}_0$
- (Non-decreasing) If $0 \leq s \leq t$, $N_s \leq N_t$
- (Counting)
- (Right continuous)

A **counting process associatedt the sequence** $(J_n)_{n\in\mathbb{N}_0}$

2 Poisson Processes

A **Poisson process**, denoted $\{N_t\}_{t\geq 0}$, is a non-decreasing stochastic process with nonnegative values satisfying

- $N_0 = 0$
- The increments are independent, $0 \leq t_0 \leq t_1 \leq \ldots \leq t_n$, the random variables $N_{t_0}, N_{t_1} - N_{t_0}, \ldots, N_{t_n} - N_{t_{n-1}}$ are independent
- The increments are stationary

$$\Pr(N_t - N_s = k) = \Pr(N_{t-s} = k)$$

- There is a single arrival (only one arrives in a small interval), for all $t \geq 0$ and $\delta > 0$, $\delta \to 0$

$$\begin{aligned}
&\Pr(N_{t+\delta} - N_t = 1) = \lambda\delta + o(\delta) \\
&\Pr(N_{t+\delta} - N_t \geq 2) = o(\delta) \\
&\Pr(N_{t+\delta} - N_t = 0) = 1 - \lambda\delta + o(\delta)
\end{aligned}$$

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An equivalent definition replaces the last condition