Calculus Formulae

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1 Some Integrals

1.1 Integrating Hyperbolic Functions

$$\int sinh(ax)dx = \frac{1}{a}cosh\ ax + C$$

$$\int sinh^2(ax)dx = \frac{1}{4a}sinh\ 2ax - \frac{x}{2} + C$$

$$\int sinh^n(ax)dx = \frac{1}{na}(sinh^{n-1}\ 2ax)(coshax) - \frac{n-1}{n}\int sinh^{n-2}(ax)dx$$

$$\int x\ sinh(ax)dx = \frac{1}{a}x\ cosh\ ax - \frac{1}{a^2}sinh\ ax + C$$

$$\int cosh(ax)dx = \frac{1}{a}sinh\ ax + C$$

$$\int cosh^2(ax)dx = \frac{1}{4a}sinh\ 2ax - \frac{x}{2} + C$$

$$\int cosh^n(ax)dx = \frac{1}{na}(cosh^{n-1}\ 2ax)(sinhax) + \frac{n-1}{n}\int cosh^{n-2}(ax)dx$$

$$\int x\ cosh(ax)dx = \frac{1}{a}x\ sinh\ ax - \frac{1}{a^2}cosh\ ax + C$$

$$\int tanh(ax)dx = \frac{1}{a}ln(cosh\ ax) + C$$

$$\int coth(ax)dx = \frac{1}{a}ln(sinh\ ax) + C$$

1.2 Integrating to Hyperbolic Functions

$$\int \frac{1}{\sqrt{a^2+x^2}}dx = arcsinh(\frac{x}{a}) + C$$

$$\int \frac{1}{\sqrt{x^2-a^2}}dx = arccosh(\frac{x}{a}) + C$$

$$\int \frac{1}{a^2-x^2}dx = \frac{1}{a}\ arctanh(\frac{x}{a}) + C \qquad (x^2 < a^2)$$

$$\int \frac{1}{a^2-x^2}dx = \frac{1}{a}\ arccoth(\frac{x}{a}) + C \qquad (x^2 > a^2)$$

$$\int \frac{1}{x\sqrt{a^2-x^2}}dx = -\frac{1}{a}\ arcsech(\frac{x}{a}) + C$$

$$\int \frac{1}{x\sqrt{a^2+x^2}}dx = -\frac{1}{a}\ arcsech|\frac{x}{a}| + C$$

1.3 Integrating Trigs

$$\int sin^2(ax)dx = \frac{x}{2} - \frac{1}{4a} sin\ 2ax + C$$

$$\int x\ sin(ax)dx = \frac{sin\ ax}{a^2} - \frac{x\ cos\ ax}{a} + C$$

$$\int cos^2(ax)dx = \frac{x}{2} + \frac{1}{4a} sin\ 2ax + C$$

$$\int x\ cos(ax)dx = \frac{cos\ ax}{a^2} - \frac{x\ sin\ ax}{a} + C$$

2 Some Vector Calculus

Tensors

$$\varepsilon_{ijk}\varepsilon_{klm} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$$

$$\varepsilon_{ijk}\varepsilon_{ilm} = \delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}$$

Grad, Div, Curl

Directional Derivative

$$\begin{aligned}\frac{\partial\phi}{\partial s} &= \frac{\partial\phi}{\partial n}(\hat{n}\cdot\hat{s}) \\ &= \hat{\mathbf{s}}\cdot\nabla\phi\end{aligned}$$

Laplacian

$$\begin{aligned}\nabla^2\phi &= div(\nabla\phi)\\ &= \frac{\partial^2\phi}{\partial x_i^2}\end{aligned}$$

3 Differential Equations

Euler-Cauchy

$$\mathcal{L}[y] = \beta_k x^k \frac{d^k y}{dx^k} + \cdots + \beta_1 x \frac{dy}{dx} = f(x)$$

Try substitution $x = e^z$:

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{x}\frac{dy}{dz}\\ \frac{d^2y}{dx^2} &= \frac{1}{x^2}[\frac{d^2y}{dz^2} - \frac{dy}{dz}]\\ \frac{d^3y}{dx^3} &= \frac{1}{x^3}[\frac{d^3y}{dz^3} - 3\ \frac{d^2y}{dz^2} + 2\ \frac{dy}{dz}]\end{aligned}$$

Note the $a(a-1)(a-2)$ factorial like pattern here.