

Calculus Formulae

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1 Some Integrals

Integrating Hyperbolic Functions

$$\int \sinh(ax)dx = \frac{1}{a}\cosh ax + C$$

$$\int \sinh^2(ax)dx = \frac{1}{4a}\sinh 2ax - \frac{x}{2} + C$$

$$\int \sinh^n(ax)dx = \frac{1}{na}(\sinh^{n-1} 2ax)(\cosh ax) - \frac{n-1}{n} \int \sinh^{n-2}(ax)dx$$

$$\int x \sinh(ax)dx = \frac{1}{a}x \cosh ax - \frac{1}{a^2}\sinh ax + C$$

$$\int \cosh(ax)dx = \frac{1}{a}\sinh ax + C$$

$$\int \cosh^2(ax)dx = \frac{1}{4a}\sinh 2ax - \frac{x}{2} + C$$

$$\int \cosh^n(ax)dx = \frac{1}{na}(\cosh^{n-1} 2ax)(\sinh ax) + \frac{n-1}{n} \int \cosh^{n-2}(ax)dx$$

$$\int x \cosh(ax) dx = \frac{1}{a} x \sinh ax - \frac{1}{a^2} \cosh ax + C$$

$$\int \tanh(ax) dx = \frac{1}{a} \ln(\cosh ax) + C$$

$$\int \coth(ax) dx = \frac{1}{a} \ln(\sinh ax) + C$$

Integrating to Hyperbolic Functions

$$\int \frac{1}{\sqrt{a^2 + x^2}} dx = \operatorname{arcsinh}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \operatorname{arccosh}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{a^2 - x^2} dx = \frac{1}{a} \operatorname{arctanh}\left(\frac{x}{a}\right) + C \quad (x^2 < a^2)$$

$$\int \frac{1}{a^2 - x^2} dx = \frac{1}{a} \operatorname{arccoth}\left(\frac{x}{a}\right) + C \quad (x^2 > a^2)$$

$$\int \frac{1}{x\sqrt{a^2 - x^2}} dx = -\frac{1}{a} \operatorname{arcsech}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{x\sqrt{a^2 + x^2}} dx = -\frac{1}{a} \operatorname{arcsech}\left|\frac{x}{a}\right| + C$$

Integrating Trigs

$$\int \sin^2(ax)dx = \frac{x}{2} - \frac{1}{4a}\sin 2ax + C$$

$$\int x \sin(ax)dx = \frac{\sin ax}{a^2} - \frac{x \cos ax}{a} + C$$

$$\int \cos^2(ax)dx = \frac{x}{2} + \frac{1}{4a}\sin 2ax + C$$

$$\int x \cos(ax)dx = \frac{\cos ax}{a^2} - \frac{x \sin ax}{a} + C$$

2 Some Vector Calculus

Tensors

$$\varepsilon_{ijk}\varepsilon_{klm} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$$

$$\varepsilon_{ijk}\varepsilon_{ilm} = \delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}$$

Grad, Div, Curl

Directional Derivative

$$\begin{aligned}\frac{\partial\phi}{\partial s} &= \frac{\partial\phi}{\partial n}(\hat{n} \cdot \hat{s}) \\ &= \hat{s} \cdot \nabla\phi\end{aligned}$$

Laplacian

$$\begin{aligned}\nabla^2\phi &= \text{div}(\nabla\phi) \\ &= \frac{\partial^2\phi}{\partial x_i^2}\end{aligned}$$

Green's Theorem

Closed curve C , L and M are continuously differentiable over R .

$$\begin{aligned}\oint_C (L \, dx + M \, dy) &= \int_R \left(\frac{\partial M}{\partial x} - \frac{\partial L}{\partial y} \right) \, dx dy \\ \oint_C \mathbf{F} \cdot \hat{\mathbf{n}} \, ds &= \int_R \text{div} \mathbf{F} \, dx dy\end{aligned}$$

Divergence Theorem

Volume τ , closed surface S , outward normal $\hat{\mathbf{n}}$ continuous derivatives.

$$\int_S \mathbf{A} \cdot \hat{\mathbf{n}} \, dS = \int_\tau \operatorname{div} \mathbf{A}$$

Gauss' Flux Theorem

S closed surface with outward unit normal $\hat{\mathbf{n}}$, O is the origin of the coordinate system.

$$\int_S \frac{\mathbf{r} \cdot \hat{\mathbf{n}}}{r^3} \, dS = \begin{cases} 0, & \text{if } O \text{ is outside} \\ 4\pi, & \text{otherwise} \end{cases} \quad (1)$$

Stokes Theorem

S an open surface, simple closed curve γ , \mathbf{A} with continuous partial derivatives, outward normal $\hat{\mathbf{n}}$ determined by right-hand rule

$$\oint_\gamma \mathbf{A} \cdot d\mathbf{r} = \int_S \operatorname{curl} \mathbf{A} \cdot \hat{\mathbf{n}} \, dS$$

3 Curvilinear coordinates

4 Differential Equations

Euler-Cauchy

$$\mathcal{L}[y] = \beta_k x^k \frac{d^k y}{dx^k} + \cdots + \beta_1 x \frac{dy}{dx} = f(x)$$

Try substitution $x = e^z$:

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{x} \frac{dy}{dz} \\ \frac{d^2 y}{dx^2} &= \frac{1}{x^2} \left[\frac{d^2 y}{dz^2} - \frac{dy}{dz} \right] \\ \frac{d^3 y}{dx^3} &= \frac{1}{x^3} \left[\frac{d^3 y}{dz^3} - 3 \frac{d^2 y}{dz^2} + 2 \frac{dy}{dz} \right]\end{aligned}$$

Note the $a(a-1)(a-2)$ factorial like pattern here.