Preliminaries

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0.1 Dynamical and Equilibrium Equations

- **Dynamical equations** invovle the time derivative $\frac{\partial}{\partial t}$
- Examples of dynamical equations include the heat equation, wave equation, and the Schrödinger equation
- (**Heat equation**)

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$$

- (**Schrödinger equation**)

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V\psi$$

where the $\hbar$ is the Planck constant, m is the mass of the particle, and V is the potential energy of the particle.

- **Equilibrium equations** do not involve the time derivative $\frac{\partial}{\partial t}$
- Examples of equilibrium equations include the Poisson equation and the Laplace equation
- (**Poisson equation**)

$$\nabla^2 \phi = f$$

- When $f = 0$, the Poisson equation becomes the Laplace equation

0.2 Linear and Nonlinear Equations

- Linear equations are equations that are linear in the dependent variable and its derivatives
- e.g. the equation $\frac{\partial u}{\partial t} = e^x \frac{\partial^2 u}{\partial x^2} + \cos(x-t)u$ is linear (despite the product term of independent variable with its derivative)
- The **Burger's equation** is an example of a nonlinear equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \frac{\partial^2 u}{\partial x^2}$$

A **homogeneous linear differential equation** has the form:

$$\mathcal{L}u = 0$$

where $\mathcal{L}$ is a linear differential operator.

- Examples of $\mathcal{L}$ include: $\mathbf{div}, \mathbf{curl}, \nabla, \nabla^2, \frac{\partial}{\partial t}$.

The **Superposition Principle** states that if u_1 and u_2 are solutions to $\mathcal{L}u = 0$, then $u = c_1 u_1 + c_2 u_2$ is also a solution to $\mathcal{L}u = 0$ for any constants c_1 and c_2..

- A consequence is that the solutions form a vector space.

- For nonhomogeneous linear differential equations, there is a similar result.