Week 11 MATH50006 Mock

2022/03/27

Q1

Let $f : \mathbb{R}^d \to \mathbb{R}$ in , and let $\mathbb{R}^d = A \cup B$.
Show that f is measurable if and only if $f|_A$ and $f|_B$ are both measurable.

Q2

Prove the following statements or give a counter example:

1. There is no such f, s.t. f is not measurable but f^2 is measurable.
2. If f is measurable, then f^2 is measurable.
3. If on some $(X, \mathcal{B}(X), \lambda)$, $\int_X f d\lambda < \infty$, then $\int_X f^2 d\lambda < \infty$

Q3

Evaluate the following expression and show your work

$$\lim_{n\to\infty} \int_0^n (1 + \frac{x}{n})^n e^{-2x} dx$$