8 Discrete Mathematics (MPF)

(*a*) (*i*) Calculate two integers x and y satisfying $177\,x + 78\,y = 3$. [3 marks]

(*ii*) Describe all the integer pairs (x, y) that satisfy the above equation. [3 marks]

(*b*) Let $\mathbb{N}_{\geq 5} = \{n \in \mathbb{N} \mid n \geq 5\}$. Prove that: $\forall\, n \in \mathbb{N}_{\geq 5}.\, 2^n > n^2$. [6 marks]

(*c*) Let $\mathrm{C}(X, Y) = \{\, S \subseteq X \mid S \cong Y \,\}$.

(*i*) For finite sets X and Y, what is the cardinality of $\mathrm{C}(X, Y)$ in terms of that of X and Y? [2 marks]

(*ii*) For elements a and b, and sets A and B such that $\{a, b\} \cap (A \cup B) = \emptyset$, consider the functions

- $f : \mathrm{C}\big(A \cup \{a\}, B \cup \{b\}\big) \longrightarrow \mathrm{C}(A, B) \uplus \mathrm{C}\big(A, B \cup \{b\}\big)$ given by

$$f(S) = \begin{cases} \big(0, S \setminus \{a\}\big) & , a \in S \\ (1, S) & , a \notin S \end{cases}$$

and

- $g : \mathrm{C}(A, B) \uplus \mathrm{C}\big(A, B \cup \{b\}\big) \longrightarrow \mathrm{C}\big(A \cup \{a\}, B \cup \{b\}\big)$ given by

$$g(i, S) = \begin{cases} S \cup \{a\} & , i = 0 \\ S & , i = 1 \end{cases}$$

Prove either that $g \circ f = \mathrm{id}$ or that $f \circ g = \mathrm{id}$. [6 marks]