

10 Discrete Mathematics (AMP)

- (a) For each symbol x in the alphabet $\Sigma = \{a, b, c\}$, let O_x be the language over Σ consisting of all strings that contain an odd number of occurrences of the symbol x ; and let E_x be the language of strings over Σ containing an even number of occurrences of the symbol x .
- (i) Give a deterministic finite automaton whose language of accepted strings is O_a . [2 marks]
- (ii) Give a regular expression whose language of matching strings is O_a . [2 marks]
- (iii) Give a deterministic finite automaton whose language of accepted strings is $O_a \cap E_b$. [4 marks]
- (b) $M = (Q, \Sigma, \delta, s, F)$ is a deterministic finite automaton whose set of states Q has ℓ elements. Suppose that M accepts a string $w \in \Sigma^*$ whose length $|w|$ satisfies $|w| \geq \ell$.
- (i) Show that $w = u_1vu_2$ for some strings $u_1, v, u_2 \in \Sigma^*$ such that $|u_1| < \ell$, $1 \leq |v| \leq \ell$ and M accepts $u_1v^nu_2$ for all $n \in \mathbb{N} = \{0, 1, 2, \dots\}$. [7 marks]
- (ii) Hence show that if infinitely many strings are accepted by M , then it must accept some string w' with $\ell \leq |w'| < 2\ell$. [5 marks]