

CST Part IA: Digital Design, SV 1
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1 Ex1

		a	b	\bar{a}	\bar{b}	\overline{ab}	$\overline{\bar{a}\bar{b}}$	$\overline{\overline{ab}\bar{a}\bar{b}}$, x
0	0	1	1	1	1	0	0	
0	1	1	0	0	1	1	1	
1	0	0	1	1	0	1	1	
1	1	0	1	0	1	0	0	
a	b	\bar{a}	\overline{ab}	\bar{b}	$\overline{a+b}$	$\overline{\bar{a}+b}$	$\overline{\overline{a+b}+\bar{a}+b}$	x
0	0	1	0	1	0	1	1	
0	1	1	0	0	1	0	0	
1	0	0	1	1	0	0	0	
1.1.1	1	1	0	0	0	0	1	

$$1.2.1 \quad a.b.c + a.c.\bar{b} = a.b.(c + \bar{b}) = ab$$

$$1.2.2 \quad a(\bar{a} + b) = a\bar{a} + ab = 0 + ab = ab$$

$$1.2.3 \quad (a + c)(\bar{a} + b)$$

$$\bar{a}c + ab + cb$$

$$\bar{a}c + ab + (\bar{a} + a)cb$$

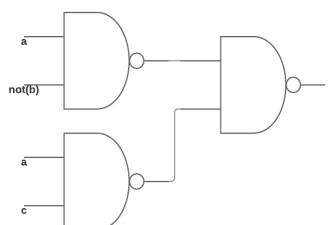
$$\bar{a}c + ab \text{ (adsorption) btw. this is consensus}$$

$$1.2.4 \quad (a + c)(a + d)(b + c)(b + d)$$

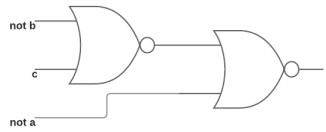
$$\frac{\overline{ac} + \overline{ad} + \overline{bc} + \overline{bd}}{\overline{a} + \overline{b} + \overline{c} + \overline{d}}$$

$$ab + cd$$

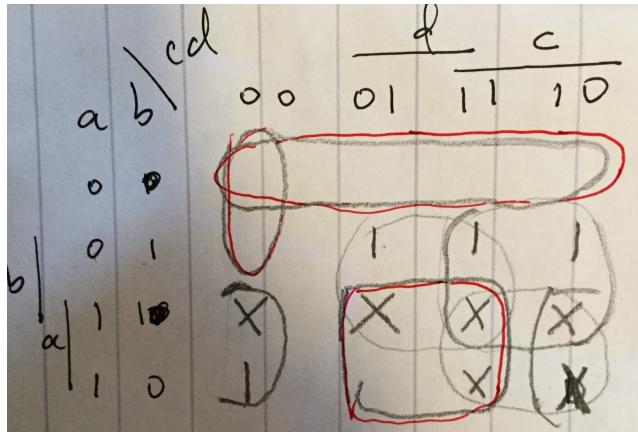
$$1.7.a \quad a\bar{b} + ac = \overline{\overline{a}\bar{b}.ac}$$



$$1.7.b \quad a(\bar{b} + c) = \overline{\bar{b} + c + \bar{a}}$$



$$1.8 \quad g = 0101 + 0110 + 0111 + 1000$$



$$g = bd + bc + a\bar{d}$$

$$\bar{g} = \bar{a}\bar{b} + \bar{a}\bar{c}\bar{d} + ad$$

$$g = (a+b)(a+c+d)(\bar{a}+\bar{d})$$

$$1.11 \quad (a) 100 \ 50 \ 25 \ 12 \ 6 \ 3 \ 1$$

reverse 0010011 is 01100100

38 19 9 4 2 1

reverse 011001 is 100110

not 00100110 is 11011001

then +1 is 11011010

so 100 = 01100100₂ and -38 = 11011010₂

01100100 + 11011010 = 100111110

remove head 1 and this is 00111110 or (58)₁₀ or (3E)₁₆ which is the attempted answer.

0010 0110 0111 0101 0100 1100 1101 1111 1110 1010

actually they will go through a properly drawn 4-4 K-map in a zigzag way.

2 Ex2

3 Ex3

(a)

$$(b) \bar{a} = \overline{0+a}$$

$$\text{notice } \overline{\bar{a} + \bar{b}} = a.b$$

$$\text{so } ab = \overline{\overline{0+a} + \overline{0+b}}$$

$$(c) \bar{a} = (0 \equiv a) = (1 \oplus a)$$

$$a + b = (ab) \oplus (a \oplus b)$$

4 Ex4

$$(a) \overline{abc\dots} = \bar{a} + \bar{b} + \bar{c}\dots$$

$$a + b + c\dots = \bar{a}\bar{b}\bar{c}\dots$$

(b) Minterms: the junction of all variables in complemented or uncomplemented form

Essential terms: covers a term that no other term covers it. (Do you mean Essential Prime terms or not?)

prime terms: cannot be further combined.

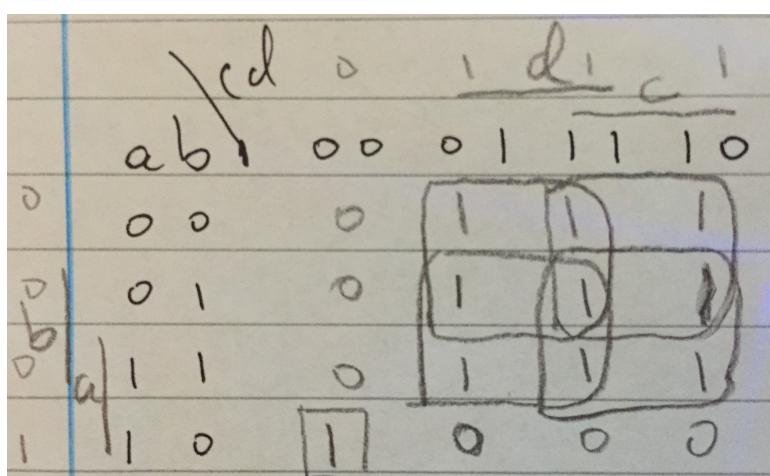
(c) sum of products form: disjunctions of junction (OR of AND variables).

minimised sum of products form: simplified SOP by K-map or Q-M method

disjunctive normal form: the disjunction of its minterms

(d) All circles are essential prime term.

minimum SOP: $bd + bc + \bar{a}d + \bar{a}c + a\bar{b}\bar{c}\bar{d}$



$$(e) \bar{a}d + \bar{a}c + a\bar{c}\bar{d}$$

$$(f) \bar{f} = \bar{a}\bar{c}\bar{d} + b\bar{c}\bar{d} + a\bar{b}d + a\bar{c}d$$

minimum POS: $f = (a + c + d)(\bar{b} + c + d)(\bar{a} + b + \bar{d})(\bar{a} + c + \bar{d})$

5 Ex5

$$(a) (X + Y)(X + Z)$$

$$\overline{XY} + \overline{XZ}$$

$$\overline{X}(\overline{Z} + \overline{Y})$$

$$X + YZ$$

(b) The same as 1.2.3

(c) Tristate Buffer

$$(d) (A + B + \bar{C}DE)(A + \bar{D} + E)(\bar{A} + C)$$

$$(AC + \bar{A}(B + \bar{C}DE))(A + \bar{D} + E)$$

$$(AC + \bar{A}B + \bar{A}\bar{C}DE)(A + \bar{D} + E)$$

$$AC + AC\bar{D} + ACE + 0 + \bar{A}\bar{D}B + \bar{A}BE + 0 + 0 + \bar{A}\bar{C}DE$$

$$AC + \bar{A}\bar{D}B + \bar{A}BE + \bar{A}\bar{C}DE$$

(e) cell adjacency: 010 011 001 000 100 101 111 110

method: a(n-1) and reversed a(n-1) then add 0 or 1 to head.

	a	b	c	d	e	f
b	0	1	0	1	1	0
c	0	1	1	1	1	0
	0	0	1	0	0	0
	0	0	0	0	0	0
i	1	0	0	0	0	0
c	1	0	1	1	1	1
b	1	1	1	1	0	1
	1	1	0	0	0	0

(f)

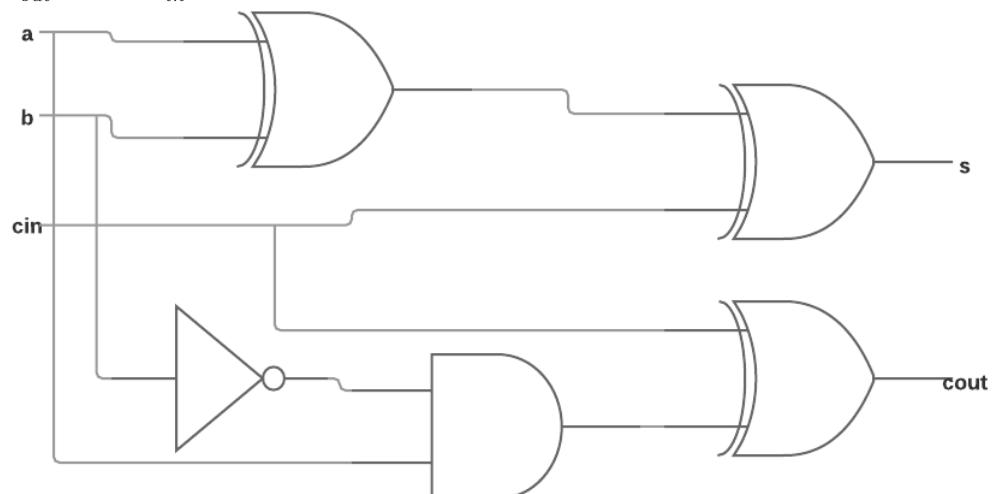
	Ac + $\bar{A}\bar{B}B + \bar{A}BE + \bar{A}\bar{C}DE$
3	00011v ① 01000v ② 00100 - ③ 01-0-*
8	01000v 01-00v
9	01001v ② 00011v ② 01--1* ② 1-1--*
11	01011v 01001v ② 0-011* 101--- 1-1-
12	01100v 01100v 010-1v 01---1*
13	01101v 10100v 01-0v 101---*
15	01111 101-0v 1-10-v
20	10100v ③ 01011v 1010-v 1-1-0v
21	10101v 01101v 1-100v Lost here.
22	10110v 10101v ④ 1110- ④ -+++
23	10111 10110v ④ 011-11v -11-1*
24	11100v 11100v 011-1v 1-1-1v
25	11101 11101v -1101* 1-11-v
26	11110 ④ 01111v 101-1v -++-
31	11111 10111v 1-101v 11101v 1011-v 11110v 1-110v
	01-0- ⑤ 11111v ⑥ 1111-v 111-1v 1-111v
	3 8 9 11 12 13- 115- 20 21 22 23 28 29 30 31
ACDE	0-011 ⑦ X
	-1101
ABE	01-1- ⑧
	101--
	-11-1
AC	-1--
let $\bar{A}B$ be 0	01-0- 1-1 ⑨ ⑩ ⑪ ⑫

6 Ex6

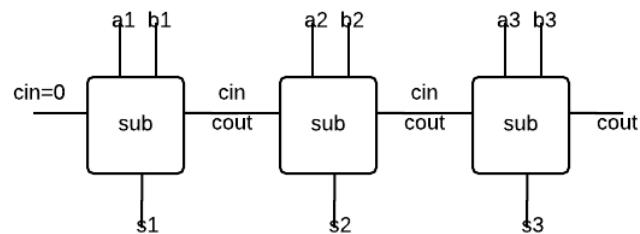
<i>a</i>	<i>b</i>	<i>c_{in}</i>	<i>s</i>	<i>c_{out}</i>
0	0	0	0	0
0	1	0	1	0
1	0	0	1	1
1	1	0	0	0
0	0	1	1	1
0	1	1	0	0
1	0	1	0	1
1	1	1	1	1

$$s = a \oplus b \oplus c_{in}$$

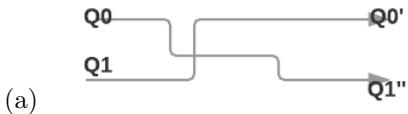
$$c_{out} = a \bar{b} \oplus c_{in}$$



The circuit above have input *a* *b* *cin* and output *s* and *cout*.
cin: 1 means need to subtract 1 0 means not.
cout: 1 means need to borrow 1 from higher digital 0 means not.



7 Ex7



- (b) The 0 0 means clear so once the first input is 0 0 then the output H should also be clear. For C, once b is logical 0 then it must be 0 (no need for add 1 to next digit) otherwise it is also clear 0 0.

A1	A0	B1	B0	H1	H0	C1	C0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	1	0
0	0	1	0	0	0	0	0
0	1	0	0	0	0	1	0
0	1	0	1	0	1	0	1
0	1	1	0	1	0	1	1
1	0	0	0	0	0	0	0
1	0	0	1	1	0	0	1
1	0	1	0	0	1	0	0

$$\begin{array}{r}
 \begin{array}{c} B_1 B_0 \\ \hline A_1 & A_0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{array} \\
 \begin{array}{c} B_0 H_1 \\ \hline B_1 \end{array}
 \end{array}$$

0	0	0	0	X	0		
A ₀	0	1	0	0	X	1	
A ₁	1	0	X	X	X	X	
	1	0	0	1	X	0	

$$H_1 = A_1 B_0 + A_0 B_1$$

$$\begin{array}{r}
 \begin{array}{c} B_1 B_0 \\ \hline A_1 & A_0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{array} \\
 \begin{array}{c} B_0 H_1 \\ \hline B_1 \end{array}
 \end{array}$$

0	0		X				
A ₀	0	1		1	X		
A ₁	1	0	X	X	X	X	
	1	0	0	1	X	1	

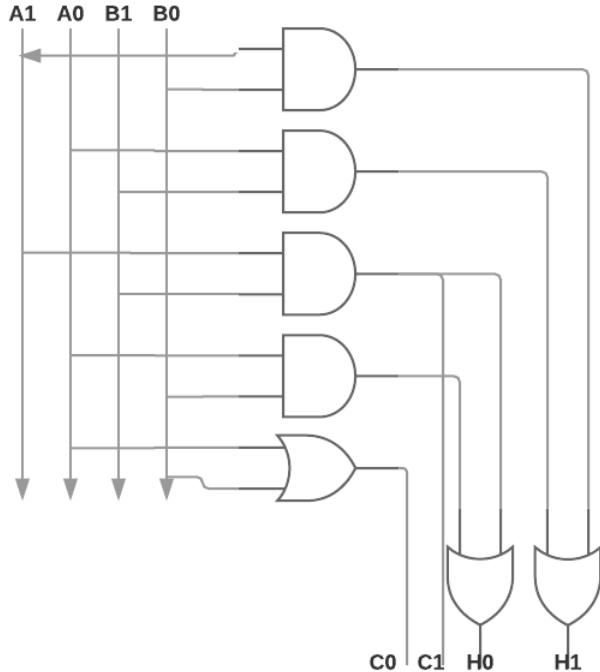
$$H_0 = A_0 B_0 + A_1 B_1$$

B_0	B_1	C_1
A_1	A_0	C_1
0	0	X
0	1	X
1	0	X
1	1	1

$$C_1 = A_1 B_1$$

B_0	B_1	C_0	C_1
A_1	A_0	C_0	C_1
0	0	1	X
0	1	1	X
1	0	X	X
1	1	1	X

$$C_0 = A_0 + B_0$$



(c) This needs 7 gates. Original one needs 3 gates.

(d) They both have $2T$ (ps) delay.

8 Ex8

notes

$$x + y = z$$

$\dots = \dots$

a label

$$ab/c$$

EXCHANGE rule

$$p < 5$$

(note that $p \in \mathbb{R}$)

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fun fib 0 = 1
| fib n = fib (n-1) + fib(n-2);
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