CST Part IA: Digital Design, SV 1
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2016-11-13

1 Ex1

1.1.1

A	B	$\bar{a}$	$\overline{\bar{a}+b}$	$\bar{b}$	$\overline{a+\bar{b}}$	x
0	0	1	1	1	1	0
0	1	1	0	0	1	1
1	0	0	1	1	0	1
1	1	0	1	0	1	0
0	0	1	0	1	0	1
0	1	1	0	0	1	0
1	0	0	1	1	0	0
1	1	0	0	0	0	1

The first circuit is called XOR.
the second circuit is called NOR.

1.2.1 $a.b.c + a.c.\bar{c} = a.b.(c+\bar{c}) = ab$

1.2.2 $a(\bar{a}+b) = a\bar{a} + ab = 0 + ab = ab$

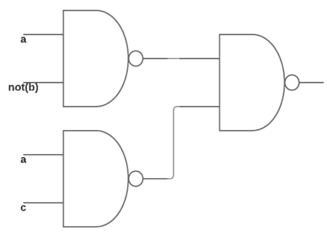
1.2.3 $(a+c)(\overline{a}+b)$
$\overline{a}c + ab + cb$
$\overline{a}c + ab + (\overline{a}+a)cb$
$\overline{a}c + ab$ (adsorbtion) btw. this is consensus

1.2.4 $(a+c)(a+d)(b+c)(b+d)$
$\overline{\overline{ac} + \overline{a}\overline{d} + \overline{b}\overline{c} + \overline{bd}}$
$\overline{\overline{a}+\overline{b}} + \overline{\overline{c}+\overline{d}}$
$ab + cd$

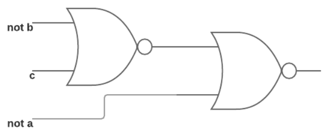
1.2.x $x + (\bar{x}y) = (x+\bar{x})(x+y) = 1.(x+y) = x+y$
To prove the second equation,take complement on both side. $\overline{x+(\bar{x}y)} = \overline{x+y}$
Then the LHS will be:$\bar{x}(x+\bar{y} = \bar{x}x + \bar{x}\bar{y} = \bar{x}\bar{y})$

So proved:$\overline{x+y} = \bar{x}\bar{y}$
To prove the third one, just replace x and y with $\bar{x}$ and $\bar{y}$
and take implement on the both side on the second equation.
so proved:$\bar{x}+\bar{y} = \overline{xy}$

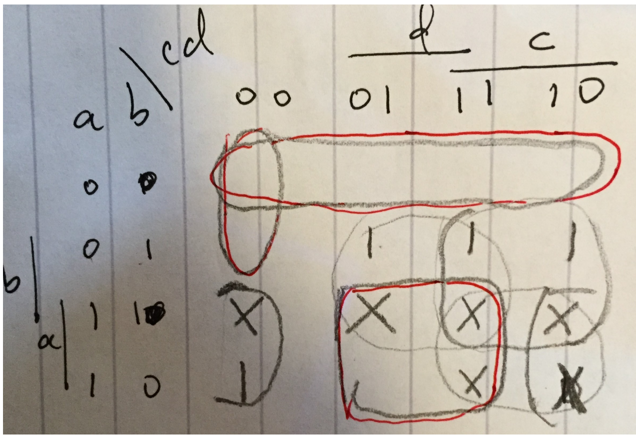
1.7.a $a\bar{b} + ac = \overline{\overline{a\bar{b}}.\overline{ac}}$



1.7.b $a(\bar{b}+c) = \overline{\overline{\bar{b}+c}+\bar{a}}$

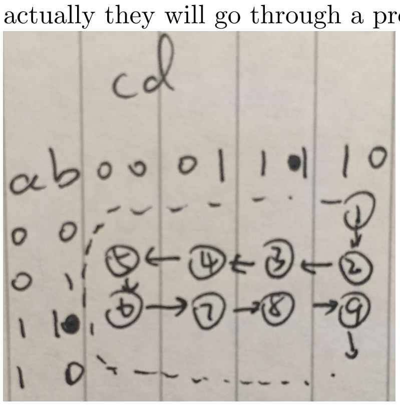


1.8 $g = 0101 + 0110 + 0111 + 1000$



$g = bd + bc + a\bar{d}$
$\bar{g} = \bar{a}\bar{b} + \bar{a}\bar{c}\bar{d} + ad$
$g = (a+b)(a+c+d)(\bar{a}+\bar{d})$

1.11 (a)100 50 25 12 6 3 1
reverse 0010011 is 01100100
38 19 9 4 2 1
reverse 011001 is 100110
not 00100110 is 11011001
then +1 is 11011010
so 100 = $01100100|_2$ and -38 = $11011010|_2$
01100100 + 11011010= 100111110
remove head 1 and this is 00111110 or $(58)_{10}$ or $(3E)_{16}$ which is the attempted answer.
0010 0110 0111 0101 0100 1100 1101 1111 1110 1010
actually they will go through a properly drawn 4-4 K-map in a zigzag way.



2 Ex2

I guess those are lists of binary code but no idea how to decode..

3 Ex3

(a) There are 16 possibility.

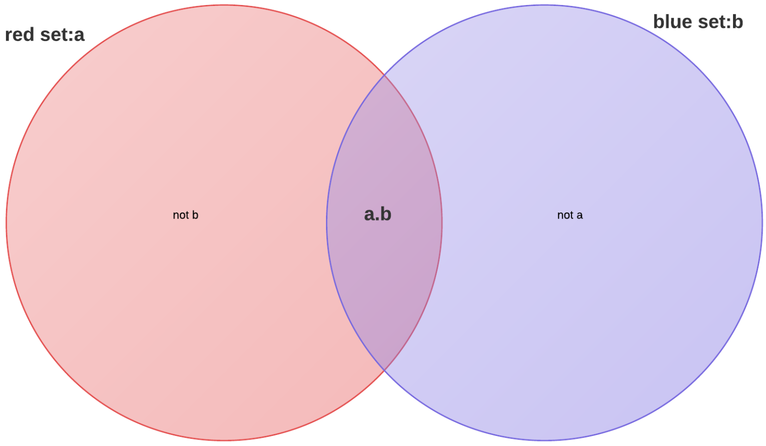
A	B	$output$
0	0	x
0	1	x
1	0	x
1	1	x

One truth table means one possible function.Then to fill each x with 0 or 1.
There are 2^4 possibility.

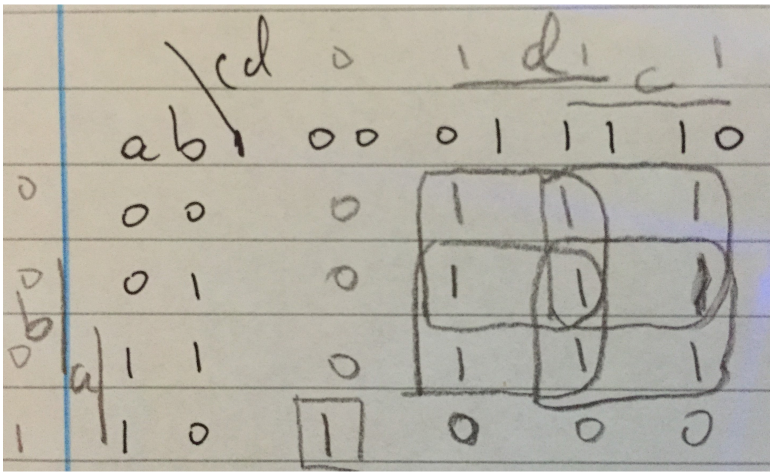
(b) $\bar{a} = \overline{0+a}$
notice $\overline{\bar{a}+\bar{b}} = a.b$
so $ab = \overline{\overline{0+a}+\overline{0+b}}$

(c) $\bar{a} = (0 \equiv a) = (1 \oplus a)$
$a + b = (ab) \oplus (a \oplus b)$

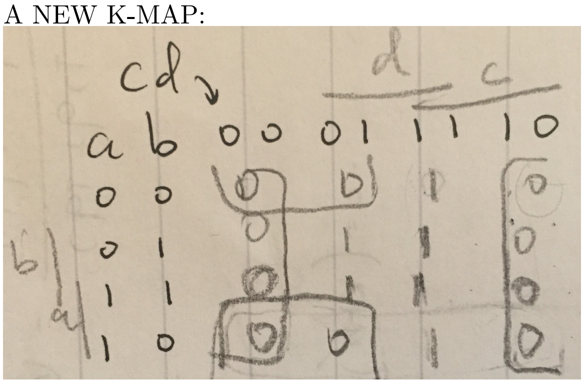
4 Ex4



(a)The common demonstration of DeMorgan theorem can be shown on a Venn diagram.
If we have two set a and b as the Venn diagram shown.
Then we have:
1.For $\overline{ab}$ we can see it exactly cover the area of $\overline{a}+\overline{b}$. So they are equal.
$\overline{abc...} = \overline{a}+\overline{b}+\overline{c}...$
2.For $\overline{a+b}$ this actually means neither of a or b is in the set.
In this way, we can also say both of not a or not b is in the set.
$\overline{a+b+c...} = \overline{a}\overline{b}\overline{c}...$
(b)Minterms:the junction of all variables in complemented or uncomplemented form
Essential terms: covers a term that no other term covers it. (Do you mean Essential Prime terms or not?)
prime terms: cannot be further combined.
(c)sum of products form: disjunctions of junction (OR of AND variables).
minimised sum of products form: simplified SOP by K-map or Q-M method
disjunctive normal form: the disjunction of its minterms
(d)All circles are essential prime term.
minimum SOP:$bd + bc + \bar{a}d + \bar{a}c + a\bar{b}\bar{c}\bar{d}$



(e)Don't care ab means we can put x in (a,b) = (1,1) row on the K-map.
In this way, Only 3 circle is left and all of them are prime and essential. $\bar{a}d + \bar{a}c + a\bar{c}\bar{d}$
(f)For this question:
$\bar{f} = \bar{a}\bar{c}\bar{d} + b\bar{c}\bar{d} + a\bar{b}d + a\bar{c}d$
minimum POS:$f = (a + c + d)(\bar{b} + c + d)(\bar{a} + b + \bar{d})(\bar{a} + c + \bar{d})$
Obviously,they have the same number of terms and literals.
A NEW K-MAP:



SOP:$g = bd + cd$ 4 terms (3 gates)
POS:$g = d(b + c)$ 3 terms (2 gates)
Note under this situation, POS gets 1 less term than SOP.

5 Ex5

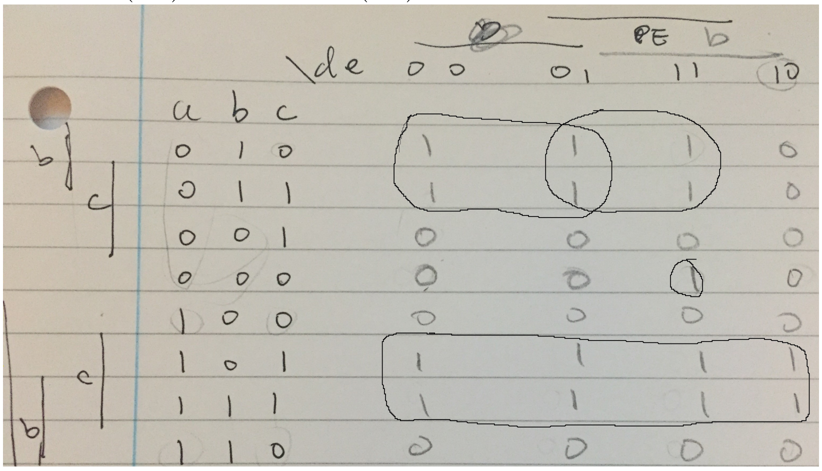
(a) $(X + Y)(X + Z)$
$\overline{\overline{\bar{X}\bar{Y} + \bar{X}\bar{Z}}}$
$\overline{\bar{X}(\bar{Z} + \bar{Y})}$
$X + YZ$

(b) The same as 1.2.3.

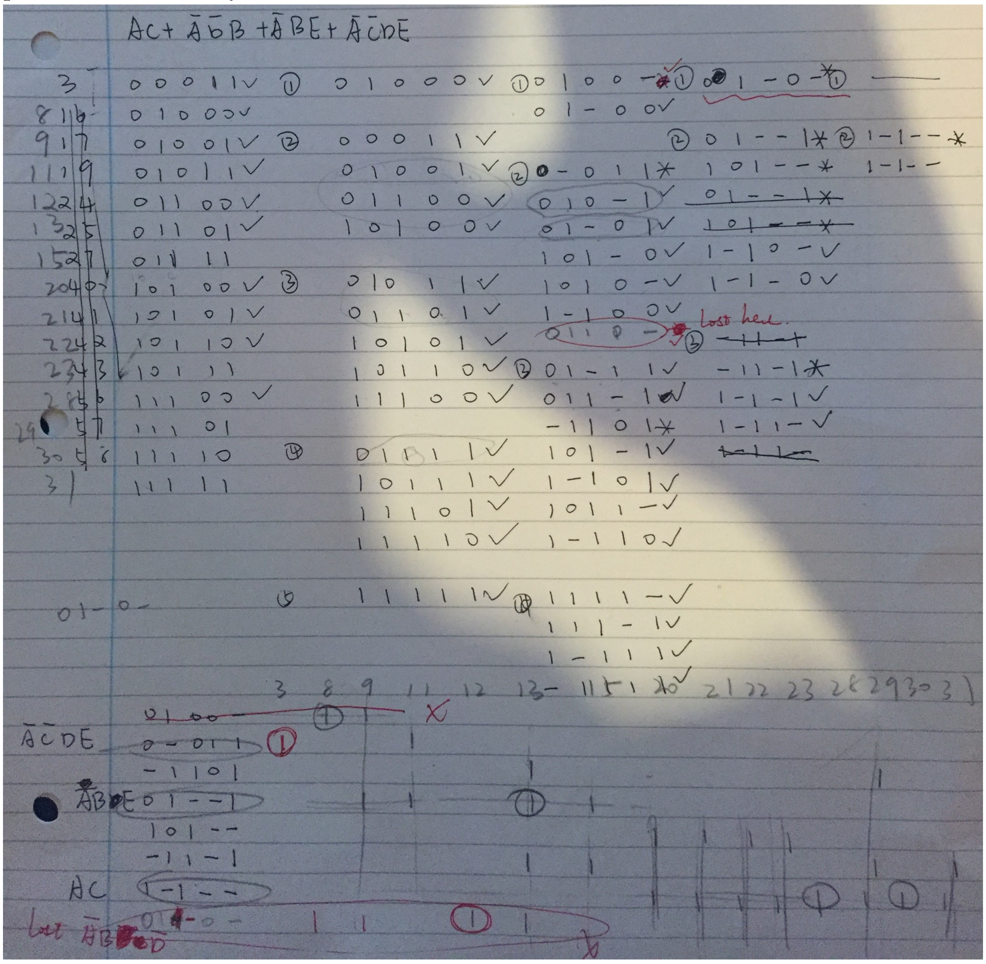
(c) Tristate Buffer

(d) $(A + B + \bar{C}DE)(A + \bar{D} + E)(\bar{A} + C)$
$(AC + \bar{A}(B + \bar{C}DE))(A + \bar{D} + E)$
$(AC + \bar{A}B + \bar{A}\bar{C}DE)(A + \bar{D} + E)$
$AC + AC\bar{D} + ACE + 0 + \bar{A}\bar{D}B + \bar{A}BE + 0 + 0 + \bar{A}\bar{C}DE$
$AC + \bar{A}\bar{D}B + \bar{A}BE + \bar{A}\bar{C}DE$

(e) cell adjacency:010 011 001 000 100 101 111 110
method: a(n-1) and reversed a(n-1) then add 0 or 1 to head.



(f) picture shows the Q-M method.



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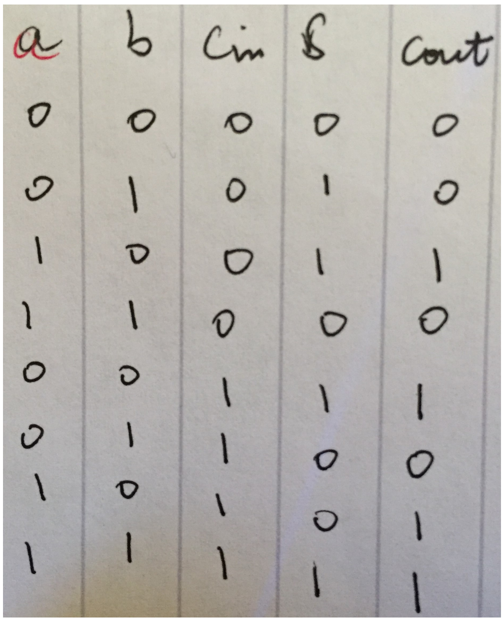
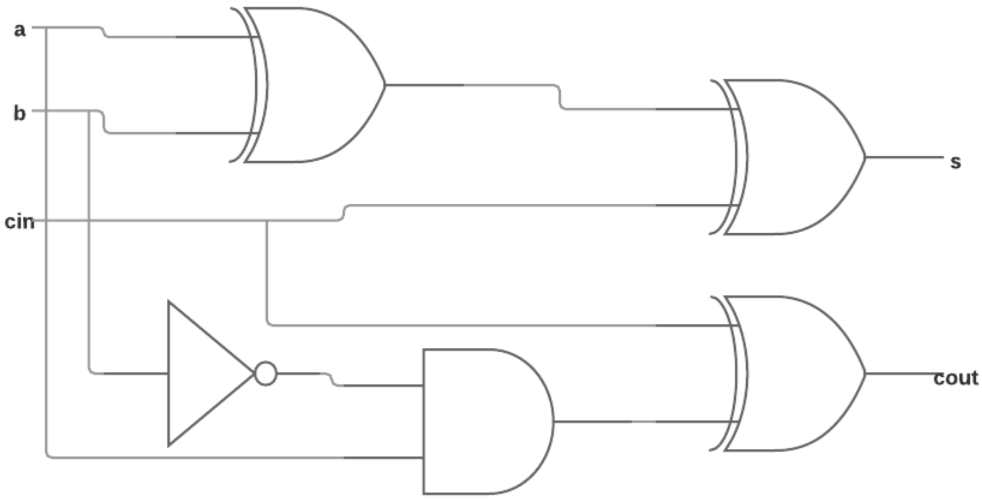
6 Ex6



a	b	Cin	S	Cout
0	0	0	0	0
0	1	0	1	0
1	0	0	1	1
1	1	0	0	0
0	0	1	1	1
0	1	1	0	0
1	0	1	0	1
1	1	1	1	1

$s = a \oplus b \oplus c_{in}$
$c_{out} = a.\bar{b} \oplus c_{in}$

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The circuit above have input a b cin and output s and cout.
cin: 1 means need to subtract 1 0 means not.
cout: 1 means need to borrow 1 from higher digital 0 means not.
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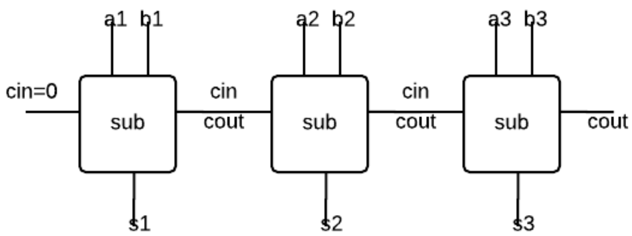

7 Ex7

(a) 

(b) The 0 0 means clear so once the first input is 0 0 then the output H should also be clear. For C, once b is logical 0 then it must be 0 (no need for add 1 to next digit) otherwise it is also clear 0 0.

```
A1 A0 B1 B0 H1 H0 C1 C0
0 0 0 0 0 0 0 0
0 0 0 1 0 0 0 1
0 0 1 0 0 0 0 0
0 1 0 0 0 0 0 1
0 1 0 1 0 1 0 1
0 1 1 0 1 0 0 1
1 0 0 0 0 0 0 0
1 0 0 1 1 0 0 1
1 0 1 0 0 1 1 0
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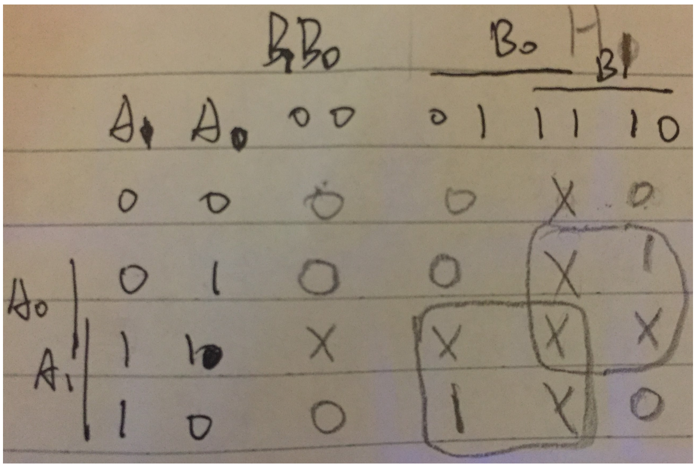
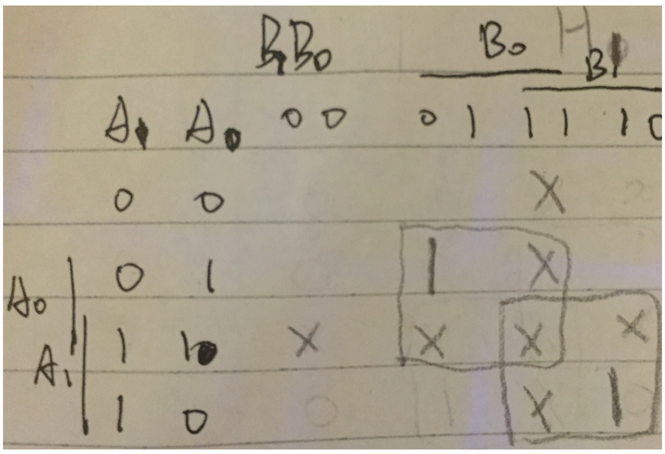
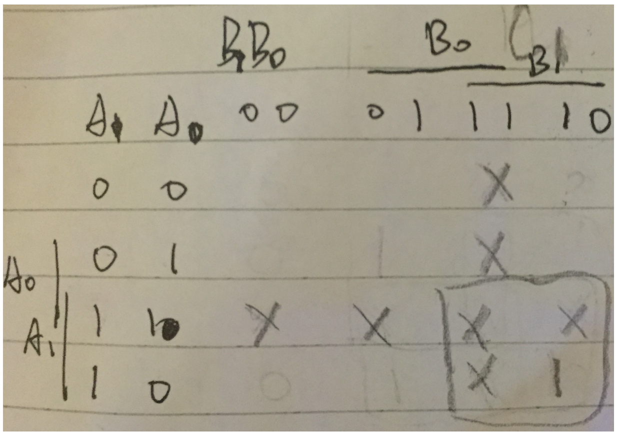


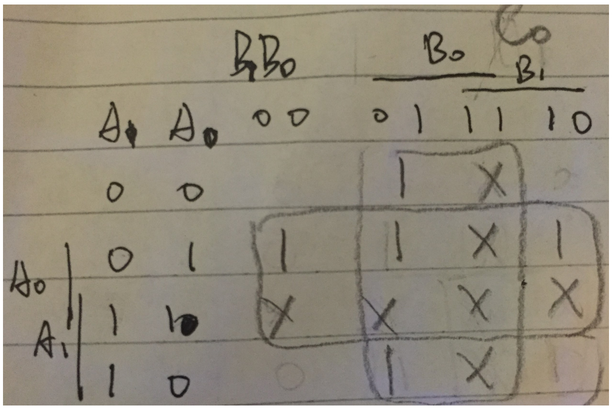
$H_1 = A_1B_0 + A_0B_1$



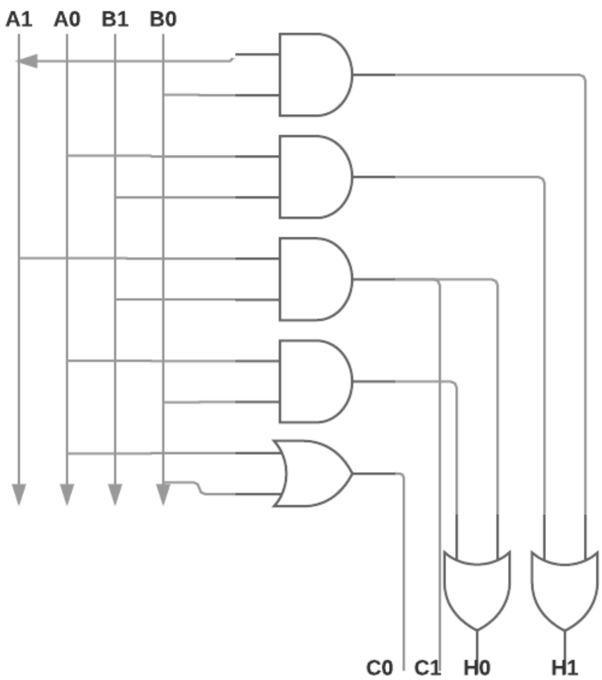
$H_0 = A_0B_0 + A_1B_1$



$C_1 = A_1 B_1$



$C_0 = A_0 + B_0$



(c) This needs 7 gates. Original circuit needs 3 gates.

(d) They both have $2T$ (ps) delay.

8 Ex8

(a) $\bar{U} = U\bar{X} + \bar{U}\bar{W}\bar{Y} + \bar{U}\bar{W}\bar{Z} + \bar{U}\bar{Z}Y + X\bar{W}\bar{U}$

(b) This is exactly the same as Ex5(e).
As change A to U, B to W, C to X, D to Y, E to X.
so $P = UX + \bar{U}\bar{Y}W + \bar{U}WZ + \bar{U}\bar{X}YZ$

(c) picture