COMPUTER SCIENCE TRIPOS Part IA − 2015 − Paper 2

8   Discrete Mathematics (MPF)

(*a*) Prove that, for all natural numbers n,

$$n^{13} \equiv n \pmod{1365}$$

You may use any standard results provided that you state them clearly.

[5 marks]

(*b*) For n ranging over the natural numbers $\mathbb{N}$, let

$$\text{Even}(n) \text{ be the predicate } \exists\, k \in \mathbb{N}.\ n = 2 \cdot k$$

and let

$$\text{Odd}(n) \text{ be the predicate } \exists\, l \in \mathbb{N}.\ n = 2 \cdot l + 1$$

Prove that
$$\forall\, n \in \mathbb{N}.\ \text{Even}(n) \vee \text{Odd}(n)$$

by the Principle of Induction.   [5 marks]

(*c*) Let $F : A \longrightarrow\!\!\!\!\!\rightarrow B$ be a relation, from a set A to a set B.

(*i*)   Define what it means for F to be a (total) function.   [2 marks]

(*ii*) Prove that F is a function if, and only if, there exists a relation $G : B \longrightarrow\!\!\!\!\!\rightarrow A$ such that $\text{id}_A \subseteq G \circ F$ and $F \circ G \subseteq \text{id}_B$.   [8 marks]