

10 Discrete Mathematics (IML)

- (a) Let  $\Sigma = \{a, b, c\}$ . Consider each of the subsets of  $\Sigma^*$  defined by the following groups of axioms and rules, and for each prove or disprove that  $\#_a(u) \geq \#_b(u)$  for all  $u \in \Sigma^*$ , where  $\#_x(u)$  is the number of occurrences of the symbol  $x$  in the string  $u$ .

$$(i) \quad \frac{}{\epsilon} \quad , \quad \frac{u}{aauab} \quad , \quad \frac{u,v}{ucv} \quad \text{for all } u, v \in \Sigma^*$$

$$(ii) \quad \frac{}{a} \quad , \quad \frac{u}{au} \quad , \quad \frac{u}{ubc} \quad \text{for all } u \in \Sigma^*$$

[6 marks]

- (b) For each of the subsets in part (a), indicate with justification whether they are regular languages.

*Note:* Complete proofs are not necessary but you should clearly outline any proof strategy.

[10 marks]

- (c) For two regular expressions  $r$  and  $s$  and an alphabet  $\Sigma$ , define  $r \& s$  to match a string in  $\Sigma^*$  if both  $r$  and  $s$  do. Given Kleene's Theorem, sketch a proof that the set of strings matched by  $r \& s$  is a regular language for any regular expressions  $r$  and  $s$ .

[4 marks]