CST Part IA: Digital Design, SV 1
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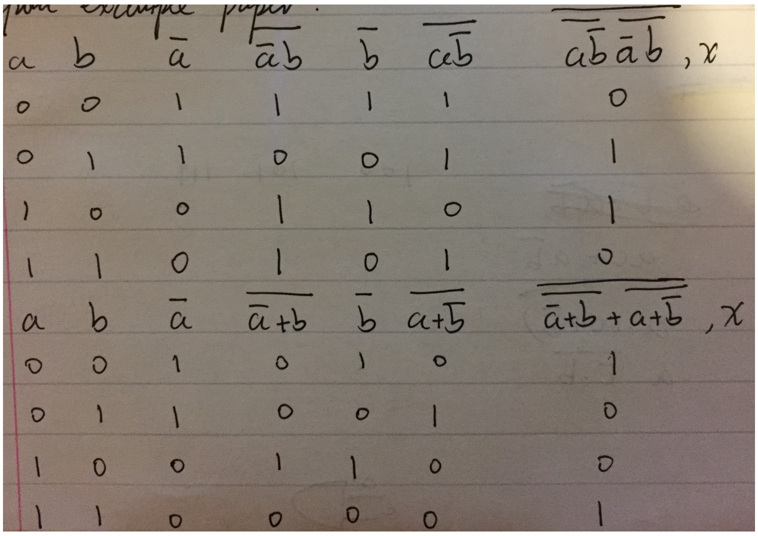
1 Ex1

1.1.1 

a	b	$\bar a$	$\overline{\bar a b}$	$\bar b$	$\overline{a\bar b}$	$\overline{\overline{a\bar b}\,\overline{\bar a b}}, x$
0	0	1	1	1	1	0
0	1	1	0	0	1	1
1	0	0	1	1	0	1
1	1	0	1	0	1	0

a	b	$\bar a$	$\overline{\bar a + b}$	$\bar b$	$\overline{a+\bar b}$	$\overline{\overline{\bar a+b}+\overline{a+\bar b}}, x$
0	0	1	0	1	0	1
0	1	1	0	0	1	0
1	0	0	1	1	0	0
1	1	0	0	0	0	1

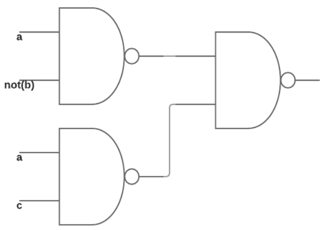
1.2.1 $a.b.c + a.c.\bar c = a.b.(c + \bar c) = ab$

1.2.2 $a(\bar a + b) = a\bar a + ab = 0 + ab = ab$

1.2.3 $(a + c)(\overline{a} + b)$
$\overline{a}c + ab + cb$
$\overline{a}c + ab + (\overline{a} + a)cb$
$\overline{a}c + ab$ (adsorbtion) btw. this is consensus

1.2.4 $\overline{(a + c)(a + d)(b + c)(b + d)}$
$\overline{\overline{ac} + \overline{ad} + \overline{bc} + \overline{bd}}$
$\overline{\overline{a} + \overline{b} + \overline{c} + \overline{d}}$
$ab + cd$

1.7.a $a\bar b + ac = \overline{\overline{a\bar b}.\overline{ac}}$



1.7.b $a(\bar b + c) = \overline{\overline{\bar b + c} + \bar a}$

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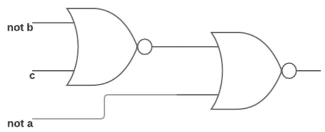
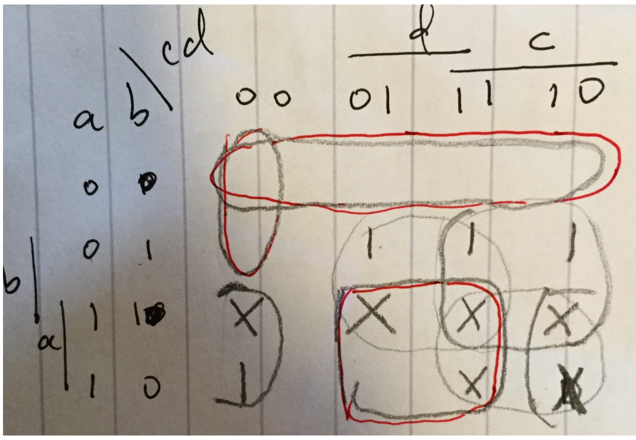


1.8 $g = 0101 + 0110 + 0111 + 1000$



$g = bd + bc + a\bar{d}$
$\bar{g} = \bar{a}\bar{b} + \bar{a}\bar{c}\bar{d} + ad$
$g = (a + b)(a + c + d)(\bar{a} + \bar{d})$

1.11 (a)100 50 25 12 6 3 1
reverse 0010011 is 01100100
38 19 9 4 2 1
reverse 011001 is 100110
not 00100110 is 11011001
then +1 is 11011010
so 100 = $01100100|_2$ and -38 = $11011010|_2$
01100100 + 11011010= 100111110
remove head 1 and this is 00111110 or $(58)_{10}$ or $(3E)_{16}$ which is the attempted answer.
0010 0110 0111 0101 0100 1100 1101 1111 1110 1010
actually they will go through a properly drawn 4-4 K-map in a zigzag way.

2 Ex2

3 Ex3

(a)

(b) $\bar{a} = \overline{0 + a}$
notice $\overline{\bar{a} + \bar{b}} = a.b$
so $ab = \overline{\overline{0 + a} + \overline{0 + b}}$

(c) $\bar{a} = (0 \equiv a) = (1 \oplus a)$
$a + b = (ab) \oplus (a \oplus b)$

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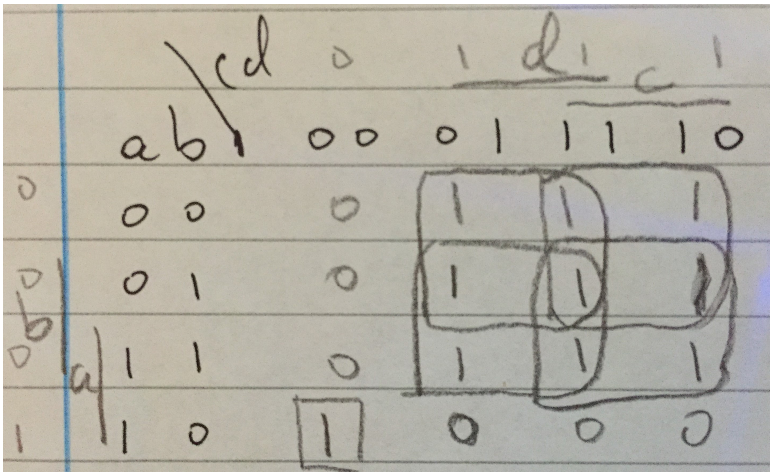
4 Ex4

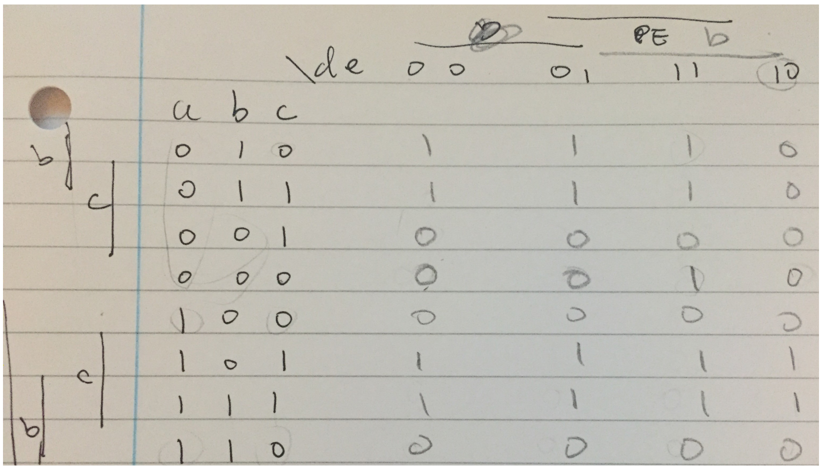
(a)$\overline{abc...} = \bar{a} + \bar{b} + \bar{c}...$
$\overline{a + b + c...} = \bar{a}\bar{b}\bar{c}...$
(b)Minterms:the junction of all variables in complemented or uncomplemented form
Essential terms: covers a term that no other term covers it. (Do you mean Essential Prime terms or not?)
prime terms: cannot be further combined.
(c)sum of products form: disjunctions of junction (OR of AND variables).
minimised sum of products form: simplified SOP by K-map or Q-M method
disjunctive normal form: the disjunction of its minterms
(d)All circles are essential prime term.
minimum SOP:$bd + bc + \bar{a}d + \bar{a}c + a\bar{b}\bar{c}\bar{d}$



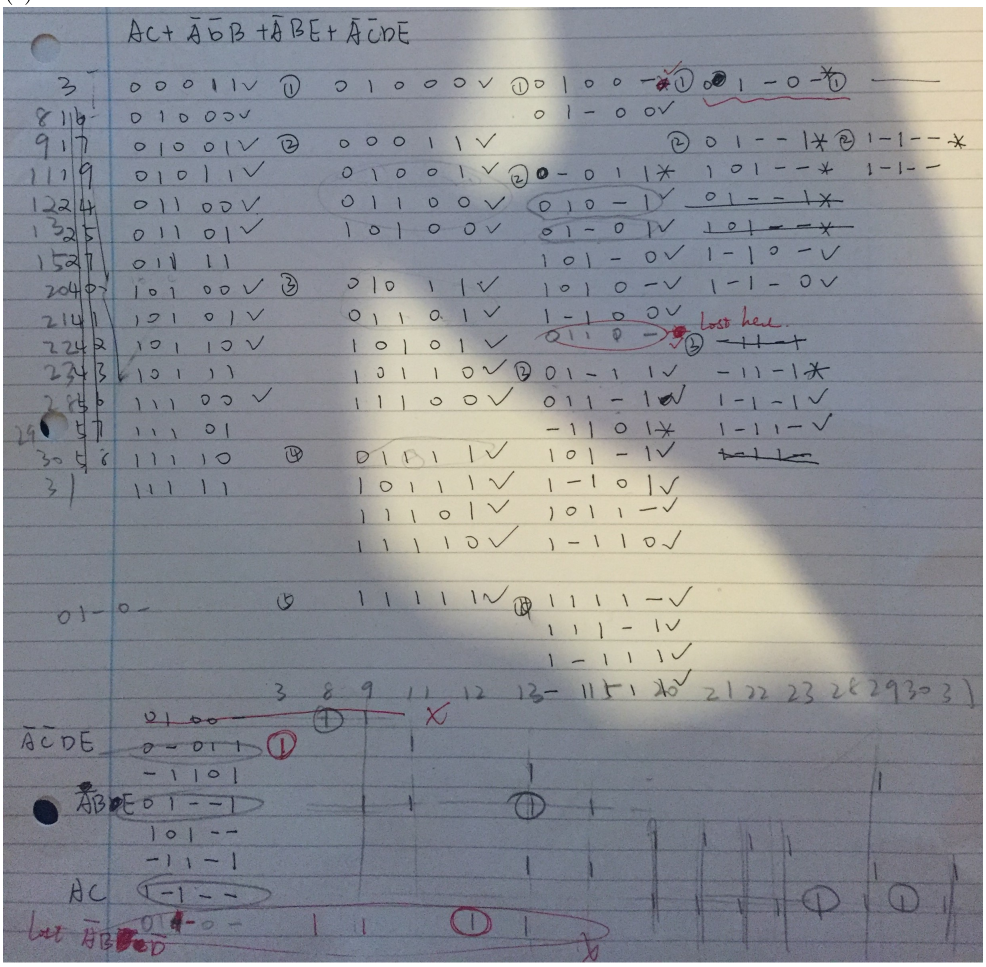
(e)$\bar{a}d + \bar{a}c + a\bar{c}\bar{d}$
(f)$\bar{f} = \bar{a}\bar{c}\bar{d} + b\bar{c}\bar{d} + a\bar{b}d + a\bar{c}d$
minimum POS:$f = (a + c + d)(\bar{b} + c + d)(\bar{a} + b + \bar{d})(\bar{a} + c + \bar{d})$

5 Ex5

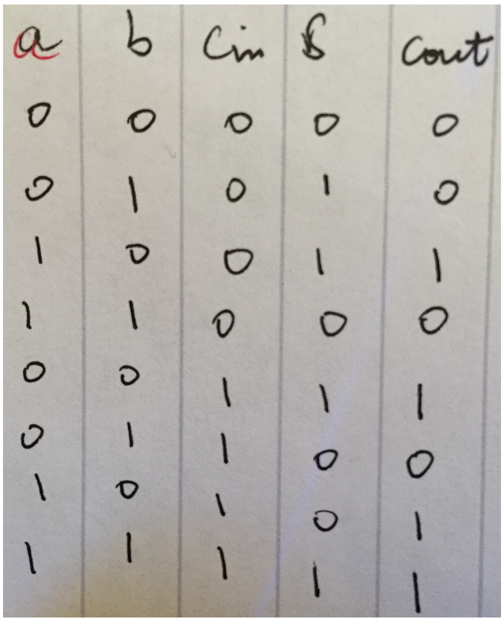
(a)$(X + Y)(X + Z)$
$\overline{\bar{X}\bar{Y} + \bar{X}\bar{Z}}$
$\overline{\bar{X}(\bar{Z} + \bar{Y})}$
$X + YZ$
(b)The same as 1.2.3
(c)Tristate Buffer
(d)$(A + B + \bar{C}DE)(A + \bar{D} + E)(\bar{A} + C)$
$(AC + \bar{A}(B + \bar{C}DE))(A + \bar{D} + E)$
$(AC + \bar{A}B + \bar{A}\bar{C}DE)(A + \bar{D} + E)$
$AC + AC\bar{D} + ACE + 0 + \bar{A}\bar{D}B + \bar{A}BE + 0 + 0 + \bar{A}\bar{C}DE$
$AC + \bar{A}\bar{D}B + \bar{A}BE + \bar{A}\bar{C}DE$
(e)cell adjacency:010 011 001 000 100 101 111 110
method: a(n-1) and reversed a(n-1) then add 0 or 1 to head.



(f)

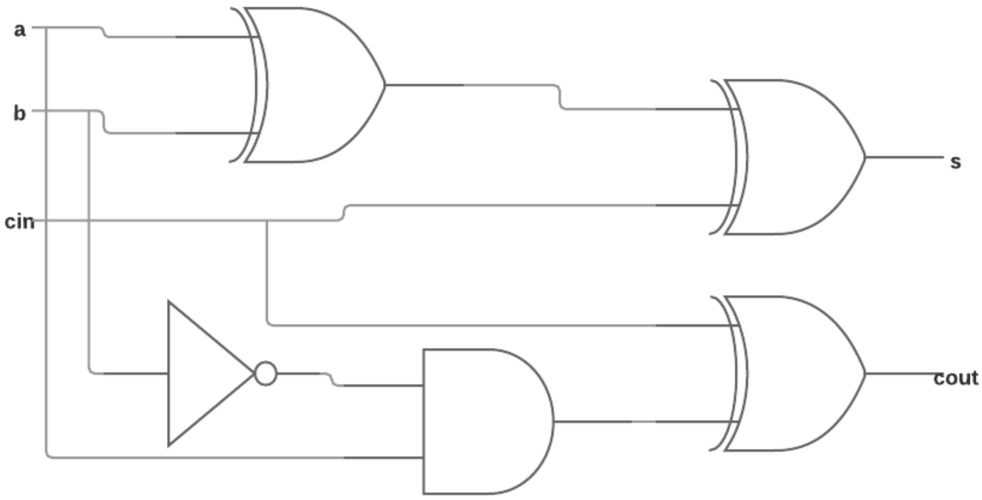


6 Ex6



a	b	cin	s	cout
0	0	0	0	0
0	1	0	1	0
1	0	0	1	1
1	1	0	0	0
0	0	1	1	1
0	1	1	0	0
1	0	1	0	1
1	1	1	1	1

$s = a \oplus b \oplus c_{in}$
$c_{out} = a.\bar{b} \oplus c_{in}$

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The circuit above have input a b cin and output s and cout.
cin: 1 means need to subtract 1 0 means not.
cout: 1 means need to borrow 1 from higher digital 0 means not.
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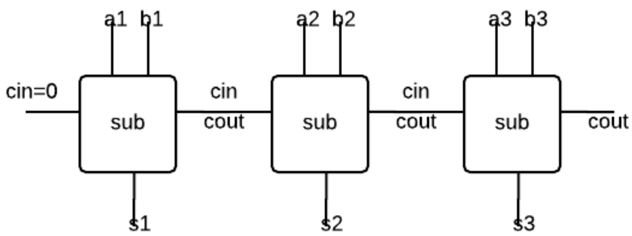
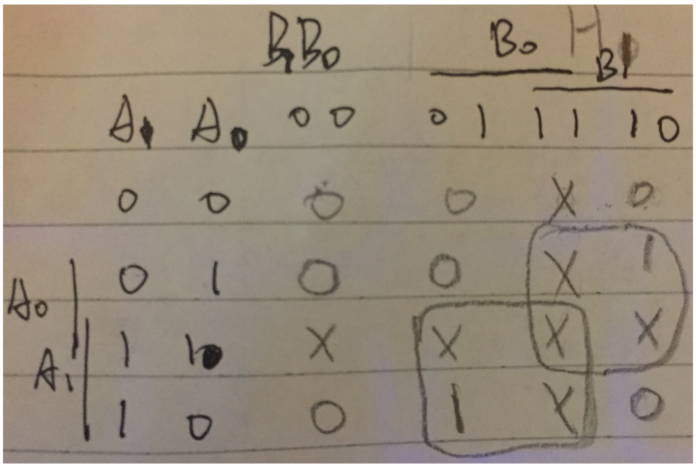


7 Ex7

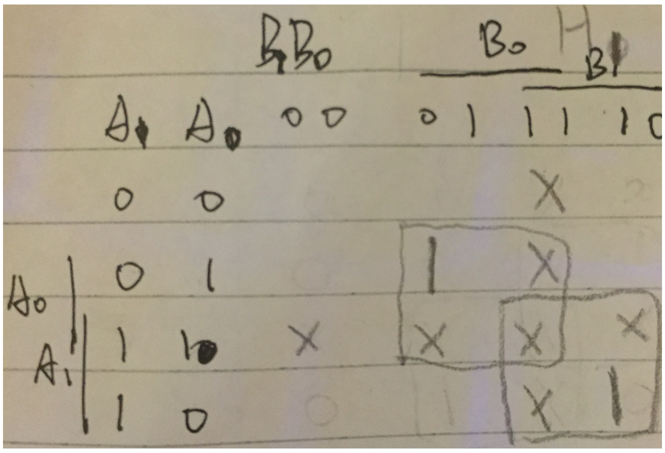
(a) 

(b) The 0 0 means clear so once the first input is 0 0 then the output H should also be clear.
For C, once b is logical 0 then it must be 0 (no need for add 1 to next digit)
otherwise it is also clear 0 0.

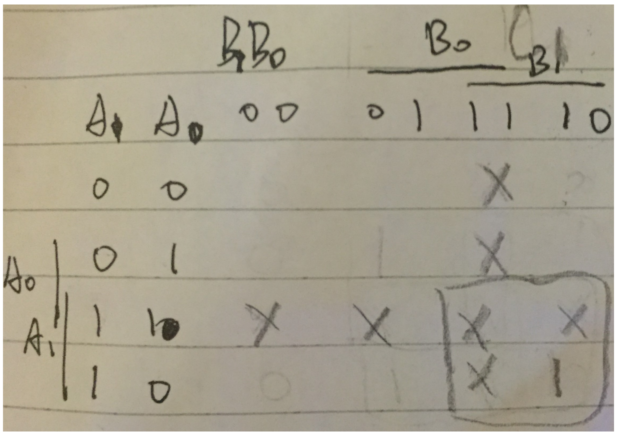
A1	A0	B1	B0	H1	H0	C1	C0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	0	0
0	1	0	0	0	0	0	1
0	1	0	1	0	1	0	1
0	1	1	0	1	0	0	1
1	0	0	0	0	0	0	0
1	0	0	1	1	0	0	1
1	0	1	0	0	1	1	0



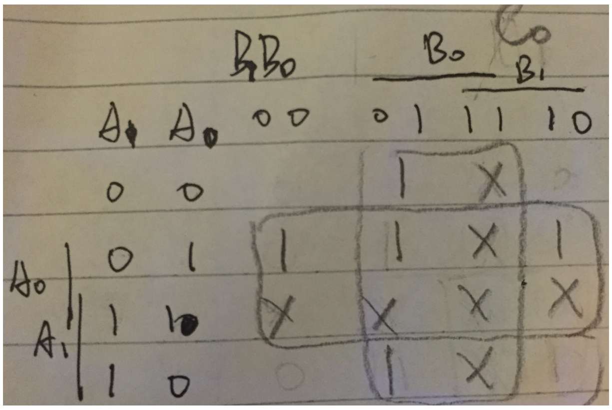
$H_1 = A_1B_0 + A_0B_1$



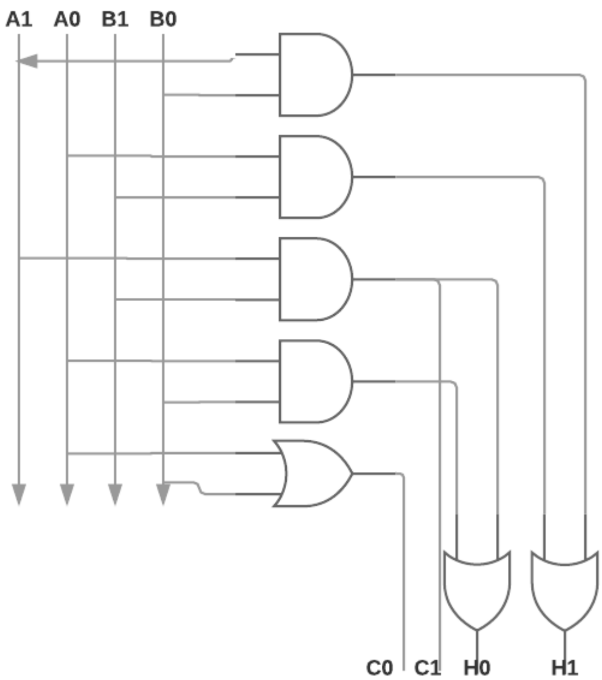
$H_0 = A_0B_0 + A_1B_1$



$C_1 = A_1 B_1$



$C_0 = A_0 + B_0$



(c) This needs 7 gates. Original one needs 3 gates.

(d) They both have $2T$ (ps) delay.

8 Ex8

notes

$$
\begin{aligned}
x + y &= z \\
\ldots &= \ldots && \text{a label} \\
ab/c & && \text{EXCHANGE rule} \\
p &< 5 && \text{(note that } p \in \mathbb{R}\text{)}
\end{aligned}
$$

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fun fib 0 = 1
  | fib n = fib (n-1) + fib(n-2);
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