UNSW Sydney

SCHOOL OF RISK AND ACTUARIAL STUDIES

ACTL2111: FINANCIAL MATHEMATICS FOR ACTUARIES

TERM 1 2025 ASSIGNMENT

DUE DATE: 15^{th} OF APRIL 2025 11:59pm

1 The Assignment Context

The Australian Superannuation system is now at maturity stage implying that cohorts approaching retirement will have accumulated substantial savings which can take them through the retirement phase given proper planning and guidance. The bulk of existing research on life-cycle modelling has been devoted to the pre-retirement/ accumulation phase with a single dimensional objective of maximising retirement wealth and there is substantial work on asset allocation of superannuation contributions (either guaranteed from employers or voluntary contributions by individuals). However, there is a significant gap when it comes to the post retirement phase where individuals are faced with multi-dimensional complexities when decision-making as they need to ensure:

· continued growth of retirement savings at a time they are no longer working,

· periodically drawing down retirement savings to cover daily living expenses,

· budgeting for health expenses which increase with age,

· eventual bequest for dependence upon death.

In trying to alleviate this challenge, the Australian government introduced the Retirement Income Covenant on the 1^{st} of July 2022 which mandates Superannuation trustees to develop strategies for guiding their members in (i) understanding and managing key risks in retirement, (ii) maximising retirement income and (iii) understanding potential income given individual circumstances.

Another dimensional challenge when it comes to the retirement phase is associated with the global shift from Defined Benefit (DB) pension schemes, where risks are fully borne by pension fund providers, to Defined Contribution (DC) pension schemes, where risks are fully borne by individual retirees. Irrespective of the maturing superannuation system, it is imperative for individuals to constantly plan for complimentary income streams beyond superannuation savings for a comfortable retirement.

Sharawata and Hamunyari met a decade ago in their very first actuarial studies course lecture (ACTL1101) on campus and to their amazement, they are a replica of each other - sharing the birthday, got similar scores in high school and have same hobbies, the rest is history!!! They finished their Bachelor of Actuarial Studies in 2018 and have been gainfully employed ever since in downtown Sydney. They will be having their one year wedding anniversary on the 1^{st} of May 2025.

The couple has just signed a contract to purchase their very first owner occupier residential home which will be settled on the 1^{st} of May 2025 coinciding with their joint 30^{th} birthdays (*lucky!!*). Their first home is a four-bedroom house sitting on a 300 square metre block of land in Box Hill, NSW 2765 which they have negotiated for $1,260,000, consistent with the current median house price in Box Hill. The lucky couple has been gifted an equivalence of 20% deposit by their parents towards purchase of their home implying that they will borrow 80% from the bank over a 30 year loan tenure.

The couple will be making monthly mortgage repayments in arrears with interest on the outstanding loan credited to their joint loan account at the end of each month. To hedge the interest rate uncertainty on both parties, the couple and the bank have agreed for a mortgage "interest rate collar" where interest rates are generally variable but floored at 2.5% p.a. and capped at 7.5% p.a. So, each time the variable interest rate goes below the floor rate, their loan account will be charged interest at a rate of 2.5% p.a. convertible monthly and whenever the variable rate is above the cap rate, their loan account will be credited with an interest rate of 7.5% p.a. convertible monthly. The bank has set the initial mortgage rate for May 2025 at 6.14% p.a convertible monthly. The interest rate dynamics applicable to the loan is specified in the next section. In addition to the mortgage repayments, the couple will be required to pay $350 per quarter as rates to The Hills Shire and $300 per quarter for the water (to Sydney Water) for as long as they remain invested in the property.

1.1 Evolution of Mortgage Rates

The mortgage rates dynamics from June 2025 onwards evolve according to the famous CIR[1] model which is represented by the following stochastic differential equation

$$dr_t = \kappa_r(\theta_r - r_t)dt + \sigma_r\sqrt{r_t}dW_1(t), \tag{1}$$

with r_t being the nominal mortgage rate in month, t, for $t =$ June 2025, July 2025, $\cdots$. Here κ_r is the speed of mean reversion of the interest rate process, θ_r is the long-run mean of r_t, σ_r is the volatility of the interest rate process, while $W_1(t)$ is a so-called "Brownian motion process" which introduces some noise to the dynamics of the interest rate process. The discretised version of Equation (1) can be represented as

$$r_{n+1} = r_n + \kappa_r(\theta_r - r_n)\Delta_n + \sigma_r\sqrt{r_n}\Delta W_1(n), \quad \text{for} \quad n = 0, 1, 2, \cdots, N-1 \tag{2}$$

with $\Delta_n = t_{n+1} - t_n$ and $\Delta W_1(n) = W_1(n+1) - W_1(n)$. Here, Brownian motion increments, $\Delta W_1(n)$, are Normally distributed with mean of zero and variance of Δ_n, that is, $\Delta W_1(n) \sim N(0, \Delta_n)$. When simulating the Brownian motion increments in excel, you can make use of the following approximation

$$\Delta W_1(n) = \sqrt{\Delta_n} \times \text{NORMINV}(\text{RAND}(), 0, 1),$$

where NORMINV and RAND are inbuilt excel functions for generating normally distributed random numbers.

Note that the discrete time approximation is

$$0 = t_0 < t_1 < \cdots < t_n < \cdots < t_N = T,$$

where T corresponds to the tenure of the loan.

Tables 1 provides the constant parameters to be used when simulating Equation (2).

Note that mortgage rates generated from Equation (2) are per annum rates convertible monthly. These rates will be reset every monthly as detected by the CIR process. The couple will make monthly mortgage repayments in arrears until their 60^{th} birthday which coincides with their loan maturity. Here, we are assuming that the couple will not refinance or redraw the loan before their 60^{th} birthday.

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[1]Cox, J.C., Ingersoll, J.E., & Ross, S.A. (1985) A theory of the term structure of interest rates. Econometrica 53, 385-407

	Δ_n	θ_r	r_0	κ_r	σ_r
Daily	1/365	0.02956	0.059594	0.40569	0.02956
Weekly	1/52	0.029567	0.059623	0.40705	0.029567
Fortnightly	1/26	0.029576	0.059657	0.408643	0.029576
Monthly	1/12	0.029595	0.059737	0.412392998	0.029595
Quarterly	0.25	0.029668	0.060035	0.426727679	0.029668
Semi-Annually	0.5	0.029778	0.060485	0.449489743	0.029778
Annually	1	0.03	0.0614	0.5	0.03

Table 1: Constant parameters for the investment return process. All interest rates are nominal per annum.

Over the next century from May 2025, monthly house prices are predicted evolve according to the following system[2]

$$dS_t = \mu S_t dt + \sqrt{v_t} S_t dW_2(t), \tag{3}$$

$$dv_t = \kappa_v(\theta_v - v_t)dt + \rho\sigma_v\sqrt{v_t}dW_2(t) + \sigma_v\sqrt{1-\rho^2}\sqrt{v_t}dW_3(t). \tag{4}$$

Equation (3) is the house price dynamics with the instantaneous rate of return given by μ and instantaneous volatility given by the square root of the variance process, v_t, which evolves according to Equation (4). In (4), κ_v is the speed of mean reversion of the variance process, θ_v being the corresponding long-run average and σ_v is the volatility of volatility of the variance process. The processes, $W_2(t)$ and $W_3(t)$ are Brownian motion processes whose increments are correlated such that $\mathbf{E}[dW_2(t)dW_3(t)] = \rho dt$, with dt being the time-step between two points and ρ being the correlation coefficient.

The discretised version of Equations (3) and (4) can be represented as

$$S_{n+1} = S_n + \mu S_n \Delta_n + \sqrt{v_{n+1}} S_n \Delta W_2(n), \tag{5}$$

$$v_{n+1} = v_n + \kappa_v(\theta_v - v_n)\Delta_n + \rho\sigma_v\sqrt{v_n}\Delta W_2(n) + \sigma_v\sqrt{1-\rho^2}\sqrt{v_n}\Delta W_3(n). \tag{6}$$

The Brownian motion increments are approximated in a similar fashion to that of $\Delta W_1(n)$. Table 2 provides parameters for the house price and variance processes.

When the couple reaches the retirement age of 67, they are planning to enhancing their retirement income by taking a "reverse mortgage", a scheme by which the Australian government allows homeowners in accessing up to 30% of their home equity for enhancing

[2]Heston, S.L., 1993. A closed-form solution for options with stochastic volatility with applications to bond and currency options. The Review of Financial Studies, 6(2), pp.327-343.

	μ	θ_v	v_0	κ_v	σ_v
Daily	0.095322625	0.041144262	0.03922282	0.4056904	0.019803164
Weekly	0.09539758	0.041158223	0.039235508	0.407050013	0.019806398
Fortnightly	0.095485086	0.041174512	0.03925031	0.408643183	0.01981017
Monthly	0.095689685	0.041212551	0.039284877	0.412392998	0.019818976
Quarterly	0.096454756	0.041354253	0.039413626	0.426727679	0.019851726
Semi-Annually	0.097617696	0.041568025	0.039607805	0.449489743	0.019900988
Annually	0.10	0.042	0.04	0.5	0.02

Table 2: Constant parameters for the house price and variance processes. All rates are nominal per annum. For all compounding frequencies, $S_0 = \$1,260,000$ and $\rho = -0.5$

their retirement income through the Home Equity Access Scheme (HEAS)[3]. Advantages of a reverse mortgage include:

- The lender will only get paid when the borrowers die. If the borrowers sell the home or refinance, the bank will also receive outstanding payment at that time.
- The income or money released from the home can be used for any purpose, there are no restrictions on how the borrowers spend the released equity.
- No income tax need to be paid by the borrowers on the equity released.
- Any equity remaining after the bank is paid will be available to beneficiaries.

The couple will start of by withdrawing \$2,500 per fortnight during the first year of retirement and this amount will increase annually on their birthday anniversary by \$100 until they hit the 30% equity ceiling of the prevailing market value of the property. For simplicity, assume that they will both survive to the maximum attainable age.

2 Task

Given the above information:

1. Simulate the nominal monthly mortgage rates corresponding to the repayment period until retirement. You must also show how to obtain the associated effective monthly rates.
2. Develop a mortgage calculator on a spreadsheet (or any other software application of your choice) showing the monthly repayments from inception of the loan. The

[3] `https://www.servicesaustralia.gov.au/home-equity-access-scheme`

spreadsheet/(other) should include all components of a loan schedule such as the simulated mortgage rates together with the corresponding interest payments, principal repayments and the outstanding balance for each month. Note that the simulated rates must be stochastic and each time the spreadsheet/(other) is refreshed, new set of interest rates and, hence, repayment schedules should be generated.

3. Modify your calculator developed in Question 2 above so that it is compatible with all payment frequencies provided in Table 1, that is, for the cases when repayments are made either daily, weekly, fortnightly, quarterly, semi-annually or annually. Your calculator should re-simulate the mortgage rate for each repayment frequency automatically. In short, your calculator should automatically update all fields if there is any change in input parameters such as interest rates, tenure of the loan, frequency of repayments, initial loan amount, among others. You may find it more helpful and convenient to develop a single calculator for Questions 2 and 3.

4. Show how the home equity will evolve through time and how long it will take to hit the 30% equity withdraw cap. For this part, assume that the bank will charge a fixed interest rate of 3.95% per annum convertible fortnightly on the outstanding balance of the released equity. You can take the median of 1,000 simulations at any given point in time to be the indicative house price at that time. The outstanding balance will be settled as a lump sum at the time of death which we assume to be the maximum attainable age of 95 for convenience.

5. Determine the expected home value at the time of the couple's death and the amount of money that will be paid to the bank. Assuming that any excess after paying the bank's outstanding balance will be paid to couple's beneficiaries, determine how much money will be credited to the beneficiaries.

3 Submission Requirements

You are expected to submit two files:

1. A PDF file summarising your findings in Questions 1-5 above. You must also interpret the effects of changing any of the input variables on your calculator. Your report may include graphs and/or tables where necessary. The page limit for this file is 4 pages excluding cover page and appendices (Times News Roman font, 12pt font size, 1.5 line spacing).

2. An Excel spreadsheet (or any output file if using any other programming languages) containing your thought process. Marks will be awarded based on the presentation and clarity of your PDF file and your Excel spreadsheet/ (other) as detailed in the assessment criteria below. Make sure that your responses in either the spreadsheet or any alternative application are easy to follow. Your spreadsheet/(other) should

    - include all the steps of calculations, including assumptions, inputs, intermediate calculations and outputs;
    - be well structured and documented.

4 Assignment Submission Procedure

Assignment reports must be submitted via the Moodle submission box which will be activated on the Course Website. Students are reminded of the risk that technical issues may delay or even prevent their submission (such as internet connection and/or computer breakdowns). Students may consider allowing enough time (at least 24 hours is recommended) between their submission and the due time.

Late Submission

The submission deadline is 11:59pm of the 15^{th} of April 2025. Late submissions will be dealt with according to the school policy as detailed in the course outline.

5 Assessment Criteria

Your assignment report will be assessed using the following criteria:

1. Clear and concise justification of your approach to and summary of key findings in the PDF file. [25 marks]

2. Accurate presentation of results in excel or any programming language of your choice. [50 marks]

3. Data visualisation in excel or other. [20 marks]

4. Follow the formatting and page limit requirements. [5 marks]