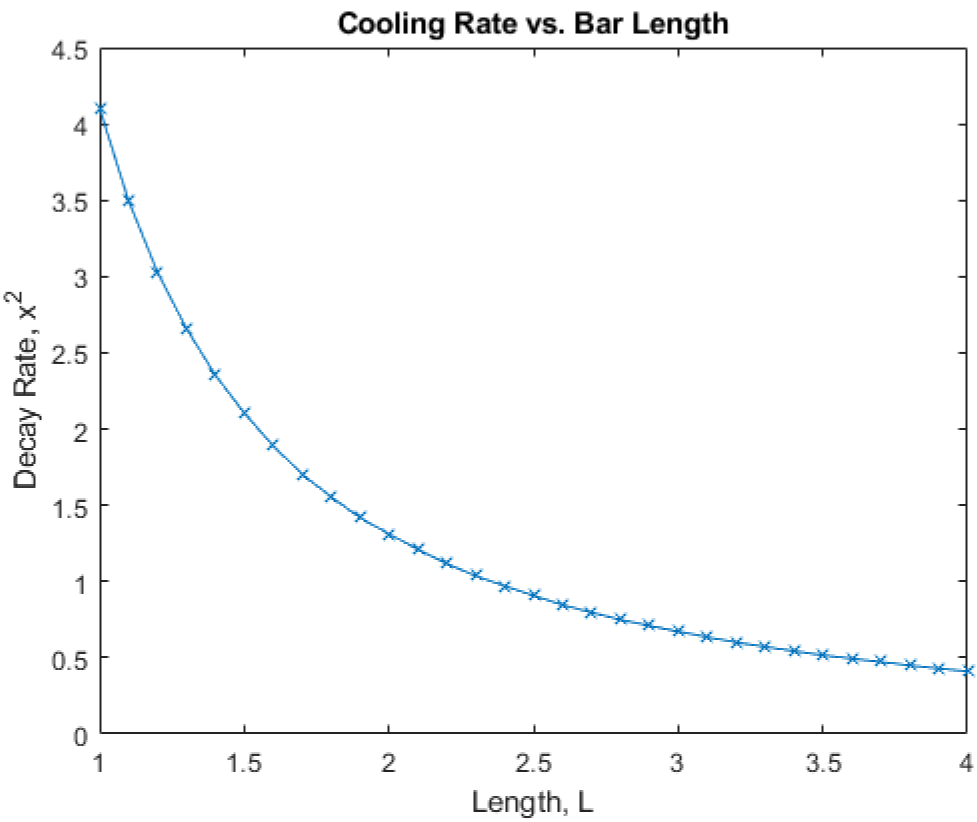
```
% Pierce Zhang, CMOR220, Fall 2023, Project 2: Root finding methods
% cooldrive.m
% Implements the Newton's method and the bisection method to find the
% square of the least positive root of a cooling function to 1) simulate
% heat dissipation from a metal rod and 2) compare the efficiencies (# of
% iterations) between the two algorithms on different epsilon (tolerance)
% levels.
% Last modified: 18 September 2023

% Driver function
function cooldrive()
```

SECTION ONE: BISECTION FOR BAR COOLING

Set bounds of bisection search [a,b], epsilon (t), and range of L from the cooling equation to solve for x.

```
a = 0.1; b = 3; t = 0.01;
L = 1 : 0.1 : 4;

% The right bound must tighten towards the left bound as L increases
% because the bisection method only returns the greatest root within
% a given range, but we want the smallest root.
x = zeros(0,length(L)); % Preallocate blank vector
for cnt = 1:length(L)
    [x(cnt), iter] = debis(a,b,t,L(cnt));
    b = b - 0.05; % Reduce b to tighten range; b will reach a at end
end

% Plot x^2 (the square of the roots from bisection) vs. L
figure()
plot(L, x.^2, "-x")
title("Cooling Rate vs. Bar Length")
xlabel("Length, L")
ylabel("Decay Rate, x^2")

clear
```

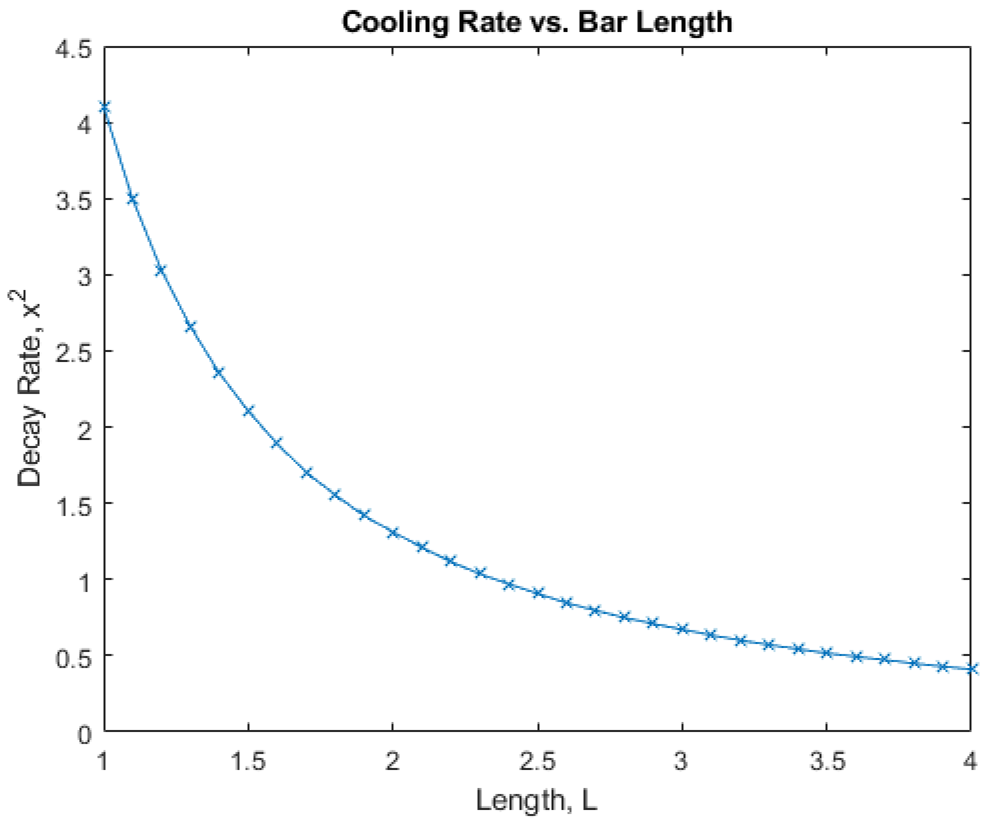


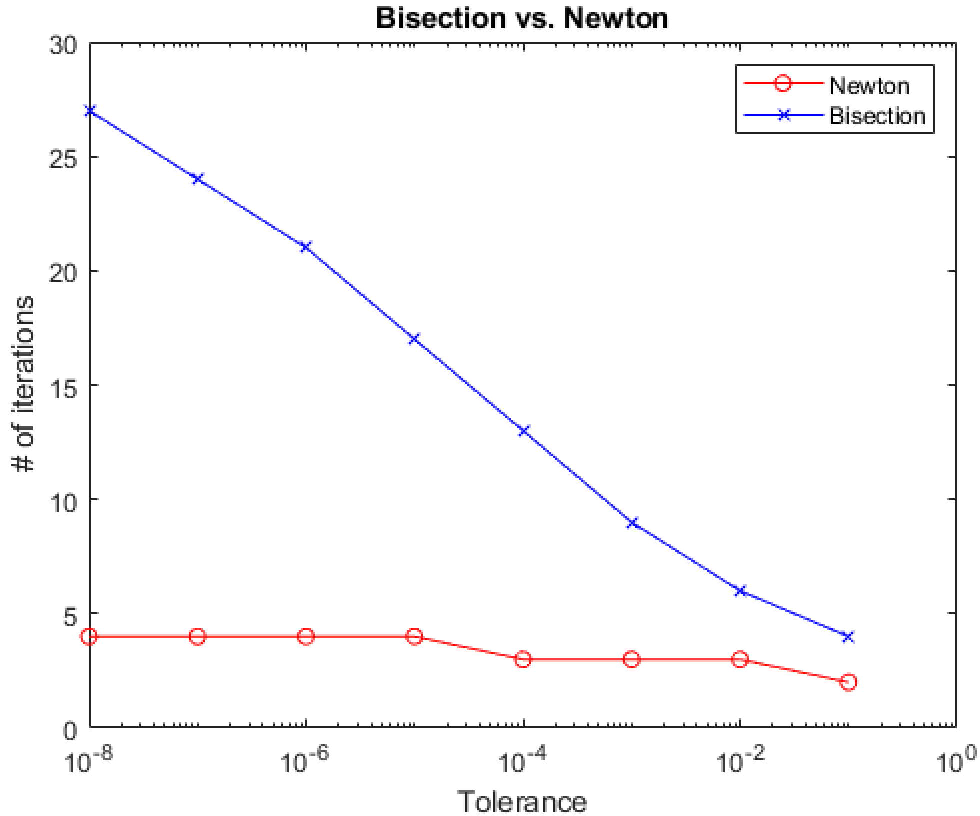
SECTION TWO: NEWTON VS BISECTION

Set bounds of search [a,b], L, and x0, initial value for using the Newton's method. Sets t to an array of tolerances from 10^-1 to 10^-8, dividing by 10 each time.

```
L = 1; a = 0.1; b = 3; x0 = (a+b)/2;
t = 10.^-(1:8);

% Preallocates two vectors, x_bis for bisection results and x_newt for
% Newton's method results
x_bis = zeros(0,length(t));
x_newt = zeros(0,length(t));
for cnt = 1:length(t)
    [~,x_bis(cnt)] = debis(a,b,t(cnt),L); % Store only # of iterations
    [~,x_newt(cnt)] = denewt(x0,t(cnt),L);
end

% Plot number of iterations of each of Newton's method and bisection
% method against t, the given episilon (tolerance)
figure()
semilogx(t, x_newt, "-o","Color",'r');
hold on
semilogx(t, x_bis, "-x","Color",'b');
hold on

title("Bisection vs. Newton")
```

```
xlabel("Tolerance")
ylabel("# of iterations")
legend("Newton","Bisection")
```






```
end

% Inputs: x (x0), the value at which to begin searching for the root, t, the
% tolerance, and L, the length of the bar which is just a scalar inside the
% cooling equation.
% Outputs: x, the root estimation, iter, number of iterations
function[x,iter] = denewt(x,t,L)
%Newtons method, calls coolfun to find value and coolfundx to find
%derivative
iter = 0;
while (abs(coolfun(x,L)) > t)
    x = x - coolfun(x,L)/coolfundx(x,L);
    iter = iter + 1;
end
end

% Inputs: a, the left bound of the range to search for the root, b, the
% right bound of the same range, forming [a,b], tol, the epsilon
% (tolerance), and L, the length of the bar for the equation
% Outputs: x, the root estimation, iter, number of iterations
% program ran
function [x,iter] = debis(a,b,tol,L)
% Perform the bisection method
iter = 0;
x = (a + b) / 2;
while abs(coolfun(x,L)) > tol
```

```
    if coolfun(a,L) * coolfun(x,L) < 0
        b = x; % Same side -> look on right side of pivot
    else
        a = x; % Left side of pivot
    end
    iter = iter + 1;
    x = (a + b) / 2;
end
end

% Inputs, x, arbitrary scalar number, and L, length of the bar used in
% cooling equation
% Outputs: val, value of the function returned when x and L are inputted.
function val = coolfun(x,L) %for a given x and L evaluate "cool"
val = sin(x*L) + x*cos(x*L);
end

% Inputs: arbitrary scalar number, and L, length of the bar used in
% cooling equation
% Outputs: val, value of the derivative of the function returned when x
% and L are inputted.
function val = coolfundx(x,L)
% evaluate the derivative, with respect to x, of coolfun
val = L*cos(x*L) + cos(x*L) - x*L*sin(x*L);
end
```

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