Review of Probability Distributions

Combination and permutation

combination and permutation are two different ways of grouping elements of a set into subsets. when we select k elements from a set of n objects:

- if the order of selection is NOT important, we have a combination
  - yes repetition --> C=(n+r-1)!/(r!(n-1)!)
  - no repetition --> C=n!/(r!(n-r)!)
- if the order matters, we have a permutation
  - yes repetition --> P=n^r
  - no repetition --> P=n!/(n-r)!

Probability Distributions

- discrete distribution : finite or countable set of possible outcomes of the random variable
  • continuous distribution : a random variable can have outcomes in an interval of the real line

$F(x) = P(X \leq x)$ cumulative distribution fucntion
$E[X] = \Sigma x_i p(x_i)$ or $E[X] = \int x f(x) dx$ --> expectation
$var(X) = E[X - E[X]]^2 = E[X^2] - (E[X])^2$ variance
$\mu_k = E[x^k] = \int x^k f(x) dx$ or $\Sigma x_j^k p_j$ moment of order k

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**Standard Discrete Distributions

1- Bernoulli process

it is a process with only two possible outcomes: success with probability p and failure with probability 1 − p (also called q, since q = 1 − p)
if we call the two outcomes, 0 and 1, we can define x ∈ [0, 1], and:

- P(X = 1) = p
- P(X = 0) = 1 − p = q
- E[x] = p
- Var(x) = p(1 − p)

the sum of n independent Bernoulli trials, follows a Binomial distribution
$Bn(xp, n) = \binom{n}{x} p(1-p)^{n-x}$, it gives the probability of x successes in n independent Bernoulli trials. E[x] = np and Var(x) = np(1 − p).
when n becomes large, the distribution tends to a gaussian

Examples
• multiple toss of a coin, or coins
• draw of dice
• drawing x red balls from an urn with n red and white balls (the fraction of red balls is p). Draws are done with replacement (p remains constant)

2-Geometric distribution
In the drunk man experiment the geometric distribution gives the number of trials to get the first success
$Geo(x|p) = p(1-p)^x$

- $E[x] = 1/p$
- $Var(x) = \sigma^2 = \frac{1-p}{p^2}$
- P(x $\leq$ r) = q^r

3-Multinomial distribution
it is a generalization of the binomial distribution to the case with more than 2 possible outcomes

- disjoint outcomes $A_1, A_2, \ldots, A_r$
- $P(A_j) = p_j$
- n independent trials
- x_j = number of times that A_j occurs
- $P = \frac{n!}{x_1!x_2!\ldots x_r!} p_1^{x_1} p_2^{x_2} \cdot \cdot p_r^{x_r}$
- expectation for class A_j is $E[x_j] = np_j$
- the variance for class A_j is $Var(x_j) = np_j(1-p_j)$
- n--> large, the distribution tends to a multinormal distribution

4-Poisson process
let’s consider an event that might happen at a given time with the following conditions:
- the probability of 1 count in Δt is proportional to Δt itself, with Δt a ’small’ value
- calling r, the intensity of the process, p = P(1 count in Δt) = rΔt
- P(≥ 2 counts) ≪ P(1 count)

- the Poisson distribution can be derived by the Binomial distribution, in the limit where the rate of success, p, is very small: np = rT = λ
- Poi(r|λ) = λ^r/r! exp(−λ)
- E[x] = λ and Var(x) = λ

Pascal or Negative Binomial distribution
the probability of obtaining the r-th success in n trials, is given by the Negative Binomial, or Pascal, distribution

Since in n − 1 trials we had r − 1 successesthe probability is given by the Binomial distribution; but we got the r-th success at the n-th trial, therefore:
$Bneg(r|n, p) = \binom{n-1}{r-1} p^r (1 - p)^{n-r}$