

A.7 Advanced simulations of a 2D Ising model

Wolff cluster algorithm Implement the Wolff cluster algorithm for the simulation of a 2D Ising model, with periodic boundary conditions.

Cluster size statistics By assuming $k_B = 1$ and $J = 1$, simulate the 2D Ising model ($L = 50$) for different temperatures: the critical temperature $T_c = \frac{2}{\ln(1+\sqrt{2})}$, a high temperature $T_h = 2T_c$, and a low temperature $T_l = T_c/2$. Collect the cluster sizes of the Wolff algorithm and plot their histograms for the three temperatures. What can one note?

Autocorrelation time For some values of L , compute the autocorrelation time τ at T_c of the magnetization time series for (a) the single-spin flip algorithm, and (b) the Wolff algorithm. Plot them in log-log scale vs L . Do they scale with L as expected?

Multiple Markov chains Implement multiple Markov chains for the Ising model. Check if the energy distributions of adjacent chains are overlapped. Record the swapping rate between chains. Compare the autocorrelation times of the chains when the swapping is allowed and when it is not (thus making the Markov chains effectively single ones).