Practical Principled FRP

Forget the past, change the future, *FRPNow*! Extra information

Atze van der Ploeg Koen Claessen

Chalmers University of Technology {atze,koen}@chalmers.se

1. when Just, when and snapshot

 $snapshot^{\dagger}$ and $when^{\dagger}$ can be defined in terms of from $whenJust^{\dagger}$

```
snapshot^{\dagger} :: Behavior \ a \to Event \ () \to Event \ a

snapshot^{\dagger} \ b \ e =

whenJust^{\dagger} \ (pure \ Nothing `switch` \ (Just <>> b) <> e)

when^{\dagger} :: Behavior \ Bool \to Event \ () \to Event \ ()

when^{\dagger} \ b \ e = whenJust^{\dagger} \ (boolToMaybe <>> b) \ e
```

Alternatively, $whenJust^{\dagger}$ could be defined in terms of $snapshot^{\dagger}$ and $when^{\dagger}$:

when
$$Just^{\dagger}b\ e = snapshot^{\dagger}\ (from Just \ll b)$$

 $(pred Leak\ (is Just \ll b)\ e)$

2. Proofs

Lemma 1. switch is forgetful.

Proof. To prove:

$$\forall b_1 \, b_2 \, e_1 \, e_2 \, n.b_1 \stackrel{\textstyle >}{=}^n \, b_2 \wedge e_1 \stackrel{\textstyle >}{=}^n \, e_2 \rightarrow switch \, b_1 \, e_1 \stackrel{\textstyle >}{=}^n \, switch \, b_2 \, e_2$$
 Let $e_1 \stackrel{\textstyle \circ}{=} (t_1, s_1)$ and $e_2 \stackrel{\textstyle \circ}{=} (t_2, s_2)$. To prove:
if $n < t1$ then b_1 'at' n else $s1$ 'at' $n \stackrel{\textstyle >}{=}^n$
if $n < t2$ then b_2 'at' n else $s2$ 'at' n

We first prove that comparison $n < t_1$ gives the same result as the comparison $n < t_2$, and hence if chooses the same branch. Because $e_1 \stackrel{\Longrightarrow}{=} t_2$ we know that $t_s = \max \ t_1 \ n = \max \ t_2 \ n$. Suppose $t_s = n$, then $n \geqslant t_1$ and $n \geqslant t_2$ resulting in the same outcome of the comparison. If $t_s \neq n$ then by definition of t_s , we know that $t_1 = t_2$, again resulting in the same outcome of the comparison.

We know that $s_1 \stackrel{>}{=}^n s_2$ and $b_1 \stackrel{>}{=}^n b_2$, and hence in both branches of the **if** expression the outcome will be the same up to time observation from n. Since the same branch of **if** will be chosen and the result is equal up time observation from n.

Lemma 2. \gg for events is forgetful.

Permission to make digital or hard copies of all or part of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. Copyrights for components of this work owned by others than ACM must be honored. Abstracting with credit is permitted. To copy otherwise, or republish, to post on servers or to redistribute to lists, requires prior specific permission and/or a fee. Request permissions from permissions @acm.org.

Proof. To prove:

$$\forall e_1 \ e_2 \ f_1 \ f_2 \ t.e_1 \stackrel{\supseteq}{=}^t e_2 \land f_1 \stackrel{\supseteq}{=}^t f_2 \rightarrow e_1 \gg f_1 \stackrel{\supseteq}{=}^n e_2 \gg f_2$$

Let $e_1 \triangleq (t_1, a_1)$ and $e_2 \triangleq (t_2, a_2)$. If $t_1 = \infty$ then t_2 must also be ∞ by max t_1 $t = \max$ t_2 t. Since two never occurring events are always equal up to time observation, and the resulting event must occur at ∞ because ∞ is the absorbing element for max, the result is equal up to observation from time t. Otherwise, Let $(t'_1, x_1) \triangleq f_1 a_1$ and $(t'_2, x_2) \triangleq f_2 a_2$. Since \max $t'_1 t = \max$ $t'_2 t$, we know that $\max(\max t_1 t'_1) t = \max(\max t_2 t'_2) t$. Furthermore we know that $t_1 \stackrel{\supseteq}{=} t_2 t$, because $t_1 \stackrel{\supseteq}{=} t_2 t$, and hence the result is again equal up to observation from time t.

The other proofs *never*, return and \gg for behaviors follow almost directly from the definition of equality up to time observation.