Practical Principled FRP

Forget the past, change the future, *FRPNow*!
Extra information

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1. *whenJust*, *when* and *snapshot*

$snapshot^\dagger$ and $when^\dagger$ can be defined in terms of from $whenJust^\dagger$

$$
\begin{array}{l}
snapshot^\dagger :: Behavior\ a \to Event\ () \to Event\ a \\
snapshot^\dagger\ b\ e = \\
\quad whenJust^\dagger\ (pure\ Nothing\ `switch`\ (Just \mathbin{<\!\$\!>} b) \mathbin{<\!\$} e) \\
when^\dagger :: Behavior\ Bool \to Event\ () \to Event\ () \\
when^\dagger\ b\ e = whenJust^\dagger\ (boolToMaybe \mathbin{<\!\$\!>} b)\ e
\end{array}
$$

Alternatively, $whenJust^\dagger$ could be defined in terms of $snapshot^\dagger$ and $when^\dagger$:

$$
\begin{array}{l}
whenJust^\dagger\ b\ e = snapshot^\dagger\ (fromJust \mathbin{<\!\$\!>} b) \\
\qquad\qquad\qquad\qquad\qquad (predLeak\ (isJust \mathbin{<\!\$\!>} b)\ e)
\end{array}
$$

2. Proofs

Lemma 1. *switch is forgetful.*

Proof. To prove:

$$\forall b_1\, b_2\, e_1\, e_2\, n. b_1 \overrightarrow{\equiv}^n b_2 \wedge e_1 \overrightarrow{\equiv}^n e_2 \to switch\, b_1\, e_1 \overrightarrow{\equiv}^n switch\, b_2\, e_2$$

Let $e_1 \mathrel{\mathring{=}} (t_1, s_1)$ and $e_2 \mathrel{\mathring{=}} (t_2, s_2)$. To prove:

$$
\begin{array}{l}
\mathbf{if}\ n < t1\ \mathbf{then}\ b_1\ `at`\ n\ \mathbf{else}\ s1\ `at`\ n \overrightarrow{\equiv}^n \\
\mathbf{if}\ n < t2\ \mathbf{then}\ b_2\ `at`\ n\ \mathbf{else}\ s2\ `at`\ n
\end{array}
$$

We first prove that comparison $n < t_1$ gives the same result as the comparison $n < t_2$, and hence **if** chooses the same branch. Because $e_1 \overrightarrow{\equiv}^t e_2$ we know that $t_s = \max\ t_1\ n = \max\ t_2\ n$. Suppose $t_s = n$, then $n \geqslant t_1$ and $n \geqslant t_2$ resulting in the same outcome of the comparison. If $t_s \neq n$ then by definition of t_s, we know that $t_1 = t_2$, again resulting in the same outcome of the comparison.

We know that $s_1 \overrightarrow{\equiv}^n s_2$ and $b_1 \overrightarrow{\equiv}^n b_2$, and hence in both branches of the **if** expression the outcome will be the same up to time observation from n. Since the same branch of **if** will be chosen and the result is equal up time observation from n. □

Lemma 2. $\gg\!=$ *for events is forgetful.*

Proof. To prove:

$$\forall e_1\ e_2\ f_1\ f_2\ t. e_1 \overrightarrow{\equiv}^t e_2 \wedge f_1 \overrightarrow{\equiv}^t f_2 \to e_1 \gg\!= f_1 \overrightarrow{\equiv}^n e_2 \gg\!= f_2$$

Let $e_1 \mathrel{\mathring{=}} (t_1, a_1)$ and $e_2 \mathrel{\mathring{=}} (t_2, a_2)$. If $t_1 = \infty$ then t_2 must also be ∞ by $\max\ t_1\ t = \max\ t_2\ t$. Since two never occurring events are always equal up to time observation, and the resulting event must occur at ∞ because ∞ is the absorbing element for $\max$, the result is equal up to observation from time t . Otherwise, Let $(t_1', x_1) \mathrel{\mathring{=}} f_1\, a_1$ and $(t_2', x_2) \mathrel{\mathring{=}} f_2\, a_2$. Since $\max\ t_1'\, t = \max\ t_2'\, t$, we know that $\max(\max\ t_1\, t_1')\, t = \max(\max\ t_2\, t_2')\, t$. Furthermore we know that $x_1 \overrightarrow{\equiv}^t x_2$, because $f_1 \overrightarrow{\equiv}^t f_2$, and hence the result is again equal up to observation from time t. □

The other proofs $never$, $return$ and $\gg\!=$ for behaviors follow almost directly from the definition of equality up to time observation.

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