

# Notes on Fracture - Cubic Law and Linear Law

Ivan Marin

August 22, 2012

## 1 Cubic Law

### 1.1 Deduction from Navier-Stokes

This model of the fracture assumes that the flow is viscous and slow, so the inertia terms in the Navier-Stokes can be neglected. The fracture is saturated with water, and are thin.

### 1.2 Cubic Law for Discharge

The cubic law for discharge in the direction of the fracture is

$$Q_s(s) = -\beta^* b^3 \frac{\partial \phi(s)}{\partial s} \quad (1)$$

where  $b$  is the total aperture of the fracture and  $\beta^*$  is equal to

$$\beta^* = \frac{\rho g}{12\mu} \quad (2)$$

where  $\rho$  is the density of water,  $[kg/m^3]$ ;  $g$  is the gravity acceleration  $[m^2/s]$ , and  $\mu$  is the dynamic viscosity,  $[Pa.s]$ ,  $[kg/(m.s^2)]$ .

### 1.3 Cubic Law for the Potential

$$Q_s(s) = -\frac{\beta^* b^3}{k} \frac{\partial \Phi}{\partial s} \quad (3)$$

## 2 Linear Law

### 2.1 Linear Law Approximations

The Linear Law for flow in the fractures assumes that the fracture is filled with a material of hydraulic conductivity  $k^*$ , that the flow is viscous and slow and incompressible.

## 2.2 Linear Law for Discharge

The discharge is equal to

$$Q_s(s) = -k^*b \frac{\partial \phi(s)}{\partial s} \quad (4)$$

## 2.3 Linear Law for Potential

$$Q_s(s) = \frac{k^*b}{k} \frac{\partial \Phi}{\partial s} \quad (5)$$