OPTIMAL PWM METHODS FOR ACTIVE POWER FILTERS

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Abstract. Active power filters are the systems which can compensate the harmonics in the power system using a power converter and a reactive storage element (an inductor or a capacitor). There are two types of active power filters, one, the current source type, and the other, the voltage source type. In both types of active power filters, the PWM control of the power converter plays an important role for the performance and the efficiency of the harmonics compensation. In this paper, new PWM methods for each type of active power filters are developed, intended particularly for obtaining the maximum efficiency of the compensation. The approach is basically based upon the harmonic elimination principle. Using these new methods, twice as many lower order harmonics as the number of pulses per one cycle can be completely eliminated by the active power filter. So, these methods are much more efficient than any other existing method.

Keywords. Electric power systems; harmonic analysis; power converters; fourier analysis; nonlinear equations; optimization; microprocessor control; active power filters.

INTRODUCTION

Recently, the harmonics problem in the power system, mainly caused by power electronic loads, appears severe. To compensate these harmonics. passive shunt filters composed of inductors, capacitors, and resistors can be used. But it is not easy to compensate the lower order harmonics with passive shunt filters. On the other hand, active power filters composed of a power converter and a reactive storage element (an inductor or a capacitor) can easily compensate the lower order harmonics. So, the combination of the active power filter and the high pass passive filter can be a good solution.

There are two types of active power filters. One is the current source type (Gyugyi, 1976; Kawahira, 1983), and the other is the voltage source type (Akagi, 1986). In both types of active power filters, the PWM control of the power converter plays an important role for the performance and the efficiency of the active power filters. So there have been many studies on the PWM methods for the active power filters (Akagi, 1986; Choe, 1988; Gyugyi, 1976; Hayafune, 1984; Kawahira, 1983; Kim, 1987).

In this paper, new PWM methods for both the current source type and the voltage source type active power filter are developed, intended particularly for obtaining the maximum efficiency of the compensation. Using these methods, twice as many lower order harmonics of the power system as the number of pulses per one cycle can be completely eliminated by the active power filter. So, these methods are much more efficient than any other existing method.

In this paper, methods only for the single phase case are presented for simplicity. But it is possible to extend these methods to the three phase case without difficulty.

BASIC OPERATION PRINCIPLE OF ACTIVE POWER FILTERS

Figure 1 shows the power system with a nonlinear load. In this case, the source current i_S is not sinusoidal because of the nonlinearity of the load. Moreover, the load voltage v_L is distorted by the harmonic voltage drop in the source impedance L_S. The active power filter can be explained as a harmonic current source for compensating these current and voltage distortions of the power system. In Fig. 2, suppose that the non-sinusoidal source current i_S is expanded by the fourier series.

$$i_S(t) = \sum_{n=1}^{\infty} a_n \sin(n\omega t) + \sum_{n=1}^{\infty} b_n \cos(n\omega t) . \tag{1}$$

Then, the harmonic current i_H is

$$i_H(t) = \sum_{n=2}^{\infty} a_n \sin(n\omega t) + \sum_{n=2}^{\infty} b_n \cos(n\omega t) . \tag{2}$$

For eliminating this harmonic current exactly, the compensation current i_C must be equal to this harmonic current, that is,

$$i_C(t) = i_H(t) . \tag{3}$$

Consider that i_C is expanded by the fourier series.

$$i_C(t) = \sum_{n=1}^{\infty} c_n \sin(n\omega t) + \sum_{n=1}^{\infty} d_n \cos(n\omega t) . \tag{4}$$

Then, equation (3) can be expressed as follows

$$c_n = a_n \quad (n = 2, 3, \cdots, \infty) \tag{5}$$
$$d_n = b_n \quad (n = 2, 3, \cdots, \infty) .$$

If this compensation current is injected into the power system, then the compensated source current i_{SC} becomes pure sinusoidal, and the voltage drop in the source impedance also becomes pure sinusoidal. Thus, the load voltage v_L becomes pure sinusoidal, too.

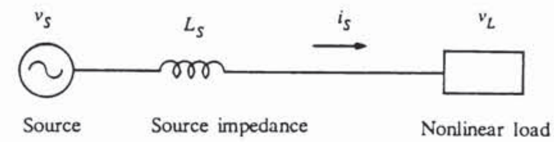


Fig. 1. Power System with a Nonlinear Load.

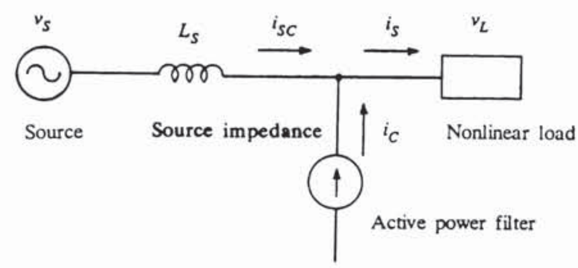


Fig. 2. Harmonics Compensation by the Active Power Filter.

OPTIMAL PWM METHOD FOR CURRENT SOURCE TYPE ACTIVE POWER FILTERS

Current Source type Active Power Filters

Figure 3 shows a current source type active power filter. This filter is composed of an inductor and a current-fed converter. The inductor acts as a constant d.c. current source, and the current-fed converter acts as a harmonic current-fed inverter to generate the compensation current i_C. Moreover, to compensate the loss of the inductor and the switching devices, this current-fed converter should act not only as a harmonic current-fed inverter, but also as a voltage rectifier.

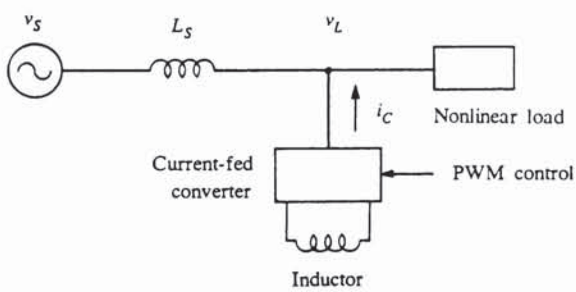


Fig. 3. Simplified diagram for the Current Source type Active Power Filter.

Analysis of the Current-fed Converter

Figure 4 shows a single phase current-fed converter. In the figure, as far as the d.c. side current i_1 is maintained positive, this current-fed converter can be analyzed using a time-varying gain $G(t)$ as follows.

$$i_2(t) = G(t)\, i_1(t) \tag{6}$$
$$v_1(t) = G(t)\, v_2(t) ,$$

where $G(t)$ has values of TABLE 1 according to the switching combination of TRs (power transistors).

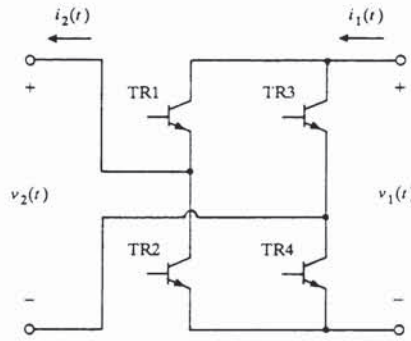


Fig. 4. A Single phase Current-fed Converter.

TABLE 1 Values of $G(t)$

Conducting TRs	Value of $G(t)$
TR1, TR4	+1
TR2, TR3	−1
TR1, TR2	0
TR3, TR4	0

Figure 5 shows an example of $G(t)$. Consider that $G(t)$ is expanded by the Fourier series.

$$G(t) = \sum_{n=1}^{\infty} g_n \sin(n\omega t) + \sum_{n=1}^{\infty} h_n \cos(n\omega t) . \tag{7}$$

Then, the Fourier coefficients g_n, h_n can be expressed by the switching angles ($\alpha_i s$ and $\beta_i s$) as follows.

$$g_n = -\frac{1}{n\pi} \sum_{i=1}^{N} [\cos(n\beta_i) - \cos(n\alpha_i)] \tag{8}$$
$$h_n = +\frac{1}{n\pi} \sum_{i=1}^{N} [\sin(n\beta_i) - \sin(n\alpha_i)]$$
$$(n = 1, 2, \cdots, \infty) .$$

(When $\alpha_i < \beta_i$, the pulse is (+) pulse, and when $\alpha_i > \beta_i$, the pulse is (−) pulse).

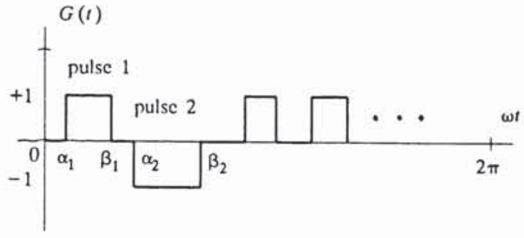


Fig. 5. An example of $G(t)$.

Optimal PWM Method

Figure 6 shows the equivalent circuit for the current source type active power filter. In the figure, R is the equivalent resistance of the inductor and the switching devices. Assume that the filter operates in the normal state, so that the load voltage v_L is pure sinusoidal.

$$v_L(t) = V_L \sin(\omega t) . \tag{9}$$

And also assume that the inductor current i_D is maintained constant.

$$i_D(t) = I_D . \tag{10}$$

Then, the compensation current i_C can be expressed as follows.

$$\begin{aligned} i_C(t) &= G(t)\, i_D(t) \\ &= G(t)\, I_D \\ &= \sum_{n=1}^{\infty} I_D g_n \sin(n\omega t) + \sum_{n=1}^{\infty} I_D h_n \cos(n\omega t) . \end{aligned} \tag{11}$$

So, comparing (11) with (4),

$$\begin{aligned} c_n &= I_D g_n \quad (n = 1, 2, \cdots, \infty) \\ d_n &= I_D h_n \quad (n = 1, 2, \cdots, \infty) . \end{aligned} \tag{12}$$

To eliminate the harmonic current exactly, comparing (12) with (5),

$$\begin{aligned} I_D g_n &= a_n \quad (n = 2, 3, \cdots, \infty) \\ I_D h_n &= b_n \quad (n = 2, 3, \cdots, \infty) . \end{aligned} \tag{13}$$

Now, in the d.c. side, the inductor voltage v_D is

$$v_D(t) = G(t)\, v_L(t) . \tag{14}$$

And the inductor current i_D is

$$R i_D(t) + L \frac{di_D(t)}{dt} = -v_D(t) . \tag{15}$$

In the steady state, with large L, the ripple of i_D is negligible, and i_D can well be approximated by its d.c. component. So, let V_D, I_D be the d.c. component of v_D, i_D, respectively, then

$$R I_D = -V_D . \tag{16}$$

V_D can be expressed using equation (9) and (14) as follows

$$\begin{aligned} V_D &= \frac{1}{2\pi} \int_0^{2\pi} v_D(t)\, dt \\ &= \frac{1}{2\pi} \int_0^{2\pi} G(t) V_L \sin(\omega t)\, dt \\ &= \frac{V_L}{2} g_1 . \end{aligned} \tag{17}$$

So, to maintain the inductor current at the value I_D, g_1 should satisfy the following equation.

$$-\frac{V_L}{2R} g_1 = I_D . \tag{18}$$

Moreover, to compensate the reactive current by the amount I_R, h_1 should satisfy the following equation.

$$I_D h_1 = I_R . \tag{19}$$

In conclusion, the following equations are obtained.

$$\begin{aligned} -\frac{V_L}{2R} g_1 &= I_D \\ I_D h_1 &= I_R \\ I_D g_n &= a_n \quad (n = 2, 3, \cdots, \infty) \\ I_D h_n &= b_n \quad (n = 2, 3, \cdots, \infty) . \end{aligned} \tag{20}$$

Now, in equations (8), the control variables are $\alpha_i s$ and $\beta_i s$. And if the number of pulses per one cycle is N, then the number of control variables is 2N. Therefore, it is not possible to satisfy equations (20) with these 2N control variables because the number of equations (20) is infinite. But it is possible to make 2N equations by giving a performance index to equations (20). One reasonable performance index is eliminating the lower order harmonics up to the N-th order. Then, equations (20) are changed into the following 2N-equations.

$$\begin{aligned} -\frac{V_L}{2R} g_1 &= I_D \\ I_D h_1 &= I_R \\ I_D g_n &= a_n \quad (n = 2, 3, \cdots, N) \\ I_D h_n &= b_n \quad (n = 2, 3, \cdots, N) . \end{aligned} \tag{21}$$

Substituting equations (8) into equations (21), a set of nonlinear simultaneous equations is obtained in the following form.

$$\mathbf{f}(\mathbf{x}) = \mathbf{b} . \tag{22}$$

By solving equations (22), the switching angles $\alpha_i s$ and $\beta_i s$ are determined. But equations (22) are not always solved. So it is desirable to change (22) into the optimization problem.

$$\min \sum_{k=1}^{2N} [f_k(\mathbf{x}) - b_k]^2 . \tag{23}$$

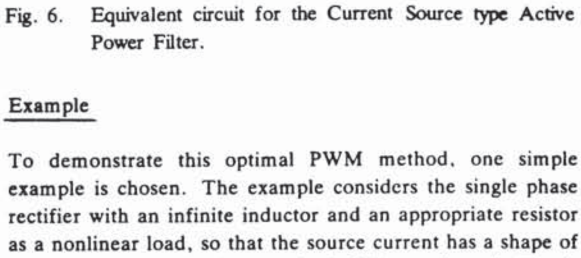


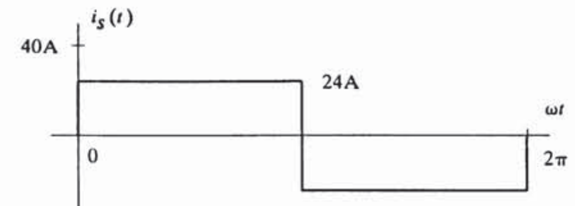
Fig. 6. Equivalent circuit for the Current Source type Active Power Filter.

Example

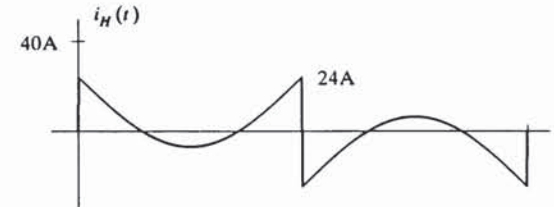
To demonstrate this optimal PWM method, one simple example is chosen. The example considers the single phase rectifier with an infinite inductor and an appropriate resistor as a nonlinear load, so that the source current has a shape of the square wave with the magnitude 24A, as shown in Fig. 7. (a). The inductor current I_D is set to 20A, the magnitude of the load voltage V_L is set to 311V, the system frequency is set to 60Hz, the number of pulses per one cycle N is set to 14, and the loss and the reactive current requirements are assumed to be zero (only for simplicity). Then, the solutions of equations (21) for this example are obtained as shown in TABLE 2 by nonlinear equations solving. In this example, the lower order harmonics up to the 14-th order are completely eliminated by the active power filter.

TABLE 2 Solutions for the example case

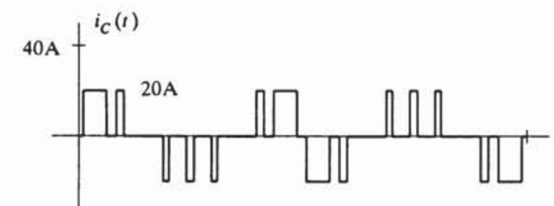
α_i (rad.)	β_i (rad.)	$G(t)$
0.06533	0.38995	+1.
0.53204	0.63915	+1.
1.19232	1.27287	−1.
1.51462	1.62697	−1.
1.86872	1.94927	−1.
2.50244	2.60955	+1.
2.75164	3.07626	+1.
3.20692	3.53154	−1.
3.67363	3.78074	−1.
4.33391	4.41446	+1.
4.65622	4.76856	+1.
5.01032	5.09087	+1.
5.64404	5.75115	−1.
5.89324	6.21786	−1.



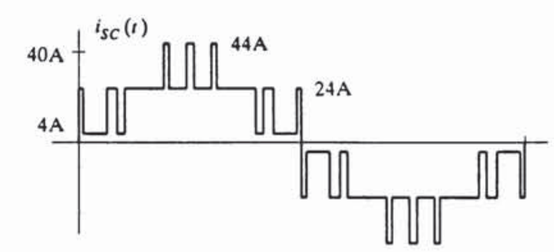
(a) Source current i_S.



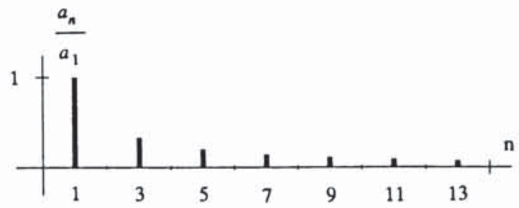
(b) Harmonic current i_H.



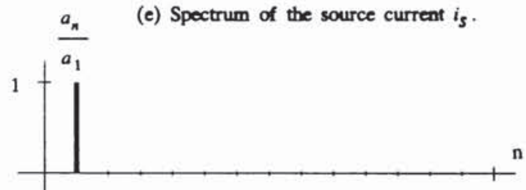
(c) Compensation current i_C.



(d) Compensated source current i_{SC}.



(e) Spectrum of the source current i_S.



(f) Spectrum of the compensated source current i_{SC}.

Fig. 7. An example of Optimal PWM Method for the Current Source type Active Power Filter.

OPTIMAL PWM METHOD FOR VOLTAGE SOURCE TYPE ACTIVE POWER FILTERS

Voltage Source type Active Power Filters

Figure 8 shows a voltage source type active power filter. This filter is composed of a capacitor, a voltage-fed converter, and a compensation inductor L_C. The capacitor acts as a constant d.c. voltage source, and the voltage-fed converter acts as a harmonic voltage-fed inverter. Then, the compensation current i_C is made by the voltage difference $v_C - v_L$ through the compensation inductor L_C. To compensate the loss of the capacitor and the switching devices, the voltage-fed converter should act not only as a harmonic voltage-fed inverter, but also as a current rectifier.

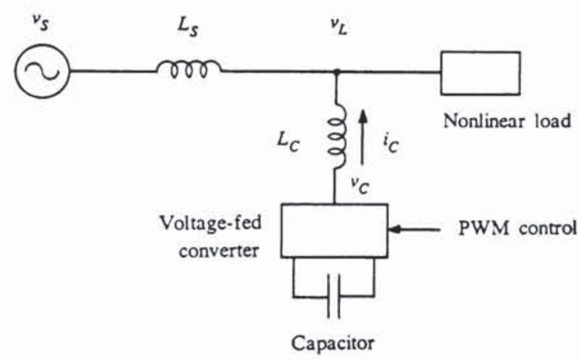


Fig. 8. Simplified diagram for the Voltage Source type Active Power Filter.

Analysis of the Voltage-fed Converter

Figure 9 shows a single phase voltage-fed converter. In the figure, as far as the d.c. side voltage v_1 is maintained positive, this voltage-fed converter can be analyzed using a time-varying gain $G(t)$ as follows.

$$v_2(t) = G(t)\, v_1(t) \qquad (24)$$
$$i_1(t) = G(t)\, i_2(t)\ .$$

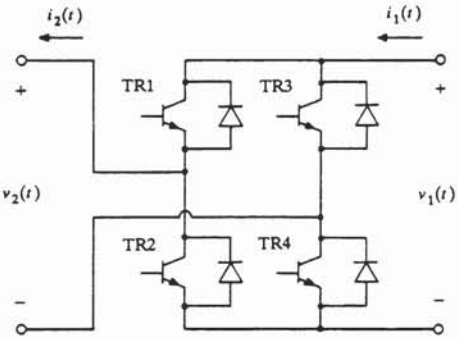


Fig. 9. A Single phase Voltage-fed Converter.

Optimal PWM Method

Figure 10 shows the equivalent circuit for the voltage source type active power filter. In the figure, R is the equivalent resistance of the capacitor and the switching devices. Assume that the filter operates in the normal state, so that the load voltage v_L is pure sinusoidal.

$$v_L(t) = V_L \sin(\omega t)\ . \qquad (25)$$

And also assume that the capacitor voltage v_D is maintained constant.

$$v_D(t) = V_D\ . \qquad (26)$$

Then, the compensation voltage v_C can be expressed as follows.

$$v_C(t) = G(t)\, v_D(t) = G(t)\, V_D\ . \qquad (27)$$

In the a.c. side, by KVL,

$$L\frac{di_C(t)}{dt} = v_C(t) - v_L(t)\ . \qquad (28)$$

The left hand side of (28) is

$$L\frac{di_C(t)}{dt} = -\sum_{n=1}^{\infty} n\omega L d_n \sin(n\omega t) + \sum_{n=1}^{\infty} n\omega L c_n \cos(n\omega t)\ . \qquad (29)$$

The right hand side of (28) is

$$v_C(t) - v_L(t) = \sum_{n=1}^{\infty} V_D g_n \sin(n\omega t) + \sum_{n=1}^{\infty} V_D h_n \cos(n\omega t) - V_L \sin(\omega t)\ . \qquad (30)$$

So, equating (29) with (30),

$$c_n = \frac{V_D}{n\omega L} h_n \quad (n = 1, 2, \cdots, \infty)$$
$$d_1 = -\frac{V_D}{\omega L} g_1 + \frac{V_L}{\omega L} \qquad (31)$$
$$d_n = -\frac{V_D}{n\omega L} g_n \quad (n = 2, 3, \cdots, \infty)\ .$$

To eliminate the harmonic current exactly, comparing (31) with (5),

$$\frac{V_D}{n\omega L} h_n = a_n \quad (n = 2, 3, \cdots, \infty) \qquad (32)$$
$$-\frac{V_D}{n\omega L} g_n = b_n \quad (n = 2, 3, \cdots, \infty)\ .$$

Now, in the d.c. side, the capacitor current i_D is

$$i_D(t) = G(t)\, i_C(t)\ . \qquad (33)$$

And the capacitor voltage v_D is

$$\frac{v_D(t)}{R} + C\frac{dv_D(t)}{dt} = -i_D(t)\ . \qquad (34)$$

In the steady state, with large C, the ripple of v_D is negligible, and v_D can well be approximated by its d.c. component. So, let I_D, V_D be the d.c. component of i_D, v_D, respectively, then,

$$\frac{V_D}{R} = -I_D\ . \qquad (35)$$

I_D can be expressed using equation (31) and (33) as follows

$$\begin{aligned} I_D &= \frac{1}{2\pi}\int_0^{2\pi} i_D(t)\, dt \\ &= \frac{1}{2\pi}\int_0^{2\pi} G(t)\,[\sum_{n=1}^{\infty} c_n \sin(n\omega t) + \sum_{n=1}^{\infty} d_n \cos(n\omega t)]\, dt \\ &= \frac{1}{2}[\sum_{n=1}^{\infty} c_n g_n + \sum_{n=1}^{\infty} d_n h_n] \\ &= \frac{V_L}{2\omega L} h_1\ . \end{aligned} \qquad (36)$$

So, to maintain the capacitor voltage at the value V_D, h_1 should satisfy the following equation.

$$-\frac{RV_L}{2\omega L} h_1 = V_D\ . \qquad (37)$$

Moreover, to compensate the reactive current by the amount I_R, g_1 should satisfy the following equation.

$$-\frac{V_D}{\omega L} g_1 + \frac{V_L}{\omega L} = I_R\ . \qquad (38)$$

In conclusion, the following equations are obtained.

$$\begin{aligned} -\frac{V_D}{\omega L} g_1 + \frac{V_L}{\omega L} &= I_R \\ -\frac{RV_L}{2\omega L} h_1 &= V_D \\ -\frac{V_D}{n\omega L} g_n &= b_n \quad (n = 2, 3, \cdots, \infty) \\ \frac{V_D}{n\omega L} h_n &= a_n \quad (n = 2, 3, \cdots, \infty)\ . \end{aligned} \qquad (39)$$

Now, by giving a performance index to equations (39), a set of nonlinear simultaneous 2N-equations can be obtained as the case of the current source type. And by solving these nonlinear equations, the switching angles $\alpha_i s$ and $\beta_i s$ are determined.

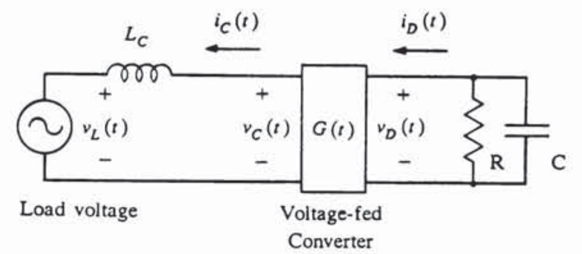
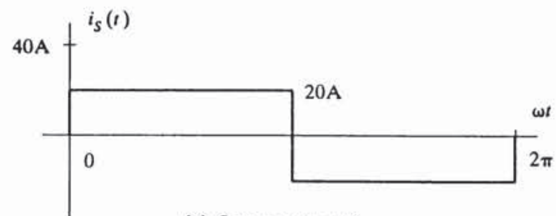


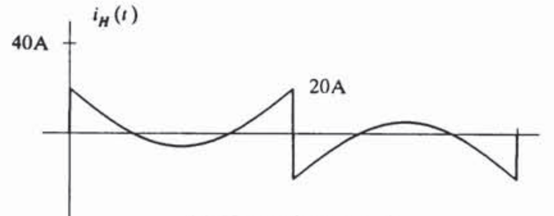
Fig. 10. Equivalent circuit for the Voltage Source type Active Power Filter.

<u>Example</u>

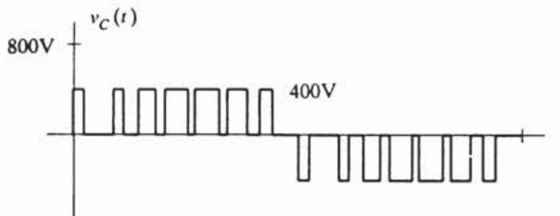
To demonstrate this optimal PWM method, one simple example is chosen, as is the case of the current source type. The magnitude of the load voltage V_L is set to 311V, the system frequency is set to 60Hz, the capacitor voltage V_D is set to 400V, the inductance of the compensation inductor L_C is set to 5mH, the number of pulses per one cycle N is set to 14, and the loss and the reactive current requirements are assumed to be zero (only for simplicity). In this example, too, the lower order harmonics up to the 14-th order are completely eliminated by the active power filter.



(a) Source current i_S.



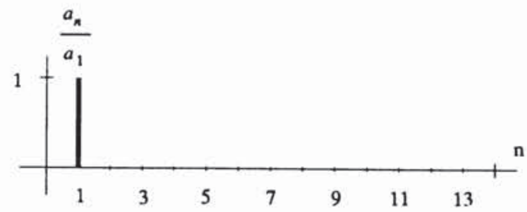
(b) Harmonic current i_H.



(c) Compensation voltage v_C.

(d) Compensation current i_C.

(e) Compensated source current i_{SC}.



(f) Spectrum of the compensated source current i_{SC}.

Fig. 11. An example of Optimal PWM Method for the Voltage Source type Active Power Filter.

CONCLUSION

In this paper, new PWM methods for both the current source type and the voltage source type active power filter are developed, intended particularly for obtaining the maximum efficiency of the compensation. Using these methods, twice as many lower order harmonics of the power system as the number of pulses per one cycle can be completely eliminated by the active power filter. So, these methods are much more efficient than any other existing method. The PWM controller is implemented using a micro-processor. But the PWM methods can not operate instantaneously because of the computational burden. So, these methods can be adapted only to the steady state load condition, for the time being.

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