Practice quiz on Bayes Theorem and the Binomial Theorem
Cuestionario Práctico • 25 min

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Practice quiz on Bayes Theorem and the Binomial Theorem

PUNTOS TOTALES DE 9

1. A jewelry store that serves just one customer at a time is concerned about the safety of its isolated customers.

   1 / 1 puntos

   The store does some research and learns that:

   - 10% of the times that a jewelry store is robbed, a customer is in the store.
   - A jewelry store has a customer on average 20% of each 24-hour day.
   - The probability that a jewelry store is being robbed (anywhere in the world) is 1 in 2 million.

   What is the probability that a robbery will occur while a customer is in the store?

   - ○ $\frac{1}{500000}$
   - ○ $\frac{1}{2000000}$
   - ◉ $\frac{1}{4000000}$
   - ○ $\frac{1}{5000000}$

Correcto

What is known is:

A: "a customer is in the store," $P(A) = 0.2$

B: "a robbery is occurring," $P(B) = \frac{1}{2{,}000{,}000}$

$P(\text{a customer is in the store} \mid \text{a robbery occurs}) = P(A \mid B)$

$P(A \mid B)$ = 10%

What is wanted:

$P(\text{a robbery occurs} \mid \text{a customer is in the store}) = P(B \mid A)$

By the product rule:

$$P(B \mid A) = \frac{P(A, B)}{P(A)}$$

and $P(A, B) = P(A \mid B)P(B)$

Therefore:

$$P(B \mid A) = \frac{P(A \mid B)P(B)}{P(A)} = \frac{(0.1)\left(\frac{1}{2000000}\right)}{0.2} = \frac{1}{4000000}$$

2. If I flip a fair coin, with heads and tails, ten times in a row, what is the probability that I will get exactly six heads?

1 / 1 puntos

- ○ 0.021
- ○ 0.187
- ◉ 0.2051
- ○ 0.305

✓ **Correcto**

By Binomial Theorem, equals

$$\binom{10}{6}\left(0.5^{10}\right)$$

$$= \left(\frac{10!}{4! \times 6!}\right)\left(\frac{1}{1024}\right)$$
$$= 0.2051$$

3. If a coin is bent so that it has a 40% probability of coming up heads, what is the probability of getting *exactly* 6 heads in 10 throws?

1 / 1 puntos

- ○ 0.0974
- ○ 0.1045
- ◉ 0.1115
- ○ 0.1219

✓ **Correcto**

$\binom{10}{6} \times 0.4^6 \times 0.6^4 = 0.1115$

4. A bent coin has 40% probability of coming up heads on each independent toss. If I toss the coin ten times, what is the probability that I get at least 8 heads?

1 / 1 puntos

- ○ 0.0213
- ◉ 0.0123
- ○ 0.0132
- ○ 0.0312

✓ **Correcto**

The answer is the sum of three binomial probabilities:

$$(\binom{10}{8} \times (0.4^8) \times (.6^2)) + (\binom{10}{9} \times (0.4^9) \times (0.6^1))+$$

$$(\binom{10}{10}) \times (0.4^{10}) \times (0.6^0))$$

5. Suppose I have a bent coin with a 60% probability of coming up heads. I throw the coin ten times and it comes up heads 8 times.

1 / 1 puntos

What is the value of the “likelihood” term in Bayes’ Theorem -- the conditional probability of the data given the parameter.

- ○ 0.122885
- ○ 0.043945
- ○ 0.168835
- ◉ 0.120932

Correcto

Bayesian “likelihood” --- the p(observed data | parameter) is

p(8 of 10 heads | coin has p = .6 of coming up heads)

$$\binom{10}{8} \times (0.6^8) \times (0.4^2) = 0.120932$$

6. We have the following information about a new medical test for diagnosing cancer.

1 / 1 puntos

Before any data are observed, we know that 5% of the population to be tested actually have Cancer.

Of those tested who do have cancer, 90% of them get an accurate test result of “Positive” for cancer. The other 10% get a false test result of “Negative” for Cancer.

Of the people who do not have cancer, 90% of them get an accurate test result of “Negative” for cancer. The other 10% get a false test result of “Positive” for cancer.

What is the conditional probability that I have Cancer, if I get a “Positive” test result for Cancer?

**Formulas in the feedback section are very long, and do not fit within the standard viewing window. Therefore, the font is a bit smaller and the word "positive test" has been abbreviated as PT.

- ○ 67.9%
- ○ 4.5%
- ○ 9.5%
- ◉ 32.1% probability that I have cancer

Correcto

I still have a more than $\frac{2}{3}$ probability of not having cancer

Posterior probability:

p(I actually have cancer | receive a "positive" Test)

By Bayes Theorem:

$$= \frac{\text{(chance of observing a PT if I have cancer)(prior probability of having cancer)}}{\text{(marginal likelihood of the observation of a PT)}}$$

$$= \frac{p(\text{receiving positive test}|\text{ has cancer})p(\text{has cancer }[\text{before data is observed}])}{p(\text{positive }|\text{ has cancer})p(\text{has cancer})+p(\text{positive }|\text{ no cancer })p(\text{no cancer})}$$

= (90%)(5%) / ((90%)(5%) + (10%)(95%)

=32.1%

7. We have the following information about a new medical test for diagnosing cancer.

1 / 1 puntos

Before any data are observed, we know that 8% of the population to be tested actually have Cancer.

Of those tested who do have cancer, 90% of them get an accurate test result of "Positive" for cancer.

The other 10% get a false test result of "Negative" for Cancer.

Of the people who do not have cancer, 95% of them get an accurate test result of "Negative" for cancer.

The other 5% get a false test result of "Positive" for cancer.

What is the conditional probability that I have cancer, if I get a "Negative" test result for Cancer?

- ○ 88.2%
- ○ 99.1%
- ○ $.80\%$
- ◉ 0.9%

Correcto

$p(\text{cancer} \mid \text{negative test}) =$

$$\frac{p(\text{negative test} \mid \text{Cancer})\, p(\text{Cancer})}{p(\text{negative test} \mid \text{cancer})\, p(\text{cancer}) + p(\text{negative test} \mid \text{no cancer})\, p(\text{no cancer})}$$

$$\frac{(10\%)(8\%)}{(10\%)(8\%)+(95\%)(92\%)}$$

$$\frac{0.8\%}{0.8\%+87.4\%}$$

$$\frac{0.8\%}{88.2\%}$$

$$= 0.9\%$$

8. An urn contains 50 marbles – 40 blue and 10 white. After 50 draws, exactly 40 blue and 10 white are observed.

0 / 1 puntos

You are not told whether the draw was done “with replacement” or “without replacement.”

What is the probability that the draw was done with replacement?

- (selected) 13.98%
- 87.73%
- 1
- 12.27%

Incorrecto

This is the likelihood of
getting exactly 40 white and 10 blue with replacement. But we also need to take into account the likelihood (which equals 1) of getting 40 white and 10 blue without replacement).

This is the probability that the draw was done without replacement.

p(40 blue and 10 white | draws without replacement) = 1 [this is the only possible outcome when 50 draws are made without replacement]

p(40 blue and 10 white | draws with replacement)

S = 40

N = 50

P = .8 [for draws with replacement] because 40 blue of 50 total means p(blue) = 40/50 = .8

$$\left(\binom{50}{40}\right)(0.8^{40})(0.2^{10})$$

$$= 13.98\%$$

By Bayes' Theorem:

p(draws with replacement | observed data) =

$$\frac{13.98\%(.5)}{(13.98\%)(.5)+(1)(.5)}$$

$$= \frac{0.1398}{1.1398}$$

$$= 12.27\%$$

9. According to Department of Customs Enforcement Research: 99% of people crossing into the United States are not smugglers.

1 / 1 puntos

The majority of all Smugglers at the border (65%) appear nervous and sweaty.

Only 8% of innocent people at the border appear nervous and sweaty.

If someone at the border appears nervous and sweaty, what is the probability that they are a Smuggler?

- (•) 7.58%
- ( ) 92.42%
- ( ) 8.57%
- ( ) 7.92%

✓ **Correcto**

By Bayes' Theorem, the answer is

$$\frac{(.65)(.01)}{((.65)(.01)+(.08)(.99))}$$

$$= 7.58\%$$