15.32

lördag 24 december 2022 12:54

$$f(0) = 0$$

$$f(x) = x + \int_0^x \frac{2 + f(t)}{1+t^2}\,dt \iff f'(x) = 1 + \frac{2x f(x)}{1+x^2} \iff$$

$$\iff f'(x) - \frac{2x}{1+x^2} f(x) = 1$$

$$g(x) = -\frac{2x}{1+x^2} \qquad G(x) = -\ln(1+x^2) \qquad IF = e^{-\ln(1+x^2)} = \frac{1}{1+x^2}$$

$$f(x) \cdot e^{-\ln(1+x^2)} = \int e^{-\ln(1+x^2)}\,dx = \int \frac{1}{1+x^2}\,dx = \arctan x + C$$

$$f(x) = (1+x^2)\arctan x + C(1+x^2)$$

$$f(0) = 0 + C \qquad C = 0$$

Sv: $f(x) = (1+x^2)\arctan x$