15.41

torsdag 5 januari 2023 10:22

$$y'(0) = y'(l) = 0 \quad , \quad l > 0$$

$$y'' + \lambda y = 0$$

$$P(r) = r^2 + \lambda = 0 \qquad r = \pm\sqrt{-\lambda}$$

$\lambda > 0$:

$$r = \pm i\sqrt{\lambda}$$

$$y = A\cos\sqrt{\lambda}x + B\sin\sqrt{\lambda}x$$

$$y' = -\sqrt{\lambda}A\sin\sqrt{\lambda}x + \sqrt{\lambda}B\cos\sqrt{\lambda}x$$

$$y'(0) = \underbrace{-\sqrt{\lambda}A\sin 0}_{0} + \sqrt{\lambda}B\underbrace{\cos 0}_{1} = 0 \qquad \boxed{B = 0}$$

$$y'(l) = -\sqrt{\lambda}A\sin\sqrt{\lambda}l + \underbrace{\sqrt{\lambda}B\cos\sqrt{\lambda}l}_{0} = 0$$

$A \neq 0$, $\sqrt{\lambda} > 0$

$$\sqrt{\lambda}l = 0 \implies \sin\sqrt{\lambda}l = 0$$

$$\sqrt{\lambda}l = n\pi \,, \; n = \text{heltal} > 0 \iff \lambda = \frac{(n\pi)^2}{l^2}$$

$$y = A\cos\frac{n\pi}{l}x$$

$\lambda = 0$:

$$r = 0$$

$$y = (C_1 x + C_2)e^0 = C_1 x + C_2$$

$$y' = C_1$$

$$y'(0) = C_1 = 0 \qquad \boxed{C_1 = 0}$$

$$y'(l) = C_1 = 0$$

$y = C_2 \neq y(x)$ → har ej x dvs ej en lösning på y som beror av x

$\lambda < 0$:

$$r = \pm\sqrt{-\lambda} \in \mathbb{R}$$

$$y = C_1 e^{\sqrt{-\lambda}x} + C_2 e^{-\sqrt{-\lambda}x}$$

$$y' = \sqrt{-\lambda}C_1 e^{\sqrt{-\lambda}x} - \sqrt{-\lambda}C_2 e^{-\sqrt{-\lambda}x} =$$

$$= \sqrt{-\lambda}\left(C_1 e^{\sqrt{-\lambda}x} - C_2 e^{-\sqrt{-\lambda}x}\right)$$

$$y'(0) = \sqrt{-\lambda}\left(C_1 - C_2\right) = 0$$

$$y'(l) = \sqrt{-\lambda}\left(C_1 e^{\sqrt{-\lambda}l} - C_2 e^{-\sqrt{-\lambda}l}\right) =$$

$$= \sqrt{-\lambda}C_1\left(e^{\sqrt{-\lambda}l} - e^{-\sqrt{-\lambda}l}\right) = 0$$

$$\boxed{C_1 = 0}$$

$y = 0$ ←

$$C_1 - C_2 = 0$$
$$C_1 = C_2$$

$y = 0$ → trivial → $\lambda \neq 0$

Sv: $y = A\cos\frac{n\pi}{l}x$, $n = \text{heltal} > 0$, $\lambda = \frac{(n\pi)^2}{l^2}$