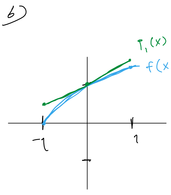
11.03

$f(x) = \sqrt{1+x}$

a) $P_1(x) = f(0) + f'(0)x = 1 + \frac{1}{2}x$

$P_1(x) \approx \lim_{x \to 0} f(x)$

b)



- $P_1(x) = 1 + 1/2\,x$
- $f(x) = \sqrt{1+x}$

c) $R_2(x) = f(x) - P_1(x)$

$f''(x) = -\frac{1}{4} \cdot \frac{1}{(1+x)^{3/2}}$

$\frac{f''(\theta x)}{2!}x^2 = \frac{1}{2}\left(-\frac{1}{4}\right) \cdot \frac{1}{(1+\theta x)^{3/2}} x^2 \quad 0 \le \theta \le 1$

Sv: $R_1(x) = -\frac{1}{8(1+\theta x)^{3/2}} \cdot x^2$

d) $|R_2(x)| = \left|-\frac{1}{8(1+\theta x)^{3/2}}x^2\right| = \frac{1}{|8(1+\theta x)^{3/2}|} \cdot x^2 \le$

$\le \frac{1}{|8(1+0\cdot x)^{3/2}|}x^2 = \frac{1}{8}x^2$ V.S.V

$0 \le \theta \quad 0 \le x$

e) $|R_2(x)| \le 5 \cdot 10^{-3} \quad 0 \le x \le 0{,}1 \quad 0 \le \theta \le 1$

$|R_2(x)| = \frac{1}{|8(1+\theta x)^{3/2}|}x^2 \le \frac{1}{8}x^2 \le \frac{1}{8} \cdot \frac{1}{10^2} =$ ($x \le 0{,}1$)

$= \frac{5}{40} \cdot \frac{1}{10^2} = 5 \cdot \frac{1}{4} \cdot \frac{1}{10^3} < 5 \cdot 10^{-3}$ V.S.V

f) $|R_2(x)| \le 5 \cdot 10^{-4} \quad 0 \le x < a \quad 0 \le \theta \le 1$

$|R_2(x)| \le \frac{1}{8(1+\theta x)^{3/2}}x^2 \le 5 \cdot 10^{-4}$

$|R_2(x)| \le \frac{1}{8}x^2 = \frac{5}{40} \cdot x^2 = 5 \cdot 10^{-1} \cdot \frac{1}{4}x^2 \le 5 \cdot 10^{-4} \cdot \frac{1}{4} < 5 \cdot 10^{-4}$ ($x < a$, $a = 10^{-3/2}$)

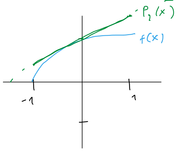
Sv: a kan vara $10^{-3/2}$ $\quad 10^{-3/2} = \sqrt{0{,}001} \approx 0{,}0316$

g) $f(x) = \sqrt{1+x} \quad f'(x) = \frac{1}{2} \cdot \frac{1}{\sqrt{1+x}} \quad f''(x) = -\frac{1}{4} \cdot \frac{1}{(1+x)^{3/2}}$

$P_2(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2$

Sv: $P_2(x) = 1 + 1/2\,x - \frac{1}{8}x^2$

h)



I) $R_3(x) = f(x) - P_2(x)$

$f'''(x) = \frac{3}{8} \cdot \frac{1}{(1+x)^{5/2}}$

$\frac{f'''(\theta x)}{3!}x^3 = \frac{1}{16(1+\theta x)^{5/2}}x^3 \quad 0 \le \theta \le 1$

j) $|R_3(x)| \le 1/16\,x^3, \quad x \ge 0$

$|R_3(x)| = \left|\frac{1}{16(1+\theta x)^{5/2}}x^3\right| = \frac{1}{16(1+\theta x)^{5/2}}x^3 \le \frac{1}{16(1+0\cdot x)^{5/2}}x^3 = \frac{1}{16}x^3$ V.S.V

($0 \le \theta$, $0 \le x$)

k) $|R_3(x)| \le \frac{1}{16}x^3 \quad 0 \le x \le 0{,}1$

$|R_3(x)| \le \frac{1}{16}(10^{-1})^3 = \frac{1}{16} \cdot 10^{-3} = 6{,}25 \cdot 10^{-2} \cdot 10^{-3} = 6{,}25 \cdot 10^{-5}$

($x \le 0{,}1$)

$1/4 = 0{,}25$

$1/8 = 0{,}125$

$1/16 = 0{,}0625$