12.32

onsdag 21 december 2022 13:06

a)

$$\int \sin x\,dx = \int \frac{\sin\left(2\cdot\frac{x}{2}\right)}{1}dx = \int \frac{2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}{\cos^2\left(2\frac{x}{2}\right)+\sin^2\left(2\frac{x}{2}\right)}dx = \int \frac{2\,\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}}{1+\frac{\sin^2\left(\frac{x}{2}\right)}{\cos^2\left(\frac{x}{2}\right)}}dx = \int \frac{2\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}dx =$$

$$= \left[\begin{array}{ll} t=\tan\frac{x}{2} & x=2\arctan t \\ \frac{dx}{dt}=\frac{2}{1+t^2} & dx=\frac{1}{1+t^2}dt \end{array}\right] = \int \frac{2t}{1+t^2}\cdot\frac{2}{1+t^2}dt = 2\int \frac{t}{\cancel{1+t^2}}\cdot\frac{\cancel{1+t^2}}{(1+t^2)^2}dt = 2\int\frac{t}{(1+t^2)^2}dt = \frac{1}{1+t^2}+C = \frac{1}{1+\tan^2\left(\frac{x}{2}\right)}+C$$

V.S.V

b)

$$\int \frac{3}{4+5\sin x}dx = \left[\begin{array}{ll} t=\tan(x/2) & x=2\arctan t \\ \frac{dx}{dt}=\frac{2}{1+t^2} & dx=\frac{2}{1+t^2}dt \end{array}\right] = \int \frac{3}{4+5\cdot\frac{2t}{1+t^2}}\cdot\frac{2}{1+t^2}dt = \int\frac{3\cancel{(1+t^2)}}{4t^2+10t+4}\cdot\frac{2}{\cancel{1+t^2}}dt = \int\frac{3}{2t^2+5t+2}dt =$$

$$= \int \frac{3}{(t+1/2)(t+2)}dt = \int \frac{A}{t+1/2}+\frac{B}{t+2}dt$$

$$3 = At+2A+Bt+\tfrac{1}{2}B = (A+B)t+2A+\tfrac{1}{2}B$$

$A+B=0$ $\quad B=-A$ $\quad \boxed{B=-2}$

$2A+\frac{1}{2}B=3$ $\quad 2A-\frac{1}{2}A=3$ $\quad \frac{3}{2}A=3$ $\quad \boxed{A=2}$

$$\int \frac{2}{t+1/2}-\frac{2}{t+2}dt = 2\ln\left|t+\tfrac{1}{2}\right| - 2\ln|t+2| + C = 2\ln\left|\frac{t+1/2}{t+2}\right|+C = 2\ln\left|\frac{\tan(x/2)+1/2}{\tan(x/2)+2}\right|+C$$