15.50

måndag 26 december 2022 14:30

$$y'' + 2y' + y = e^{x} + e^{-x}$$

(1) $y'' + 2y' + y = e^{x}$ (2) $y'' - 3y' + y = e^{-x}$

$$y = y_h + y_p = y_h + (y_p^1 + y_p^2)$$

- y_h: $p(r) = r^2 + 2r + 1 = 0$ $r = -1 \pm 0$

$r_1 = r_2 = -1$

$$y_h = (C_1 x + C_2) e^{-x}$$

(1)

- y_p^1:

$y_p = z e^{x}$ $y_p' = z' e^{x} + z e^{x} = e^{x}(z' + z)$

$y_p'' = e^{x}(z' + z) + e^{x}(z'' + z') = e^{x}(z'' + 2z' + z)$

$e^{x}(z'' + 2z' + z) + 2e^{x}(z' + z) + z e^{x} = e^{x} \iff$

$\iff z'' + 4z' + 4z = 1$

- $z = A$

$0 + 0 + 4z = 1 \iff A = 1/4 \iff y_p^1 = 1/4\, e^{x}$

$y_p^1 = 1/4\, e^{x}$

(2)

- y_p^2:

$y = z e^{-x}$, $y' = z' e^{-x} - e^{-x} z = e^{-x}(z' - z)$,

, $y'' = -e^{-x}(z' - z) + e^{-x}(z'' - z') = e^{-x}(z'' - 2z' + z)$

$e^{-x}(z'' - 2z' + z) + 2e^{-x}(z' - z) + z e^{-x} = e^{-x} \iff$

$\iff z = Ax^2$, $z' = 2Ax$, $z'' = 2A$

$2A = 1 \iff y_p^2 = 1/2\, x^2 e^{-x}$

$y_2 = C_1 + C_2 e^{-2x} + 1/2\, x^2 e^{-x}$

$$y = y_h + y_p^1 + y_p^2 = C_1 + C_2 e^{-2x} + 1/4\, e^{x} + 1/2\, x^2 e^{-x}$$

$$= C_1 + C_2 e^{-2x} + 1/4\, e^{x} + 1/2\, x^2 e^{-x}$$

Sv: $y = (C_1 x + C_2) e^{-x} + 1/4\, e^{x} + 1/2\, x^2 e^{-x}$