15.08 c

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$$y' - \frac{2x}{1+x^2} y = \arctan x \quad , \quad y(1) = 2$$

$$g(x) = -\frac{2x}{1+x^2} \qquad G_1(x) = -\ln(1+x^2) \qquad IF = e^{-\ln(1+x^2)} = \frac{1}{1+x^2}$$

$$y \cdot e^{-\ln(1+x^2)} = \int \arctan x \cdot e^{-\ln(1+x^2)} dx = \arctan^2 x - \int \frac{1}{1+x^2} \cdot \arctan x$$

$$\int \arctan x \cdot e^{-\ln(1+x^2)} = \tfrac{1}{2} \arctan^2 x + C$$

$$y = \frac{(1+x^2)}{2} \arctan^2 x + C(1+x^2)$$

$$y(1) = \arctan^2 1 + 2C = 2 \iff 2C = 2 - \left(\frac{\pi}{4}\right)^2$$

$$C = \frac{32 - \pi^2}{32}$$

Sv: $y = \frac{1+x^2}{2} \arctan^2 x + \frac{(32-\pi^2)(1+x^2)}{32} = (1+x^2)\left(\tfrac{1}{2}\arctan^2 x + 1 - \pi^2/32\right)$