15.40

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$y'' + \lambda y = 0 \qquad y(0) = y(\ell) = 0 \qquad \ell > 0$

$P(r) = r^2 + \lambda \qquad r = 0 \pm \sqrt{-\lambda}$

$\lambda > 0$:

$r = i\sqrt{\lambda}$

$y = A\cos\sqrt{\lambda}\,x + B\sin\sqrt{\lambda}\,x$

$y(0) = A \underset{1}{\cos 0} + \underbrace{B\sin 0}_{0} = 0 \qquad A = 0$

$y(\ell) = A\cos\sqrt{\lambda}\,\ell + B\sin\sqrt{\lambda}\,\ell = 0 \qquad B \neq 0$ (trivial)

$$\boxed{\begin{array}{l} \sqrt{\lambda}\,\ell = 0 \Rightarrow \sin\sqrt{\lambda}\,\ell = 0 \qquad n = \text{heltal} > 0 \text{ (anners trivial)} \\ \sqrt{\lambda}\,\ell = n\pi \overset{\ell>0}{\Leftrightarrow} \lambda = \dfrac{(n\pi)^2}{\ell^2} \end{array}}$$

Pröva $n=0$ o se vad som händer med $y = B\sin\sqrt{\lambda}\,x$

$$\boxed{y = B\sin\frac{n\pi}{\ell}x \qquad n > 0}$$

$\lambda = 0$:

$r = 0$

$y = (C_1x + C_2)$

$y(0) = C_2 = 0 \qquad C_2 = 0$

$y(\ell) = C_1\ell = 0 \qquad C_1 = 0$

$y = 0 \rightarrow$ trivial $\rightarrow \lambda \neq 0$

$\lambda < 0$: $\Rightarrow \sqrt{-\lambda} \in \mathbb{R}$

$r = \pm\sqrt{-\lambda}$

$y = C_1 e^{\sqrt{-\lambda}\,x} + C_2 e^{-\sqrt{-\lambda}\,x}$

$y(0) = C_1 + C_2 = 0$

$y(\ell) = C_1 e^{\sqrt{-\lambda}\,\ell} + C_2 e^{-\sqrt{-\lambda}\,\ell} = 0$

$\boxed{C_2 = -C_1}$

$\boxed{\ell > 0 \quad \lambda < 0}$

$y(\ell) = C_1\left(e^{\sqrt{-\lambda}\,\ell} - e^{-\sqrt{-\lambda}\,\ell}\right) = 0 \qquad \boxed{\begin{array}{l} C_1 = 0 \\ C_2 = 0 \end{array}}$

$y = 0 \rightarrow$ trivial $\rightarrow \lambda \not< 0$

Sv: $y = B\sin\frac{n\pi}{\ell}x, \quad n > 0, \quad \lambda = \frac{n^2\pi^2}{\ell^2}$