14.29

fredag 23 december 2022 08:19

$y = \ln(1 - x^2) \qquad 0 \le x \le 1/2$

$$L = \int \sqrt{1 + f'(x)^2}\, dx$$

$$L = \int_0^{1/2} \sqrt{1 + \left(-\frac{2x}{1-x^2}\right)^2}\, dx = \int_0^{1/2} \sqrt{1 + \frac{4x^2}{(1-x^2)^2}}\, dx =$$

$$= \int_0^{1/2} \sqrt{\frac{x^4 + 2x + 1}{(1-x^2)^2}}\, dx = \int_0^{1/2} \frac{1 + x^2}{1 - x^2}\, dx \overset{\text{polydiv}}{=} \int_0^{1/2} -1 + \frac{2}{1 - x^2}\, dx =$$

$(1-x)(1+x)$

$$\begin{array}{l} \quad -1 \\ \hline 1 + \cancel{x}^2 \;\big|\; 1 - x^2 \\ -(-1 + \cancel{x}^2) \\ \hline \quad 2 \end{array}$$

$$= \Big[-x\Big]_0^{1/2} + \int_0^{1/2} \frac{2}{(1-x)(1+x)}\, dx =$$

$$= -1/2 + \int_0^{1/2} \frac{A}{1-x} + \frac{B}{1+x}\, dx =$$

$2 = A + Ax + B - Bx = (A-B)x + A + B$

$A - B = 0 \qquad A = B$

$A + B = 2 \qquad \boxed{A = 1 \quad B = 1}$

$$= -\frac{1}{2} + \Big[-\ln|1-x| + \ln|1+x|\Big]_0^{1/2} = -\frac{1}{2} + \left[\ln\left|\frac{1+x}{1-x}\right|\right]_0^{1/2} = \underline{\underline{\ln 3 - 1/2}}$$