11.40

lördag 7 januari 2023 01:22

$$S(x) = \int_0^x \ln(\cos t)\,dt \quad , \quad |x| < \pi/2$$

a)

$$S'(x) = \ln(\cos x)$$

$$S''(x) = -\frac{\sin x}{\cos x} = -\tan x$$

$$S'''(x) = -\frac{1}{\cos^2 x} = -(1+\tan^2 x)$$

D $-\cos^{-2} x = 2\cos^{-3} x \cdot (-\sin x)$

$$S^{(4)}(x) = -\frac{2\sin x}{\cos^3 x}$$

$$S^{(5)}(x) = -\frac{2\cos^4 x - 2\sin x \cdot 3\cos^2 x \cdot (-\sin x)}{\cos^6 x} = -\frac{2\cos^4 x + 2\sin^2 x \cdot 3\cos^2 x}{\cos^6 x} =$$

$$= -\frac{2\cos^2 x + 6\sin^2 x}{\cos^4 x} = -\frac{2}{\cos^2 x} - 6\tan^2 x \cdot \frac{1}{\cos^2 x} = -2\left((1+\tan^2 x) + 3\tan^2 x \cdot (1+\tan^2 x)\right) =$$

$$= -2\,(1+3\tan^2 x)(1+\tan^2 x)$$

b)

$$|R_5(x)| \le \frac{|x|^5}{3} \qquad 0 \le \theta \le 1 \qquad |x| \le \pi/4$$

$$|R_5(x)| = \left| \frac{-2(1+3\tan^2(\theta x))(1+\tan^2(\theta x))}{120} x^5 \right| \le \frac{(1+3)(1+1)}{60}|x|^5 = \frac{4\cdot 2}{4\cdot 15}|x|^5 = \frac{2}{15}|x|^5 \le \frac{1}{3}|x|^5$$

$|x| \le \pi/4$, $\theta \le 1$

$$R_5(x) = S(x) - P_4(x)$$

$$P_4(x) = S(0) + S'(0)x + S''(0)\frac{1}{2}x^2 + S'''(0)\frac{1}{6}x^3 + S^{(4)}(0)\frac{1}{24}x^4 =$$

$$= 0 + 0x + 0\cdot\tfrac{1}{2}x^2 - 1\cdot\tfrac{1}{6}x^3 + 0\cdot\tfrac{1}{24}x^4 =$$

$$= -\tfrac{1}{6}x^3$$