12.33 a

onsdag 21 december 2022 14:12

a)
$$\int \frac{1}{2+\sin x}\,dx = \left[\begin{matrix} t = \tan x/2 & x = 2\arctan t \\ dx = \frac{2}{1+t^2}\,dt & \end{matrix}\right] = \int \frac{1}{2+\frac{2t}{1+t^2}} \cdot \frac{2}{1+t^2}\,dt = \int \frac{\cancel{1+t^2}}{\cancel{2}(t^2+t+1)} \cdot \frac{\cancel{2}}{\cancel{1+t^2}}\,dt = \int \frac{1}{t^2+t+1}\,dt = \int \frac{1}{(t+1/2)^2+3/4}\,dt =$$

$$= \left[\begin{matrix} s = t+1/2 & t = s-1/2 \\ ds = dt & \end{matrix}\right] = \int \frac{1}{s^2+3/4}\,ds = \frac{4}{3}\int \frac{1}{\left(\frac{2s}{\sqrt{3}}\right)^2+1}\,ds = \frac{4}{3}\arctan\left(\frac{2s}{\sqrt{3}}\right)+C = \frac{4}{3}\arctan\left(\frac{2}{\sqrt{3}}\left(t+1/2\right)\right)+C = \frac{4}{3}\arctan\left(\frac{2}{\sqrt{3}}\left(\tan\left(\frac{x}{2}\right)+\frac{1}{2}\right)\right)+C = \frac{4}{3}\arctan\left(\frac{2\tan(x/2)+1}{\sqrt{3}}\right)+C$$