12.34

onsdag 21 december 2022 14:54

a)

$$\int \frac{\sin x}{\sqrt{\cos^2 x + 2\cos x + 3}}\,dx = \left[\begin{array}{l} t = \cos x \\ \frac{dt}{dx} = -\sin x \quad -dt = \sin x\,dx \end{array}\right] = \int -\frac{1}{\sqrt{t^2+2t+3}}\,dt = \int -\frac{1}{\sqrt{(t+1)^2+2}}\,dt = \left[\begin{array}{l} s = t+1 \\ ds = dt \end{array}\right] = \int -\frac{1}{\sqrt{s^2+2}} = -\ln\left|s + \sqrt{s^2+2}\right| + C =$$

$$= -\ln\left|t+1+\sqrt{(t+1)^2+2}\right| + C = -\ln\left|\cos x + 1 + \sqrt{\cos^2 x + 2\cos x + 3}\right| + C$$

b)

$$\int \frac{\sin 2x}{\cos^3 x}\,dx = \int \frac{2\cancel{\cos x}\cdot \sin x}{\cos^2 x \cdot \cancel{\cos x}}\,dx = 2\int \frac{\sin x}{\cos^2 x}\,dx = \left[\begin{array}{l} t = \cos x \\ \frac{dt}{dx} = -\sin x \quad -dt = \sin x\,dx \end{array}\right] = -2\int \frac{1}{t^2}\,dt = 2\cdot\frac{1}{t} + C = \frac{2}{\cos x} + C$$