15.54

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b) $y'' + 4y = 2\sin x - \cos 2x$

$y = y_h + y_p = y_h + (y_p^1 + y_p^2)$

$\underline{y_h}$: $p(r) = r^2 + 4 = 0 \quad r = 0 \pm 2i$

$$\boxed{y_h = A\cos 2x + B\sin 2x}$$

1) $y'' + 4y = 2\sin x$

- hjälp ekv. $y'' + 4y = 2e^{ix} = 2(\cos x + \underbrace{i\sin x}_{Im})$
- $y_p^1 = ze^{ix}$, $y' = z'e^{ix} + ize^{ix} = e^{ix}(z' + iz)$,

$y'' = ie^{ix}(z' + iz) + e^{ix}(z'' + iz') = e^{ix}(z'' + 2iz' - z)$

$\cancel{e^{ix}}(z'' + 2iz' - z) + 4z\cancel{e^{ix}} = 2\cancel{e^{ix}} \Leftrightarrow z'' + 2iz' + 3z = 2$

- $z = A$

$0 + 0 + 3A = 2 \Leftrightarrow z = 2/3 \Leftrightarrow y_p^1 = 2/3\, e^{ix} = 2/3(\cos x + i\sin x) = 2/3\cos x + i\underline{(2/3\sin x)}$

2) $y'' + 4y = -\cos x$

- hjälp ekv. $y'' + 4y = -e^{ix} = -(\underbrace{\cos x}_{re} + i\sin x)$
- $y_p^2 = ze^{ix}$, $y' = ize^{ix} + z'e^{ix} = e^{ix}(z' + iz)$, $y'' = ie^{ix}(z' + iz) + e^{ix}(z'' + iz') = e^{ix}(z'' + 2iz' - z)$

$\cancel{e^{ix}}(z'' + 2iz' - z) + 4z\cancel{e^{ix}} = -\cancel{e^{ix}} \Leftrightarrow z'' + 2iz' + 3z = -1$

- $z = A$

$0 + 0 + 3A = -1 \Leftrightarrow A = -1/3 \Leftrightarrow z = -1/3 \Leftrightarrow y_p^2 = -1/3\, e^{ix} = \underline{-1/3\cos x} + i\sin x) = -1/3\cos x$

Sv: $y = A\cos 2x + B\sin 2x + 2/3\sin x - 1/3\cos x$

c) $y'' + 4y = 1 + \cos 2x$

$y = y_h + y_p = y_h + (y_p^1 + y_p^2)$

$\underline{y_h} = A\cos 2x + B\sin 2x$ (se b))

1) $y'' + 4y = 1$

- $y_p^1 = A$

$0 + 4A = 1 \Leftrightarrow y_p^1 = 1/4$

2) $y'' + 4y = \cos 2x$

- hjälp ekv. $y'' + 4y = e^{i2x} = \underbrace{\cos 2x}_{Re} + i\sin 2x$
- $y_p^2 = ze^{i2x}$, $y' = z'e^{i2x} + 2ize^{i2x} = e^{i2x}(z' + 2iz)$,

$y'' = 2ie^{i2x}(z' + 2iz) + e^{i2x}(z'' + 2iz') = e^{i2x}(z'' + 4iz' - 4z)$

$\cancel{e^{i2x}}(z'' + 4iz' - \cancel{4z}) + \cancel{4z}\cancel{e^{i2x}} = \cancel{e^{i2x}} \Leftrightarrow z'' + 4iz' = 1$

- $z = Ax$

$0 + 4iA = 1 \Leftrightarrow A = 1/4i = -i/4 \Leftrightarrow y_p^2 = -i/4\, x e^{i2x} = -\frac{i}{4}x(\cos 2x + i\sin 2x) = i\left(-\frac{1}{4}x\cos 2x\right) + \underbrace{\frac{1}{4}x\sin 2x}_{re}$

Sv: $y = A\cos 2x + B\sin 2x + 1/4 + 1/4\, x\sin 2x$