15.47

lördag 24 december 2022 15:15

a) $y'' - 3y' + 2y = 6$ $\quad y = y_h + y_p$

y_h: $p(r) = r^2 - 3r + 2 = 0$ $\quad r = 3/2 \pm \sqrt{9/4 - 8/4} = 3/2 \pm 1/2$

$r_1 = 2 \quad r_2 = 1$

$y_h = C_1 e^{2x} + C_2 e^x$

y_p: $y = A$

$0 - 0 + 2A = 6 \quad A = 3 \quad y_p = 3$

Sv: $y = C_1 e^{2x} + C_2 e^x + 3$

b) $y'' - 3y' + 2y = x^2$

y_h: $y_h = C_1 e^{2x} + C_2 e^x$ (se a))

y_p: $y_p = Ax^2 + Bx + C$, $y' = 2Ax + B$, $y'' = 2A$

$2A - 3(2Ax + B) + 2(Ax^2 + Bx + C) = x^2 \Leftrightarrow$

$\Leftrightarrow 2Ax^2 + (2B - 6A)x + 2C + 2A - 3B = x^2$

$2A = 1 \quad A = 1/2$

$2B - 6A = 0 \quad B = 3/2$

$2C + 2A - 3B = 0 \quad C = 7/4$

$y_p = 1/2 x^2 + 3/2 x + 7/4$

Sv: $y = C_1 e^{2x} + C_2 e^x + 1/2 x^2 + 3/2 x + 7/4$

c) $y''' + 3y' + 2y = x^3 + x + 1$

y_h: $p(r) = r^2 + 3r + 2 = 0 \quad r = -3/2 \pm \sqrt{9/4 - 8/4} = -3/2 \pm 1/2$

$r_1 = -1 \quad r_2 = -2$

$y_h = C_1 e^{-2x} + C_2 e^{-x}$

y_p: $y_p = Ax^3 + Bx^2 + Cx + D$, $y' = 3Ax^2 + 2Bx + C$,

$y'' = 6Ax + 2B$

$6Ax + 2B + 3(3Ax^2 + 2Bx + C) + 2(Ax^3 + Bx^2 + Cx + D) =$

$= x^3 + x + 1 \Leftrightarrow$

$\Leftrightarrow 2Ax^3 + (2B + 9A)x^2 + (2C + 6B + 6A)x + 2B + 3C + 2D =$

$= x^3 + x + 1$

$2A = 1 \quad A = 1/2$

$2B + 9A = 0 \quad B = -9/4$

$2C + 6B + 6A = 1 \quad C = 23/4$

$2B + 3C + 2D = 1 \quad D = -47/8$

$y_p = 1/2 x^3 - 9/4 x^2 + 23/4 x - 47/8$

$y = C_1 e^{-2x} + C_2 e^{-x} + 1/2 x^3 - 9/4 x^2 + 23/4 x - 47/8$

d) $y'' + 2y' = x^2 + 1$

y_h: $p(r) = r^2 + 2r = 0 \quad r = -1 \pm 1 \quad r_1 = 0$

$r_2 = -2$

$y_h = C_1 + C_2 e^{-2x}$

y_p: $y_p = (Ax^2 + Bx + C)x = Ax^3 + Bx^2 + Cx$

$y' = 3Ax^2 + 2Bx + C$, $y'' = 6Ax + 2B$

$6Ax + 2B + 2(3Ax^2 + 2Bx + C) = x^2 + 1$

$6A = 1 \quad A = 1/6$

$6A + 4B = 0 \quad B = -1/4$

$2B + 2C = 1 \quad C = 3/4$

$y_p = 1/6 x^3 - 1/4 x^2 + 3/4 x$

Sv: $y = C_1 + C_2 e^{-2x} + 1/6 x^3 - 1/4 x^2 + 3/4 x$