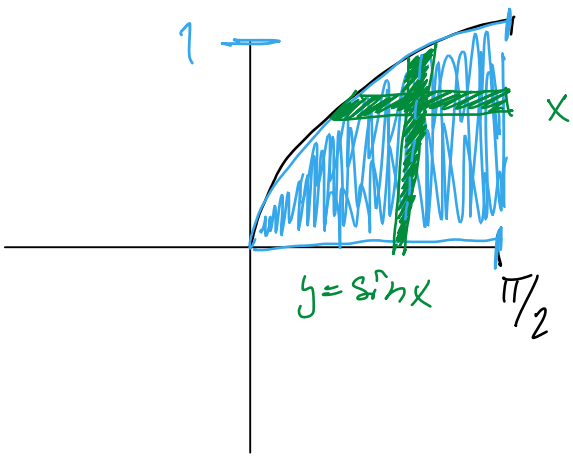
14.17

tisdag 3 januari 2023 05:18



$$X_T = \frac{1}{m}\int_K x\,dm = \frac{\int x \rho\,dv}{\int \rho\,dv} = \frac{\int x\,dv}{\int dv}$$

$$X_T = \int_0^{\pi/2} x \sin x\,dx = \underbrace{\left[-x\cos x\right]_0^{\pi/2}}_{0} + \int_0^{\pi/2}\cos x\,dx =$$

$$= \left[\sin x\right]_0^{\pi/2} = 1$$

$$y_T = \int_0^1 y\,\arcsin y\,dx = \left[\begin{matrix} t = \arcsin y & y = \sin t \\ dy = \cos t\,dt & \end{matrix}\right] =$$

$y = 0 \quad t = 0 \qquad y = 1 \quad t = \pi/2$

$$= \int_0^{\pi/2} \sin t \cdot t \cdot \cos t\,dt = \frac{1}{2}\int_0^{\pi/2} 2\sin t\cos t \cdot t\,dt =$$

$$= \frac{1}{2}\int_0^{\pi/2} t \cdot \sin 2t\,dt = \frac{1}{2}\left[-\frac{\cos 2t}{2}\cdot t\right]_0^{\pi/2} + \frac{1}{2}\int_0^{\pi/2}\frac{\cos 2t}{2}\,dt =$$

$$= \frac{1}{2}\left(\frac{1}{2}\cdot\frac{\pi}{2}\right) + \underbrace{\frac{1}{2}\left[\frac{\sin 2t}{4}\right]_0^{\pi/2}}_{0} =$$

$$= \pi/8$$

$$(X_T, y_T) = (1, \pi/8)$$