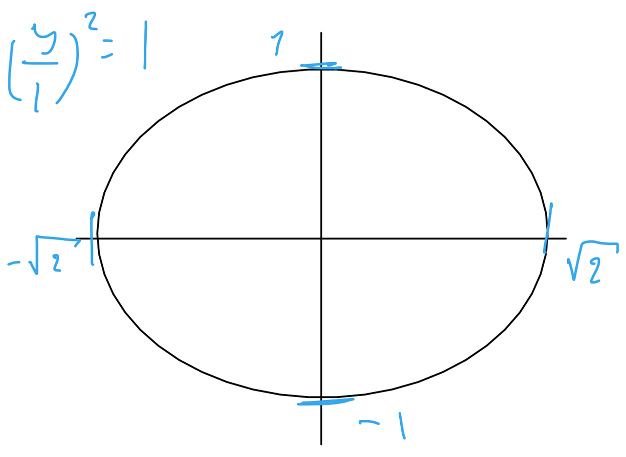
14.38

fredag 23 december 2022 10:50

$$\frac{x^2}{2}+y^2=1 \qquad \left(\frac{x}{\sqrt{2}}\right)^2+\left(\frac{y}{1}\right)^2=1$$



$$y=(\pm)\sqrt{1-\frac{x^2}{2}}$$

$$y'=-\frac{x}{2\sqrt{1-\frac{x^2}{2}}}$$

$$A=\int_{-\sqrt{2}}^{\sqrt{2}} 2\pi\sqrt{1-\frac{x^2}{2}}\sqrt{1+\left(-\frac{x}{2\sqrt{1-\frac{x^2}{2}}}\right)^2}\,dx=$$

$$=\int_{-\sqrt{2}}^{\sqrt{2}} 2\pi\sqrt{1-\frac{x^2}{2}}\sqrt{1+\frac{x^2}{4(1-x^2/2)}}\,dx=$$

$$=2\pi\int_{-\sqrt{2}}^{\sqrt{2}}\sqrt{\left(\frac{2-x^2}{2}\right)\left(\frac{4-x^2}{4-2x^2}\right)}\,dx=2\pi\int_{-\sqrt{2}}^{\sqrt{2}}\sqrt{\frac{\cancel{(2-x^2)}(4-x^2)}{4\cancel{(2-x^2)}}}\,dx=$$

$$=\cancel{2}\pi\int_{-\sqrt{2}}^{\sqrt{2}}\frac{\sqrt{4-x^2}}{\cancel{2}}\,dx=\pi\int_{-\sqrt{2}}^{\sqrt{2}}1\cdot\sqrt{4-x^2}\,dx=$$

$$=\pi\Big[x\sqrt{4-x^2}\Big]_{-\sqrt{2}}^{\sqrt{2}}-\pi\int_{-\sqrt{2}}^{\sqrt{2}}x\cdot\left(-\frac{x}{\sqrt{4-x^2}}\right)dx=$$

$$=\pi\Big[x\sqrt{4-x^2}\Big]_{-\sqrt{2}}^{\sqrt{2}}+\pi\int_{-\sqrt{2}}^{\sqrt{2}}x^2\,\frac{1}{2\sqrt{1-\left(\frac{x}{2}\right)^2}}\,dx=$$

$$=\pi\Big[x\sqrt{4-x^2}\Big]_{-\sqrt{2}}^{\sqrt{2}}+\frac{\pi}{2}\int_{-\sqrt{2}}^{\sqrt{2}}x^2\,\frac{1}{\sqrt{1-(x/2)^2}}\,dx=$$

$$=\pi\Big[x\sqrt{4-x^2}\Big]_{-\sqrt{2}}^{\sqrt{2}}+\frac{\pi}{2}\int_{-\sqrt{2}}^{\sqrt{2}}\frac{(x^2-4)+4}{\sqrt{1-x^2/4}}\,dx=$$

$$=\pi\Big[x\sqrt{4-x^2}\Big]_{-\sqrt{2}}^{\sqrt{2}}+\cancel{\frac{\pi}{2}}\int_{-\sqrt{2}}^{\sqrt{2}}\cancel{4}^{2}\cdot\frac{(x^2/4-1)+1}{\sqrt{1-x^2/4}}\,dx=$$

$$=\pi\Big[x\sqrt{4-x^2}\Big]_{-\sqrt{2}}^{\sqrt{2}}+2\pi\int_{-\sqrt{2}}^{\sqrt{2}}-\frac{(1-x^2/4)}{\sqrt{1-x^2/4}}+\frac{1}{\sqrt{1-x^2/4}}\,dx=$$

$$=\underbrace{\pi\Big[x\sqrt{4-x^2}+4\arcsin x/2\Big]_{-\sqrt{2}}^{\sqrt{2}}}_{*}-2\pi\int_{-\sqrt{2}}^{\sqrt{2}}\sqrt{1-x^2/4}\,dx=$$

$$=*-\pi\int_{-\sqrt{2}}^{\sqrt{2}}\sqrt{4-x^2}\,dx$$

$$\pi\int_{-\sqrt{2}}^{\sqrt{2}}\sqrt{4-x^2}\,dx=\pi/2\Big[x\sqrt{4-x^2}+4\arcsin x/2\Big]_{-\sqrt{2}}^{\sqrt{2}}=$$

$$=\frac{\pi}{2}\left(\underbrace{\sqrt{2}\cdot\sqrt{2}}_{2}+\underbrace{4\arcsin 1/\sqrt{2}}_{\pi}-\left(\underbrace{-\sqrt{2}\cdot\sqrt{2}}_{-2}+\underbrace{4\arcsin -1/\sqrt{2}}_{-\pi}\right)\right)=$$

$$=\frac{\pi}{2}\left(2+\pi+2+\pi\right)=\underline{\underline{2\pi+\pi^2\text{ a.e}}}$$