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måndag 26 december 2022 13:54

a) $y'' - 3y' + 2y = e^{5x}$

y_h: $p(r) = r^2 - 3r + 2 = 0$ $\quad r = 3/2 \pm \sqrt{\frac{9}{4} - \frac{8}{4}}$

$r_1 = 2 \quad r_2 = 1$

$y_h = C_1 e^x + C_2 e^{2x}$

$y_p = z e^{5x} = (z_h + z_p) e^{5x}$

$y' = 5z e^{5x} + z' e^{5x} = e^{5x}(5z + z')$

$y'' = 5e^{5x}(5z + z') + e^{5x}(5z' + z'') = e^{5x}(z'' + 10z' + 25z)$

$e^{5x}(z'' + 10z' + 25z) - 3e^{5x}(5z + z') + 2e^{5x} z = e^{5x} \Leftrightarrow$

$\Leftrightarrow z'' + 7z' + 12z = 1$

z: $z = A$

$0 + 0 + 12A = 1 \quad A = 1/12, \quad z = 1/12$

Sv: $y = C_1 e^x + C_2 e^{2x} + 1/12\, e^{5x}$

b) $y'' - 3y' + 2y = e^{2x}$

$y_h = C_1 e^x + C_2 e^{2x}$

$y_p = z e^{2x}, \quad y' = e^{2x}(2z + z'), \quad y'' = e^{2x}(z'' + 4z' + 4z)$

$z'' + 4z' + 4z - 3(2z + z') + 2z = 1 \Leftrightarrow z'' + z' = 1$

$z = Ax$

$0 + A = 1 \Leftrightarrow A = 1 \Leftrightarrow z = x$

Sv: $y = C_1 e^x + C_2 e^{2x} + x e^{2x}$

c) $y'' + 6y' + 9y = 4e^{-x}$ $\quad y(0) = 2 \quad y'(0) = -2$

$y_p = z e^{-x}$

$y' = z' e^{-x} - z e^{-x} = e^{-x}(z' - z)$

$y'' = -e^{-x}(z' - z) + e^{-x}(z'' - z') = e^{-x}(z'' - 2z' + z)$

$e^{-x}(z'' - 2z' + z) + 6e^{-x}(z' - z) + 9e^{-x} z = 4e^{-x} \Leftrightarrow$

$\Leftrightarrow z'' + 4z' + 4z = 4$

$z = A$

$0 + 0 + 4A = 4 \Leftrightarrow A = 1 \Leftrightarrow z = 1 \Leftrightarrow y_p = e^{-x}$

y_h: $p(r) = r^2 + 6r + 9 = 0 \quad r = -3 \pm \sqrt{0} \quad r_1 = r_2 = -3$

$y_h = (C_1 x + C_2) e^{-3x}$

$y = (C_1 x + C_2) e^{-3x} + e^{-x}$

$y' = (C_1) e^{-3x} - 3e^{-3x}(C_1 x + C_2) - e^{-x}$

$y(0) = C_2 + 1 = 2 \quad C_2 = 1$

$y'(0) = C_1 - 3(1) - 1 = C_1 - 4 = -2 \quad C_1 = 2$

Sv: $y = (2x + 1) e^{-3x} + e^{-x}$, $y(0) = 2 \quad y'(0) = -2$