15.22a

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$$y' = (y^2 - 1)x \qquad \frac{dy}{dx} = (y^2 - 1)x$$

$$dy \cdot \frac{1}{(y^2-1)} = x\,dx$$

$$\int \left(\frac{1}{y^2-1}\right) dy = \int x\,dx$$

$$\int \frac{A}{y+1} + \frac{B}{y-1}\,dy = \frac{x^2}{2} + B$$

$$1 = A(y-1) + B(y+1) = (A+B)y + B - A$$

$$A+B=0 \qquad A=-B$$

$$2B=1 \qquad B=1/2 \qquad A=-1/2$$

$$B-A=1$$

$$\frac{1}{2}\int \frac{1}{y-1} - \frac{1}{y+1}\,dy = \frac{1}{2}\left(\ln\frac{|y-1|}{|y+1|}\right) + A =$$

$$\frac{1}{2}\left(\ln\left|\frac{y-1}{y+1}\right|\right) + A = \frac{x^2}{2} + B$$

$$\ln\left|\frac{y-1}{y+1}\right| = x^2 + C$$

$$\ln\left|\frac{y-1}{y+1}\right| = x^2 + C \qquad \ln\left|-\frac{1}{1}\right| = 0 + C \qquad C = 0$$

$$\ln\left|\frac{y-1}{y+1}\right| = x^2$$

$$\frac{y-1}{y+1} = (\pm 1)\,e^{x^2}$$

$$y-1 = e^{x^2}(y+1) \qquad y-1 = e^{x^2}y + e^{x^2}$$

$$y - e^{x^2}y = e^{x^2} + 1 \qquad y(1-e^{x^2}) = e^{x^2} + 1$$

Sv: $y = \dfrac{e^{x^2}+1}{1-e^{x^2}}$