

15.56 a

måndag 26 december 2022 18:47

$$y'' - 6y' + 10y = e^{3x} \cos x \quad , \quad y = y_h + y_p$$

$$\bullet \underline{y_h}: \quad p(r) = r^2 - 6r + 10 = 0 \quad r = 3 \pm \sqrt{9 - 10} = 3 \pm i$$

$$y_h = e^{3x} (A \cos x + B \sin x)$$

$$\bullet \underline{y_p}: \quad \text{hjälp} \quad e^{(3+i)x}: \quad y'' - 6y' + 10y = e^3 \cdot e^{ix} = e^{(3+i)x} = e^3 (\cos x + i \sin x)$$

$$y_p = z e^{(3+i)x} \quad \underline{y'} = (3+i) z e^{(3+i)x} + z' e^{(3+i)x} = e^{(3+i)x} (z' + (3+i)z)$$

$$\underline{y''} = (3+i) e^{(3+i)x} (z' + (3+i)z) + e^{(3+i)x} (z'' + (3+i)z') = e^{(3+i)x} (z'' + (6+2i)z' + \underbrace{(3+i)^2}_{8+6i} z)$$

$$\cancel{e^{(3+i)x}} (z'' + (6+2i)z' + (8+6i)z) - 6 \cancel{e^{(3+i)x}} (z' + (3+i)z) + 10 z \cancel{e^{(3+i)x}} = \cancel{e^{(3+i)x}} \Leftrightarrow$$

$$\Leftrightarrow z'' + (6+2i)z' + (8+6i)z - 6z' + (-18-6i)z + 10z = 1 \Leftrightarrow$$

$$\Leftrightarrow z'' + \cancel{6z'} + 2iz' + \cancel{8z} + \cancel{6iz} - \cancel{6z'} - \cancel{18z} - \cancel{6iz} + \cancel{10z} = 1 \Leftrightarrow$$

$$\Leftrightarrow z'' + 2iz' = 1$$

$$\bullet \quad z = Ax$$

$$0 + 2iA = 1 \Leftrightarrow A = -\frac{i}{2} \Leftrightarrow z = -\frac{i}{2}x \Leftrightarrow y_p = -\frac{i}{2}x e^{(3+i)x}$$

$$y_p = -\frac{i}{2}x e^{(3+i)x} = -\frac{i}{2}x e^3 (\cos x + i \sin x) = i \left(-\frac{1}{2}x e^3 \cos x \right) + \underbrace{\left(\frac{1}{2}x e^3 \sin x \right)}_{rc}$$

$$y_p = \frac{1}{2}x e^3 \sin x$$

$$\text{Sv: } y = e^3 \left(A \cos x + B \sin x + \frac{1}{2}x \sin x \right)$$