13.24

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a) $$\lim_{K\to\infty} \int_0^K \frac{x}{1+x^2}\,dx = \frac{1}{2}\Big[\ln(1+x^2)\Big]_0^K = \frac{1}{2}\big(\ln(1-K^2) - \ln 1\big) = \frac{1}{2}\ln(1-K^2) = \text{divergent då } K\to\infty$$

b) $$\lim_{K\to\infty} \int_0^K \underset{\downarrow}{x}\, \overset{\uparrow}{e^{-x}}\,dx = \Big[-e^{-x}\cdot x\Big]_0^K + \int_0^K e^{-x}\,dx = \Big[-e^{-x}\cdot x - e^{-x}\Big]_0^K = -e^{-K}\cdot K - e^{-K} - \big(\cancel{-e^{0}\cdot 0} - e^{0}\big) = -\underbrace{\frac{K}{e^K}}_{0} - \underbrace{\frac{1}{e^K}}_{0} + 1 = \underline{\underline{1}}$$

exponent växer snabbare än potens

c) $$\boxed{\lim_{K\to\infty}} \int_1^K \frac{\ln(2x-1)}{x^2}\,dx = \int_1^K \overset{\uparrow}{\frac{1}{x^2}}\cdot \underset{\downarrow}{\ln(2x-1)}\,dx = \underbrace{\Big[-\frac{1}{x}\cdot\ln(2x-1)\Big]_1^K}_{0} - \int_1^K -\frac{1}{x}\cdot\frac{2}{2x-1}\,dx = \int_1^K \frac{2}{x(2x-1)}\,dx$$

$$= \int_1^K \frac{A}{x} + \frac{B}{2x-1}\,dx = \int_1^K -\frac{2}{x} + \frac{4}{2x-1}\,dx = \Big[-2\ln|x| + 2\ln|2x-1|\Big]_1^K = 2\Big[\ln\Big|\frac{2x-1}{x}\Big|\Big]_1^K = 2\Big(\ln\frac{2K-1}{K}\Big) = 2\cdot\ln\Big(\underbrace{\frac{K}{K}}_{1}\cdot\underbrace{\frac{2-1/K}{1}}_{2}\Big) = \underline{\underline{2\ln 2}}$$

(vid nedre gränsen: $\to 0$)

$$2 = 2Ax - A + Bx = (2A+B)x - A, \quad \boxed{\begin{aligned} A &= -2 \\ B &= 4 \end{aligned}}$$

d) $$\boxed{\lim_{K\to\infty}} \int_2^\infty \frac{x}{x^4-1}\,dx = \left[\begin{aligned} &t = x^2 \quad x = \sqrt{t} \\ &dt = 2x\,dx \\ &x=2\ t=4 \quad x=K\ t=T \end{aligned}\right] = \int_4^T \frac{1}{\underbrace{t^2-1}_{(t-1)(t+1)}}\cdot\frac{1}{2}\,dt = \frac{1}{2}\int_4^T \frac{A}{t-1} + \frac{B}{t+1}\,dt$$

$$\boxed{\begin{aligned} &1 = Ax + A + Bx - B = (A+B)x + A - B \\ &A + B = 0 \qquad A = -B \\ &A - B = 1 \qquad -2B = 1 \qquad \boxed{B = -\tfrac{1}{2} \quad A = \tfrac{1}{2}} \end{aligned}}$$

$$\frac{1}{4}\int_4^T \frac{1}{t-1} - \frac{1}{t+1}\,dt = \frac{1}{4}\Big[\ln\Big|\frac{t-1}{t+1}\Big|\Big]_4^T = \frac{1}{4}\Big(\underbrace{\ln\Big(\underbrace{\frac{T}{T}}_{1}\cdot\underbrace{\frac{1-1/T}{1+1/T}}_{1}\Big)}_{0} - \ln\Big|\frac{3}{5}\Big|\Big) = \underline{\underline{\frac{1}{4}\ln\frac{5}{3}}}$$