13.28

torsdag 22 december 2022 16:14

a) $\lim_{k\to\infty} \int_1^k \frac{1}{\sqrt{x}(x+4)}\,dx = \left[\begin{matrix} t=\sqrt{x} & x=t^2 \\ dx = 2t\,dt & \\ x=1\ t=1 & x=k\ t=k \end{matrix}\right] = \int_1^k \frac{1}{\cancel{t}(t^2+4)} \cdot 2\cancel{t}\,dt = \int_1^k \frac{2}{t^2+4}\,dt = \frac{1}{2}\int_1^k \frac{1}{\left(\frac{t}{2}\right)^2+1}\,dt =$

$= \left[\arctan\left(\frac{t}{2}\right)\right]_1^k = \left(\underbrace{\arctan\left(\frac{k}{2}\right)}_{\pi/2} - \arctan\left(\frac{1}{2}\right)\right) = \frac{\pi}{2} - \arctan\left(\frac{1}{2}\right)$

ej $\frac{1}{2}\left[\arctan t/2\right]_1^k$, testa $D \arctan(t/2)$

b) $\int_0^1 \frac{1}{\sqrt{x}(x+4)}\,dx = \left[\begin{matrix} t=\sqrt{x} & x=t^2 \\ dx = 2t\,dt & \\ x=0\ t=0 & x=1\ t=1 \end{matrix}\right] = \int_0^1 \frac{1}{\cancel{t}(t^2+4)} \cdot 2\cancel{t}\,dt = \left[\arctan t/2\right]_0^1 = \left(\arctan 1/2 - 0\right) = \arctan 1/2$

c) $\frac{\pi}{2} - \cancel{\arctan} 1/2 + \cancel{\arctan} 1/2 = \pi/2$

$\int_0^K \frac{1}{\sqrt{x}(x+4)}\,dx = \left[\begin{matrix} t=\sqrt{x} & x=t^2 \\ dt = 2t\,dt & \end{matrix}\right] = \int_0^K \frac{1}{\cancel{t}(t^2+4)} \cdot 2\cancel{t} = \int_0^K \frac{2}{t^2+4}\,dt = \frac{1}{2}\int_0^K \frac{1}{\left(\frac{t}{2}\right)^2 \cdot 1}\,dt = \left[\arctan(t/2)\right]_0^K = \underbrace{\arctan \frac{K}{2}}_{\pi/2} - \underbrace{\arctan 0}_{0}$