15.31

onsdag 4 januari 2023 21:06

$y(t)$ = antalet individer vid tiden t

$y' = ry(K-y)$ $\quad r, K$ = positiva konstanter

- $y(0) = 10^4$
- $y(1) = 2\cdot 10^4$
- $y(\alpha) = 10^5 \quad \alpha \to \infty$

$$y' = ry(K-y) \Leftrightarrow y' \cdot \frac{1}{y(K-y)} = r \Leftrightarrow$$

$$\Leftrightarrow \int \frac{1}{y(K-y)}\,dy = \int r\,dt \Leftrightarrow \int \frac{A}{y} + \frac{B}{K-y}\,dy = rt + c \Leftrightarrow$$

$$\boxed{1 = AK - Ay + By \quad AK = 1 \quad B - A = 0 \quad A = 1/K \quad B = 1/K}$$

$$\frac{1}{K}\int \frac{1}{y} + \frac{1}{K-y}\,dy = rt + c \Leftrightarrow \frac{1}{K}\left(\ln|y| - \ln|K-y|\right) = rt + c \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{K}\left(\ln\left|\frac{y}{K-y}\right|\right) = rt + c \Leftrightarrow \ln\left|\frac{y}{K-y}\right| = Krt + \underset{}{\overset{c\cdot K}{D}} \Leftrightarrow$$

$$\Leftrightarrow \frac{y}{K-y} = (\pm)\, e^{Krt}\cdot \overset{e^{D}}{E} \Leftrightarrow y = (K-y)(e^{Krt}\cdot E) \Leftrightarrow$$

$$\Leftrightarrow y = KEe^{Krt} - yEe^{Krt} \Leftrightarrow y(1 + Ee^{Krt}) = KEe^{Krt} \Leftrightarrow$$

$$\Leftrightarrow y = \frac{KEe^{Krt}}{1 + Ee^{Krt}}$$

$$\lim_{\alpha\to\infty} y(\alpha) = \frac{e^{Krt}}{e^{Krt}} \cdot \frac{KE}{\underbrace{1/e^{Krt}}_{0} + E} = 1 \cdot K = 10^5$$

$$\boxed{K = 10^5}$$

$$y(0) = \frac{10^5 E}{1+E} = 10^5 \cdot \frac{E}{1+E} = 10^4 \Leftrightarrow$$

$$\Leftrightarrow y(0) = \frac{E}{1+E} = 10^{-1} \Leftrightarrow \text{Poly div.}$$

$$\Leftrightarrow y(0) = 1 - \frac{1}{1+E} = \frac{10}{10} - \frac{1}{1+E} = \frac{1}{10} \Leftrightarrow$$

$$\Leftrightarrow y(0) = \frac{9}{10} = \frac{1}{1+E} \Leftrightarrow y(0) = 9(1+E) = 10$$

$$\boxed{E = 1/9}$$

$$y(1) = \frac{10^5 \cdot 1/9 \cdot e^{10^5\cdot r}}{\underbrace{1 + 1/9\, e^{10^5\cdot r}}_{\frac{9}{9} + \frac{1}{9}e^{10^5\cdot r}}} = \cancel{10^5} \cdot \frac{e^{10^5\cdot r}}{9 + e^{10^5\cdot r}} = 2 \cdot \overset{10^{-1}}{\cancel{10^4}} \Leftrightarrow$$

$$\Leftrightarrow y(1) = \frac{e^{10^5\cdot r}}{9 + e^{10^5\cdot r}} = \frac{1}{5} \Leftrightarrow y(1) = \overset{4}{\cancel{5}}\, e^{10^5\cdot r} = 9 + \cancel{e^{10^5\cdot r}} \Leftrightarrow$$

$$\Leftrightarrow y(1) = e^{10^5\cdot r} = 9/4 \Leftrightarrow y(1) = r = 10^{-5}\cdot \ln 9/4$$

$$\boxed{r = 10^{-5}\cdot \ln 9/4}$$

Sv: $r = 10^{-5} \ln(9/4)$; $K = 10^5$