13.23

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a) $$\int_1^\infty \frac{1}{e^x - e^{-x}}\,dx = \left[\begin{matrix} t = e^x & x = \ln t \\ dx = \frac{1}{t}\,dt & \end{matrix}\right]_{x=1\ t=e \quad x\to\infty\ t\to\infty} = \int_1^\infty \frac{1}{t - t^{-1}} \cdot \frac{1}{t}\,dt = \int_1^\infty \frac{1}{t^2-1}\,dt = \int_1^\infty \frac{1}{(t-1)(t+1)}\,dt = \int_1^\infty \frac{A}{t-1} + \frac{B}{t+1}\,dt$$

$$1 = At + A + Bt - B = (A+B)t + A - B$$

$A + B = 0 \qquad A = -B$

$A - B = 1 \qquad -2B = 1 \qquad \boxed{B = -1/2 \quad A = 1/2}$

$k \to \infty$

$$\frac{1}{2}\int_e^k \frac{1}{t-1} - \frac{1}{t+1}\,dt = \frac{1}{2}\Big[\ln|t-1| - \ln|t+1|\Big]_e^k = \frac{1}{2}\left(\ln\left|\frac{k-1}{k+1}\right| - \ln\left|\frac{e-1}{e+1}\right|\right) = \frac{1}{2}\left(\underbrace{\ln\left|\frac{1-1/k}{1+1/k}\right|}_{\ln 1} + \ln\left|\frac{e+1}{e-1}\right|\right) = \frac{1}{2}\ln\frac{e+1}{e-1}$$

b) $$\int_0^{x}\frac{\cos x}{3+\sin x}\,dx \quad (x\to\infty) = \left[\begin{matrix} t = \sin x & x = \arcsin x \\ dt = \cos x\,dx & \end{matrix}\right]_{x=0\ t=0 \quad x\to\infty\ T\to\infty} = \int_0^T \frac{1}{3+t}\,dt = \Big[\ln|3+t|\Big]_0^T = \ln|3+T| - \ln|3| = \ln\left|1+\frac{T}{3}\right| = T \to \infty \text{ då } x\to\infty$$

divergent