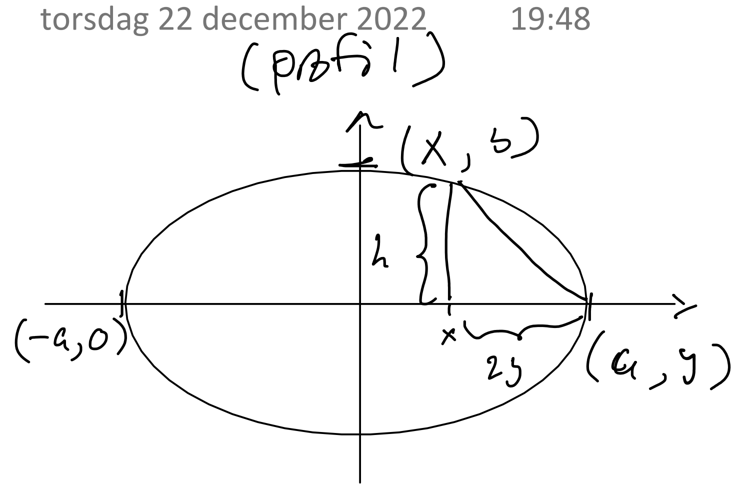
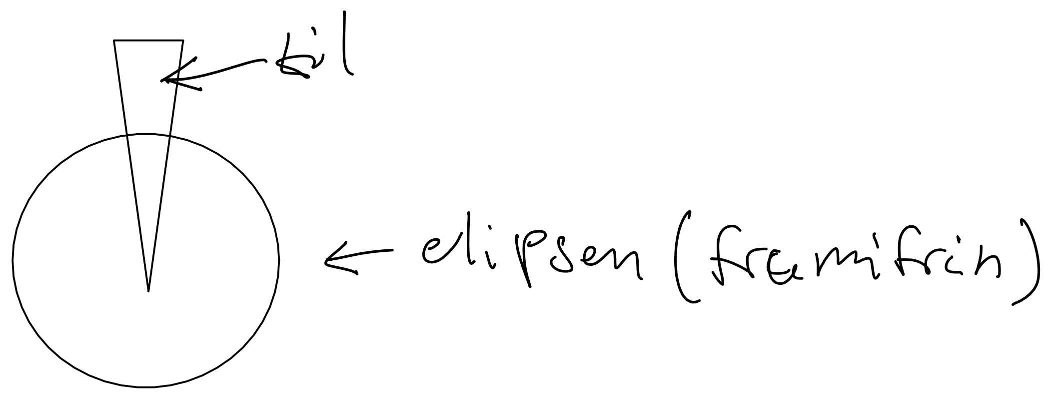
14.04

torsdag 22 december 2022 19:48



$$\left(\frac{y}{b}\right)^2 + \left(\frac{x}{a}\right)^2 = 1$$

$$y = b\sqrt{1-(x/a)^2}$$



$$\text{Kilens area} = A(x) = \frac{h\cdot \cancel{2}y}{\cancel{2}} = \frac{h\cdot \cancel{2}b\sqrt{1-(x/a)^2}}{\cancel{2}}$$

$$V = \int_{-a}^{a} \frac{h\cdot 2y}{2}\,dy = \int_{-a}^{a} \frac{h\cdot \cancel{2}b\sqrt{1-(x/a)^2}}{\cancel{2}}\,dx = hb\int_{-a}^{a}\sqrt{1-(x/a)^2}\,dx = \begin{bmatrix} t = \frac{x}{a} & x = at \\ dx = a\,dt & \end{bmatrix}_{x=-a\ t=-1 \quad x=a\ t=1} =$$

$$= hba\int_{-1}^{1}\sqrt{1-t^2}\,dt = \begin{bmatrix} t = \sin u & u = \arcsin t \\ dt = \cos u\,du & \end{bmatrix}_{t=-1\ u=-\pi/2 \quad t=1\ u=\pi/2} = abh\int_{\frac{\pi}{2}}^{\pi/2}\underbrace{\sqrt{1-\sin^2 u}}_{\cos u}\cdot\cos u\,du = abh\int_{-\frac{\pi}{2}}^{\pi/2}\cos^2 u\,du = abh\int_{-\frac{\pi}{2}}^{\pi/2}\frac{1+\cos 2t}{2}\,du =$$

$$= abh\left[\frac{1}{2}u + \frac{\sin 2u}{4}\right]_{-\frac{\pi}{2}}^{\pi/2} = abh\left(\frac{\pi}{4} + 0 - \left(-\frac{\pi}{4}+0\right)\right) = abh\left(\frac{\pi}{4}+\frac{\pi}{4}\right) = abh\,\frac{\pi}{2}$$