12.35

onsdag 21 december 2022 15:46

a)

$$\int \sin^3 x \, dx = \int \underbrace{\sin^2 x \cdot \sin x}_{(1-\cos^2 x)^2 \cdot \sin x} \, dx = \left[\begin{array}{l} t = \cos x \\ \frac{dt}{dx} = -\sin x \quad -dt = \sin x \, dx \end{array} \right] = -\int (1-t^2)^2 \, dt = \int t^4 - 2t^2 + 1 \, dt = -\frac{t^5}{5} + 2\frac{t^3}{3} - t + C = -\frac{\cos^5 x}{5} + 2\,\frac{\cos^3 x}{3} - \cos x + C$$

b)

$$\int \sin^4 x \, dx = \int (\sin^2 x)^2 \, dx = \int \frac{(1-\cos 2x)^2}{4} \, dx = \int \frac{\cos^2 2x - 2\cos 2x + 1}{4} \, dx =$$

$$= -\frac{\sin 2x}{4} + \frac{1}{4}x + \frac{1}{4}\int \cos^2 2x \, dx = \frac{1}{4}x - \frac{1}{4}\sin 2x + \frac{1}{4}\int \frac{1+\cos 4x}{2} \, dx = \frac{1}{4}x - \frac{1}{4}\sin x + \frac{1}{8}x + \frac{1}{32}\sin 4x + C$$

c)

$$\int e^{2x} \sin 3x \, dx = \int e^{2x} \cdot \frac{e^{i3x} - e^{-i3x}}{2i} \, dx = \int \frac{e^{(2+3i)x} - e^{(2-3i)x}}{2i} \, dx = \frac{1}{2i(2+3i)} \cdot e^{(2+3i)x} - \frac{1}{2i(2-3i)} \cdot e^{(2-3i)x} + C = \frac{e^{2x}}{2i}\left(\frac{e^{i3x}}{2+3i} - \frac{e^{-i3x}}{2-3i} \right) + C =$$

$$= \frac{e^{2x}}{2i}\left(\frac{\cos 3x + i \sin 3x}{2+3i} - \frac{\cos(3x) \overset{-}{(+)} i \sin(-3x)}{2-3i} \right) + C$$

$$= \frac{e^{2x}}{2i}\left(\frac{\cancel{2\cos(3x)} - 3i\cos(3x) + 2i\sin(3x) + \cancel{3\sin(3x)}}{(2+3i)(2-3i)} - \frac{-\cancel{2\cos(3x)} \overset{-}{(+)} 3i\cos(3x) \overset{+}{(-)} 2i\sin(3x) \overset{-}{(+)} \cancel{3\sin(3x)}}{(2+3i)(2-3i)} \right) + C$$

$$= \frac{e^{2x}}{2i}\left(\frac{-6i\cos(3x) + 4i\sin(3x)}{13} \right) + C = \frac{e^{2x}}{13}\left(2\sin(3x) - 3\cos(3x) \right) + C$$