13.21 a

torsdag 22 december 2022 12:00

$$\int_{-1}^{3} \frac{x+3}{\sqrt{x^2+2x+10}}\,dx = \int \frac{x+2}{\sqrt{(x+1)^2+9}}\,dx = \left[\begin{matrix} t=x+1 & x=t-1 \\ dt=dx & \end{matrix}\right] = \int_0^4 \frac{t}{\sqrt{t^2+9}} + \frac{2}{\sqrt{t^2+9}}\,dt =$$

$x=1 \quad t=0 \quad x=3 \quad t=4$

$$= \left[\begin{matrix} p = t+\sqrt{t^2+9} & t=\frac{p^2-9}{2p} \\ p-t=\sqrt{t^2+9} & \\ p^2-2tp+\cancel{t^2}=\cancel{t^2}+9 & \\ dt=\frac{p^2+9}{2p^2}\,dp & \end{matrix}\right] = \int_3^9 \frac{\frac{p^2-9}{2p}+2}{\sqrt{\left(\frac{p^2-9}{2p}\right)^2+9}} \cdot \frac{p^2+9}{2p^2}\,dp = \int_3^9 \frac{\frac{p^2-9}{2p}+2}{\sqrt{\frac{p^4-18p^2+81}{4p^2}+\frac{36p^2}{4p^2}}} \cdot \frac{p^2+9}{2p^2}\,dp =$$

$t=0 \quad p=3 \qquad t=4 \quad p=9$

$$= \int_3^9 \frac{\frac{p^2-9}{2p}+2}{\sqrt{\frac{p^4+18p^2+81}{4p^2}}} \cdot \frac{p^2+9}{2p^2}\,dp = \int_3^9 \frac{p^2+4p-9}{\cancel{2p}\cdot\frac{\cancel{p^2+9}}{\cancel{2p}}} \cdot \frac{\cancel{p^2+9}}{2p^2}\,dp = \int_3^9 \frac{p^2+4p-9}{2p^2}\,dp$$

Poly. div

$$\begin{array}{l|l} \multicolumn{1}{l}{\tfrac{1}{2}} & \\ \hline \cancel{p^2}+4p-9 & 2p^2 \\ -(\cancel{p^2}) & \\ \hline 4p-9 & \end{array}$$

$$= \int_3^9 \frac{1}{2} + \frac{4p-9}{2p^2}\,dp = \left[\frac{1}{2}p\right]_3^9 + \int_3^9 \frac{2}{p} - \frac{9}{2p^2}\,dp = \left[\frac{1}{2}p + 2\ln|p| + \frac{9}{2p}\right]_3^9$$

$$= \frac{1}{2}\cdot 9 + 2\ln|9| + \frac{1}{2} - \left(\frac{1}{2}3 + 2\ln|3| + \frac{3}{2}\right) = \frac{10}{2} + 2\ln|9| - \cancel{\frac{3}{2}} - 2\ln|3| - \cancel{\frac{3}{2}} = \underline{\underline{\frac{1}{2} + 2\ln|3|}}$$

alt. lösning (direkt primitiv)

$$\left[\begin{matrix} t=x+1 & x=t-1 \\ dx=dt & \end{matrix}\right] = \int_0^4 \frac{t}{\sqrt{t^2+9}} + \frac{2}{\sqrt{t^2+9}}\,dt = \left[\sqrt{t^2+9} + 2\ln\left|t+\sqrt{t^2+9}\right|\right]_0^4 =$$

$x=-1 \quad t=0 \quad x=3 \quad t=4$

$$= 5 + 2\ln|9| - \left(3 + 2\ln|3|\right) = \underline{\underline{2 + 2\ln|3|}}$$