12.24

tisdag 20 december 2022 18:44

a)

$$\int \frac{1}{x(x-3)^2}\,dx = \int \frac{A}{x} + \frac{B}{x-3} + \frac{C}{(x-3)^2}\,dx$$

$$1 = A(x-3)^2 + Bx(x-3) + Cx =$$

$$= Ax^2 - 6Ax + 9A + Bx^2 - 3Bx + Cx =$$

$$= (A+B)x^2 - (6A + 3B + C)x + 9A$$

$A = 1/9$

$B = -1/9$

$A + B = 0$

$-6A - 3B + C = 0$

$9A = 1$

$-\frac{6}{9} + \frac{3}{9} + \frac{3}{9}$

$C = 3/9 = 1/3$

$$\int \frac{1}{9x} - \frac{1}{9(x-3)} + \frac{1}{3(x-3)^2}\,dx = \frac{1}{9}\left(\ln|x| - \ln|x-3|\right) - \frac{1}{3(x-3)} + C = \frac{1}{9}\left(\ln\left|\frac{x}{x-3}\right| - \frac{3}{(x-3)}\right) + C$$

d)

$$\int \frac{1}{x(x+1)^2}\,dx = \int \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}\,dx$$

$$1 = A(x+1)^2 + Bx(x+1) + Cx = Ax^2 + 2Ax + A + Bx^2 + Bx + Cx$$

$A + B = 0$

$2A + B + C = 0$

$A = 1$

$B = -1$

$C = -1$

$$\int \frac{1}{x} - \frac{1}{x+1} - \frac{1}{(x+1)^2}\,dx = \ln|x| - \ln|x+2| + \frac{1}{(x+1)} + C = \ln\left|\frac{x}{x+2}\right| + \frac{1}{x+1} + C$$