11.07

tisdag 27 december 2022 15:58

$$\left| \ln(1+x) - x + \frac{x^2}{2} \right| \le \frac{8|x|^3}{3} \quad \text{om } |x| \le 1/2$$

$$f(x) = \ln(1+x) \quad f'(x) = 1/1+x \quad f''(x) = -\frac{1}{(1+x)^2} \quad f^3(x) = \frac{2}{(1+x)^3}$$

$$\left| \ln(1+x) - x + \frac{x^2}{2} \right| = \left| f(x) - P_2(x) \right| = \left| R_3(x) \right| = \left| \frac{f^{(3)}(\beta)}{3!} x^3 \right| = \frac{2}{3!(1+\beta)^3} \cdot x^3 = \frac{1}{3(1+\beta)^3} \cdot x^3 \underset{\beta \le 1/2}{\le} \frac{1}{3(1+1/2)^3} \cdot x^3 = \frac{1}{3(1/8)} x^3 = \frac{8}{3} x^3 \quad V.S.V$$

$$0 \le \beta \le x \quad |x| \le 1/2$$