

15.21

lördag 24 december 2022

11:08

$$a) \int y \cdot dy = - \int x \, dx$$

$$\frac{y^2}{2} = - \frac{x^2}{2} + C \quad y^2 = -x^2 + C$$

$$\underline{\underline{y = \sqrt{-x^2 + C}}}$$

$$b) y' \cdot e^{-y} = e^x$$

$$\int e^{-y} dy = \int e^x dx$$

$$-e^{-y} = e^x + C$$

$$\underline{\underline{y = -\ln(-e^x + C)}} \quad \begin{matrix} -C \\ \uparrow \end{matrix}$$

$$c) \int \frac{1}{y^2} dy = \int 1 \, dx$$

$$-\frac{1}{y} = x + C \quad y = -\frac{1}{x+C} \quad 1 = -\frac{1}{1+C} \quad C = -2$$

$$\text{Sv: } y = -\frac{1}{x-2}$$

$$d) \int \frac{1}{y^2} dy = \int 1 \, dx$$

$$-\frac{1}{y} = x + C \quad y = -\frac{1}{x+C} \quad 0 = -\frac{1}{1+C} \quad C = \infty$$

$$\text{Sv: } y = 0$$

$$e) x^2 y \cdot y' = 1 + x^2$$

$$y \cdot y' = \frac{1+x^2}{x^2} = \frac{1}{x^2} + 1$$

$$\int y \, dy = \int \frac{1}{x^2} + 1 \, dx$$

$$\frac{y^2}{2} = -\frac{1}{x} + x + C$$

$$y^2 = -\frac{2}{x} + 2x + 2C \quad y = \sqrt{-\frac{2}{x} + 2x + 2C}$$

$$2 = \sqrt{-1 + 4 + 2C} \quad C = \frac{1}{2}$$

$$\underline{\underline{y = \sqrt{-\frac{2}{x} + 2x + 1}}}$$