13.27

torsdag 22 december 2022 16:01

a) $\lim_{\varepsilon \to 0^+} \int_\varepsilon^1 \ln x \, dx = \left[x \cdot \ln x\right]_\varepsilon^1 - \int_\varepsilon^1 1 \, dx = \left[x \ln x - x\right]_\varepsilon^1 = 0 - 1 - (\underbrace{\varepsilon \ln \varepsilon}_{} - \underbrace{\varepsilon}_{0}) = \underline{\underline{-1}}$

$\lim_{\varepsilon \to 0^+} \varepsilon \ln \varepsilon = 0$

standardgränsvärde

b) $\lim_{\varepsilon \to 0^+} \int_{1+\varepsilon}^2 \frac{1}{x^2-1} \, dx = \int_{1+\varepsilon}^2 \frac{A}{x-1} + \frac{B}{x+1} \, dx = \frac{1}{2}\int_{1+\varepsilon}^2 \frac{1}{x-1} - \frac{1}{x+1} \, dx = \frac{1}{2}\left[\ln \left|\frac{x-1}{x+1}\right|\right]_{1+\varepsilon}^2 = \frac{1}{2}\left(\ln\left|\frac{1}{3}\right| - \underbrace{\ln\left(\frac{\varepsilon}{\varepsilon} \cdot \frac{1}{1/\varepsilon + 1}\right)}_{\infty}\right) = \text{divergent}$

$1 = Ax + A + Bx - B = (A+B)x + A - B$

$A + B = 0 \qquad A = -B$

$A - B = 1 \qquad -2B = 1 \qquad B = -1/2 \quad A = 1/2$