12.39

onsdag 21 december 2022 17:06

$$\int \frac{1}{(4x+3)\sqrt{x^2+1}}\,dx = \left[\begin{array}{ll} t = x+\sqrt{x^2+1} & x = \frac{t^2-1}{2t} \\ dx = \frac{t^2+1}{2t^2}\,dt & \end{array}\right] =$$

$$= \int \frac{1}{\left(\frac{4(t^2-1)}{2t}+3\right)\cdot \frac{t^2+1}{2t}} \cdot \frac{t^2+1}{2t^2}\,dt =$$

$t-x$

$$= \int \frac{1}{\left(\frac{4(t^2-1)+6t}{2t}\right)\left(\frac{\cancel{t^2+1}}{\cancel{2t}}\right)} \cdot \frac{\cancel{t^2+1}}{\cancel{2t^2}}\,dt =$$

t

$$= \int \frac{1}{\frac{\overset{2}{\cancel{4}}(t^2-1)+\overset{3}{\cancel{6}}t}{\cancel{2t}}} \cdot \frac{1}{\cancel{t}}\,dt =$$

$$= \int \frac{1}{2t^2-2+3t}\,dt = \int \frac{1}{2t^2+3t-2}\,dt =$$

$$= \frac{1}{2}\int \frac{1}{t^2+\frac{3}{2}t-1}\,dt = \frac{1}{2}\int \frac{1}{(t-\frac{1}{2})(t+2)}\,dt = \frac{1}{2}\int \frac{A}{t-\frac{1}{2}} + \frac{B}{t+2}$$

$$t = -\frac{3}{4} \pm \sqrt{\frac{9}{16}+\frac{16}{16}} = -\frac{3}{4} \pm \frac{5}{4} \quad t_1 = \frac{1}{2} \quad t_2 = -2$$

$$1 = At + 2A + Bt - \tfrac{1}{2}B = (A+B)t + 2A - \tfrac{1}{2}B$$

$A+B=0$ $\quad B = -A$

$2A - \frac{1}{2}B = 1$ $\quad 2A + \frac{1}{2}A = 1 \quad \frac{5}{2}A = 1 \quad \boxed{A = \frac{2}{5}}$

$\boxed{B = -\frac{2}{5}}$

$$= \frac{1}{\cancel{2}}\int \frac{\cancel{2}}{5(t-\frac{1}{2})} - \frac{\cancel{2}}{5(t+2)}\,dt = \frac{1}{5}\int \frac{1}{t-\frac{1}{2}} - \frac{1}{t+2}\,dt =$$

$$= \frac{1}{5}\left(\ln\left|t-\tfrac{1}{2}\right| - \ln|t+2|\right)+C = \frac{1}{5}\ln\left|\frac{t-\frac{1}{2}}{t+2}\right| + C = \frac{1}{5}\ln\left|\frac{x+\sqrt{x^2+1}-\frac{1}{2}}{x+\sqrt{x^2+1}+2}\right| + C$$