

a) $y'''' + 6y'' + 11y' + 6y = 0$

$$P(r) = r^4 + 6r^2 + 11r + 6 = 0$$

gissning: $r = \pm 1, \pm 2, \pm 3$

$$r_1 = -1 \quad r_2 = -2 \quad r_3 = -3$$

S: $y = A e^{-x} + B e^{-2x} + C e^{-3x}$

b) $y^{(4)} - 2y'' + 2y' - y = e^x$

J_h: $P(r) = r^4 - 2r^3 + 2r - 1 = 0$

gissning: $r = \pm 1, \pm 2, \pm 3$

$$r_1 = 1 \quad r_2 = -1$$

$$r_{3,4} = 1$$

$$(r-1)(r+1) = r^2 - 1$$

$$(r-1)(r+1)(r^2 - 2r + 1) = (r-1)(r+1)(r-1)^2$$

$r^2 - 2r + 1$	Poly div.
$\cancel{r^4} - 2r^3 + 2r - 1$	$\overline{r^2 - 1}$
$-(\cancel{r^4} - r^2)$	
$\cancel{-2r^3} + r^2 + 2r - 1$	
$-(-\cancel{2r^3} + 2r)$	
$\cancel{r^2} - 1$	
$-(\cancel{r^2} - 1)$	
0	

$$r_{1,3,4} = 1$$

$$r_2 = -1$$

$$y_h = (Ax^2 + Bx + C)e^x + De^{-x}$$

J_p: $y_p = ze^x$

$$y' = ze^x + z'e^x = e^x(z' + z)$$

$$y'' = e^x(z' + z) + e^x(z'' + z') = e^x(z'' + 2z' + z)$$

$$y''' = e^x(z'' + 2z' + z) + e^x(z''' + 2z'' + z') =$$

$$= e^x(z''' + 3z'' + 3z' + z)$$

$$y^{(4)} = e^x(z^{(4)} + 4z''' + 6z'' + 4z' + z)$$

$$\cancel{e^x}(z^{(4)} + 4z''' + 6z'' + 4z' + z) - 2\cancel{e^x}(z'' + 3z' + z) + 2\cancel{e^x}(z' + z) - \cancel{ze^x} = \cancel{e^x} \Leftrightarrow$$

$$\Leftrightarrow z^{(4)} + 2z'' = 1$$

$$z = Ax^3 \quad z' = 3Ax^2 \quad z'' = 6Ax \quad z''' = 6A \quad z^{(4)} = 0$$

$$0 + 2 \cdot 6A = 1$$

$$A = 1/12$$

$$y_p = 1/12 x^3 e^x$$

J: $y = (Ax^2 + Bx + C)e^x + De^{-x} + 1/12 x^3 e^x$