11.22

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a)

$$\lim_{x\to 0} \frac{\cos x - e^{x^2}}{x \sin x} = \lim_{x\to 0} \frac{\cancel{1} - \frac{1}{2}x^2 + \frac{1}{24}x^4 + x^6B(x) - (\cancel{1} + x^2 + \frac{1}{2}x^4 + x^6B(x))}{x(x - \frac{1}{6}x^3 + x^5B(x))} = \lim_{x\to 0} \frac{-\frac{3}{2}x^2 - \frac{11}{24}x^4 + x^6B(x)}{x^2 - \frac{1}{6}x^4 + x^6B(x)} =$$

$$= \lim_{x\to 0} \frac{\cancel{x^2}}{\cancel{x^2}} \cdot \frac{-3/2 - 11/24\, x^2 + x^4B(x)}{1 - 1/6\, x^2 + x^4B(x)} = \underline{-3/2 \text{ då } x \to 0}$$

$\cos x = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + x^6B(x)$

$e^t = 1 + t + 1/2\, t^2 + t^4B(t)$ $\quad e^{x^2} = 1 + x^2 + 1/2\, x^4 + x^8B(x) = 1 + x^2 + 1/2\, x^4 + \underbrace{x^6(x^2B(x))}_{x^6B(x)}$

$\sin x = 0 + x - 0 - \frac{1}{6}x^3 + x^5B(x)$

b)

$$\lim_{x\to 0} \frac{x + \ln(1-x)}{1 - \sqrt{1-x^2}} = \lim_{x\to 0} \frac{\cancel{x} - \cancel{x} - 1/2\, x^2 + 1/3\, x^3 + x^4B(x)}{\cancel{1} - \cancel{1} + 1/2\, x^2 - x^4B(x)} =$$

$$= \lim_{x\to 0} \frac{\cancel{x^2}}{\cancel{x^2}} \cdot \frac{-1/2 + 1/3\, x + x^2B(x)}{1/2 - x^2B(x)} = \underline{-1 \text{ då } x \to 0}$$

$\ln(1-x) = 0 - x - \frac{1}{2}x^2 + \frac{1}{3}x^3 + x^4B(x)$

$\sqrt{1-t} = 1 - 1/2\, t + t^2B(x)$

$\sqrt{1-x^2} = 1 - 1/2\, x^2 + x^4B(x)$

c)

$$\lim_{x\to 0} \frac{\sin x - \arctan x}{x(\cos 2x - 1)} = \lim_{x\to 0} \frac{\cancel{x} - 1/6\, x^3 + -(\cancel{x} - 1/3\, x^3) + x^5B(x)}{x(\cancel{1} - 2x^2 + 2/3\, x^4 + x^6B(x) - \cancel{1})} = \lim_{x\to 0} \frac{1/6\, x^3 + x^5B(x)}{-2x^3 + 2/3\, x^5 + x^7B(x)} =$$

$$= \lim_{x\to 0} \frac{\cancel{x^3}}{\cancel{x^3}} \cdot \frac{1/6 + x^2B(x)}{-2 + 2/3\, x^2 + x^4B(x)} = \underline{-1/12 \text{ då } x \to 0}$$

$\sin x = 0 + x - 0x^2 - \frac{1}{6}x^3 + x^5B(x)$

$\cos t = 1 + 0x - 1/2\, t^2 + 0t^3 + 1/24\, t^4 + t^6B(x)$

$\cos 2x = 1 - 2x^2 + 2/3\, x^4 + x^6B(x)$

$\arctan x = x + x^3/3 + x^5B(x)$