6.51

måndag 19 december 2022   19:48

$$p(z) = z^4 - 2z^3 + 12z^2 - 14z + 35 = 0$$

$$z_1 = 1 + bi \qquad z_2 = 1 - bi$$

$$p(z_1) = (1 + b^4 - 6b^2) + i(4b - 4b^3) - 2\big((1 - 3b^2) + i(3b - b^3)\big) + 12(1 - b^2 + 2bi) - 14(1 + bi) + 35$$

$$(1+bi)^4 = 1^4 + \binom{4}{1}1^3(bi)^1 + \binom{4}{2}1^2(bi)^2 + \binom{4}{3}1(bi)^3 + (bi)^4 =$$

$$= 1^4 + 4bi - 6b^2 - 4b^3i + b^4 = (1 + b^4 - 6b^2) + i(4b - 4b^3)$$

$$(1+bi)^3 = 1^3 + \binom{3}{1}1^2 bi + \binom{3}{2}1(bi)^2 + (bi)^3 = 1 + 3bi - 3b^2 - b^3i =$$

$$= (1 - 3b^2) + i(3b - b^3)$$

$$p(z_1) = \big(1 + b^4 - 6b^2 - 2(1 - 3b^2) + 12(1 - b^2) - 14 + 35\big) + i\big(4b - 4b^3 - 2(3b - b^3) + 24b - 14b\big) =$$

$$= (32 - 12b^2 + b^4) + i(8b - 2b^3)$$

$$1 + b^4 - \cancel{6b^2} - 2 + \cancel{6b^2} + 12 - 12b^2 - 14 + 35 \qquad 4b - 4b^3 - 6b + 2b^3 + 24b - 14b$$

$$\begin{cases} 32 - 12b^2 + b^4 = 0 \\ 8b - 2b^3 = 0 \iff b(8 - 2b^2) = 0 \iff 2b(4 - b^2) = 0 \iff 2b(2-b)(2+b) = 0 \end{cases}$$

$\cancel{b_1 = 0}$ $\quad b_{2,3} = \pm 2$

falsk rot

$$z_1 = 1 + 2i \qquad z_2 = 1 - 2i$$

$$(z - z_1)(z - z_2) = \big(z - (1+2i)\big)\big(z - (1-2i)\big) = z^2 - (1-2i)z - (1+2i)z + (1+2i)(1-2i) =$$

$$= z^2 - 2z + 5$$

$$z^2 + 7$$

$$\cancel{z^4} - \cancel{2z^3} + 12z^2 - 14z + 35 = 0 \;\; | \; z^2 - 2z + 5$$

$$-(\cancel{z^4} - \cancel{2z^3} + 5z^2)$$

$$\cancel{7z^2 - 14z + 35}$$

$$-\cancel{(7z^2 - 14z + 35)}$$

$$0$$

$$p(z) = \big(z - (1+2i)\big)\big(z - (1-2i)\big)(z^2 + 7) = 0$$

$$z^2 + 7 = 0$$

$$z = 0 \pm \sqrt{-7} = \pm\sqrt{7}\,i$$

Sv: $z_{1,2} = 1 \pm 2i \qquad z_{3,4} = \pm\sqrt{7}\,i$