11.17 a

lördag 7 januari 2023 00:55

$$e^t = f(0) + f'(0)x + f''(0)\frac{1}{2}x^2 + f'''(0)\frac{1}{6}x^3 + f^{(4)}\frac{1}{24}x^4 =$$

$$= 1 + t + \tfrac{1}{2}t^2 + \tfrac{1}{6}t^3 + \tfrac{1}{24}t^4 + t^5 B_1(t)$$

$$= 1 + (x^2) + \tfrac{1}{2}(x^2)^2 + \tfrac{1}{6}(x^2)^3 + \tfrac{1}{24}(x^2)^4 + (x^2)^5 B_1(x^2) =$$

$$e^{x^2} = 1 + (x^2) + \tfrac{1}{2}(x^2)^2 + \tfrac{1}{6}(x^2)^3 + \tfrac{1}{24}(x^2)^4 + (x^2)^5 B_1(x^2) = 1 + x^2 + \tfrac{1}{2}x^4 + \tfrac{1}{6}x^6 + \tfrac{1}{24}x^8 + x^{10}B(x) =$$

$$x^6\left(\tfrac{1}{6} + \tfrac{1}{24}x^2 + x^4 B(x)\right) = x^6 B_1(x)$$

$$= 1 + x^2 + \tfrac{1}{2}x^4 + \tfrac{1}{6}x^6 + \tfrac{1}{24}x^8 + x^{10}B(x)$$

$$= 1 + x^2 + \tfrac{1}{2}x^4 + x^6 B_1(x)$$

$$\cos x = 1 - 0x - \tfrac{1}{2}x^2 + 0\,\tfrac{1}{6}x^3 + \tfrac{1}{24}x^4 + x^6 B_2(x) =$$

$$= 1 - \tfrac{1}{2}x^2 + \tfrac{1}{24}x^4 + x^6 B_2(x)$$

$f(x) = \cos x \quad f'(x) = -\sin x \quad f''(x) = -\cos x \quad f'''(x) = \sin x \quad f^{(4)}(x) = f(x)$

$$x^6\left(B_2(x)\ x^6 B_1(x)\right) = x^6 B_3(x)$$

$$e^{x^2}\cos x = \left(1 + x^2 + \tfrac{1}{2}x^4 + x^6 B_1(x)\right)\left(1 - \tfrac{1}{2}x^2 + \tfrac{1}{24}x^4 + x^6 B_2(x)\right) =$$ (Se s. 268-269)

$$= 1 - \tfrac{1}{2}x^2 + \tfrac{1}{24}x^4 - x^2 - \cancel{\tfrac{1}{2}x^4} + \cancel{\tfrac{1}{2}x^4} + x^6 B_3(x) =$$

$$= 1 - \tfrac{3}{2}x^2 + \tfrac{1}{24}x^4 + x^6 B_3(x)$$