15.60

torsdag 5 januari 2023 20:26

$$y^{(4)} - 3y'' - 4y = e^{-2x}$$

$\underline{y_h:}$ $P(r) = r^4 - 3r^2 - 4 = 0 \qquad r^2 = t$

$$t^2 - 3t - 4 = 0$$

$$t = 3/2 \pm \sqrt{9/4 + \frac{16}{4}} = 3/2 \pm 5/2$$

$$t_1 = r^2 = 4 \qquad t_2 = r^2 = -1$$

$$r_1 = 2 \quad r_2 = -2 \quad r_{3,4} = \pm i$$

$$y_h = Ae^{2x} + Be^{-2x} + C\cos x + D\sin x$$

$\underline{y_p:}$

$$y_p = z e^{-2x}$$

$$\underline{y'} = -2ze^{-2x} + z'e^{-2x} = \underline{e^{-2x}(z' - 2z)}$$

$$\underline{y''} = e^{-2x}(z'' - 2z') - 2e^{-2x}(z' - 2z) =$$

$$= \underline{e^{-2x}(z'' - 4z' + 4z)}$$

$$\underline{y^{(3)}} = -2e^{-2x}(z'' - 4z' + 4z) + e^{-2x}(z^{(3)} - 4z'' + 4z') =$$

$$= \underline{e^{-2x}(z^{(3)} - 6z'' + 12z' - 8z)}$$

$$\underline{y^{(4)}} = -2e^{-2x}(z^{(3)} - 6z'' + 12z' - 8z) + e^{-2x}(z^{(4)} - 6z^{(3)} + 12z'' - 8z') =$$

$$= \underline{e^{-2x}(z^{(4)} - 8z^{(3)} + 24z'' - 32z' + 16z)}$$

$$\cancel{e^{-2x}}(z^{(4)} - 8z^{(3)} + 24z'' - 32z' + 16z) - 3\cancel{e^{-2x}}(z'' - 4z' + 4z) - 4z\cancel{e^{-2x}} = \cancel{e^{-2x}} \Leftrightarrow$$

$$\Leftrightarrow z^{(4)} - 8z^{(3)} + 24z'' - 32z' + \cancel{16z} - 3z'' + 12z' - \cancel{12z} - \cancel{4z} = 1$$

$$\Leftrightarrow z^{(4)} - 8z^{(3)} + 21z'' - 20z' = 1$$

$$z = Ax, \quad z' = A \qquad z'' = z^{(3)} = z^{(4)} = 0$$

$$0 - 0 + 0 - 20A = 1 \qquad A = -1/20$$

$$y_p = -1/20\, x e^{-2x}$$

Sv: $y = Ae^{2x} + Be^{-2x} + C\cos x + D\sin x - \frac{1}{20} x e^{-2x}$