a)
$$P_3(x) = f(0) + f'(0)x + f''(0) \cdot \frac{1}{2}x^2 + f'''(0) \cdot \frac{1}{3!}x^3$$

$$f'(x) = f''(x) = f'''(x) = e^x$$

$$f'(0) = 1$$

$$\boxed{S_r: P_3(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3}$$

b)
$$R_4(x) = \frac{f^{(4)}(\beta)}{4!}x^4 = \frac{f^{(4)}(\theta x)}{4!}x^4 = \frac{e^{\theta x}}{4!}x^4 \qquad 0 \leq \theta \leq 1$$

c) $e < 3$

$$|R_4(x)| = \frac{e^{\theta x}}{4!}x^4 \quad, \quad |x| \leq 0.1 \quad 0 \leq \theta \leq 1$$

$$|R_4(x)| < \frac{3}{4!}(0.1)^4 = \frac{3}{24} \cdot \frac{1}{10^4} = \frac{1}{8 \cdot 10^4}$$

d)
$$|R_4(x)| = \frac{e^{\theta x}}{4!}x^4 \quad, \quad |x| \leq 0.1 \qquad 0 \leq \theta \leq 1$$

$$|R_4(x)| \leq C \cdot x^4 = \frac{1}{8} \cdot x^4 \quad (\text{se c}) \longrightarrow c \approx 3$$