15.34

lördag 24 december 2022 13:25

$x \geq 1$ $\quad f(1) = 1$

$$x f(x) = x + \int_1^x \frac{t}{1+t} f(t)\, dt$$

$$\left(x f(x)\right)' = 1 + \frac{x}{1+x} f(x) \Leftrightarrow$$

$$\Leftrightarrow f(x) + x f'(x) = 1 + \frac{x}{1+x} f(x) \Leftrightarrow$$

$$\Leftrightarrow f(x) - \frac{x}{1+x} f(x) + x f'(x) = 1 \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{1+x} f(x) + x f'(x) = 1 \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{x(1+x)} f(x) + f'(x) = 1/x \Leftrightarrow$$

partial bråk uppdelning

$$\boxed{g(x) = 1/x(1+x) \quad G(x) = \ln\left(\frac{x}{1+x}\right) \quad IF = e^{\ln\left(\frac{x}{1+x}\right)} = \frac{x}{1+x}}$$

$$f(x) \cdot e^{\ln\left(\frac{x}{1+x}\right)} = \int \frac{1}{x} \cdot e^{\ln\left(\frac{x}{1+x}\right)} dx = \int \cancel{\frac{1}{x}} \cdot \frac{\cancel{x}}{1+x} dx = \int \frac{1}{1+x} dx = \ln(1+x) + C$$

$$f(x) = \ln(1+x) \cdot e^{-\ln\left(\frac{x}{1+x}\right)} + C e^{-\ln\left(x/1+x\right)} =$$

$$= \frac{1+x}{x}\left(\ln(1+x) + C\right)$$

$$f(1) = 2 \cdot (\ln 2 + C) = 1 \Leftrightarrow \ln 2 + C = 1/2 \qquad \boxed{C = 1/2 - \ln 2}$$

$$\text{Svar: } f(x) = \frac{1+x}{x}\left(\ln(1+x) + 1/2 - \ln 2\right)$$