12.17

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a)
$$\int x^2 \ln x \, dx = \frac{x^3}{3} \cdot \ln x - \int \frac{\cancel{x^3}^{\,2}}{3} \cdot \frac{1}{\cancel{x}} \, dx =$$
$$= \frac{x^3 \cdot \ln x}{3} - \int \frac{1}{3} x^2 \, dx = \frac{x^3 \ln x}{3} - \frac{x^3}{9} + C$$

b)
$$\int x e^{-x} \, dx = -x e^{-x} - \int -1 \cdot e^{-x} \, dx = -x e^{-x} - \left(e^{-x} + C\right) = -x e^{-x} - e^{-x} - C$$

c)
$$\int \sqrt{x} \ln x \, dx = x^{3/2} \cdot \frac{2}{3} \cdot \ln x - \int \underbrace{\frac{2}{3} x^{3/2} \cdot \frac{1}{x}}_{\frac{2}{3} \cdot x^{1/2}} \, dx = \frac{2}{3} x^{3/2} \ln x - \frac{4}{9} x^{3/2} + C$$

d)
$$\int \arctan x \, dx = \int 1 \cdot \arctan x \, dx = x \arctan x - \int x \cdot \frac{1}{1+x^2} \, dx = x \arctan x - \frac{1}{2} \ln(1+x^2) + C$$

e)
$$\int x \arctan x \, dx = \frac{x^2}{2} \arctan x - \int \frac{x^2}{2} \cdot \frac{1}{1+x^2} = \frac{x^2}{2} \arctan x - \frac{1}{2} \int x^2 \cdot \frac{1}{1+x^2} \, dx = \frac{x^2}{2} \arctan x - \frac{1}{2} \int 1 - \frac{1}{1+x^2} \, dx = \frac{x^2}{2} \arctan x - \frac{1}{2}\left(x - \arctan x + C_0\right) = \frac{x^2}{2} \arctan x - \frac{x}{2} + \frac{1}{2} \arctan x + C_1 =$$
$$= \frac{1}{2}\left(x^2 \arctan x - x + \arctan x\right) + C_1$$

Poly.div

$C_0/2$

f)
$$\int \ln(x+1) \, dx = \int 1 \cdot \ln(x+1) \, dx = x \ln(x+1) - \int x \cdot \frac{1}{x+1} \, dx = x \ln(x+1) - \int 1 - \frac{1}{x+1} \, dx = x \ln(x+1) - \left(x - \ln(x+1) + C\right) = x \ln(x+1) - x + \ln(x+1) - C$$

Poly.div

g)
$$\int (\ln x)^2 \, dx = \int 1 \cdot (\ln x)^2 \, dx = x(\ln x)^2 - \int \cancel{x} \cdot 2 \ln x \cdot \frac{1}{\cancel{x}} \, dx = x(\ln x)^2 - \left(2x \ln x - \int 2\cancel{x} \cdot \frac{1}{\cancel{x}} \, dx\right) = x(\ln x)^2 - 2x \ln x + 2x + C = x\left((\ln x)^2 - 2 \ln x + 2\right) + C$$

h)
$$\int x^2 \sin x \, dx = -\cos x \cdot x^2 - \int -\cos x \cdot 2x \, dx = -\cos x \cdot x^2 - \left(-\sin x \cdot 2x - \underbrace{\int -\sin x \cdot 2 \, dx}_{F(x) = 2\cos x + C}\right) = -\cos x \cdot x^2 + 2x \sin x + 2 \cos x + C = (2 - x^2)\cos x + 2x \sin x + C$$