14.02

torsdag 22 december 2022 18:23

a)

$$\frac{2}{1+x^2} = \frac{1}{\sqrt{1+x^2}} \iff 2 = \sqrt{1+x^2} \iff \pm 2^2 = 1+x^2 \implies x = \sqrt{3}$$

$$\int_0^{\sqrt{3}} \frac{2}{1+x^2} - \int_0^{\sqrt{3}} \frac{1}{\sqrt{1+x^2}}\,dx = \left[2\arctan x - \ln\left|x+\sqrt{1+x^2}\right|\right]_0^{\sqrt{3}} = 2\arctan\sqrt{3} - \ln\left|\sqrt{3}+2\right| - \left(2\arctan 0 - \ln|1|\right) =$$

$\arctan\sqrt{3} = \frac{\pi}{3}$, $\arctan 0 = 0$, $\ln|1| = 0$

$$= \frac{2\pi}{3} - \ln\left|\sqrt{3}+2\right|$$

b)

$$\int_0^{\pi/2} \sin x - \left(1-\cos^2 x\right)dx = \int_0^{\pi/2} \sin x - \sin^2 x\,dx = \left[-\cos x\right]_0^{\pi/2} - \int_0^{\pi/2} \frac{1-\cos 2x}{2}\,dx = \left[-\cos x - \frac{1}{2}x + \frac{\sin 2x}{4}\right]_0^{\pi/2} = 0 - \frac{\pi}{4} + 0 - \left(-1 - 0 + 0\right) = 1 - \frac{\pi}{4}$$