13.14

onsdag 21 december 2022 21:47

$$\int \frac{x+1}{x^2+5x+6}\,dx = \int \frac{x+1}{(x+5/2)^2-\frac{1}{4}}\,dx = \int 4 \cdot \frac{x+1}{(2x+5)^2-1}\,dx = \left[\begin{matrix} t = 2x+5 & x = \frac{t-5}{2} \\ \frac{dx}{dt} = \frac{1}{2} & dx = \frac{1}{2}dt \end{matrix}\right] =$$

$$= 2\int \frac{\frac{t-5}{2}+1}{t^2-1}\,dt = 2\int \frac{t-3}{2(t^2-1)}\,dt = \int \frac{t}{t^2-1}\,dt - \int \frac{3}{t^2-1}\,dt = \frac{1}{2}\ln(t^2-1) - \int \frac{A}{t-1} + \frac{B}{t+1}\,dt =$$

$t-1 \quad t+1$

$$3 = A(t+1) + B(t-1) = (A+B)t + A - B$$

$$\begin{cases} A+B=0 \\ A-B=3 \end{cases} \quad 2A = 3 \quad \boxed{A = 3/2 \quad B = -3/2}$$

$$= \frac{1}{2}\ln(t^2-1) - \frac{3}{2}\left(\ln\left|\frac{t-1}{t+1}\right|\right) + C =$$

$$= \frac{1}{2}\left(\ln\frac{(t^2-1)|t+1|^3}{|t-1|^3}\right) + C = \frac{1}{2}\left(\ln\frac{|t+1|^4}{|t-1|^2}\right) + C =$$

$$= \ln\frac{(t+1)^2}{(t-1)} + C = \ln\frac{(2x+6)^2}{2x+4} + C = \ln\frac{4x^2+24x+36}{2(x+2)} + C = \ln\frac{2x^2+12x+18}{x+2} + C$$

$$\int_0^1 \frac{x+1}{x^2+5x+6}\,dx = \left[\ln\frac{2x^2+12x+18}{x+2}\right]_0^1 = \ln\frac{32}{3} - \ln\frac{18}{2} = \underline{\underline{\ln\frac{32}{27}}}$$