6.41

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a) $z^3 = i \qquad |z^3| = 1$

$z^3 = (r e^{i\theta})^3 = r^3 e^{i3\theta}$

$\begin{cases} r^3 = 1 \\ 3\theta = \pi/2 + k\,2\pi \end{cases} \qquad \begin{cases} r = 1 \\ \theta = \pi/6 + k\,2/3\pi \end{cases} \qquad k = \text{heltal}$

$z_k = e^{i(\pi/6 + k\,2\pi/3)} \qquad k = 0, 1, 2$

$z_1 = e^{i\pi/6} = \cos \pi/6 + i \sin \pi/6 = \underline{\frac{\sqrt{3}}{2} + i\,\frac{1}{2}}$

$z_2 = e^{i\,5\pi/6} = \underline{-\sqrt{3}/2 + i\,1/2}$

$z_3 = e^{i\,9\pi/6} = e^{i\,3\pi/2} = \cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2} = 0 - i = \underline{-i}$

b) $z^3 = 1 + i \qquad |z^3| = |z|^3 = r^3 = \sqrt{2}$

$z^3 = r^3 e^{i3\theta}$

$\begin{cases} r^3 = \sqrt{2} \\ 3\theta = \frac{\pi}{4} + k\,2\pi \end{cases} \qquad \begin{cases} r = 2^{1/6} \\ \theta = \frac{\pi}{12} + k\,2\pi/3 \end{cases}$

$1/6 \quad i\pi/12$

$z_1 = 2\ e$

$z_2 = 2^{1/6}\ e^{i\,3\pi/4}$

$z_3 = 2^{1/6}\ e^{i\,17\pi/12}$

c) $z^4 = 16 \qquad |z^4| = \sqrt{2^8} = 16$

$z^4 = r^4 e^{i4\theta}$

$r^4 = 16 \qquad r = 2$

$4\theta = 0 + k\,2\pi \qquad \theta = 0 + k\,\pi/2$

$z_1 = 2e^0 = 2 \qquad z_2 = 2e^{i\pi/2} = 2i \qquad z_3 = 2e^{i\pi} = -2 \qquad z_4 = 2e^{i\,3\pi/2} = -2i$

d) $z^3 = i\sqrt{3} - 1 \qquad |z^3| = 2$

$r^3 = 2 \qquad r = 2^{1/3}$

$3\theta = 2\pi/3 + k\,2\pi \qquad \theta = 2\pi/9 + k\,2\pi/3$

$z_1 = 2^{1/3}\ e^{i\,3\pi/9} \qquad z_2 = 2^{1/3}\ e^{i\,8\pi/9} \qquad z_3 = 2^{1/3}\ e^{i\,14\pi/9}$

f) $z^4 = -1 \qquad |z^4| = 1$

$r^4 = 1 \qquad r = 1$

$4\theta = \pi + k\,2\pi \qquad \theta = \pi/4 + k\,\pi/2$

$z_1 = e^{i\pi/4} = \frac{1}{\sqrt{2}} + i\,\frac{1}{\sqrt{2}} \qquad z_2 = e^{i\,3\pi/4} = -\frac{1}{\sqrt{2}} + i\,\frac{1}{\sqrt{2}} \qquad z_3 = e^{i\,5\pi/4} = -\frac{1}{\sqrt{2}} - i\,\frac{1}{\sqrt{2}}$

$z_2 = e^{i\,7\pi/4} = e^{-i\pi/4} = \frac{1}{\sqrt{2}} - i\,\frac{1}{\sqrt{2}}$

e) $z^5 = 4i \qquad |z^5| = \sqrt{4^2} = 4$

$r^5 = 4 \qquad r = (4)^{1/5}$

$5\theta = \pi/2 + k\,2\pi \qquad \theta = \pi/10 + k\,2\pi/5$

$z_1 = 4^{1/5}\ e^{i\pi/10} \qquad z_2 = 4^{1/5}\ e^{i\pi/2} \qquad z_3 = 4^{1/5}\ e^{i\,9\pi/10} \qquad z_4 = 4^{1/5}\ e^{i\,13\pi/10}$

$z_5 = 4^{1/5}\ e^{i\,17\pi/10}$

$4^{1/5}\ i$