13.15

onsdag 21 december 2022 22:31

a)

$$\int \ln(1+x^2)\,dx = \int 1\cdot \ln(1+x^2)\,dx = x\cdot\ln(x^2+1) - \int 2x^2\cdot\frac{1}{1+x^2}\,dx = x\cdot\ln(1+x^2) - 2\int 1-\frac{1}{1+x^2}\,dx = x\ln(1+x^2) - 2x + 2\arctan x + C$$

$$\int_0^1 \ln(1+x^2)\,dx = \Big[x\ln(1+x^2) - 2x + 2\arctan x\Big]_0^1 = \ln 2 - 2 + \frac{\pi}{2}$$

b)

$$\int e^x\ln(1+e^x)\,dx = e^x\ln(1+e^x) - \int e^x\left(\frac{e^x}{1+e^x}\right)dx = e^x\ln(1+e^x) - \int e^x\left(1-\frac{1}{1+e^x}\right)dx = e^x\ln(1+e^x) - \left(e^x - \ln(1+e^x)\right) =$$

$$= e^x\ln(1+e^x) - e^x + \ln(1+e^x) + C = (e^x+1)\ln(1+e^x) - e^x + C$$

$$\int_0^1 e^x\ln(1+e^x)\,dx = \Big[(e^x+1)\ln(1+e^x) - e^x\Big]_0^1 = (e+1)\ln(1+e) - e - (2\ln 2 - 1) = \underline{(e+1)\ln(1+e) - e - 2\ln 2 + 1}$$