15.57

måndag 26 december 2022 19:12

$y'' + \frac{1}{4} y = -\sin x \quad y(0) = y(\pi) = 0$

$y = y_h + y_p$

- y_h: $p(r) = r^2 + \frac{1}{4} = 0 \quad r = 0 \pm \sqrt{-\frac{1}{4}} = \pm \frac{1}{2} i$

$y_h = A \cos \frac{1}{2} x + B \sin \frac{1}{2} x$

- y_p: hjälp ekv: $y'' + \frac{1}{4} y = -e^{ix} = -(\cos x + i \sin x)$ (Im)

$y_p = z e^{ix}$

$y' = z' e^{ix} + i z e^{ix} = e^{ix}(z' + iz)$

$y'' = i e^{ix}(z' + iz) + e^{ix}(z'' + iz') = e^{ix}(z'' + 2iz' - z)$

$e^{ix}(z'' + 2iz' - z) + \frac{1}{4} z e^{ix} = -e^{ix}$

$\Leftrightarrow \quad z'' + (\frac{1}{4} + 2i) z' - \frac{3}{4} z = -1$

- $z = A$

$0 + 0 - \frac{3}{4} A = -1 \quad , A = \frac{4}{3} \quad , y_p = \frac{4}{3} e^{ix} = \frac{4}{3}(\cos x + i \sin x)$ (Im)

$y_p = \frac{4}{3} \sin x$

$y = A \cos \frac{1}{2} x + B \sin \frac{1}{2} x + \frac{4}{3} \sin x$

$y(0) = A \cos 0 + B \sin 0 + \frac{4}{3} \sin 0 \quad , A = 0$

($\cos 0 = 1$; $B \sin 0 = 0$; $\frac{4}{3} \sin 0 = 0$)

$y(\pi) = A \cos \frac{\pi}{2} + B \sin \frac{\pi}{2} + \frac{4}{3} \sin \pi \quad , B = 0$

($A \cos \frac{\pi}{2} = 0$; $\sin \frac{\pi}{2} = 1$; $\frac{4}{3} \sin \pi = 0$)

Sv: $y = \frac{4}{3} \sin x$