

$$f(x) = \sin x$$

a)

$$P_1(x) = f(0) + f'(0)x = x$$

$$f(0) = 0 \quad f'(x) = \cos x \quad f'(0) = 1$$

$$P_2(x) = P_1(x) + \frac{f''(0)}{2!} \cdot x^2 = P_1(x)$$

$$f''(x) = -\sin x \quad f''(0) = 0$$

$$\text{S: } P_2(x) = P_1(x) = x$$

b)

$$R_2(x) = \frac{f''(\beta)}{2!} x^2 \quad 0 \leq \beta \leq x \quad \text{alt. } R_2(x) = \frac{f''(\theta x)}{2!} x^2, \quad 0 \leq \theta \leq 1$$

$$R_3(x) = \frac{f^{(3)}(\beta)}{3!} x^3 \quad 0 \leq \beta \leq x \quad f^{(3)}(x) = -\cos x$$

$$\text{S: } f(x) = x - \frac{\sin \beta}{2} x^2$$

$$f(x) = x - \frac{\cos \beta}{6} x^3$$

$$c) |R_2(x)| = \left| -\frac{\sin \theta x}{2} \cdot x^2 \right| = \frac{|\sin \theta x|}{2} x^2 \leq \frac{1}{2} x^2 \leq \frac{1}{2} \cdot \frac{1}{10^2} = \frac{1}{200}$$

$\boxed{-1 \leq \sin \theta x \leq 1}$

$\sin \theta x \leq 1$

$x < 0.1$

$$|R_3(x)| = \left| -\frac{\cos \theta x}{6} x^3 \right| \leq \frac{1}{6} x^3 \leq \frac{1}{6} \cdot \frac{1}{10^3} = \frac{1}{6000}$$

$\cos \theta x \leq 1$

$x < 0.1$

$$\boxed{-1 \leq \cos \theta x \leq 1}$$

$$\text{S: } R_2(x) \text{ är minst 2 gånger bättre}$$