15.53

måndag 26 december 2022 17:03

$y'' + 4y = \sin 2x$

y_h: $p(r) = r^2 + 4 = 0 \quad r = 0 \pm \sqrt{-4} \quad r = \pm 2i$

$y_h = (A\cos 2x + B\sin 2x)$

y_p: hjälp ekv. $y'' + 4y = e^{i2x} = (\cos 2x + i\sin 2x)$

Im

$y_p = z e^{i2x}$, $y' = z' e^{i2x} + 2iz e^{i2x} = e^{i2x}(z' + 2iz)$

$y'' = 2i e^{2ix}(z' + 2iz) + e^{i2x}(z'' + 2iz') = e^{i2x}(z'' + 4iz' - 4z)$

$e^{i2x}(z'' + 4iz' - 4z) + 4z e^{i2x} = e^{i2x} \Leftrightarrow$

$\Leftrightarrow \quad z'' + 4iz' = 1$

a $z = Ax$

$0 + 4iA = 1 \Leftrightarrow A = 1/4i = -i/4$, $y_p = -i/4\, e^{i2x}$

$y_p = -i/4\, e^{i2x} = -i/4(\cos 2x + i\sin 2x) = -1/4\, i\cos 2x + 1/4 \sin 2x$

Im

Sv: $y = (A\cos 2x + B\sin 2x) - 1/4\, i\cos 2x$