6.17

måndag 19 december 2022 11:33

$z + \frac{1}{z} = \mathbb{R}$, $z = a + bi$

$$\frac{z^2+1}{z} = \frac{z^2\bar{z} + \bar{z}}{|z|^2} = \frac{(a+bi)^2(a-bi)+(a-bi)}{a^2+b^2} =$$

$$= \frac{(a^2+b^2)(a+bi)+(a-bi)}{a^2+b^2} = \frac{a^3+a^2bi+b^2a+b^3i+a-bi}{a^2+b^2} =$$

$$= \frac{(a^3+a+ab^2)+(a^2b+b^2a+b^3-b)i}{a^2+b^2}$$

$\mathrm{Im}\left(z+\frac{1}{z}\right) = 0$ $\quad \mathrm{Re}\left(z+\frac{1}{z}\right) \in \mathbb{R}$

Sv: alla tal på reella axeln, utom origo, och enhetscirkeln