12.38

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a)

$$D\ln\left(x+\sqrt{x^2+1}\right)+C=\frac{1}{x+\sqrt{x^2+1}}\cdot\left(1+\frac{1}{\cancel{2}}\cdot\frac{1}{\sqrt{x^2+1}}\cdot\cancel{2}x\right)=$$

$$=\frac{1}{\cancel{x+\sqrt{x^2+1}}}\cdot\frac{\cancel{\sqrt{x^2+1}+x}}{\sqrt{x^2+1}}=\frac{1}{\sqrt{x^2+1}}\quad V.S.V$$

b) c)

V.S.V c)

$$\int\frac{1}{\sqrt{x^2+1}}\,dx=\left[\begin{array}{ll} t-x=\sqrt{x^2+1} & x=\dfrac{t^2-1}{2t}\\ dx=\dfrac{t^2+1}{2t^2} & \end{array}\right]=$$

$$\sqrt{x^2+1}=t-\frac{t^2-1}{2t}=\frac{2t^2-t^2+1}{2t}=\frac{t^2+1}{2t}$$

c)

$$t-x=\sqrt{x^2+1}\Rightarrow$$

$$(t-x)^2=x^2+1\Leftrightarrow$$

$$t^2-2tx+\cancel{x^2}=\cancel{x^2}+1$$

$$t^2-1=2tx\Leftrightarrow x=\frac{t^2-1}{2t}$$

$$=\int\frac{1}{\frac{t^2+1}{2t}}\cdot\frac{t^2+1}{2t^2}\,dt=\int\frac{\cancel{2t}}{\cancel{t^2+1}}\cdot\frac{\cancel{t^2+1}}{\cancel{2t^2}_{\,t}}\,dt=\int\frac{1}{t}\,dt=$$

$$=\ln|t|+C=\ln\left|x+\sqrt{x^2+1}\right|+C\overset{x>-1}{=}\ln\left(x+\sqrt{x^2+1}\right)+C$$