

6.53

torsdag 29 december 2022 20:06

$$P(x) = x^5 - x^4 + 4x - 4$$

gissning:  $x = \pm 1, \pm 2, \pm 3$

$$P(1) = 1 - 1 + 4 - 4 = 0$$

$x - 1$  är därav en faktor till  $P(x)$

$$\begin{array}{r} x^4 + 4 \\ \hline \cancel{x^5} - \cancel{x^4} + 4x - 4 \quad | \quad x - 1 \\ -(\cancel{x^5} - \cancel{x^4}) \\ \hline \cancel{4x} - 4 \\ -(\cancel{4x} - 4) \\ \hline 0 \end{array}$$

$$P(x) = (x - 1)(x^4 + 4)$$

$$x^4 + 4 = 0$$

$$x^4 = -4 \quad x^4 = r^4 e^{i4\theta} = 4 e^{i\pi}$$

$$\begin{cases} r^4 = 4 \\ 4\theta = \pi + 2\pi n \end{cases} \quad \begin{cases} r = \sqrt{2} \\ \theta = \pi/4 + \pi/2 n \end{cases} \quad n = 0, 1, 2, 3$$

$$x_0 = \sqrt{2} e^{i\pi/4} = 1 + i \quad x_1 = \sqrt{2} e^{i3\pi/4} = -1 + i \quad x_2 = \sqrt{2} e^{i5\pi/4} = -1 - i$$

$$x_3 = \sqrt{2} e^{i7\pi/4} = 1 - i$$

$$P(x) = (x - 1)(x^4 + 4) = (x - 1)(x - x_0)(x - x_1)(x - x_2)(x - x_3) =$$

$$= (x - 1)(x - (1 + i))(x - (-1 + i))(x - (-1 - i))(x - (1 - i)) =$$

$$= (x - 1)(x^2 - (1 - i)x - (1 + i)x + (1 + i)(1 - i))(x^2 - (-1 - i)x - (-1 + i)x + (-1 + i)(-1 - i)) =$$

$$= (x - 1)(x^2 - \cancel{x + xi} - \cancel{x - ix} + 2)(x^2 + \cancel{x + ix} + \cancel{x - ix} + 2) =$$

$$= (x - 1)(x^2 - 2x + 2)(x^2 + 2x + 2)$$

$$\text{Sv: } P(x) = (x - 1)(x^2 - 2x + 2)(x^2 + 2x + 2)$$