6.38

a)

$$z^2 = -i \qquad |z^2| = |z|^2 = \sqrt{(-1)^2} = 1$$

$$\downarrow \atop a^2 + b^2$$

$$z = a + bi$$

$$a^2 - b^2 + 2abi = -i$$

$$\begin{cases} 1) \ a^2 - b^2 = 0 \\ 2) \ 2ab = -1 \\ 3) \ a^2 + b^2 = 1 \end{cases} \quad \widehat{1+3} \begin{cases} 2a^2 = 1 \\ 2ab = -1 \end{cases} \quad a = \pm \tfrac{1}{\sqrt{2}} \quad b = \pm \sqrt{\tfrac{1}{2}}$$

$$\text{Sv: } z_1 = \tfrac{1}{\sqrt{2}} - i\,\tfrac{1}{\sqrt{2}} \ , \ z_2 = -\tfrac{1}{\sqrt{2}} + i\,\tfrac{1}{\sqrt{2}}$$

b)

$$z^2 = 1 + i\sqrt{3} \qquad |z^2| = |z|^2 = \sqrt{1+3} = 2$$

$$\downarrow \atop a^2 + b^2$$

$$z = a + bi$$

$$a^2 - b^2 + 2abi = 1 + i\sqrt{3}$$

$$\begin{cases} 1) \ a^2 - b^2 = 1 \\ 2) \ 2ab = \sqrt{3} \\ 3) \ a^2 + b^2 = 2 \end{cases} \quad \widehat{1+3} \begin{cases} 2a^2 = 3 \\ 2ab = \sqrt{3} \end{cases} \quad a = \pm \sqrt{\tfrac{3}{2}} \quad b = \pm \sqrt{\tfrac{1}{2}}$$

$$\text{Sv: } z_1 = \sqrt{\tfrac{3}{2}} + i\sqrt{\tfrac{1}{2}} \ , \ z_2 = -\sqrt{\tfrac{3}{2}} - \sqrt{\tfrac{1}{2}}\,i$$

c)

$$z^2 = 3 + 4i \qquad |z^2| = |z|^2 = \sqrt{9+16} = \sqrt{25} = 5$$

$$\downarrow \atop a^2 + b^2$$

$$z = a + bi$$

$$a^2 - b^2 + 2abi = 3 + 4i$$

$$\begin{cases} a^2 - b^2 = 3 \\ 2ab = 4 \\ a^2 + b^2 = 5 \end{cases} \quad \begin{cases} 2a^2 = 8 \\ 2ab = 4 \end{cases} \quad a = \pm 2 \quad b = \pm 1$$

$$\text{Sv: } z_1 = 2 + i \ , \ z_2 = -2 - i$$