7.04*

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a) Ex 5.3 s.64

$Y = \mathcal{L}(0y)$

$\mathcal{L}(0y') = s\cdot\mathcal{L}(0y) - y(0) = sY - y(0) = sY$ ($y(0)=0$)

$\mathcal{L}(0y'') = s^2\cdot\mathcal{L}(0y) - sy(0) - y'(0) = s^2Y$ ($y(0)=0$, $y'(0)=0$)

$\mathcal{L}(0y''') = s^3\cdot\mathcal{L}(0y) - s^2y(0) - sy'(0) - y''(0) = s^3Y$ ($y(0)=0$, $y'(0)=0$, $y''(0)=0$)

$F = \mathcal{L}(0f)$

$\mathcal{L}(0f') = s\mathcal{L}(0f) - f(0) = sF - f(0) = sF$

$s^3Y + 3s^2Y + 2Y = sF + 3F \Leftrightarrow \frac{Y(s)}{F(s)} = \frac{s+3}{s(s^2+3s+2)} =$

$= \frac{s+3}{s(s+2)(s+1)}$

b) Satz 7.7 s.94

$H(s) = \frac{s+3}{s(s+2)(s+1)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+1} =$

$= \frac{3}{2}\cdot\frac{1}{s} + \frac{1}{2}\cdot\frac{1}{s+2} - \frac{2}{s+1}$

$h(t) = \frac{3}{2}\theta(t) + \frac{1}{2}e^{-2t}\theta(t) - 2e^{-t}\theta(t) =$

$= \frac{1}{2}(3 + e^{-2t} - 4e^{-t})\theta(t) =$ impulssvaret

c) Satz 7.5 s.91

$S(0) = \int_0^t \frac{1}{2}(3 + e^{-2T} - 4e^{-T})\,dT\cdot\theta(t) =$

$= \left(\frac{1}{2}[3T]_0^t + \frac{1}{2}\left[-\frac{e^{-2T}}{2}\right]_0^t + \frac{1}{2}[4e^{-T}]_0^t\right)\theta(t) =$

$= \frac{1}{2}\left(3t - \frac{e^{-2t}}{2} + \frac{1}{2} + 4e^{-t} - 4\right)\theta(t) =$

$= \frac{1}{4}(6t - e^{-2t} + 8e^{-t} - 7)\theta(t)$