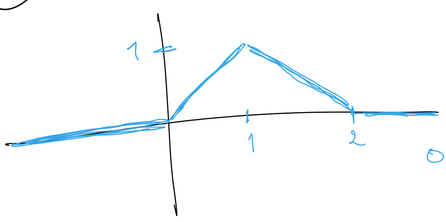
5.13**

tisdag 13 februari 2024 17:23

a)



$f(t)=0 \quad , t<0 \qquad f(t)=-t \quad , 1<t<2$

$f(t)=t \quad , 0<t<1 \qquad f(t)=0 \quad , t>2$

$+\theta(t)-t\theta(t-1) \qquad -t\theta(t-1)+t\theta(t-2)$

$$f(t)=t(\theta(t)-\theta(t-1))-t(\theta(t-1)-\theta(t-2))=$$

$$=t\theta(t)-t\theta(t-1)-t\theta(t-1)+t\theta(t-2)=$$

$$=t\theta(t)-2t\theta(t-1)+t\theta(t-2)=$$

$$F(s)=\frac{1}{s^2}-\frac{2}{s^2}e^{-s}+\frac{1}{s^2}e^{-2s}=$$

$$=\frac{1}{s^2}\left(1-2e^{-s}+e^{-2s}\right)$$

$$\mathcal{L}(f')=\frac{1}{s}\left(1-2e^{-s}+e^{-2s}\right)$$

$$\mathcal{L}(f'')=1-2e^{-s}+e^{-2s}$$

$$\mathcal{L}^{-1}(f'')=\delta(t)-2\delta(t-1)+\delta(t-2)$$

Så: $f''(t)=\delta(t)-2\delta(t-1)+\delta(t-2)$

b)

$$Y=\mathcal{L}(y) \qquad \mathcal{L}(y')=sY \qquad \mathcal{L}(y'')=s^2Y$$

$$s^2Y+2sY+2Y=1-2e^{-s}+e^{-2s}$$

$$\Leftrightarrow \quad Y(s^2+2s+2)=1-2e^{-s}+e^{-2s} \quad \Leftrightarrow$$

$$\Leftrightarrow \quad Y(s)=\frac{1}{(s+1)^2+1}-\frac{2e^{-s}}{(s+1)^2+1}+\frac{e^{-2s}}{(s+1)^2+1}$$

$$y(t)=e^{-t}\sin(t)\theta(t)-2e^{-(t-1)}\sin(t-1)\theta(t-1)+e^{-(t-2)}\sin(t-2)\theta(t-2)$$