7.03*

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a) Kap 7.4

ja, $h(t) = 0$ då $t < 0$

b) Kap 7.6

definitionen för stegsvaret för δ:

$$s(t) = \int_{-\infty}^{t} h(\tau)\, d\tau \iff$$

$$\iff s(t) = \int_{0}^{t} e^{-2\tau}\, d\tau\, \theta(t) \iff$$

$$\Rightarrow s(t) = \left[-\tfrac{1}{2} e^{-2\tau}\right]_0^t \cdot \theta(t) \iff$$

$$\iff s(t) = \left(-\tfrac{1}{2} e^{-2t} + \tfrac{1}{2}\right)\theta(t) = \tfrac{1}{2}\left(1 - e^{-2t}\right)\theta(t)$$

$$\underline{s(\theta) = \tfrac{1}{2}\left(1 - e^{-2t}\right)}$$

c) Kap 7.7

$$\mathcal{L}(h)(s) = \frac{1}{s+2}$$

d) Kap 7.5

Systemet är stabilt då $\int_{-\infty}^{\infty} |h(t)|\, dt =$

$$= \int_{-\infty}^{\infty} e^{-2t}\theta(t)\, dt = \begin{cases} \int_0^\infty e^{-2t}\, dt, & t \geq 0 \\ 0, & t < 0 \end{cases} \Rightarrow$$

$$\Rightarrow \int_0^\infty e^{-2t}\, dt = \tfrac{1}{2}\left[-e^{-2t}\right]_0^\infty = \underline{\tfrac{1}{2}} \text{ (konvergent)}$$

e) Kap 7.8

$$\underline{H(i\omega) = \frac{1}{i\omega + 2}}$$

f)

$$\operatorname{im}\left(H(i2) \cdot e^{i2t}\right)$$

$$H(i2) = \frac{1}{i2+2} = \frac{1\,(2-2i)}{(i2+2)(2-2i)} =$$

$$= \frac{2(1-i)}{2^2 - (2i)^2} = \frac{1-i}{4} = \frac{1}{4} - \frac{i}{4} =$$

$$= \frac{\sqrt{2}}{4}\left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right) = \frac{\sqrt{2}}{4} e^{-i\pi/4}$$

$$\operatorname{im}\left(\frac{\sqrt{2}}{4} e^{-i\pi/4} \cdot e^{i2t}\right) =$$

$$= \operatorname{im}\left(\frac{\sqrt{2}}{4} e^{i(2t - \pi/4)}\right) =$$

$$= \underline{\frac{\sqrt{2}}{4} \sin(2t - \pi/4)}$$

g)

$$H(s) \cdot \mathcal{L}(f) = \frac{1}{s+2} \cdot \frac{2}{s^2 + 2^2} =$$

$A = 1/4$ $B = -1/4$ $C = 1/2$

$$= \frac{2}{(s+2)(s^2+4)} = \frac{A}{s+2} + \frac{Bs + C}{s^2+4} =$$

$$= \frac{1}{4}\frac{1}{s+2} + \frac{1}{4} \cdot \frac{2-s}{s^2+4} = \underbrace{\frac{1}{4}\frac{1}{s+2}}_{Z(s)} + \underbrace{\frac{1}{4}\frac{2}{s^2+2^2}}_{F(s)} - \underbrace{\frac{1}{4}\frac{s}{s^2+2^2}}_{G(s)}$$

$$z(t) = 1/4 \cdot e^{-2t}\theta(t)$$

$$f(t) = 1/4 \sin 2t\, \theta(t)$$

$$g(t) = \frac{1}{4} \cos 2t\, \theta(t)$$

$$\underline{y(t) = \frac{1}{4}\left(e^{-2t} + \sin 2t - \cos 2t\right)\theta(t)}$$