

10.06*

söndag 3 mars 2024

23:01

a) sats 10.2

$$Q^T Q = I$$

b)

matris A är symmetrisk d: $A = A^T$

c)

$$\det(\lambda I - A) = \begin{vmatrix} \lambda + 1 & -12 \\ -12 & \lambda - 6 \end{vmatrix} = (\lambda + 1)(\lambda - 6) - 144 =$$

$$= \lambda^2 - 5\lambda - 150 = 0 \quad \lambda_1 = -10 \quad \lambda_2 = 15$$

$$\underline{(-10I - A)x = 0:}$$

$$\begin{cases} -9x_1 - 12x_2 = 0 \\ -12x_1 - 16x_2 = 0 \end{cases} \Leftrightarrow \begin{cases} -3x_1 = 4x_2 \\ 0 = 0 \end{cases} \Leftrightarrow s_1 = t_1 \begin{pmatrix} -4/3 \\ 1 \end{pmatrix}, t_1 \neq 0$$

$$\underline{(15I - A)x = 0:}$$

$$\begin{cases} 16x_1 - 12x_2 = 0 \\ -12x_1 + 9x_2 = 0 \end{cases} \Leftrightarrow \begin{cases} 4x_1 = 3x_2 \\ 0 = 0 \end{cases} \Leftrightarrow s_2 = t_2 \begin{pmatrix} 3/4 \\ 1 \end{pmatrix}, t_2 \neq 0$$

$$t_1 = \frac{1}{\sqrt{\left(\frac{16}{9}\right) + 1}} = \frac{1}{5/3} = \frac{3}{5}$$

$$t_2 = \frac{1}{\sqrt{\frac{9}{16} + 1}} = \frac{1}{5/4} = \frac{4}{5}$$

$$Q = \begin{pmatrix} -4/5 & 3/5 \\ 3/5 & 4/5 \end{pmatrix} = \frac{1}{5} \begin{pmatrix} -4 & 3 \\ 3 & 4 \end{pmatrix}$$

$$Q^T Q = \frac{1}{25} \begin{pmatrix} -4 & 3 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} -4 & 3 \\ 3 & 4 \end{pmatrix} =$$

$$= \frac{1}{25} \begin{pmatrix} 25 & 0 \\ 0 & 25 \end{pmatrix} = I$$

$$Q^{-1} A Q = \frac{1}{25} \begin{pmatrix} -4 & 3 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} -1 & 12 \\ 12 & 6 \end{pmatrix} \begin{pmatrix} -4 & 3 \\ 3 & 4 \end{pmatrix} =$$

$$= \frac{1}{25} \begin{pmatrix} 40 & -30 \\ 45 & 60 \end{pmatrix} \begin{pmatrix} -4 & 3 \\ 3 & 4 \end{pmatrix} = \frac{1}{25} \begin{pmatrix} -250 & 0 \\ 0 & 375 \end{pmatrix} =$$

$$= \begin{pmatrix} 10 & 0 \\ 0 & 15 \end{pmatrix}$$