9.16*

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a) s. 31

$$e^{at}f(t) = \int_{-\infty}^{\infty} e^{-st}\left(e^{at}\cdot f(t)\right) dt =$$

$$= \int_{-\infty}^{\infty} e^{-(s-a)t} \cdot f(t)\, dt =$$

$$= F(s-a)$$

b) s. 135

$$\mathcal{L}\left(e^{tA}\theta(t)\right) = (sI - A)^{-1}$$

c) s. 136

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$$

$$sI - A = \begin{pmatrix} s-2 & 1 \\ -1 & s \end{pmatrix}$$

$$(sI - A)^{-1} = \frac{1}{\det(sI-A)} \cdot \operatorname{adj}(sI - A) =$$

$$= \frac{1}{(s-1)^2}\begin{pmatrix} s & -1 \\ 1 & s-2 \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{s}{(s-1)^2} & -\frac{1}{(s-1)^2} \\ \frac{1}{(s-1)^2} & \frac{s}{(s-1)^2} - \frac{2}{(s-1)^2} \end{pmatrix} = \begin{pmatrix} \frac{1}{s-1} + \frac{1}{(s-1)^2} & -\frac{1}{(s-1)^2} \\ \frac{1}{(s-1)^2} & \frac{1}{s-1} - \frac{1}{(s-1)^2} \end{pmatrix} =$$

$$\mathcal{L}^{-1}(sI - A)^{-1} = \begin{pmatrix} e^t\theta(t) + te^t\theta(t) & -te^t\theta(t) \\ te^t\theta(t) & e^t\theta(t) - te^t\theta(t) \end{pmatrix} =$$

$$= \begin{pmatrix} e^t(1+t) & -te^t \\ te^t & e^t(1-t) \end{pmatrix}\theta(t) = e^{tA}\theta(t) \Leftrightarrow$$

$$\Leftrightarrow \quad e^{tA} = \begin{pmatrix} e^t(1+t) & -te^t \\ te^t & e^t(1-t) \end{pmatrix}$$