6.09*

S.75

a)

$$Y = \mathcal{L}(\theta y)$$

$$\mathcal{L}(\theta y') = sY$$

$$\mathcal{L}(\theta y'') = s^2 Y$$

$$y'' + y = f(t), \quad t > 0 \quad \Leftrightarrow \quad y''\theta + y\theta = \theta f(t), \quad t > 0$$

$$\Leftrightarrow \quad Y s^2 + Y = F, \quad t > 0 \quad \Leftrightarrow \quad Y(s) = F \cdot \boxed{\frac{1}{s^2 + 1}} \; H$$

b)

$$t > 0, \quad \theta(t-T) \cdot \theta(T) \to 1$$

$$y(t) = \int_0^t \theta(t-T) \cdot f(t-T) \cdot \sin T \cdot \theta(T) \, dT =$$

$$= \int_0^t \sin T \cdot f(t-T) \, dT$$