8.12*

torsdag 29 februari 2024 20:49

$$\det(\lambda I - A) =$$

$$= \begin{pmatrix} \lambda - 5 & 2 \\ -6 & \lambda + 2 \end{pmatrix} = (\lambda-5)(\lambda+2) + 12 =$$

$$= \lambda^2 - 3\lambda + 2 \ , \ \lambda_1 = 2 \ , \ \lambda_2 = 1$$

$$(2I - A)x = 0 :$$

$$-2 \cdot \begin{cases} -3x_1 + 2x_2 = 0 \\ -6x_1 + 4x_2 = 0 \end{cases} \iff \begin{cases} -3x_1 + 2x_2 = 0 \\ 0 = 0 \end{cases} \iff$$

$$\iff \begin{cases} x_1 = t \\ x_2 = 3/2\, t \end{cases} \iff x = t\tfrac{1}{2}\begin{pmatrix} 2 \\ 3 \end{pmatrix} , \ t \neq 0$$

$$\underline{(I - A)x = 0 :}$$

$$\begin{array}{l} \frac{1}{2} \\ -\frac{1}{3} \end{array} \begin{cases} -4x_1 + 2x_2 = 0 \\ -6x_1 + 3x_2 = 0 \end{cases} \iff \begin{cases} -2x_1 + x_2 = 0 \\ 0 = 0 \end{cases} \iff$$

$$\iff \begin{cases} x_1 = t \\ x_2 = 2t \end{cases} \iff x = t\begin{pmatrix} 1 \\ 2 \end{pmatrix} , \ t \neq 0$$

a)

$$S = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \qquad S^{-1} = \begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}$$

$$\underline{\underline{D = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}}}$$

b)

$$A^{10} = S D^{10} S^{-1} = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 2^{10} & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix} =$$

$$= \begin{pmatrix} 2^{11} & 1 \\ 3 \cdot 2^{10} & 2 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 2^{12} - 3 & -2^{11} + 2 \\ 3 \cdot 2^{11} - 6 & -3 \cdot 2^{10} + 4 \end{pmatrix}}}$$

$$\underline{\underline{A^n = \begin{pmatrix} 2^{n+2} - 3 & -2^{n+1} + 2 \\ 3 \cdot 2^{n+1} - 6 & -3 \cdot 2^{n} + 4 \end{pmatrix}}}$$