5.01*

tisdag 13 februari 2024 17:22

$$\begin{cases} y''(t) + 2y'(t) + 5y(t) = e^{-t}, & t > 0 \\ y(0) = 1 & y'(0) = 2 \end{cases}$$

multiplicera med $\theta(t)$

$$\begin{cases} \theta y'' + 2\theta y' + 5\theta y = \underbrace{e^{-t}\theta(t)}_{\frac{1}{s+1}} \end{cases}$$

$Y = \mathcal{L}(\theta y)$

- $\mathcal{L}(\theta y') = s\mathcal{L}(\theta y) - y(0) = s\mathcal{L}(\theta y) - 1 =$
  $= sY - 1$
- $\mathcal{L}(\theta y'') = s^2\mathcal{L}(\theta y) - sy(0) - y'(0) =$
  $= s^2 Y - s - 2$

$$s^2Y - s - 2 + 2(sY - 1) + 5Y = \frac{1}{s+1} \iff$$

$$\iff s^2Y - s - 2 + 2sY - 2 + 5Y = \frac{1}{s+1} \iff$$

$$\iff Y(s^2 + 2s + 5) - s - 4 = \frac{1}{s+1} \iff$$

$$\iff Y(s) = \frac{1 + s(s+1) + 4(s+1)}{(s+1)(s^2+2s+5)} = \frac{s^2 + 5s + 5}{(s+1)(s^2+2s+5)} =$$

$$= \frac{A}{s+1} + \frac{Bs + C}{s^2 + 2s + 5} = \frac{1}{4(s+1)} + \frac{3}{4} \cdot \frac{s+5}{s^2+2s+5} =$$

$A = 1/4$ $B = 3/4$

$C = \frac{15}{4}$

$$= \frac{1}{4}\left(\frac{1}{s+1} + 3\frac{s+1}{(s+1)^2+2^2} + 6 \cdot \frac{2}{(s+1)^2+2^2}\right)$$

inversa Laplacetransformation:

$\theta(t) = 1$ $t > 0$

$$y(t) = 1/4\left(e^{-t} + e^{-t} \cdot 3\sin(2t) + 6\cos(2t)\right) =$$

$$= 1/4 \cdot e^{-t}\left(1 + 3\sin(2t) + 6\cos(2t)\right), \ t \geq 0$$

$5A + C = 5$