9.10*

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a)

$$\det(\lambda I - A) = \begin{vmatrix} \lambda - 1 & 1 \\ -2 & \lambda - 4 \end{vmatrix} =$$

$$= (\lambda - 1)(\lambda - 4) + 2 = \lambda^2 - 5\lambda + 6$$

$$\lambda_1 = 3 \quad \lambda_2 = 2 \qquad s_1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \quad s_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

b)

$$\frac{dx}{dt} = Ax \qquad D = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$$

$$x = Sy$$

$$S\frac{dy}{dt} = ASy = Dy$$

$$\begin{cases} y_1'(t) = 3y_1 \\ y_2'(t) = 2y_2 \end{cases} \qquad \begin{cases} y_1 = c_1 e^{3t} \\ y_2 = c_2 e^{2t} \end{cases}$$

$$x = Sy = \begin{pmatrix} 1 & 1 \\ -2 & -1 \end{pmatrix} \begin{pmatrix} c_1 e^{3t} \\ c_2 e^{2t} \end{pmatrix} =$$

$$= \begin{pmatrix} c_1 e^{3t} + c_2 e^{2t} \\ -2c_1 e^{3t} - c_2 e^{2t} \end{pmatrix}$$

$$\begin{cases} x_1(t) = c_1 e^{3t} + c_2 e^{2t} \\ x_2(t) = -2c_1 e^{3t} - c_2 e^{2t} \end{cases}$$