5.06**

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$Y = \mathcal{L}(y)$

$\mathcal{L}(y') = s\mathcal{L}(y) - y(0) = sY$

$\mathcal{L}(y'') = s^2\mathcal{L}(y) - sy(0) - y'(0) = s^2Y$

$\mathcal{L}(y^{(3)}) = s^3\mathcal{L}(y) - s^2y(0) - sy'(0) - y''(0) = s^3Y$

$\mathcal{L}(y^{(4)}) = s^4\mathcal{L}(y) - s^3y(0) - s^2y'(0) - sy''(0) - y'''(0) =$

$= s^4Y - 1$

$y^{(4)}(t) = y(t) \iff s^4Y - 1 = Y \iff s^4Y - Y = 1 \iff$

$\iff Y(s^4 - 1) = 1 \iff Y(s) = \frac{1}{s^4-1} = \frac{1}{(s^2-1)(s^2+1)} =$

$= \frac{1}{(s^2+1)(s-1)(s+1)} = \frac{As+B}{s^2+1} + \frac{C}{s-1} + \frac{D}{s+1}$

$(As+B)(s^2-1) + C(s^3+s^2+s+1) + D(s^3-s^2+s-1) =$

$= s^3(A+C+D) + s^2(B+C-D) + s(-A+C+D) +$

$+ (-B+C-D)$

$$\begin{cases} A+C+D=0 & (1) \\ B+C-D=0 & (2) \\ -A+C+D=0 & (3) \\ -B+C-D=1 & (4) \end{cases}$$

$D = B + C$

$\Updownarrow$

(4) $-B+C-B+C = 1 \iff \boxed{B = -1/2}$

(1)+(3)

$2C + 2D = 0 \iff \boxed{C = -D} \overset{(1)+(3)}{\iff} \boxed{A = 0}$

(2)

$B + C - D = -\frac{1}{2} - 2D = 0 \iff \boxed{D = -\frac{1}{4}}$

$\boxed{C = \frac{1}{4}}$

$Y = -\frac{1}{2(s^2+1)} + \frac{1}{4(s-1)} - \frac{1}{4(s+1)} = \frac{1}{2}\left(-\frac{1}{s^2+1} + \frac{1}{2}\cdot\frac{1}{s-1} - \frac{1}{2}\cdot\frac{1}{s+1}\right)$

$y(t) = \frac{1}{2}\left(-\sin(t)\theta(t) + \frac{1}{2}\cdot e^t\theta(t) - \frac{1}{2}e^{-t}\theta(t)\right) =$

$= \frac{1}{4}\left(-2\sin(t)\theta(t) + e^t\theta(t) - e^{-t}\theta(t)\right)$

$\theta(t) = 1, \quad t > 0$