5.03

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$$Y = \mathcal{L}(\theta y)$$

$$\begin{cases} \mathcal{L}(\theta y') = s\mathcal{L}(\theta y) - y(0) = sY \\ \mathcal{L}(\theta y'') = s^2\mathcal{L}(\theta y) - sy(0) - y'(0) = s^2 Y \end{cases}$$

$$y''\theta(t) + 2y'\theta(t) + y\theta(t) = e^{-2t}\theta(t) \iff s^2Y + 2sY + Y = \frac{1}{s+2} \iff$$

$$\iff Y(s) = \frac{1}{(s+2)(s+1)^2} = \frac{A}{s+2} + \frac{B}{s+1} + \frac{C}{(s+1)^2} =$$

$$A = 1 \qquad B = -1 \qquad C = 1$$

$$= \frac{1}{s+2} - \frac{1}{s+1} + \frac{1}{(s+1)^2}$$

$$\mathcal{L}^{-1}(Y(s)) = e^{-2t}\theta(t) - e^{-t}\theta(t) + te^{-t}\theta(t)$$

$$\underline{\underline{y(t) = e^{-t}(e^{-t} - 1 + t), \quad t \geq 0}}$$