9.17*

lördag 2 mars 2024 23:47

a)

$$sI - A = \begin{pmatrix} s-3 & 1 \\ -1 & s-1 \end{pmatrix}$$

$$\det(sI - A) = (s-3)(s-1) + 1 = s^2 - 4s + 4 =$$
$$= (s-2)^2$$

$$\text{adj}(sI - A) = \begin{pmatrix} s-1 & -1 \\ 1 & s-3 \end{pmatrix}$$

$$(sI - A)^{-1} = \frac{1}{(s-2)^2} \begin{pmatrix} s-1 & -1 \\ 1 & s-3 \end{pmatrix}$$

b)

$$(sI - A)^{-1} = \begin{pmatrix} \frac{s-1}{(s-2)^2} & -\frac{1}{(s-2)^2} \\ \frac{1}{(s-2)^2} & \frac{s-3}{(s-2)^2} \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{1}{s-2} + \frac{1}{(s-2)^2} & -\frac{1}{(s-2)^2} \\ \frac{1}{(s-2)^2} & \frac{1}{s-2} - \frac{1}{(s-2)^2} \end{pmatrix}$$

$$\mathcal{L}^{-1}\left((sI - A)^{-1}\right) = e^{tA}\theta(t) =$$

$$= \begin{pmatrix} e^{2t}\theta(t) + te^{2t}\theta(t) & -te^{2t}\theta(t) \\ te^{2t}\theta(t) & e^{2t}\theta(t) - te^{2t}\theta(t) \end{pmatrix} =$$

$$= e^{2t} \begin{pmatrix} 1+t & -t \\ t & 1-t \end{pmatrix} \theta(t) \iff$$

$$\iff \underline{e^{tA} = e^{2t} \begin{pmatrix} 1+t & -t \\ t & 1-t \end{pmatrix}}$$

c) Ex 9.4 s.134

$$x(t) = e^{tA} \cdot x(0) + \underbrace{\int_0^t e^{(t-\tau)A} \cdot \overset{0}{f(\tau)}\, d\tau}_{0} =$$

$$= e^{2t} \begin{pmatrix} 1+t & -t \\ t & 1-t \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} =$$

$$= \underline{e^{2t} \begin{pmatrix} 2+t \\ 1+t \end{pmatrix}}$$