9.05***

söndag 3 mars 2024 17:29

$$\begin{cases} \frac{du_1}{dt} = u_1 + 2u_2 \\ \frac{du_2}{dt} = 4u_1 + 3u_2 \end{cases}$$

a)

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$$

$$\det(\lambda I - A) = \begin{vmatrix} \lambda - 1 & -2 \\ -4 & \lambda - 3 \end{vmatrix} =$$

$$= (\lambda - 1)(\lambda - 3) - 8 = \lambda^2 - 4\lambda - 5$$

$$\lambda_1 = 5 \quad \lambda_2 = -1$$

$$D = \begin{pmatrix} 5 & 0 \\ 0 & -1 \end{pmatrix} \qquad S = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$$

$$u = Sv \Leftrightarrow \frac{du}{dt} = S\frac{dv}{dt}$$

$$\frac{du}{dt} = Au \Leftrightarrow S\frac{dv}{dt} = ASv \Leftrightarrow$$

$$\Leftrightarrow \frac{dv}{dt} = S^{-1}ASv = Dv$$

$$\begin{cases} \frac{dv_1}{dt} = 5v_1 \\ \frac{dv_2}{dt} = -v_2 \end{cases} \Rightarrow \begin{cases} v_1(t) = C_1 e^{5t} \\ v_2(t) = C_2 e^{-t} \end{cases}$$

$$v_1(0) = C_1 \quad v_2(0) = C_2$$

$$v(0) = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$$

$u = Sv$:

$$\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} C_1 e^{5t} \\ C_2 e^{-t} \end{bmatrix}$$

$$\begin{cases} u_1(t) = C_1 e^{5t} + C_2 e^{-t} \\ u_2(t) = 2C_1 e^{5t} - C_2 e^{-t} \end{cases}$$

b)

$$u(0) = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \qquad S^{-1} = \frac{1}{3}\begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$$

$$u = Sv \Leftrightarrow v = S^{-1}u$$

$$v(0) = \frac{1}{3}\begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} a_1 + a_2 \\ 2a_1 - a_2 \end{pmatrix}$$

$$\begin{cases} u_1(t) = \frac{1}{3}(a_1 + a_2)e^{5t} + \frac{1}{3}(2a_1 - a_2)e^{-t} \\ u_2(t) = \frac{2}{3}(2a_1 - a_2)e^{5t} - \frac{1}{3}(2a_1 - a_2)e^{-t} \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} u_1(t) = \frac{1}{3}(e^{5t} + 2e^{-t})a_1 + \frac{1}{3}(e^{5t} - e^{-t})a_2 \\ u_2(t) = \frac{1}{3}(4e^{5t} - 2e^{-t})a_1 + \frac{1}{3}(-2e^{5t} + e^{-t})a_2 \end{cases} \leftarrow \text{fel ?}$$