

7.16\*

torsdag 22 februari 2024

19:46

$$y'' + 2y' + 2y = f'' + f'$$

for ett kausalt system

$$\mathcal{L}(y) = Y \quad \mathcal{L}(y') = sY \quad \mathcal{L}(y'') = s^2 Y$$

$$\mathcal{L}(f) = F \quad \mathcal{L}(f') = sF \quad \mathcal{L}(f'') = s^2 F$$

$$s^2 Y + 2sY + 2Y = s^2 F + sF$$

$$Y(s^2 + 2s + 2) = (s^2 + s) F \quad H(s) = \frac{Y}{F} = \frac{s^2 + s}{s^2 + 2s + 2} = \frac{s(s+1)}{(s+1)^2 + 1}$$

$$\mathcal{L}(+e^{-t}\theta(t)) = \frac{1}{(s+1)^2} = G(s)$$

$$G(s) \cdot H(s) = \frac{1}{(s+1)^2} \cdot \frac{s(s+1)}{(s+1)^2 + 1} = \frac{s}{(s+1)^3 + (s+1)} = \frac{s}{(s+1)(s^2 + 2s + 2)}$$

$$= \frac{A}{s+1} + \frac{Bs+C}{s^2+2s+2} = -\frac{1}{s+1} + \frac{s+2}{(s+1)^2+1} = -\frac{1}{s+1} + \frac{s+1}{(s+1)^2+1} + \frac{1}{(s+1)^2+1}$$

$$A = -1 \quad B = 1 \quad C = 2$$

$$\mathcal{L}^{-1}(Q)(t) = -e^{-t}\theta(t) + e^{-t}(\sin t + \cos t)\theta(t)$$

$$q(t) = e^{-t}(\sin t + \cos t - 1)\theta(t)$$