9.06**

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$$A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} \qquad S = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$$

$$D = \begin{pmatrix} 5 & 0 \\ 0 & -1 \end{pmatrix} \qquad S^{-1} = \frac{1}{3}\begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$$

$$f = \begin{pmatrix} -2e^{t} \\ -10e^{t} \end{pmatrix} \qquad a = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$$

$u = Sv$

s. 127

$$\frac{du}{dt} = Au + f \iff \frac{dv}{dt} = Dv + S^{-1}f$$

$$S^{-1}f = \frac{1}{3}\begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} -2e^{t} \\ -10e^{t} \end{pmatrix} = \frac{1}{3}\begin{pmatrix} -12e^{t} \\ 6e^{t} \end{pmatrix} = \begin{pmatrix} -4e^{t} \\ 2e^{t} \end{pmatrix}$$

$$Dv = \begin{pmatrix} 5 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 5v_1 \\ -v_2 \end{pmatrix}$$

$$\begin{cases} \frac{dv_1}{dt} = 5v_1 - 4e^{t} \\ \frac{dv_2}{dt} = -v_2 + 2e^{t} \end{cases}$$

$$v(0) = S^{-1}u(0) \iff v(0) = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

$v_1(t)$:

$v_1' = 5v_1 - 4e^{t} \qquad v_1(0) = 2$

$v_1' - 5v_1 = -4e^{t} \qquad g(t) = -5 \quad G(t) = -5t$

IF e^{-5t}: $v_1' - 5e^{-5t}v_1 = -4e^{-4t}$

$$\frac{d}{dt}\left(e^{-5t} \cdot v_1\right) = -4e^{-4t}$$

$$e^{-5t} \cdot v_1 = \int -4e^{-4t}\,dt = e^{-4t} + C$$

$$v_1 = e^{t} + Ce^{5t} \iff v_1(0) = 2 \quad v_1 = e^{t} + e^{5t}$$

$v_2(t)$:

$$v_2' = -v_2 + 2e^{t} \iff$$

$$v_2' + v_2 = 2e^{t} \iff$$

$$e^{t}v_2' + e^{t}v_2 = 2e^{2t} \iff$$

$$\frac{d}{dt}\left(e^{t} \cdot v_2\right) = 2e^{2t} \iff$$

$$e^{t} \cdot v_2 = \int 2e^{2t}\,dt = e^{2t} + C \iff$$

$$v_2 = e^{t} + Ce^{-t} \iff v_2(0) = 4 \quad v_2 = e^{t} + 3e^{-t}$$

$$\begin{cases} v_1(t) = e^{t} + e^{5t} \\ v_2(t) = e^{t} + 3e^{-t} \end{cases}$$

$$u = Sv \iff u = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} e^{t} + e^{5t} \\ e^{t} + 3e^{-t} \end{pmatrix}$$

$$\begin{cases} u_1(t) = 2e^{t} + e^{5t} + 3e^{-t} \\ u_2(t) = e^{t} + 2e^{5t} - 3e^{-t} \end{cases}$$