10.03

söndag 3 mars 2024 23:01

a)

$$\det(\lambda I - A) = \begin{vmatrix} \lambda - 2 & -2 \\ -2 & \lambda - 5 \end{vmatrix} =$$

$$= (\lambda - 2)(\lambda - 5) - 4 = \lambda^2 - 7\lambda + 6 = 0$$

$$\lambda_1 = 1 \quad \lambda_2 = 6$$

$(I - A)x = 0$:

$$\begin{cases} -x_1 - 2x_2 = 0 \\ -2x_1 - 4x_2 = 0 \end{cases} \Leftrightarrow \begin{cases} x_1 = -2x_2 \\ 0 = 0 \end{cases}$$

$$S_1 = t_1 \begin{pmatrix} -2 \\ 1 \end{pmatrix}, \quad t_1 \neq 0$$

$(6I - A)x = 0$:

$$\begin{cases} 4x_1 - 2x_2 = 0 \\ -2x_1 + x_2 = 0 \end{cases} \Leftrightarrow \begin{cases} 2x_1 - x_2 = 0 \\ 0 = 0 \end{cases}$$

$$S_2 = t_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad t_2 \neq 0$$

b)

$$t_1 = \frac{1}{\sqrt{(-2)^2 + 1^2}} = \frac{1}{\sqrt{5}}$$

$$t_2 = \frac{1}{\sqrt{1^2 + 2^2}} = \frac{1}{\sqrt{5}}$$

$$Q = \frac{1}{\sqrt{5}} \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix}$$

$$Q^{-1} A Q = Q^T A Q = D = \operatorname{diag}(1, 6)$$

$$Q^T \cdot A \cdot Q = \frac{1}{\sqrt{5}} \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 2 & 2 \\ 2 & 5 \end{pmatrix} \cdot \frac{1}{\sqrt{5}} \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix} =$$

$$= \frac{1}{5} \begin{pmatrix} -2 & 1 \\ 6 & 12 \end{pmatrix} \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix} =$$

$$= \frac{1}{5} \begin{pmatrix} 5 & 0 \\ 0 & 30 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 6 \end{pmatrix} = D$$