10.04

söndag 3 mars 2024 23:02

a) $A = \begin{pmatrix} -1 & 8 \\ 8 & 11 \end{pmatrix}$

$$\det(\lambda I - A) = \begin{vmatrix} \lambda+1 & -8 \\ -8 & \lambda-11 \end{vmatrix} = (\lambda+1)(\lambda-11) - 64 =$$

$$= \lambda^2 - 10\lambda - 75 = 0 \qquad \lambda_1 = -5 \quad \lambda_2 = 15$$

$s_1 = t_1 \begin{pmatrix} -2 \\ 1 \end{pmatrix}, \; t_1 \neq 0 \quad (\lambda_1 = -5)$

$s_2 = t_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \; t_2 \neq 0 \quad (\lambda_2 = 15)$

b) $s_1^T s_2 = 0$ s_1 & s_2 är parvis ortogonala.

c) normering

$t_1 = \frac{1}{\sqrt{(-2)^2+1}} = \frac{1}{\sqrt{5}}$

$t_2 = \frac{1}{\sqrt{5}}$

$$Q = \frac{1}{\sqrt{5}} \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix}$$

$$Q^{-1} A Q = Q^T A Q = \frac{1}{\sqrt{5}} \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} -1 & 8 \\ 8 & 11 \end{pmatrix} \frac{1}{\sqrt{5}} \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix} =$$

$$= \frac{1}{5} \begin{pmatrix} 10 & -5 \\ 15 & 30 \end{pmatrix} \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} -5 & 0 \\ 0 & 15 \end{pmatrix} := \hat{A} =$$

$= \mathrm{diag}(-5, 15)$