2.13ac*

torsdag 25 januari 2024 14:40

a)

$$\int_{-\infty}^{+} e^{-\tau}\,\theta(\tau)\,d\tau = \begin{cases} \int_0^{+} e^{-\tau}\,d\tau, & t>0 \\ 0, & t<0 \end{cases} =$$

$$= \left(\int_0^{+} e^{-\tau}\,d\tau\right)\theta(\tau) =$$

$$= \theta(\tau)\left[-e^{-\tau}\right]_0^{+} = \theta(t)\left(-e^{-t}+e^{-0}\right) = \underline{\underline{(1-e^{-t})\,\theta(t)}}$$

c)

$$\int_{-\infty}^{+} e^{\tau}\,(1-\theta(\tau))\,d\tau =$$

$$= \int_{-\infty}^{+} e^{\tau}\,d\tau - \int_{-\infty}^{+} e^{\tau}\,\theta(t)\,d\tau =$$

$$= \left[e^{\tau}\right]_{-\infty}^{+} - \left(\int_0^{+} e^{\tau}\,d\tau\right)\theta(t) =$$

$$= e^{+} - \left[e^{\tau}\right]_0^{+}\theta(t) = e^{+} - \left(e^{+}-1\right)\theta(t) =$$

$$= e^{+} - e^{+}\theta(t) + \theta(t) = e^{+}\left(1-\theta(t)\right) + \theta(t)$$