8.01**

a) ? ? ?

$$\vec{x} = x_1 \vec{e}_1 + x_2 \vec{e}_2 = \hat{x}_1 \hat{e}_1 + \hat{x}_2 \hat{e}_2$$

b)

$$\hat{e}_1 : (3, 2), (1, 0)$$

$$\hat{e}_2 : (1, 1), (0, 1)$$

c)

$$S = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$$

d)

$$\hat{x} = S^{-1} x$$

$$\begin{cases} 3x_1 + x_2 = y_1 \\ 2x_1 + x_2 = y_2 \end{cases} \iff \begin{cases} x_1 = y_1 - y_2 \\ x_2 = -2y_1 + 3y_2 \end{cases}$$

$$S^{-1} = \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix}$$