370019 (Israel Vidal)

CK0031: Homework 04

Exercise 04.00. (Events and probability rules).

1. Let A and B be two events and let $P(A \cup B) = 0.7$ and $P(A \cap B) = 0.4$ and $P(A \mid B) = 0.6$. Calculate p(A), p(B), $p(B \mid A)$
2. Let A and B be two events and let $P(A) = 3/10$ and $P(B) = 9/10$. Is it possible that $P(A \cap B) = 2/5$? Is it possible that $P(A \cap B) = 3/10$? And, $P(A \cap B) = 1/5$?
3. Let A and B be two independent events and let $P(A) = \alpha$ and $P(B) = \beta$. Calculate $p(A \cup B)$.

Solution:

1. Sabe-se que:

   - $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
   - $P(A \cap B) = P(B \mid A)P(A) = P(A \mid B)P(B)$

   Portanto,
   $P(A \mid B)P(B) = 0.4$
   $0.6P(B) = 0.4$
   $P(B) = \frac{0.4}{0.6} \approx 0.6667$

   $P(A) = P(A \cup B) - P(B) + P(A \cap B) = 0.7 - \frac{0.4}{0.6} + 0.4 \approx 0.4333$

   $P(B \mid A) = \frac{P(A|B)P(B)}{P(A)} \approx \frac{0.6*0.6667}{0.4333} \approx 0.92$

2. Sabe-se que $P(A \cap B) = P(B \mid A)P(A) = P(A \mid B)P(B)$ e $P(A) = \frac{3}{10}$, $P(B) = \frac{9}{10}$.
   Portanto, para que as probabilidades acima sejam possveis, o resultado dessa equação deve estar entre 0 e 1.

   - $P(A \cap B) = \frac{2}{5}$
     $P(B \mid A)\frac{3}{10} = \frac{2}{5} \implies P(B \mid A) \approx 1.333.$
     Portanto não é possível que $P(A \cap B) = \frac{2}{5}$
   - $P(A \cap B) = \frac{3}{10}$
     $P(B \mid A)\frac{3}{10} = \frac{3}{10} \implies P(B \mid A) = 1$
     $P(A \mid B)\frac{9}{10} = \frac{3}{10} \implies P(A \mid B) = \frac{3}{9}.$
     Portanto, como os dois valores estão entre 0 e 1, é uma probabilidade possível
   - $P(A \cap B) = \frac{1}{5}$
     $P(B \mid A)\frac{3}{10} = \frac{1}{5} \implies P(B \mid A) \approx 0.6667$
     $P(A \mid B)\frac{9}{10} = \frac{1}{5} \implies P(A \mid B) = \frac{2}{9}.$
     Portanto, como os dois valores estão entre 0 e 1, é uma probabilidade possível

3. Sabe-se que $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, $P(A) = \alpha$, $P(B) = \beta$.
   Portando temos que $P(A \cup B) = \alpha + \beta - P(A \cap B)$
   Como A e B são eventos indepentententes, temos que $P(A \cap B) = P(A)P(B)$.
   Logo, temos que:
   $P(A \cup B) = \alpha + \beta - (\alpha * \beta)$

Exercise 04.01. Analyse the Inspector Clouseau example and as- sociated BRMLtoolbox demo (demoClouseau.m). Then, repeat the scenario with the restriction that either the Maid or the Butler or neither or both are the murderer(s). Explicitly, the probability of the Maid being the murderer and not the Butler is 0.04, the probability of the Butler being the murderer and not the maid is 0.64, the probability of neither being the murderer is 0.32 and the probability of both being the murderers is 0.00. Modify the original demo to implement the modified scenario and report on all the probabilities of being the murderer(s), assuming (given) that the knife is the used weapon.

Solution: `https://github.com/israelcvidal/Artificial_Inteligence/blob/master/AI-HW04/clouseau.m`

Exercise 04.02. Implement the soft XOR gate example using the BRMLtoolbox. You may find condpot.m of use.

Solution: `https://github.com/israelcvidal/Artificial_Inteligence/blob/master/AI-HW04/xor.m`

Exercise 04.03. Implement the hamburger/KJ-disease example using the BRML- toolbox for both scenarios (that is, p(Hamburger eater) = 0.5 and p(Hamburger eater) = 0.001). To do so you will need to define the joint distribution p(Hamburger eaters,KJ) in which we have dom(Hamburger eater) = dom(KJ) = true,false.

Solution: `https://github.com/israelcvidal/Artificial_Inteligence/blob/master/AI-HW04/hamburguer.m`

Exercise 04.04. Implement the two dice example using the BRMLtoolbox.

Solution: `https://github.com/israelcvidal/Artificial_Inteligence/blob/master/AI-HW04/dice.m`