J.P. Morgan/Reuters RiskMetrics® Monitor

RiskMetrics® News

- Version 1.2 of FourFifteen® – J.P. Morgan's system for evaluating, exploring and reporting market risk – has been released.

- Please note that on or about July 17, 1997 it is our intention to cease updating the datasets at the J.P. Morgan web site. They are now and will continue to be available but only at the Reuter's web site.

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Research, Development, and Applications

In response to reader feedback, this quarter's *Monitor* offers two articles which expand on the material presented in the last issue.

In the article *On measuring credit exposure* which appeared in the last issue of the *RiskMetrics® Monitor* we presented three simple models to calculate the credit exposure of market driven transactions. Since the publication we have received several inquiries concerning the details of the credit exposure calculation, in particular, as they pertain to the methodology that relies on monte carlo simulation. In this article, we present a step-by-step analysis of the credit exposure calculation of a foreign exchange forward contract that employs monte carlo simulation. The purpose of this article is to offer the reader a more complete understanding of the credit exposure calculation and to make the reader aware of potential shortcomings of such a simple credit exposure calculation.

In the previous RiskMetrics Monitor in *Streamlining the risk measurement process*, we described an alternative to the variance-covariance (VCV) method for portfolio risk analysis. We called this method portfolio aggregation. In this article we provide a general framework that end-users can apply to produce estimates of VaR. As a specific example of this approach, we show how to employ Monte Carlo simulation without computing a covariance matrix.
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RiskMetrics™ News

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FourFifteen® version 1.2 – another leap forward

Like all FourFifteen upgrades the new version, designated FourFifteen 1.2, will be distributed to existing clients free of charge.

In line with our commitment to provide regular enhancements and new risk tools on an ongoing basis, this is the second upgrade of the system since it was launched in April 1996. Version 1.2 offers improvements in a number of areas, including:

- scenario analysis / stress testing

- credit exposure analysis, including an export facility to CreditManager™ (the software system for CreditMetrics™)

- basket currencies

- compatibility with Excel 5.0c and Excel 7

- report link to MATLAB®

- new instruments (equity derivatives, caps and floors) and new markets

These enhancements, along with the ongoing development plan for FourFifteen, mirror the risk-management concerns and priorities of our broad customer base. Current FourFifteen clients can be found amongst supranational organizations, central banks and ministries of finance, commercial and investment banks, fund managers, hedge funds, insurance companies, and corporations.

Its use by institutions across such a wide range of industries reflects the benefits that structured risk analysis can bring to financial decision-making.

Even as version 1.2 was being developed, work had already started on the next major release of the system. In January this year it was announced that J.P. Morgan was teaming up with The MathWorks Inc., a US software company with a long-standing reputation in mathematical software and data visualization. Together J.P. Morgan and The MathWorks are developing FourFifteen version 2.0, based on The MathWorks' acclaimed MATLAB® software. This new version will offer a wider choice of alternative risk methodologies (including historical and Monte Carlo simulation, as well as the existing parametric and stress-test methods), intuitive graphics and new tools for analyzing risk.

For further information on FourFifteen, please contact your local J.P. Morgan representative, or visit our Website: *http://www.jpmorgan.com/RiskManagement/415.htm*

RiskMetrics' Datasets

Reuters has been producing the datasets in parallel for the past 2 months. Therefore J.P. Morgan intends to cease updating its web site on or about July 17, 1997. We expect that a month will allow those who currently access J.P. Morgan to make the transition to Reuters. Their web addresses are *http://www.riskmetrics.reuters.com/WDown4.htm* and *ftp://ftp.riskmetrics.reuters.com/datasets/.*

A detailed analysis of a simple credit exposure calculator

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In the article *On measuring credit exposure* which appeared in the last issue of the *RiskMetrics® Monitor* we presented three simple models to calculate the credit exposure of market driven transactions. Since the publication we have received several inquiries concerning the details of the credit exposure calculation, in particular, as they pertain to the methodology that relies on monte carlo simulation. In this article, we present a step-by-step analysis of the credit exposure calculation of a foreign exchange forward contract that employs monte carlo simulation. In so doing, we highlight some of the important assumptions underlying the calculation. The purpose of this article is to offer the reader a more complete understanding of the credit exposure calculation and to make the reader aware of potential shortcomings of such a simple credit exposure calculation.

The rest of the article is organized as follows:

- In section 1 we present the specifications of a foreign exchange forward contract where a US importer agrees to purchase German marks (DEM) with US dollars (USD) from a Bank at some future point in time. We identify the forward contract's cashflows, find its mark-to-market value and compute the forward's current credit exposure.

- Section 2 provides the details behind the forward's potential credit exposure calculation. Potential exposure calculations such as expected, maximum and average exposure, are computed via monte carlo simulation. More specifically, we

  - describe the sampling times when we compute the forward's expected and maximum exposures
  - explain how to construct forward foreign exchange and interest rate curves, and
  - explain how to produce cashflow maps at each sampling time

  Before simulating foreign exchange and interest rates we:
  - review the basic assumptions of the model used in the simulation
  - explain how forward rates are used to determine the 'drift' component in foreign exchange and interest rates, and
  - describe the technique used to generate returns at each sampling time

  Having simulated the rates, we explain how to revalue the forward contract at each sampling time and how we compute expected and maximum exposure. Finally, we compare the forward's exposure profile to the exposure profile of an interest rate swap.

- Section 3 summarizes the main results of the article and provides direction for future research.

1. A foreign exchange forward and its current credit exposure

Recall from *On measuring credit exposure* that current exposure is the exposure based on a transaction's mark-to-market value. If a transaction has a positive market value then its current exposure will be equivalent to its market value since if a counterparty defaults, the mark-to-market value of the transaction is lost. On the other hand, the current exposure of a transaction that has a negative (or zero) mark-to-market value is zero. This follows from the fact that if a party "owes" money at the time its counterpart defaults, its loss is zero.

In this article we analyze the credit exposure of a US importer who entered into a foreign exchange forward contract on January 23, 1996 that gives the importer the right to buy 1 million German marks (DEM) at a strike price of 0.6937 USD/DEM on May 2, 2001. We analyze the credit exposure of this
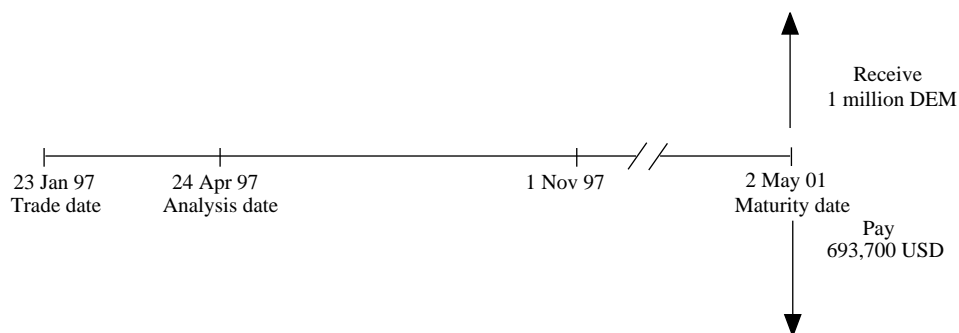
transaction on April 24, 1997 which is referred to as the analysis date. Table 1 provides the details of the contract.

Table 1
Foreign exchange forward description
Analysis date: April 24, 1997

Trade date:	January 23, 1996
Maturity date:	May 2, 2001 (4.002 years between analysis date and maturity)
Notional amount	1,000,000 DEM
Spot exchange rate	0.5824 USD/DEM
Strike price	0.6937 USD/DEM
US importer	purchase 1,000,000 DEM with 693,700 USD
Bank	sell 1,000,000 DEM with 693,700 USD

The forward contract's cashflows, from the importer's perspective, are presented in chart 1.

Chart 1
Forward contract's cashflows
US importer perspective

Receive
1 million DEM

23 Jan 97	24 Apr 97	1 Nov 97	2 May 01
Trade date	Analysis date		Maturity date

Pay
693,700 USD

To calculate the US importer's current credit exposure we must find the forward's mark-to-market (MTM) value. In general, the forward's MTM is calculated as follows:

[1]   MTM = Notional x (current forward rate - strike price) x discount factor

On April 24, 1997, the current forward USD/DEM exchange rate (from the analysis date to maturity) is 0.6379 USD/DEM. The US interest rate for the same period (4.022 years) is 6.783%. Using a continuous discounting procedure[1] we find the MTM to be -42,419 USD, that is,

[2] $1,000,000 \times (0.6379 - 0.6937) \times \exp(-0.06783 \times 4.022) = -42,419$ USD

Note that the forward's MTM is negative. This implies that at the analysis date the forward is out-of-the-money for the importer. Consequently, the US importer's **current credit exposure** is zero. Chart 2 shows the foreign exchange forward's profit and loss (P&L) profile as a function of the current forward rate.

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[1] Throughout this analysis we employ a continuous compounding convention.

Chart 2
Forward's P&L profile
P&L is reported in USD

Forward's P&L

out-of-the-money   in-the-money

0

-42,419

0.6379   0.6937(strike)

USD/DEM forward exchange rate
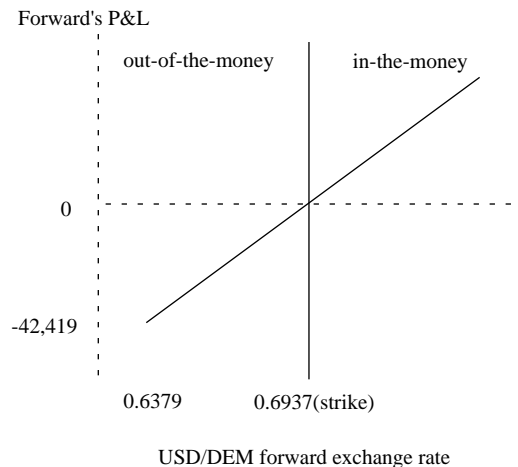
Intuitively, the foreign exchange forward's value to the US importer is negative because on the trade date, January 23, 1996, the importer contracted to pay 0.6937 USD/DEM and the forward price has since fallen. Alternatively expressed, the importer agreed to pay 693,700 USD to buy 1,000,000 DEM on May 2, 2001. At the analysis date, however, the current price for 1,000,000 DEM to be received on May 2, 2001 is 637,900 USD. Hence, the USD price of DEM has fallen relative to the USD price of DEM at the trade date. Consequently, on the analysis date it would cost the US importer 42,419 USD less to purchase DEM at the current forward price rather than at 0.6937 USD/DEM.

Now, suppose instead of decreasing to 0.6937USD/DEM, the current forward price of DEM increased relative to the strike price. In this case we would find that the US importer's forward contract would have a positive mark-to-market value (refer to chart 2) — it would be in-the-money—since the importer could buy DEM (with USD) at a discount relative to the market prices at the analysis date.

2. Calculating the foreign exchange forward's potential exposure

Recall that potential exposures, which include expected, average, maximum and peak exposures, are a function of future changes in underlying prices and rates that affect the value of a particular transaction. That is to say, potential exposure calculations recognize the probability distribution of underlying financial prices.

2.1 Background
The first step towards measuring potential credit exposures is to define sampling times. Sampling times are future dates when exposures are calculated. Table 2 presents the sampling times for the US importer's forward contract.

Table 2
Sampling times
In days and years; calculations assume 1 year = 365 days

Days from analysis date	30	192	373	557	738	922	1102	1287
Years from analysis date	0.083	0.526	1.021	1.526	2.022	2.526	3.022	3.526

Having determined the sampling times, next we need to identify the RiskMetrics interest rate and foreign exchange rate nodes that will be used to map the forward's cashflows. Remember that cashflow mapping is the process of redistributing the observed cashflows onto so-called RiskMetrics nodes, to produce RiskMetrics cashflows[2]. RiskMetrics provides volatilities and correlations for the following US and German interest rate nodes:

Table 3
RiskMetrics interest rate nodes
For US and German interest rates

Years	0.083	0.25	0.50	1.0	2.0	3.0	4.0	5.0	7.0	9.0	10.0	15.0	20.0

In addition, RiskMetrics provides volatilities and correlations associated with the logarithmic change in the USD/DEM exchange rate.

In order to compute potential exposures we will need to simulate a distribution of the forward's value at each sampling time. It follows from interest rate parity (IRP), that the simulation of the forward's future value can be done in two (equivalent) ways[3]:

1. At each sampling time, simulate *forward* (USD/DEM) exchange rates and US interest rates and compute a distribution of future values of the forward contract, or

2. At each sampling time, simulate German and US interest rates as well as *spot* foreign exchange rates and compute a distribution of future values of the forward contract.

2.2 Constructing foreign exchange and interest rate curves and cashflow mapping
As a first step, to determine the future values of the forward contract at each sampling time we must build foreign exchange forward and interest rate curves. Chart 3 shows the US and German term structure of interest rates on April 24, 1997.

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[2] See section 6.2 of the RiskMetrics Technical document, 4th edition.
[3] In cases where interest rate parity (IRP) does not hold, the appropriate premium or discount should be incorporated into the IRP equation.

Chart 3
USD and DEM zero rates
April 24, 1997
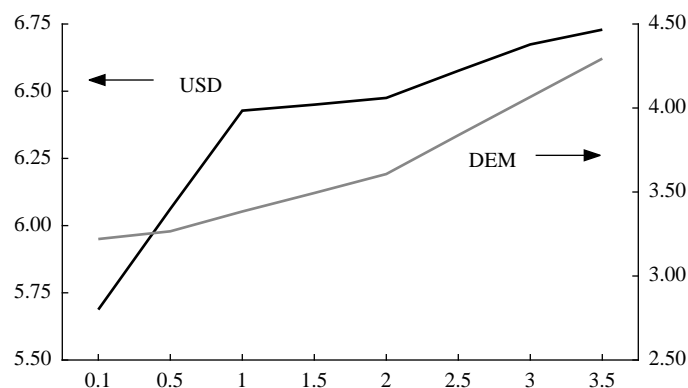
Based on these zero rates and the spot USD/DEM exchange rate we can construct USD/DEM forward rates at each sampling time. We now explain how to construct the forward USD/DEM curve. In general, if we let S_t denote the spot USD/DEM exchange rate at time t and let $r_{t,T}^{us}$ and $r_{t,T}^{de}$ denote all US and German zero rates at time t, for the period between time t and T, then it follows from interest rate parity that the forward foreign exchange rate $F_{t,T}$ is given by the expression

$$[1] \qquad F_{t,T} = S_t \times \left[\frac{\exp\left(r_{t,T}^{us} \times years \right)}{\exp\left(r_{t,T}^{de} \times years \right)} \right]$$

where years represents the times presented in table 2. Using the appropriate spot exchange rate and zero interest rates, the foreign exchange forward curve is presented in chart 4.

Chart 4
USD/DEM forward exchange rate curve on April 24, 1997
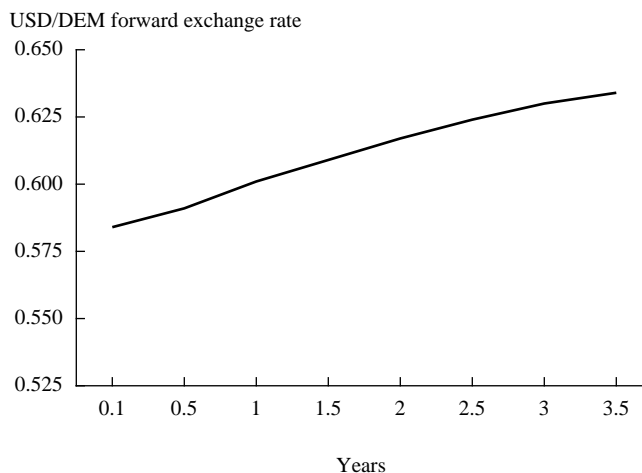*Spot USD/DEM foreign exchange rate is 0.5824.*

Table 4 provides the US and German interest rates and the corresponding foreign exchange forward curve at each of the 8 sampling times

Table 4
US and German interest rates and foreign exchange forward curve
Spot USD/DEM foreign exchange rate is 0.5824

| Years from analysis date	$r_{t,T}^{us}$ (%)	$r_{t,T}^{de}$ (%)	F_t
0.083	5.687	3.221	0.584
0.526	6.062	3.266	0.591
1.021	6.427	3.384	0.601
1.526	6.450	3.494	0.609
2.022	6.475	3.607	0.617
2.526	6.576	3.838	0.624
3.022	6.674	4.066	0.630
3.526	6.729	4.294	0.634

In addition to the foreign exchange forward rates we also need to construct US and German forward *interest* rate curves. We must construct such curves at each sampling time. Further, since we will map the forward's cashflows to the RiskMetrics nodes, the forward rates must coincide with the RiskMetrics nodes. We will now demonstrate the construction of forward interest rates with an example.

Assuming that bond yields are continuously compounded, we can relate the so-called short rate, r_0, and forward rate, r_f to the long rate, r_1 as follows:

[2] $$e^{r_0 T_0} e^{r_f T_f} = e^{r_1 T_1}$$

where T_0, T_f and T_1 are the time periods for the short, forward and long rates, respectively. Chart 5 illustrates the relationship between the time periods when the short, forward and long rates prevail.

Chart 5
Time profile of short, forward and long rates
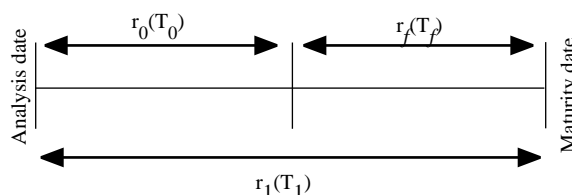
Now, to determine the forward rate, we solve [2] r_f, i.e.,

[3] $$r_f = \frac{r_1 T_1 - r_0 T_0}{T_f}$$

Given the short and long rates, we can use [3] to solve for the relevant forward rates. For example, suppose the sampling time, T_0, is 0.526 (about one-half of a year from the analysis date). The (short) US interest rate corresponding to this time period, r_0^{us}, is 6.062%. Our goal is to construct forward interest rates, r_f^{us}, at the following RiskMetrics nodes; 0.083, 0.25, 0.50, 1.0, 2.0, 3.0, and 4.0 years. We can now use the short and long rates to solve for the forward rates corresponding to the RiskMetrics nodes.

These forward rates are presented in table 5 along with the corresponding long rates, r_1^{us}, and the time at which they occur. Note that the $T_1 = T_f + T_0$.

Table 5
Constructing US interest forward rates at second sampling time
*All times are presented in years, $T_0 = 0.526$; $r_0^{us} = 6.062\%$*

T_f	T_1	r_1^{us} (%)	r_f^{us} (%)
0.083	0.609	6.126	6.531
0.250	0.776	6.254	6.659
0.50	1.026	6.427	6.813
1.00	1.526	6.450	6.654
2.00	2.526	6.576	6.712
3.00	3.526	6.729	6.846
4.00	4.526	6.819	6.918

These forward interest rates correspond to the relevant RiskMetrics nodes that are used for mapping at the second sampling time. Having created the forward rates, the next step is to map the forward's cashflows to the RiskMetrics nodes. To demonstrate the mapping procedure we continue to focus on the cashflow map that occurs at the second sampling time (i.e., at 0.526 years from the analysis date).

Since the time from the analysis date until maturity is 4.022 years, the forward's cashflows (1,000,000 DEM and -693,700 USD) occur approximately 3.50 years from the second sampling time. Chart 6 shows the relationship between the forward's cashflows and the sampling time.

Chart 6
Forward's cashflows at the second sampling time
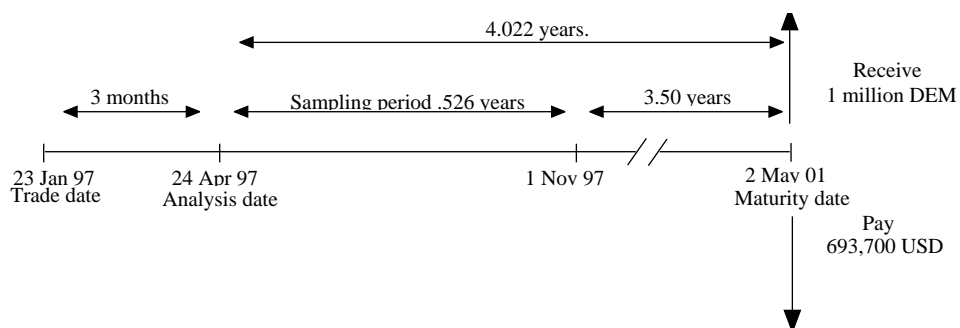
Since these cashflow's occur between years 3 and 4, we map these cashflows to the 3 and 4 year RiskMetrics nodes. Table 6 presents the cashflow maps for the DEM and USD positions at the second sampling time

Table 6
Cashflow maps and RiskMetrics nodes
Cashflow occurs in 3.50 years from 0.526 year sampling time

RiskMetrics nodes	3 year	4 year
DEM position	501,369.86	498,630.14
USD position	347,800.27	345,899.73

For a more complete picture of the cashflow maps table 7 presents the cashflow maps and the corresponding RiskMetrics nodes for each of the 8 sampling times. The column denoted "Cashflow date" represents the time at which the forward's cashflows occur, adjusting for the sampling time.

Table 7
Cashflow maps for all sampling times

Sampling time (yrs)	Cashflow date (yrs)	US position (RM node) USD		German position (RM node) DEM	
0.083	3.940	40,703 (3)	652,996 (4)	58675 (3)	941324 (4)
0.526	3.500	347,800 (3)	345,899 (4)	501,369 (3)	498,630 (4)
1.021	3.003	691,799 (3)	1,900 (4)	997,260 (3)	2,739 (4)
1.526	2.500	347,800 (2)	345,899 (3)	501,369 (2)	498,630 (3)
2.022	2.003	691,799 (2)	1,900 (3)	997,260 (2)	2,739 (3)
2.526	1.500	347,800 (1)	345,899 (2)	501,369 (1)	498,630 (2)
3.022	1.003	691,799 (1)	1,900 (2)	997,260 (1)	2,739 (2)
3.526	0.498	3,801 (0.25)	698,898 (0.50)	5,479 (0.25)	994,520 (0.50)

2.3 Performing monte carlo simulation
With the appropriate foreign exchange and interest rate forward curves as well as the cashflow map, we now demonstrate how to simulate future foreign exchange and interest rates at each of the eight sampling times. For ease of exposition we describe the simulation process for the second sampling time, i.e., simulating future rates 0.526 years from the analysis date.

First, we specify the appropriate covariance matrix of returns, or as written in *Pete Benson's piece* a matrix of underlying price and interest rate returns. These data are determined by the RiskMetrics cashflow map that were described in the last section. For example, for the simulation that occurs at the second sampling time, we require a time series of logarithmic changes (returns) on the USD/DEM foreign exchange rate as well as US and German 3 and 4 year government bond rates.

Next, we assume that the USD/DEM foreign exchange rate and the 3 and 4 year interest rates evolve according to the following process.

[4] $S_{t+1} = S_t \exp(\varepsilon_t)$

or, using natural logarithms, we get

[5] $\ln(S_{t+1}) = \ln(S_t) + \varepsilon_t$

where

ε_t is a normally distributed random variable with mean μ_t and variance σ^2_t.

S_t is a generic variable representing the USD/DEM foreign exchange rate and the relevant US and German interest rates.

Now, suppose we wanted to simulate the USD/DEM foreign exchange rate over one day. To do so, we would need estimates of the one-day mean, $\mu_{fx,t}$, and variance, $\sigma^2_{fx,t}$. We would then set $S_{fx,t}$ to the current spot foreign exchange rate (0.5824 USD/DEM, as of the analysis date) and simulate a random set of numbers, ε_t, from a normal distribution with a mean $\mu_{fx,t}$ and variance $\sigma^2_{fx,t}$. We would then substitute these normally distributed numbers as well as the current spot rate into [4] to produce a distribution of foreign exchange rates for the next day.

Extending this approach, if one were to simulate prices over, say, the next T days we would modify [4] to become

$$[6] \qquad \begin{aligned} S_{t+T} &= S_t \exp\left(\sum_{t=1}^{T} \varepsilon_t \right) \\ &= S_t \exp(z_t) \end{aligned}$$

Note that z_t is normally distributed with a mean $T\mu_t$ and variance $T\sigma^2_t$. Therefore, if we were to simulate foreign exchange rates T days from the analysis date we would simulate normal random variates with mean $T\mu_{fx,t}$ and variance $T\sigma^2_{fx,t}$, and then using the current spot rate ($S_{fx,t}$) would apply [6].

In practice, while the variance is often estimated from historical data, the mean, or drift parameter is derived from the current forward rate. We derive an expression for the mean μ_t as follows:

1. Find the expected value of the price T days forward, S_{t+T}. The mathematical expectation of [6] is given by the following expression

$$[7] \qquad E(S_{t+T}) = S_t \exp\left(T\mu_t + \frac{T\sigma^2_t}{2} \right)$$

2. Set the exchange rates forward value, $F_{t,T}$ equal to the spot rate's expected value, i.e,
    $F_{t,T} = E(S_{t+T})$ and solve for the mean $T\mu_t$. This yields the expression

$$[8] \qquad T\mu_t = \ln\left(\frac{F_{t,T}}{S_t} \right) - \frac{T\sigma^2_t}{2}$$

Therefore, for a given forward rate, which was derived in section 2.2, and an estimate of the daily variance of foreign exchange returns, we can back-out the foreign exchange rate's drift parameter. At the second sampling time, the foreign exchange forward rate, $(F_{t,T})$, is 0.591 and at the analysis date the spot rate, $(S_{fx,t})$, is 0.5824. The one-day variance, $\left(\sigma^2_{fx,t} \right)$, for the returns on the USD/DEM exchange rate is 0.0000211. Substituting this information into [8] yields $T\mu_{fx,t} = 1.4\%$.

Note that we can solve for the interest rate drift parameters in exactly the same way.

Using the drift parameters and the estimated daily variances (and covariances) we can simulate foreign exchange and interest rates, taking into account the correlation among the rates, by applying any of the techniques presented in Appendix E of the RiskMetrics Technical Document, 4th edition. Specifically,

we will demonstrate how to perform monte carlo simulation by applying the singular value decomposition of the estimated covariance matrix.

Suppose we wish to simulate 1000 foreign exchange and interest scenarios (simulations) at the second sampling time. We will generate a 1000 x 5 matrix of future prices, Y. At the second sampling time, the columns of Y represent the USD/DEM foreign exchange rate, the US 3 and 4 year rates and the German 3 and 4 year rates, respectively, 0.526 years from the analysis date. We now demonstrate how we construct Y in three steps.

- First, we assume that each row of Y, y_i, (i=1,..,1000) is distributed according to the multivariate (5-variate) normal distribution with a mean m_i and covariance matrix Σ_i. Σ_i has dimension 5 x 5. The mean m_i is defined as $m_i = \left[\mu_{fx,t} \ \mu_{3y,t}^{us} \ \mu_{4y,t}^{us} \ \mu_{3y,t}^{de} \ \mu_{4y,t}^{de} \right]$

  Decompose Σ_i, using a singular value decomposition as follows: $\Sigma_i = VDV^T$. Let $Q = VD^{1/2}$ so that $\Sigma_i = QQ^T$.

- Second, write $\hat{y}_i = m_i + \varepsilon_i Q^T$ where ε_i is a 1 x 5 vector of normal random variates with a mean 0 and a variance 1. Now, if we simulate ε_i and then construct $\hat{y}_i$, we know from its definition that $\hat{y}_i$ has a mean m_i and covariance matrix Σ_i. Therefore, we can use the definition of $\hat{y}_i$ to recover the properties of y_i.

- Third, we generate 1000 $\hat{y}_i$ from 1000 randomly generated ε_i 's, to form Y.

It is important to understand that the simulation involves only those elements of the generic RiskMetrics covariance matrix that correspond to the mapped cashflows. That is, simulation is performed only after cashflow mapping. This greatly reduces the amount of required simulation.

Applying this algorithm at the second sampling time, we simulate 5 rates (one foreign exchange and four interest rates) 0.526 years from the analysis date. This results in distributions of the USD/DEM exchange rate and the 3 and 4 year US and German (forward) interest rates at that sampling time.

Chart 7 shows the distribution of simulated USD/DEM exchange rates at the second sampling time.

Chart 7
Histogram of simulated USD/DEM exchange rates
At 0.526 years from analysis date; forward rate is USD/DEM 0.591

Frequency

USD/DEM exchange rate
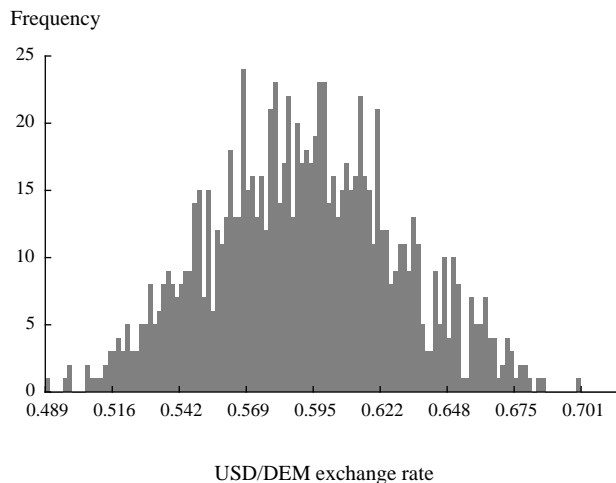
Charts 8 shows the histograms of the simulated 3 and 4 year forward interest rates, 0.526 years from the analysis date.

Chart 8
Histogram of 3 and 4 year US interest rate

Frequency

3 year US interest rate

Frequency

4 year US interest rate
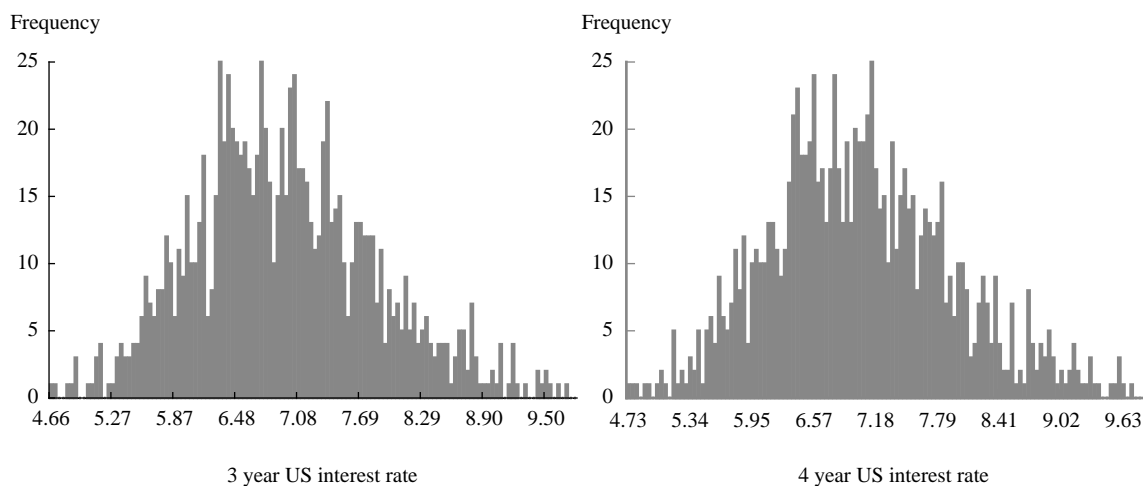
2.4 Revaluation and the credit exposure calculation
Once the foreign exchange and interest rates have been simulated, the forward contract at each sampling time must be revalued at the new rates. As noted in section 2.1, there are two ways to generate the profit and loss of the forward contract. According to the first approach, we simulate forward exchange rates at each sampling time using the simulated foreign exchange and interest rates.

For example, consider computing the forward foreign exchange rate at the second sampling time. In this case, the forward's cashflow occurs in 3.5 years from the sampling time and it is mapped to the 3 and 4 year RiskMetrics nodes. Therefore, we will calculate two forward foreign exchange rates corresponding to each node in the cashflow map. If we let $SFX_{0.526}$ represent the simulated (spot) foreign exchange rates at the second sampling time and $r^{us}_{0.526,\,3yr}$ ($r^{de}_{0.526,\,3yr}$) and $r^{us}_{0.526,\,4yr}$ ($r^{de}_{0.526,\,4yr}$) represent the simulated US (German) interest rates for each cashflow map at the 3 and 4 year nodes, then the two forward foreign exchange rates are given by the following expressions.

[9]
$$FWD_{0.526,\,3yr} = SFX_{0.526}\frac{\exp\!\left(r^{us}_{0.526,\,3yr}\times 3\right)}{\exp\!\left(r^{de}_{0.526,\,3yr}\times 3\right)} \quad \text{(corresponding to the 3 year node)}$$

and

[10]
$$FWD_{0.526,\,4yr} = SFX_{0.526}\frac{\exp\!\left(r^{us}_{0.526,\,4yr}\times 4\right)}{\exp\!\left(r^{de}_{0.526,\,4yr}\times 4\right)} \quad \text{(corresponding to the 4 year node)}$$

The distribution of profit and loss on the forward contract 0.526 years from the analysis date is (see table 7):

[11]
$$\begin{aligned}
\text{P\&L} = \ & (501,369.86 \times FWD_{0.526,\,3yr}) \times \exp\!\left(-r^{us}_{0.526,\,3yr}\times 3\right) \\
& + (498,630.14 \times FWD_{0.526,\,4yr}) \times \exp\!\left(-r^{us}_{0.526,\,4yr}\times 4\right) \\
& - 347,800.27 \times \exp\!\left(-r^{us}_{0.526,\,3yr}\times 3\right) - 345,899.73 \times \exp\!\left(-r^{us}_{0.526,\,4yr}\times 4\right)
\end{aligned}$$

Alternatively, it follows from interest rate parity that we can compute the profit and loss on the forward contract as:
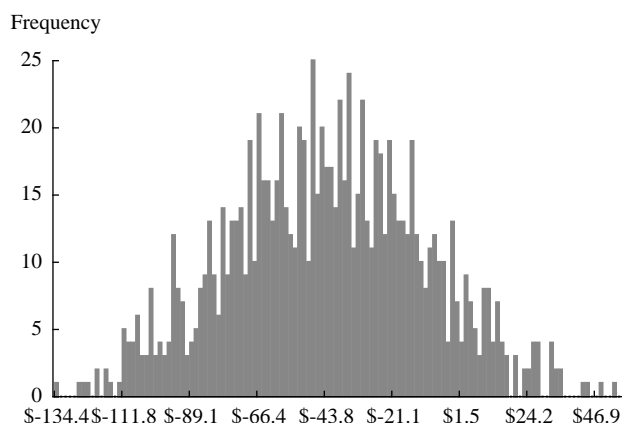
[12]
$$\begin{aligned}
\text{P\&L} = \ & (501,369.86 \times 0.5824) \times \exp\!\left(-r^{de}_{0.526,\,3yr}\times 3\right) \\
& + (498,630.14 \times 0.5824) \times \exp\!\left(-r^{de}_{0.526,\,4yr}\times 4\right) \\
& - 347,800.27 \times \exp\!\left(-r^{us}_{0.526,\,3yr}\times 3\right) - 345,899.73 \times \exp\!\left(-r^{us}_{0.526,\,4yr}\times 4\right)
\end{aligned}$$

Chart 9 presents a histogram of the foreign exchange forward contract's P&L.

Chart 9
Histogram of forward contract's P&L (in USD)
At 0.526 years from analysis date

Frequency

P&L($000)

The P&L distribution has a mean value of -44,439 USD and a standard deviation of 32,941 USD. The negative mean is the forward's mark-to-market at the second sampling time. This implies that the current exposure at the second sampling time is zero since the forward's forward value is out-of-the-money.

Recall that a party has positive credit exposure only if the contract it currently holds is in-the-money. Consequently, the distribution of the forward's credit exposure corresponds to the non-negative values of the P&L distribution. The distribution of the forward's credit exposure at the second sampling time is presented in chart 10.

Chart 10
Histogram of forward contract's credit exposure (in USD)
At 0.526 years from analysis date

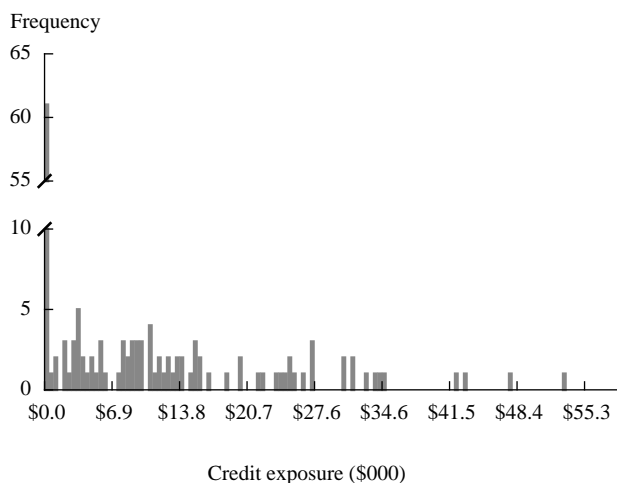Frequency

Credit exposure ($000)

Chart 10 is the same as chart 9 but with all negative values converted to zero. The mean of the credit exposure distribution which represents the expected exposure 0.526 years after the analysis date is

1,341USD. The maximum exposure of the forward contract at the second sampling time, which is the 95th percentile of the credit exposure distribution, is 9,947USD. The average exposure of the forward contract, which is a measure of how much, on average, the US importer expects to lose over the life of the forward contract, is 9,687USD. Chart 11 shows the expected and maximum exposure profiles over the life of the forward contract for the US importer.

Chart 11
US importer's foreign exchange forward exposure profile
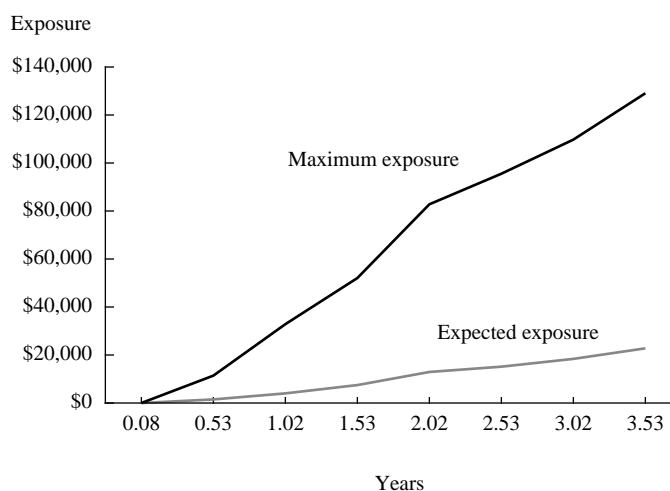
Note that the exposure profile of the forward increases over the life of the contract. This reflects the forward's single payment (at maturity) as well as the increase in the forward's volatility with time. Compare the forward's exposure profile to that of an interest rate swap's (Chart 12).

Chart 12
IR swap's exposure profile
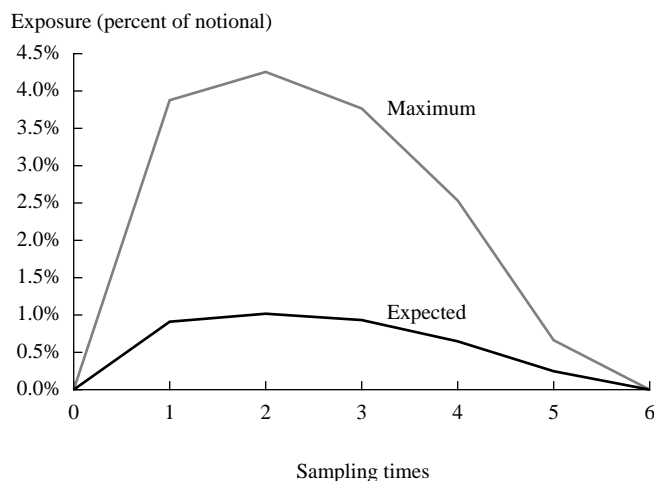
Note that whereas the interest rate swap's profile has a hump shape, the forward's exposure profile increases towards the forward's maturity. The difference in the appearance of the exposure profiles is due to the amortization of the swap's payments. In other words, unlike the forward whose cashflows remain

constant over the life of the contract, the swap's cashflows decrease (as more payments are made/received) over its life. Therefore, for the swap the "volatility effect" dominates at the beginning of the swap's life and then is overtaken by the so-called "amortization effect". However, in the case of the forward, the volatility effect dominates the forward's exposure profile since there is no amortization.

3. Conclusions and directions for future research

This article has presents the computational details behind the credit exposure calculation of a foreign exchange forward contract. After finding the forward's mark-to-market and we demonstrate how to compute potential credit exposure by way of monte carlo simulation. The potential credit exposure calculation involves six important steps:

1. Setting sampling times

2. Constructing forward foreign exchange and interest rate curves

3. Producing cashflow maps at each sampling time

4. Simulating future foreign exchange and interest rates, and

5. Revaluing the foreign exchange forward contract at the simulated rates.

In order to simulate future foreign exchange and interest rates we propose a model where the mean and variance parameters scale with time (Eq. [7]). In the case of forecasting interest rates, such a model is reasonable for short-term forecasts but can become problematic when the forecasts are extended over longer time periods. This should seem obvious since the variance of the logarithmic changes in interest rates "blows-up" over time. Consequently, when estimating the potential credit exposure of a set of instruments over longer time horizon's, we would want to model the term structure of interest rates within a formal paradigm that allows interest rates to revert to some mean value. Such a model would rid us of the problems associated with a scaled variance. We plan on presented credit exposure calculations within a complete term structure model of interest rates in upcoming *RiskMetrics Monitors*.

A general approach to calculating VaR without volatilities and correlations

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In the previous RiskMetrics Monitor[1], we described an alternative to the variance-covariance (VCV) method for portfolio risk analysis. We called this new method portfolio aggregation. Recall that portfolio aggregation involves reconstructing a time series of daily portfolio returns from a current set of portfolio positions and daily returns on individual securities. Value-at-Risk (VaR) estimates of such a portfolio are then obtained by computing the portfolio standard deviation directly from the portfolio return series instead of constructing individual volatilities and correlations.

In this note, we describe the data used in portfolio aggregation, and introduce more applications to risk analysis, including Monte Carlo simulation. We provide a general framework that end-users can use to produce estimates of VaR. As a specific example of this approach, we show how to employ Monte Carlo simulation without computing a covariance matrix. The rest of this article is organized as follows:

- In section 1 we demonstrate how to compute value-at-risk using without a covariance matrix. Specifically, we show how to calculate VaR directly from the underlying return series under equal and exponential weighting schemes. In the case where exponential weighting is applied, we show that this VaR calculation is identical to that used by RiskMetrics™ VCV methodology.

- In section 2 we explain how to compute Monte Carlo without first constructing a variance/covariance matrix.

- Section 3 briefly mentions other applications where covariance matrix is not required

- Section 4 presents conclusions and direction for future research

1. Using returns time series in place of volatilities and correlations

RiskMetrics provides volatilities and correlations for a set of benchmark securities. These securities are what we use to map actual cashflows. For example, suppose we need to compute the VaR of a cashflow denominated in US dollars that occurs in 6 years time. In order to compute VaR, we would map this cashflow to the two nearest RiskMetrics nodes which represent the 5 and 7 year US zero rates. The volatility of the log changes on the 5 and 7 year nodes as well as the correlation between the two log changes are then used to (1) find how much to allocated to the two nodes, and (2) compute VaR. Note that the 5 and 7 year nodes are what we refer to as the benchmark securities.

In these calculations, each benchmark has associated with it a time series of volatility adjusted returns, i.e., returns divided by their standard deviation. The volatilities and correlations are calculated from the set of all time series, and then these time series are discarded. Here, we suggest that rather than computing the volatilities and correlations and discarding the time series, we work directly with the time series of returns, and bypass the calculation of correlations.

This provides a number of advantages:

- Existing time series can be modified directly, and new time series added, without recalculating the correlations with other time series

- If returns are to be weighted (e.g, exponentially), the weighting scheme can be altered without replacing the dataset

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[1] See the article, *Streamlining the market risk measurement process*.

- If there are a large number of benchmarks relative to the number of returns per benchmark, the dataset of benchmark returns is smaller than the correlation matrix

- Better numerical precision

- An intuitive interpretation of how Monte Carlo returns are generated

- Fast marginal risk analysis.

We now explain how users can work directly with the return time series (RTS). Define the RTS dataset as follows. For benchmark i, let r_i be the $T \times 1$ vector of returns (with mean 0), where T is the number of returns (historical observations) for each benchmark. Define $r_{i,t}$ to be the return in period t of benchmark i. Let $R = \langle r_1, r_2, ..., r_n \rangle$, where n is the number of benchmarks. We can write the T x n matrix of returns R as follows.

$$[1] \qquad R = \begin{bmatrix} r_{11} & \cdots & \cdots & \cdots & r_{1n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & r_{JJ} & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ r_{T1} & \cdots & \cdots & \cdots & r_{Tn} \end{bmatrix}$$

Since R is a $T \times n$ matrix, the RTS dataset has Tn values. Compare this to the VCV dataset which has n standard deviations, and $n(n-1)/2$ correlations, for a total of $n(n+1)/2$ values. For situations where the number of benchmarks is more than twice the number of observations, RTS requires less storage, i.e., R requires less storage than its corresponding covariance matrix.

1.1 Relationship to the covariance matrix

The fact that we can use the returns directly to compute VaR is made obvious from the following observation: the covariance matrix under equally weighted observations is

$$[2] \qquad C = T^{-1} R^T R .$$

where R^T is the transpose of R. Hence, R provides a simple factoring of the covariance matrix. This will be useful later.

1.2 Application to VCV

If w is the vector of benchmark equivalents where each element of w, w_i, is the position value associated with one of the n benchmarks, VaR is given by the equation:

$$[3] \qquad VaR = 1.65 \sqrt{w^T C w}$$

assuming that underlying returns are distributed according to the conditional multivariate normal distribution. Now, it follows from [1] that

$$[4] \qquad VaR = 1.65 \sqrt{w^T C w} = 1.65 \sqrt{w^T T^{-1} R^T R w} = 1.65 T^{-1} \| R w \| .$$

As we can see from the right-hand side of equation [3], the VaR calculation depends only on the benchmark weights, w, the underlying return matrix, R, and the number of historical observations T. Because the computational effort varies linearly with the number of benchmarks, using the RTS matrix is faster

than using the covariance matrix C (provided the number of benchmarks is more than twice the number of observations in each benchmark).

The preceding analysis demonstrates how to compute VaR by the VCV method when the data are equally weighted. However, we note that this methodology is general and applies equally well when the data are weighted exponentially.

1.3 Applying exponential weighting

We now show how similar results to those presented in section 1.2 are obtained when applying exponential weighting. When computing the covariance and correlation matrices, use, instead of the data matrix R, the augmented data matrix $\tilde{R}$ shown in equation [5].

$$[5] \qquad \tilde{R} = \begin{bmatrix} r_{11} & \cdots & \cdots & \cdots & r_{1n} \\ \sqrt{\lambda}\,r_{21} & \cdots & \cdots & \cdots & \sqrt{\lambda}\,r_{21} \\ \cdots & \cdots & \sqrt{\lambda^{J-1}}\,r_{JJ} & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \sqrt{\lambda^{T-1}}\,r_{T1} & \cdots & \cdots & \cdots & \sqrt{\lambda^{T-1}}\,r_{Tn} \end{bmatrix}$$

Now, we can define the covariance matrix based on exponential weighting simply as

$$[6] \qquad \tilde{C} = \left(\sum_{i=1}^{T} \lambda^{i-1} \right)^{-1} \cdot \tilde{R}^{T}\tilde{R}$$

$$= \Lambda^{-1} \cdot \tilde{R}^{T}\tilde{R}$$

where

$$[7] \qquad \Lambda = \sum_{i=1}^{T} \lambda^{i-1}$$

It follows immediately from the results presented in [4] that VaR in this case is

$$[8] \qquad VaR = 1.65\Lambda^{-1}\left\| R^{T}w \right\|$$

Once a decay factor, λ, is selected in the VCV method, the correlation matrix is computed, and it is no longer possible to change the weight without recomputing the entire covariance matrix. However, since there are no assumed weights in the RTS dataset, we are free to choose different values without any additional computational burden.

2. Generating multivariate normal returns for Monte Carlo

To generate multivariate normal returns, one typically performs a Cholesky or Singular Value decomposition (SVD) on the correlation matrix[2]. The resulting matrix is then combined with a vector of random variates to produce multivariate returns.

There are drawbacks to this approach:

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[2] See Appendix E of the RiskMetrics Technical document, 4th edition.

- The decomposed matrix does not easily provide an intuitive understanding of how the deviates are generated

- Changing a single return value of one benchmark requires a new decomposition

- Cholesky decomposition requires that the correlation matrix be PD (positive definite), and SVD requires PSD (positive semi-definite).[3]

Using the RTS dataset, we can take a more direct approach that suggests an intuitive interpretation. Consider a row of the R matrix. It represents one observation interval--a snapshot of all benchmark returns. Suppose we multiply each row by a normally distributed random variate, and add the rows. The result is a vector of Monte Carlo returns that have the correct variance and correlations. In other words, Monte Carlo returns are simply the sum of random multiples of the benchmark snapshots.

We now show how to perform Monte Carlo with the RTS that avoids any volatility and correlation calculations.

Let $\varepsilon = \{\varepsilon_1, \varepsilon_2, ..., \varepsilon_T\}$ be a $T \times 1$ vector of independent, N(0,1) random variates. Then we are interested in simulating a $1 \times n$ vector of correlated returns, $\varepsilon^T R$. Consider the ith component, $\hat{r}_i = \varepsilon^T r_i$. Note that $\varepsilon^T R$ is just a sum of independent, normally distributed random variables (all with mean zero, and standard deviation vector r_i). So the mean of the sum is the sum of the means which are zero. And the variance is the sum of the variances, which is the variance of r_i. So the mean and variance of $\hat{r}_i$ is the same as that of r_i. Unlike r_i, $\hat{r}_i$ is truly normally distributed.

What about the correlation of $\hat{r}_i$ with other simulated returns? Consider another simulated return, $\hat{r}_j$. The correlation is $\hat{\rho}_{ij} = cov\left(\left(\hat{r}_i, \hat{r}_j\right)\right)/\sigma_i\sigma_j$. Since $\hat{r}_i$ and $\hat{r}_j$ have mean zero,

$$cov\,(\hat{r}_i, \hat{r}_j) \;=\; E\,[\hat{r}_i \cdot \hat{r}_j]$$

[9]
$$= E\left[\Sigma_{t=1}^T \varepsilon_t r_{i,t} \Sigma_{t=1}^T \varepsilon_t r_{j,t}\right]$$

$$= E\left[\Sigma_{t=1}^T \varepsilon^2_t r_{i,t} r_{j,t} + 2\Sigma_{t=1}^T \Sigma_{s=t+1}^m (\varepsilon_t \varepsilon_s r_{i,t} r_{j,t})\right]$$

Because the ε_i are independent, the cross terms have zero expected value. Since the expectation of the sum equals the sum of the expectation, the right-hand side becomes

[10]
$$E\left[\Sigma_{t=1}^T \varepsilon^2_t r_{i,t} r_{j,t}\right] \;=\; \Sigma_{t=1}^T r_{i,t} r_{j,t} E\varepsilon^2_t \;=\; \Sigma_{t=1}^T r_{i,t} r_{j,t} \;=\; cov\,(r_i, r_j)$$

Hence, covariance and correlations are also preserved. In general, we can show that the covariance matrix of the 1 x n vector of random variables y=$\varepsilon^T R$ is C=$R^T R$. This follows immediately from the definition of the variance of y where the mean of y is zero. Letting E(x) denote the mathematical expectation of x, we can write the variance of y as follows:

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[3] The correlation matrix cannot be PD if there are more benchmarks (columns of R) than there are observations in each benchmark (rows of R). In fact, due to roundoff errors, even PSD is rarely satisfied. These problems can be dealt with by tweaking the correlation matrix to become PSD (which introduces other errors), or by increasing the number of observations in each benchmark (which bloats the dataset without adding significant information, and may require data that is unavailable, or outside the sample of interest). The RTS dataset allows analysis with fewer observations. Of course, one must still be careful that the chosen dataset is sufficiently representative of the relationships between benchmarks.

$$[11] \quad \begin{aligned} Variance\,(y) \; &= \; E\!\left(y^T y\right) \; = \; E\!\left(R^T \varepsilon\varepsilon^T R\right) \\ &= \; R^T E\!\left(\varepsilon\varepsilon^T\right)R \\ &= \; C \end{aligned}$$

Now, certainly Monte Carlo with a covariance matrix represents a performance improvement in terms of pre-processing (i.e. in decomposing a correlation matrix), since we do not need any pre-processing. However, is it a fast way to generate deviates? Once the T independent normal deviates are generated, simulating each benchmark uses T multiples. This compares with k (where k is the rank of R) multiplications when using a Cholesky or SVD decomposition.

However, we can make return generation still faster. By randomly sampling the observation vectors (i.e. using only a subset), we can still preserve correlations and variances. In fact, we can generate our Monte Carlo returns with just one random normal deviate per trial, and one multiply per benchmark. Details are left for future discussion.

3. Other applications

The RTS dataset also allows more powerful tools for marginal analysis, and ad hoc manipulation of the benchmark returns. The Monte Carlo sampling technique discussed above, as well as marginal analysis techniques based on RTS, are employed by CreditManager™ (J.P. Morgan's credit risk calculator) for Monte Carlo simulation of credit portfolios. Details of these methods may be covered in a subsequent note.

4. Conclusions

As the number of benchmarks grows relative to the effective number of observations per benchmark, it becomes more efficient to use the RTS dataset. Moreover, using the RTS dataset provides a number of benefits over VCV-based approaches in terms of flexibility. In particular, we've shown that it provides an intuitively appealing technique for generating Monte Carlo samples.

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- The effect of EMU on risk management

- Streamlining the market risk measurement process

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