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Investors use low volatility strategies for two main reasons: to reduce the risk of their portfolio (and by doing so improve risk-adjusted returns) and to benefit from the low volatility premium. These two characteristics have made low volatility strategies increasingly popular with equity investors.

Several articles describe the historical performance of low volatility strategies and discuss potential explanations behind the low volatility premium, for example, Haugen and Baker [1991]; Ang et al. [2006]; Baker, Bradley, and Wurgler [2011]; Frazzini and Pedersen [2014]; Ang [2014]; and Muijsson, Fishwick, and Satchell [2015]. In a recent paper (Alighanbari, Doole, and Shankar [2016]), the authors focus on practical implementation issues and discuss the benefits of using a fundamental factor model and a controlled optimization process in the design of investable minimum volatility strategies.

Well-designed minimum volatility strategies seek to reduce overall portfolio volatility, which is an important dimension of risk. Minimum volatility strategies also tend to improve other key risk measures for institutional investors. For instance, in Downing et al. [2015], the authors show that a minimum volatility strategy's capabilities in mitigating tail risk is in line with strategies that directly target tail risk (minimum CVaR). They attribute this finding to the use of factor models in the minimum volatility strategy and the high level of information that is embedded in the factor models employed. Moreover, Scherer [2009] has highlighted the problems in handling explicit CVaR optimization compared with minimizing volatility (estimation error, approximation error, systematic momentum exposure).

In this article, however, we examine additional dimensions of risk that the design of minimum volatility strategies may not address explicitly but are nonetheless important to consider for many investors. We investigate whether a minimum volatility strategy implicitly addresses these additional risk considerations and, if not, how we can modify the strategy to incorporate them without compromising the main objective of reducing portfolio volatility.

Diversification is arguably the most widely used approach to manage risk. Diversified portfolios reduce concentration in order to avoid asset-specific or company-specific risks. In addition, negatively correlated or weakly correlated assets can offset each other's risk when combined in a portfolio. We assess whether a standard minimum volatility strategy captures the benefits of diversification or whether additional constraints are required.

Environmental, social, and governance (ESG) considerations, while not new, have attracted increasing attention recently.

The three pillars of ESG risk can each be interpreted as financial risk measures—good ESG ratings are not driven just by policy disclosure but also by the significant risks borne by the company's business model and its mix and location of assets and markets. Companies with poor environmental, social, and governance policies may have greater exposure to idiosyncratic events such as accidents, shutdowns, fraud, strikes, etc., which could result in both financial and reputational loss. In addition, low ESG ratings may indicate greater exposure to systematic nonfinancial risks such as weather patterns, water scarcity, data security, and skills shortages. In general, ESG characteristics of companies can be regarded as qualitative and quantitative forward-looking measures of risk, which may be, as yet, incompletely priced by equity markets. We evaluate how minimum volatility strategies can be extended to explicitly manage these ESG-related risks.

Crowding risk has been less studied but much discussed, particularly since the August 2007 "quant crisis." It arises when investors herd into similar assets as they adopt similar strategies, which may consequently become expensive. Crowding is therefore a market-level state of stress that manifests itself in a time of crisis. For example, it often becomes clear when investors try to exit the strategy by rapidly liquidating large (potentially leveraged) positions, resulting in severe losses. The effect of crowding is real and intuitive, but measuring crowding risk is difficult as it depends on many factors and, moreover, reliable and timely aggregate assessments of capital allocation are highly challenging. Measurement can be guided by assessing the capacity of one contributing strategy to absorb capital, but it is much more difficult to crowd the entire equity market compared with a particular strategy or even a specific stock.

Minimum volatility strategies, like any other active strategy, may also be susceptible to crowding risk. For example, the availability and cost-effectiveness of factor indexes have enabled more investors to adopt these strategies. Some of the assets flowing to passive minimum volatility strategies may come from active managers who were implementing similar strategies and therefore may not be impacting the net capacity of these strategies. Moreover, managed volatility strategies have been offered by institutional investors for many years, so trends in index implementation can be a poor guide to trends in aggregate capital allocation. Two recent papers (Ang, Madhavan, and Sobczyk [2017] and Ratcliffe, Miranda, and Ang [2017]) show that crowding risk is low for minimum volatility strategies, based on current levels of adoption, and low compared with other factor strategies. However, as minimum volatility becomes increasingly popular, crowding risk may increase in the future.

In Bayraktar et al. [2015], the authors discuss four different metrics to assess crowding for factors in a balanced way (although crowdedness of a pure factor does not imply that a related investable factor strategy or index need be equally stressed). One metric the paper adopts is time-series relative valuation: crowded strategies can appear materially expensive versus their history. Therefore, the valuations of a strategy may show one symptom of crowding, especially for unanchored strategies. While controlling valuations does not eliminate the risk of crowding, it may help reduce vulnerability for a particular strategy. In this article, we examine the impact of controlling valuations on the effectiveness of minimum volatility strategies to reduce portfolio volatility.

MANAGING CONCENTRATION RISK

Mean–variance portfolio construction aims to maximize expected risk-adjusted return:[1]

$$\max_{h}\left\{h'r-\frac{1}{2}\lambda h'Vh\right\}\quad s.t.\quad h'e=1 \tag{1}$$

Risk-based strategies in general can be viewed as solutions to the mean–variance problem when we have no information about expected returns (Melas, Briand, and Urwin [2011]). The three main strategies in this class are: equal weighting, which assumes no information on returns, volatilities, or correlations; risk weighting, which requires forecasts of volatility; and minimum volatility, which relies on volatility and correlation estimates to solve the mean–variance problem.

Other examples of risk-based strategies include diversity weighting (Fernholz, Garvy, and Hannon [1998]), maximum diversification (Choueifaty and Coignard [2008]), maximum Sharpe ratio (Martellini [2008]), and equal risk contribution (Maillard, Roncalli, and Teïletche [2010]). Many of these strategies could also be viewed as special cases of the mean–variance problem with additional constraints or assumptions (Exhibit 1). For example, the maximum diversification and maximum Sharpe ratio portfolios can be derived

Exhibit 1

Risk-Based Strategies and Their Implementation by Mean–Variance Optimization

Strategy	Parameters
Equal Weighted	$r = 0$; $V = I_{nxn}$ (Identity matrix)
Risk Weighted	$r = 0$; V (diagonal matrix, $v_{i,i}$ = variance of stock i)
Minimum Volatility	$r = 0$; V stock level covariance matrix
Maximum Diversification	r = vector of stock volatilities, V stock level covariance matrix

from the mean–variance solution by substituting returns with volatilities, while the equal risk contribution portfolio can be calculated by solving the minimum variance problem with an additional nonlinear constraint.

In this section, we examine the impact of adding an explicit diversification constraint to a minimum volatility strategy. We compare two approaches: the standard minimum volatility approach described in Alighanbari, Doole, and Shankar [2016], and a second version that uses a simple additional constraint to target the maximum possible decrease in the "dirty" (effective average) correlation between constituents, as calculated using the Barra Global Equity Model (GEM LT) factor risk model.[2] In the standard minimum volatility strategy, the optimization is set up to have a greater risk aversion to stock-specific rather than factor risk (so as to encourage stock-specific diversification). The analysis presented in Exhibit 2 shows that the standard minimum volatility strategy provided higher historical return while also experiencing lower realized volatility. In addition, the standard minimum volatility strategy demonstrates lower drawdowns as well as lower VaR and CVaR. These dimensions of risk aim to represent the riskiness of the portfolio for times of turbulence, when diversification and low correlation are most in demand by institutional investors. The results show that the minimum volatility portfolio demonstrates better resilience to tail-risk events, without incorporating any tail-risk control or any explicit measure of diversification.

Moreover, when we look at other intuitive and practical measures of concentration such as the effective number of stocks[3] or the weight assigned to the top 10 stocks, we see that adding the diversification constraint has a negative effect.

To better understand the reasons behind the difference in historical performance, realized volatility, and tail risk characteristics, we ran a performance factor

Exhibit 2

Key Metrics for Minimum Volatility Strategies with and without an Explicit Diversification Constraint

Key Metrics	MSCI World	Minimum Volatility	Constrained Volatility
Total Return (%)	4.6	7.6	7.1
Total Risk (%)	15.4	10.3	11.6
Sharpe Ratio	0.17	0.54	0.43
Active Return (%)	0.0	3.0	2.4
Tracking Error (%)	0.0	8.0	7.2
Information Ratio	NA	0.37	0.34
VaR @ 99%	–11.3	–8.2	–9.0
CVaR @ 99%	–15.4	–11.7	–13.2
Max Drawdown (%)	57.5	42.4	46.4
Historical Beta	1.0	0.6	0.7
Avg Num Stocks	1624	358	1616
Effective No of Stocks	319	176	125
Top10 Sec Wt (%)	11	13	16
Annual Turnover (%)	3.0	20.1	19.3

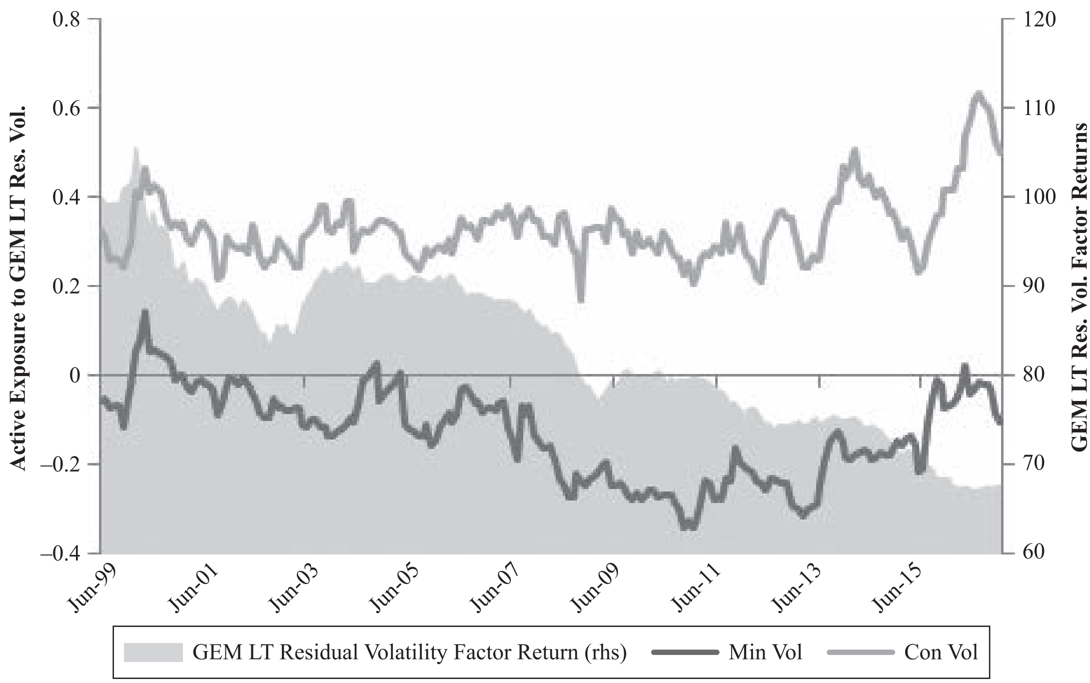
Note: Analysis over period 31-May-1999 to 30-Dec-2016, gross returns annualized in USD.

attribution using the Barra GEM LT. This analysis, presented in Exhibit 3, shows that by adding such a diversification constraint, the strategy has actually systematically gained a positive exposure to the residual volatility factor. The two lines in the charts show the exposure of the two strategies (min vol and constrained vol) to the residual volatility factor.

Modern portfolio theory suggests that only market risk should be rewarded, as specific risk can be diversified away. Empirical results over the last two decades are not consistent with this one-factor hypothesis. The residual volatility factor, a proxy for a strategy that goes long high specific risk stocks and short low risk stocks, earned negative excess returns over the last 18 years (the shaded line in Exhibit 3). In other words, betting on stocks with high specific risk has been a losing strategy over this extended period. As a result, the positive exposure of the constrained volatility strategy to the residual volatility factor made a negative contribution to performance. The constrained volatility strategy performed better than minimum volatility over certain periods when the market favored high specific risk stocks, for example, during the technology bubble of 1998–2000 and during the credit bubble of 2003–2006. But over the entire period of analysis, adding a diversification constraint to

EXHIBIT 3

Exposure to Residual Volatility in Alternative Minimum Volatility Strategies and the Return to the Residual Volatility Factor



Source: MSCI.

the minimum volatility strategy had a negative impact on historical performance and realized volatility.

By selecting weakly correlated and low volatility stocks, a minimum volatility strategy already balances reducing the overall volatility of the portfolio with achieving a good degree of diversification. Actively imposing a constraint to further increase diversification is shown to increase exposure to residual volatility, a factor that historically earned negative return and significantly weakened other key risk measures such as drawdown.

MANAGING SUSTAINABILITY RISK

Investors are actively pursuing a range of strategies to integrate ESG considerations into their investment process. ESG integration is motivated by a variety of concerns, including new regulatory or legal requirements, alignment with stakeholder values, and for some, the possibility to generate additional alpha. But many institutional investors also consider ESG policies and exposures as potential long-term risk indicators. Although this article discusses ESG risk as an additional dimension of risk and examines how it interacts with the traditional risk measure of volatility, the same analysis is relevant for ESG integration motivated by other considerations such as regulatory compliance, values alignment, and alpha generation.

One important question for minimum volatility investors who wish to build ESG improvement into their portfolios is how the ESG criteria may affect the risk reduction properties central to the strategy. To measure the effect of ESG risk, we run analyses to incorporate ESG improvement into the construction of the minimum volatility strategy. ESG improvement can be formulated as a penalty function, directly incorporated into the utility function of a minimum volatility optimization. Equally, we could normalize the ESG ratings and use them as a target characteristic. But the most straightforward and scalable approach for integrating ESG risk into a minimum volatility strategy is through explicit constraints on strategy-level sustainability (ESG) scores:[4]

$$\min_h h'Vh \quad s.t. \quad h's \geq \mu h_b's \tag{2}$$

EXHIBIT 4

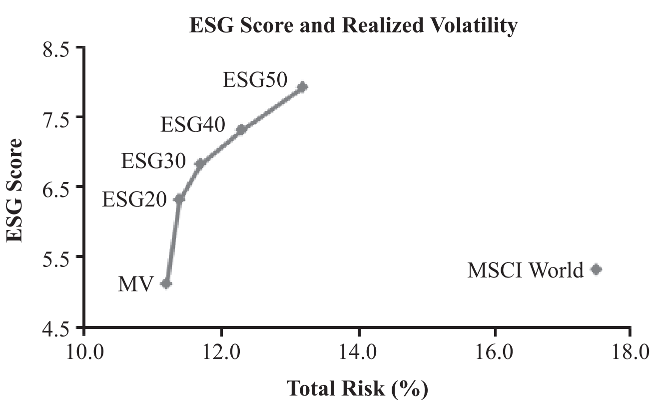
Comparison of Standard Minimum Volatility Strategies with Those Constrained to Have a Fixed Improvement in their Overall ESG Profile

Key Metrics	MSCI World	Min Vol	ESG20	ESG30	ESG40	ESG50
Total Return (%)	3.1	6.7	6.9	6.7	6.4	6.1
Total Risk (%)	17.5	11.2	11.4	11.7	12.3	13.2
Sharpe Ratio	0.14	0.55	0.56	0.53	0.47	0.42
Active Return (%)	0.0	3.6	3.8	3.7	3.3	3.1
Tracking Error (%)	0.0	9.3	8.8	8.2	7.3	6.0
Information Ratio	NA	0.39	0.43	0.45	0.45	0.51
Price To Book	1.9	2.3	2.3	2.2	2.2	2.1
Price to Earnings	16.1	16.8	16.7	16.7	16.6	16.5
Dividend Yield (%)	2.7	2.9	2.9	2.9	2.9	2.9
ESG Score	5.3	5.1	6.3	6.8	7.3	7.9
Number of Stocks	1,660	287	261	239	211	181
Historical Beta	1.00	0.56	0.58	0.62	0.66	0.72

Note: Analysis over period 31-May-1999 to 30-Dec-2016, gross returns annualized in USD.
Source: MSCI.

EXHIBIT 5

The Trade-Off between the Improvement in ESG Profile and Realized Risk Reduction



Source: MSCI.

along with all the other constraints. Exhibit 4 presents five simulated minimum volatility strategies incorporating a constraint that gradually increases from 20% to 50% in terms of the required improvement in ESG score relative to the underlying market cap-weighted benchmark ($\mu = 1.2, \ldots, 1.5$ in the earlier equation).[5] This exhibit shows that realized volatility only rose marginally by 20 basis points and 50 basis points for a 20% and 30% improvement in sustainability rating score, respectively (in a simulation from May 31, 1999, to December 30, 2016). Even for a 50% improvement, the minimum volatility portfolio experienced volatility that was still 4.3 percentage points lower than the MSCI World Index.

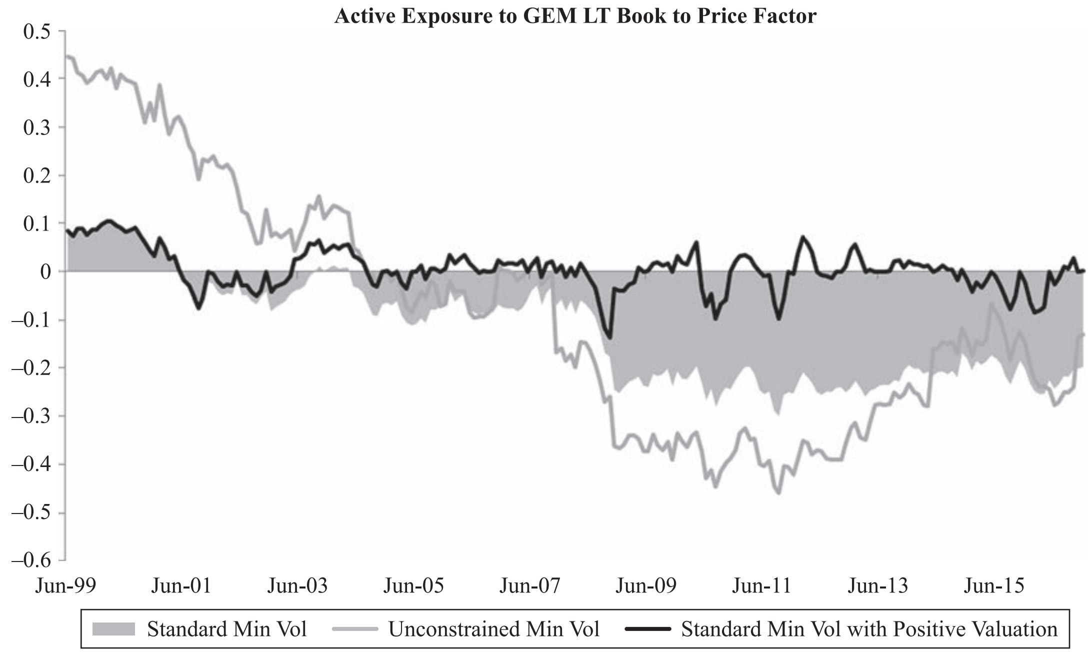
Exhibit 5 summarizes the effect of incorporating ESG risk into a minimum volatility strategy by showing an "efficient frontier" for ESG improvement versus realized volatility. The relatively flat starting part of the trade-off line suggests that substantial improvement (20%–30%) in the ESG profile of the minimum volatility portfolio can be achieved without a significant effect on its volatility outcomes. We see a similar trade-off pattern for other price volatility measures such as maximum drawdown (which we have not directly targeted). In general, these results show that ESG risk can be incorporated into a low volatility strategy as a simple linear constraint without compromising the volatility reduction properties of the factor strategy.

MANAGING CROWDING RISK

Crowding is an investment risk, and a crowded strategy may be susceptible to significant losses in times of market stress. Valuations are often used as shorthand

E X H I B I T   6

Asset-Based Valuations of Alternative Minimum Volatility Strategies



Source: MSCI.

for crowding risk, even though it has other key dimensions, and indeed time-series valuations have not been a strong predictor of factor strategy returns in general, even when valuations have been at extremes (Bayraktar et al. [2015] and Asness [2016a, 2016b]). Alighanbari, Doole, and Shankar [2016] discuss the valuations of minimum volatility strategies. While the analysis in that paper shows that recent minimum volatility strategy valuations are not extreme, crowding and overvaluation are an inherent risk of any investment strategy. In this section, we examine how the valuations of a minimum volatility strategy could be controlled to mitigate one dimension of the risks associated with investing in overvalued or "crowded" stocks within the strategy.

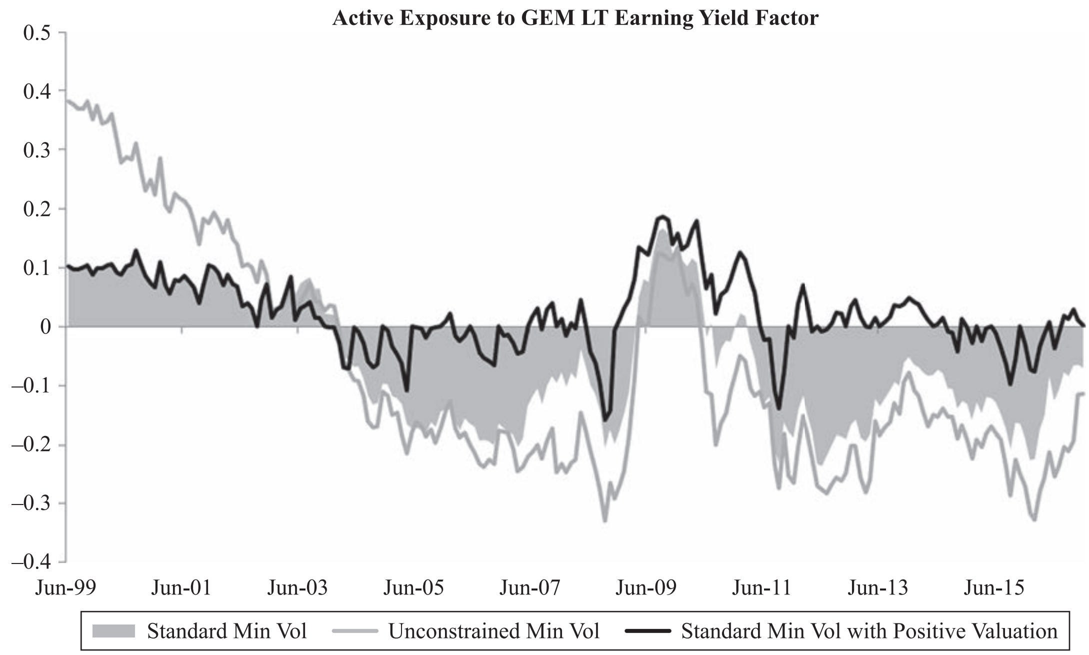
The notion of strategy crowding is, of course, much broader than just controlling the valuations of one particular implementation of the strategy. Crowding risk may be addressed at the asset allocation level, as well as through efficient portfolio construction. Here we focus on how appropriate constraints can be introduced into a minimum volatility strategy to control valuations and avoid expensive stocks by incorporating them into the portfolio construction process.

In optimized strategies such as minimum volatility, this objective can be achieved explicitly through linear constraints in the optimization. In a naïve minimum volatility strategy that uses a stock-level covariance matrix to minimize portfolio volatility, weighted average valuation ratios can be calculated and used as constraints. In more sophisticated investable strategies where a fundamental factor model is employed, value is often one of the main risk factors in the model and can therefore be accessed and controlled directly (without a risk of factor misalignment). By controlling the value factor exposure and keeping it within a narrow range of its benchmark, the strategy is not allowed to become too expensive compared with an underlying benchmark or the broader market from a cross-sectional perspective.

In our analysis, we use MSCI's Barra GEM LT. This model has two value factors, book to price and earnings yield. The latter comprises four descriptors: E/P, forecast E/P, cash E/P, and EBIT/EV. We analyze exposure to these value factors over time for three minimum volatility strategies: (1) an unconstrained strategy, (2) a strategy where exposure to value factors is constrained to be within +/−0.25 cross-sectional standard

E X H I B I T 7

Earnings-Based Valuations of Alternative Minimum Volatility Strategies



Source: MSCI.

deviations (z-score) relative to the benchmark, and (3) a strategy where value factor exposure is constrained to be positive.

Exhibits 6 and 7 show active exposure of these three strategies to the two GEM LT value factors. Value factor exposures for an unconstrained minimum volatility portfolio have deviated by up to 0.5 standard deviations from the benchmark over this period, in both positive and negative directions, indicating that an unconstrained minimum volatility portfolio can potentially become very cheap or very expensive compared with the broader market.

However, over this period, the deviation has been rather benign. Adding constraints ensures that value factor exposures remain within a close range relative to the market. Note that these constraints have been applied at each rebalancing date (here semi-annually). Value factor exposures may drift outside the range between rebalances, but they are brought back to the appropriate level at the next strategy rebalance.

When we tighten the value constraint to be positive, we effectively force the portfolio to be at least as cheap as (or no more expensive than) the broad market. But it could be argued that a positive-only value constraint adds a value factor targeting dimension to the minimum volatility strategy, taking it beyond its pure volatility reduction objective. The adoption of a two-factor strategy should be done transparently by investors.

This approach is summarized in Exhibit 8. The positive value constraint has added 40 basis points of performance to the strategy and increased the volatility by 30 basis points. Performance attribution analysis shows that the improvement in historical return is attributable to the increased contribution from value factors to performance. By forcing value exposure to be positive, the minimum volatility strategy benefited from the value premium, with only a small increase in realized volatility.

CONCLUSION

In this article, we investigated how a minimum volatility strategy can address financial and nonfinancial dimensions of risk beyond those price volatility metrics directly modeled via its portfolio construction. A minimum volatility strategy aims to reduce the volatility of the portfolio by minimizing the total expected risk or volatility. Even within the space of price vola-

Exhibit 8

Key Metrics for Minimum Volatility Strategies with Increasing Valuation Constraints

Key Metrics	MSCI World	Standard Min Vol	Value Factor Exposures > 0
Total Return (%)	4.6	7.6	8.0
Total Risk (%)	15.4	10.3	10.6
Sharpe Ratio	0.17	0.54	0.56
Active Return (%)	0.0	3.0	3.4
Tracking Error (%)	0.0	8.0	7.6
Information Ratio	NaN	0.37	0.44
Historical Beta	1.00	0.59	0.61
Turnove (%)	3.0	20.1	20.1
Price to Book	2.2	2.5	2.2
Price to Earnings	18.5	18.6	17.5
Dividend Yield (%)	2.3	2.6	2.6

Note: Analysis over period 31-May-1999 to 30-Dec-2016, gross returns annualized in USD.

Source: MSCI.

tility metrics, the resulting more defensive, low beta equity portfolio historically has experienced improved risk characteristics with respect to metrics not explicitly targeted, such as maximum drawdown.

Constraints are often added to a strategy to mitigate other factor sources of risk. For instance, country and sector exposure constraints can reduce the country/sector risks with outlier returns. Constraints on stock level weight can help with investability and can mitigate liquidity risk but also mitigate risk associated with company-specific events.

The minimum volatility strategy reduces the overall volatility by selecting for low volatility and weakly correlated stocks, and implicitly achieves the benefits of diversification. Our analysis showed that directly targeting increases in diversification by incorporating a constraint may perversely challenge the optimization to minimize the total volatility of the portfolio while picking high volatility stocks. This diversification constraint then results in the portfolio developing a positive exposure to the residual volatility factor, a factor which historically has had persistent negative returns. The resulting portfolio therefore tends to show lower historical return and higher volatility characteristics.

ESG concerns are another risk dimension that has attracted much attention from institutional investors recently. A minimum volatility strategy need not improve the ESG profile of the portfolio, depending on the region or country being studied. However, we demonstrated that ESG concerns and exposure to the low volatility factor are quite sympathetic. Adding an improvement constraint for the ESG profile does not significantly compromise the main objective of the minimum volatility strategy while delivering the sort of ESG improvement usually seen in pure ESG strategies or indexes. For example, historically, a 30% improvement on the strategy level ESG score was achieved while only a 50 basis point increase in realized volatility of the strategy was observed.

Crowding is another dimension of investment risk that is not so easily measured. A crowded strategy may experience large losses in a market crisis or when investors start withdrawing significant capital from the strategy. Controlling for valuation can help mitigate some of the symptoms associated with crowding. We show that when a fundamental factor model is used for creating the minimum volatility strategy, the value factor exposure can directly be controlled to be close to the broad market, preventing the optimization from picking low volatility but expensive stocks. We see that adding a valuation constraint only marginally increased the historical volatility of the strategy without compromising the long-term performance of the strategy or its varied risk-reduction characteristics.

Hence a well-designed and sensibly constrained minimum volatility strategy offered risk mitigation for various financial and nonfinancial dimensions. For three of those additional dimensions, we saw that the more practical concentration risk mitigation comes as a by-product of the usual minimum volatility strategy, rather than any refinement. Conversely, ESG and valuation risks can be explicitly incorporated into the methodology as constraints without compromising historical delivery on the main objectives of a minimum volatility strategy.

ENDNOTES

[1]In this equation, n-vectors h, r, and e denote portfolio weights, expected returns, and vector of ones, respectively. V is the asset by asset covariance matrix, and λ is the risk aversion coefficient.

[2]"Dirty" correlation is the effective, constant correlation between stocks in a portfolio often used in equity

derivatives trading (Bossu [2014]) and is here defined by $\rho = \left(\frac{\sigma_P}{\Sigma w_i \sigma_i} \right)^2$.

[3]Effective number of stocks (EN) is a measure of index concentration and ranges between 1 (for a single stock) and the number of stocks in the index (for an equal-weighted index). Generally, the lower the EN, the more concentrated an index.

[4]Here, s is an n-vector of ESG scores, h_b is the n-vector weight of the benchmark, and μ is the scalar defining the level of ESG improvement of the index compared with the benchmark.

[5]In the analyses, we use industry-adjusted ESG scores to measure ESG risk of the individual stocks and portfolios. These scores range from 0 to 10, 0 being the worst ESG score, and 10 the best ESG score.

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