COVARIANCE MATRIX REVERSE-ENGINEERING FOR EXTREME SCENARIO STRESS TESTING

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Covariance matrix reverse-engineering for extreme scenario stress testing

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Abstract:

The market crises over the last decades and more recently the sub-prime crisis have highlighted practitioners' need to improve investment risk management, especially through stress testing. Temporary extreme market conditions usually result in unexpected large losses and higher long horizon portfolio volatility for investors. At the same time the losses happen across many assets and even asset classes, indicating a higher correlation than during normal market times.

We show the application of a new way of manipulating the covariance matrix of a multi-factor model in order to shock correlations. We apply this method to sample historical extreme scenarios. We reverse engineer the covariance matrix that used in a stress test would replicate the large, correlated losses of the historical extreme scenario. We show that this technique opens up new possibilities for risk managers to create hypothetical stress test scenarios that include stressed correlations.

Following the 2008/2009 financial markets turmoil, asset owners and managers are looking for improved or new tools to manage and monitor risk in their multi-asset-class portfolios, especially through stress testing. Investors are no longer only at "regular" risk, most of them have extended their worries "regular and extreme" risk. The common thinking that a company or even a state is "too big to fail" tends to be questioned more often.

Temporary extreme market conditions, typically increasing asset-class co-movements and risk contagion between markets, will result in unexpected large losses and higher long horizon portfolio volatility for investors - a common definition of risk is volatility (Markowitz, 1952) -. The losses happen jointly across many assets and even asset classes indicating a higher correlation than during regular market times. The correlation and volatility shocks during extreme events do impact long horizon volatility and change the portfolio optimization scheme, as it often loses its diversification benefit.

To simulate extreme conditions and evaluate the potential losses linked to those conditions, practitioners (should) use stress-testing according Basel II. The stress tests aim to assess the impact of extreme rare, but plausible, movements of market parameters (asset returns, liquidity and volatility shocks, correlations changes, etc.) on a portfolio. The largely agreed definition is that stress-testing means choosing scenarios that are costly and rare, and then putting them to a valuation model. The tested scenarios could be hypothetical (i.e. assuming user-defined large negative returns on specific assets) or historical (i.e. historical returns that occurred a defined investment period of interest. For example: Asian Financial Crisis; Russian Economic Crisis; Terrorist Attacks 9/11, Subprime Mortgage Meltdown...). One is able to assess full stress-tests, or (un)correlated sensitivity-analysis tests.

For example, investment manager could be interested in simulating shocks that are more likely to occur than the historical returns suggest, or shocks that have never occurred. The objective of stress-testing is not to predict in any manner the next crisis, but to provide evaluation tools of possible portfolio losses under severe conditions.

As mentioned, the study will focus on the risk contagion that occurs during financial market turmoil. Then, interdependencies and correlations across assets increase during bear markets (Longin, Solnik (2001)). Using multi-factor models, we try to replicate extreme scenarios and the contagion across factors, starting from a set of leading factor shocks. To do so, we apply a new way of manipulating the covariance matrix from the multi-factor model (GEM2) in order to consistently increase correlations (Bender, Stefek, Lee (2010)). Indeed, one cannot just manipulate a correlation matrix changing correlations pairs by new values. The correlation matrix has to respect its mathematical properties: it has to remain a positive semi-definite matrix.

This correlation matrix manipulation approach has been back-tested on a sample of pre-listed extreme scenarios in BarraOne: we reverse-engineer the covariance matrix that, used in a correlated sensitivity-analysis stress test and based on the multivariate normal conditional expectation value, would replicate the correlated losses of the considered historical extreme scenario.

This technique does show improved return replication over the historical scenarios than with the traditional methodology, and opens up new possibilities for risk managers to create hypothetical stress test scenarios of current interest, that include stressed correlations.

This report aims to describe and present the research carried out during this project, and its results. The first part will deal with empirical studies and findings that have motivated our research and approach. The second part focuses on covariance reverse engineering on historical extreme scenarios (results and observations). The third part will conclude this report by presenting the next steps of this project and its possible extensions.

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I. Correlation and extreme events

A. Introductory remark:

The historical data are respectively:

- From 01-01-1986 to 26-02-2009 for both MSCI USA and MSCI Japan.
- From 01-01-1988 to 26-02-2009 for MSCI EMU.

Given the time lag between international markets, we will have to consider this bias that exists within our daily historical returns before undertaking empirical research and analysis any further.

Let's describe a typical trading day:

1. The Asian markets open first.
2. The European market places open once the Asian markets close.
3. Around 2pm in Europe, Wall Street opens. Therefore, until the end of the trading day, the European countries can observe/assess how the US market is behaving (index performances, economic indicators, etc.), and this often drives the rest of the trading activity in most of the European financial markets.

The next day, the Asian financial markets open up with the results from US and Europe financial market performance, and this often drives the activity of the day. To adjust for this time lag (1 day between the US and the Japanese and European market), we shift the MSCI Japan historical returns one day backward.

B. Extreme event correlations:

In this first section, our focus will be on the correlation across MSCI USA and MSCI Japan, conditional on extreme events, defined as simultaneous losses lower than -5%. The following chart is a scatter plot of the MSCI USA and Japan simultaneous returns.

This being said, we aim to focus on the correlations between MSCI USA and MSCI Japan during "extreme events", defined as day when large negative returns (≤ -5%) occurred in both of these equities. Therefore we are calculating the correlations between MSCI US and Japan, conditional on both of the returns at day *t* are lower than a threshold, here μ = -5%.

$$\rho_{\leq\mu}^{US/JP} = \rho[(r_t^{US} \leq \mu)\wedge(r_t^{JP} \leq \mu)]_{t\in[0;n]}$$

Similarly, we calculate extreme positive event correlation (μ= = 4%)

$$\rho_{\geq\mu}^{US/JP} = \rho[(r_t^{US} \geq \mu)\wedge(r_t^{JP} \geq \mu)]_{t\in[0;n]}$$

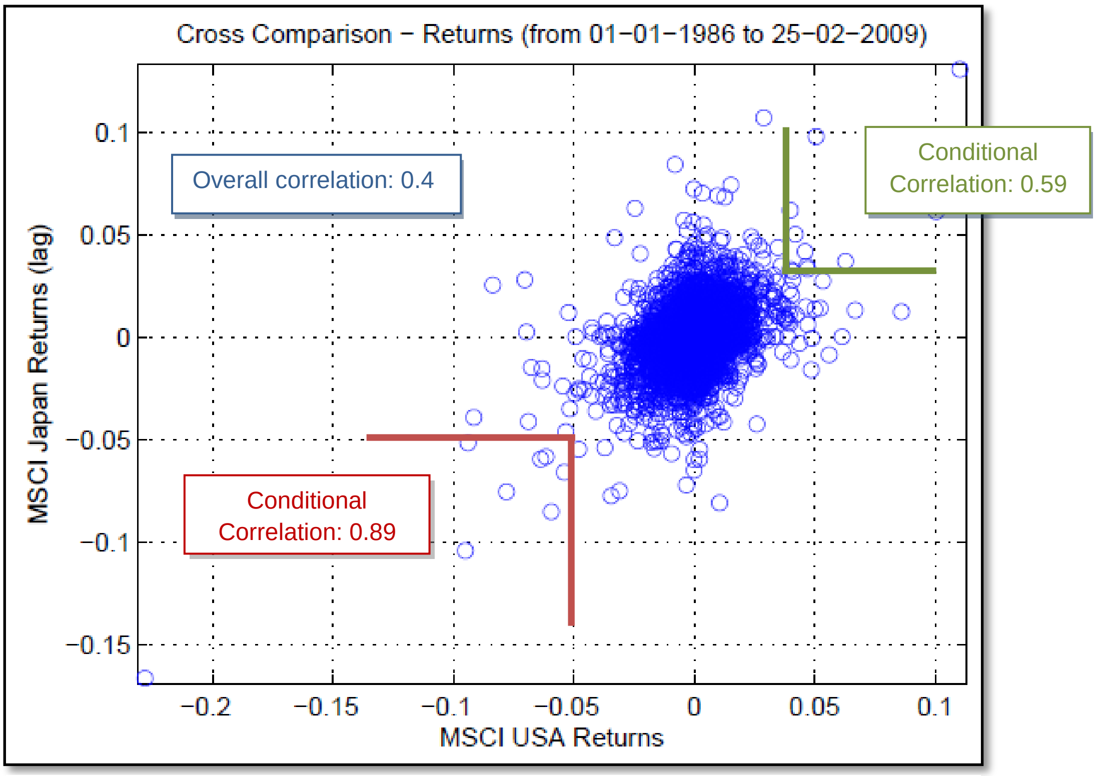


Figure 1.3: MSCI US-MSCI JP Conditional Correlations

<u>Observations (Figure 1.3):</u>

The overall correlation between MSCI USA and MSCI Japan (lag-adjusted) returns is 0.4. We define an "extreme event" filter to compute extreme correlations: an extreme negative (resp. positive) return is considered as an "extreme event" if the losses on the same day in both indices are lower than -5% (resp. greater than +4%, as the sample of returns with +5% was too short).

The "extreme event" conditional correlation during extreme negative returns is close to 0.9 (!), whereas the "extreme event" conditional correlation during extreme positive returns is close to 0.6.

We would like to compare those values with standard benchmarks. If variables x and y were normally distributed, then the correlation between them conditional on an extreme event would be given by the formula:

$$\rho_A = \rho[\rho^2 + (1-\rho^2)\frac{Var(x)}{Var(x|x \in A)}]^{-1/2}$$

As a reminder, the overall correlation is: $\rho = 0.4$. We will only assume that volatility doubles during extreme event:

$$\sigma_A = 2\sigma$$

$$\sigma_A{}^2 = (2\sigma)^2$$

$$Var(x|x \in A) = 2^2\, Var(x)$$

Then,

$$\rho_A = 0.4[0.4^2 + (1-0.4^2)\frac{Var(x)}{2^2 Var(x)}]^{-1/2}$$

$$\rho_A = 0.4[0.16 + 0.84 * \frac{1}{4}]^{-1/2}$$

$$\rho_A \sim 0.66$$

However, empirical correlations tend to increase even more when extreme events occur (ρ_A = 0.89). Then, MSCI US and MSCI Japan have had a higher correlation in extreme bear markets than the multivariate normal conditional correlation would indicate. It is consistent with Longin, Solnik (2001).

II. Stress Testing and Manipulating Correlations

C. Motivation of Using Stress Testing

During extreme events, market conditions are not "regular" anymore. The conditions turn into stressed conditions (i.e. increasing liquidity, volatility, risk aversion, bond spreads… and correlation across asset classes) and long horizon portfolio volatility is changed.

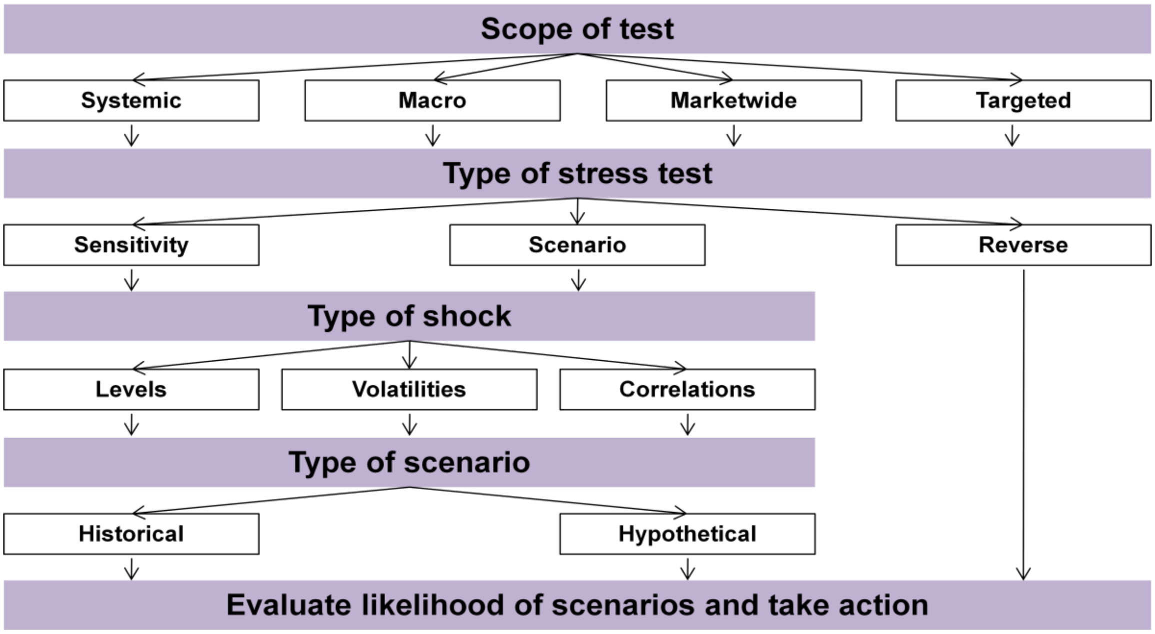
The aims of our project are to:

- Assess the impact of future extreme events on multi-asset-class portfolios
- Construct a better stress-testing framework for portfolio managers, providing functionality such as user-defined strategic asset class assumptions and stress testing scenarios.
- Contribute to new stress test standards for the asset management industry
- Implement the new MSCI latent factor methodology to stress correlations, and research its application to historical extreme scenarios

Most commonly, stress tests are used to shock the levels of underlying variables: illiquidity, correlation breakdowns, risk contagion…

We distinguish several kinds of stress tests:

- Full historical stress test: specify all factor shocks and market parameters (volatility, …) and assess the impacts on the portfolio
- Sensitivity stress test: specify one (historical or hypothetical) factor shock (and/or market parameter shock) and assess the factor losses associated with this shock.
- Correlated stress test: specify one or a set of factor shocks, use covariance matrix to assess the shock propagation or "contagion" to the other factors, and assess the impact on the portfolio



The covariance matrix used in these stress-tests is calculated from a multi-factor risk model with a specified half-life. As a first step, we will use long horizon models (48-month half-life), and in a second step use short term ones (6-month half-life).

As next steps, we will use a shorter half-life covariance matrix, to capture the latest interdependencies (prior to a crisis), that has a very short half life (21 days). To compute the replicated shocks from a sample of applied shocks, we will use the multivariate normal conditional expectation value defined as follows:

$$\boldsymbol{E(Y|S) = E(Y) + \Sigma_{SY}^{T}\Sigma_{SS}^{-1}[S - E(S)]} \quad \textbf{(1)}$$

$$For\ our\ purpose, we\ assume$$

$$E(S) = 0\ and\ E(Y) = 0$$

$$\boldsymbol{S}: \boldsymbol{vector\ of\ user - defined\ shocks}$$

$$\boldsymbol{Y}: \boldsymbol{vector\ of\ changes\ in\ returns\ of\ targets\ to\ be\ analyzed}$$

In previous research papers on this topic, such as Kupiec (1998) and Kim & Finger (2000), methods tested also included the so-called “naïve sensitivity stress-test” method, which consists of shocking one factor while leaving others unaffected. Kupiec (1998) conclusively shows that this technique roughly underestimates the loss potential: it disregards any relationships between the asset being shocked and the rest of the assets in the model. In other words, it implicitly assumes zero correlation between assets. This method does not capture any dynamics of co-movements of the assets during an extreme event. Our focus will be on correlated sensitivity-analysis stress-tests that capture, even partially, interdependencies between assets.

The method used to assess the propagation of an applied shock through other assets is based on the conditional multivariate normal distribution. Normality is acceptable here as we are not dealing with daily impacts. We are dealing with weekly, monthly, quarterly impacts (scenario horizon), and their distribution resemble the bell curve much more than with daily returns. Secondly, assuming a conditional normal distribution is quite different from the unconditional one and, in case of mixture normals, can produce fat-tail effects.

This method is the current method used in BarraOne.

D. BarraOne Stress-Testing: Results and Motivation

BarraOne lets the user stress test portfolios to assess how severely they might be affected by various market scenarios. Many historical scenarios are pre-listed and BarraOne offers the user a wide range of parameter shocks.

Then, the different shocks that can be applied to market categories: Equities (price or volatility); Interest Rates (shift in curve, shape of curve, volatility); Foreign Exchange (rates or volatility); Credit Spreads; Commodities (price or volatility). Then, the market parameter shocks can be applied in two ways:

- Uncorrelated shocks: only the specified assets that are impacted are revalued, and only the designated market conditions are shocked.
- Correlated shocks: BarraOne combines the specified shocks with the inherent relationships that exist between the Barra Integrated Model's

covariance matrix. Then, these shocks are propagated to other factors through the captured covariances across factors.

Example:

Period: August-October 2008.

Why: Assess the accuracy of a correlated back test over a historical scenario, listed in BarraOne.

Specificity of the scenario: the scenario is not a single-day crisis, but goes over two month (consistency with the theoritical conditional expectation value in BarraOne). Lehman Brothers collapsed in September 2008.

Applied shocks: historical equity returns (MSCI USA, MSCI Japan, MSCI EMU), respectively -23.41%, -29.92 and -24.85%.

To calculate the replicated shocks from a sample of applied shocks, we will use the multivariate normal expectation value:

$$\boldsymbol{E}(\boldsymbol{Y}|\boldsymbol{S}) = \boldsymbol{E}(\boldsymbol{Y}) + \boldsymbol{\Sigma}_{\mathbf{SY}}^{\mathbf{T}}\boldsymbol{\Sigma}_{\mathbf{SS}}^{-\mathbf{1}}[\mathbf{S} - \mathbf{E}(\mathbf{S})] \quad (\mathbf{1})$$

$For\ our\ purpose, we\ assume$

$E(S) = 0\ and\ E(Y) = 0$

$\boldsymbol{S}: \boldsymbol{vector\ of\ user-defined\ shocks}$

$\boldsymbol{Y}: \boldsymbol{vector\ of\ changes\ in\ returns\ of\ targets\ to\ be\ analyzed}$

As a first step, we will try to replicate historical shocks over the scenario, using the correlated shock technique, and the time-weighted covariance matrix prior to the crisis.

01/08/2008 - 30/10/2008

Shock on US_EQUITY	-25%
Portfolio	-26.06%
MSCI EMU Historical Return	-23.41%
MSCI JP Historical Return	*-29.92%*
MSCI US Historical Return	*-24.85%*

Analysis Date 01/08/2008

		test_US_EQ		
Asset Name	Initial Market Value	Final Market Value	P&L	Asset Id
Total	100.00	81.01	-18.99%	
MSCI Europe	33.33	25.76	*-22.73%*	MSEUR
MSCI Japan	33.33	30.07	*-9.80%*	MSCIJPN
MSCI USA	33.33	25.19	-24.43%	MSCIUSA

01/08/2008 - 30/10/2008

Shock on JP_EQUITY	-25%
Portfolio	-26.06%
MSCI EMU Historical Return	-23.41%
MSCI JP Historical Return	*-29.92%*
MSCI US Historical Return	*-24.85%*

Analysis Date 01/08/2008

		test_JP_EQ		
Asset Name	Initial Market Value	Final Market Value	P&L	Asset Id
Total	100.00	87.78	-12.22%	
MSCI Europe	33.33	31.29	*-6.14%*	MSEUR
MSCI Japan	33.33	24.78	-25.67%	MSCIJPN
MSCI USA	33.33	31.72	*-4.84%*	MSCIUSA

01/08/2008 - 30/10/2008

Shock on EMU_EQUITY	-25%
Portfolio	-26.06%
MSCI EMU Historical Return	-23.41%
MSCI JP Historical Return	*-29.92%*
MSCI US Historical Return	*-24.85%*

Analysis Date 01/08/2008

		test_EU_EQ		
Asset Name	Initial Market Value	Final Market Value	P&L	Asset Id
Total	100.00	83.77	-16.23%	
MSCI Europe	33.33	25.42	-23.73%	MSEUR
MSCI Japan	33.33	30.55	*-8.35%*	MSCIJPN
MSCI USA	33.33	27.79	*-16.62%*	MSCIUSA

Table 2.1: Shock Replication, using Multivariate Normal framework (48-Month Half-Life Covariance Matrix prior to the Crisis)

First try (top table), we apply a -25% on MSCI US (est. universe). We obtain the following results:

- MSCI EMU historical shock is closely replicated
- MSCI Japan historical shock is under-estimated (-20% difference between historical and estimated shock)

Second try (middle table), we apply a -25% shock on MSCI Japan (est. universe).

- MSCI US historical shock is under-estimated (-20% difference between historical and estimated shock)
- MSCI EMU historical shock is under-estimated (~ -20% difference between historical and estimated shock)

Last try (bottom table), we apply a -25% shock on MSCI EMU (est. universe).

- MSCI US historical shock is under-estimated (~ -10% difference between historical and estimated shock)
- MSCI Japan historical shock is under-estimated (~ -20% difference between historical and estimated shock)

<u>Observations (Table 2,1)</u>:

BarraOne does not fully replicate the three historical equity shocks from a single one, using historical covariance prior to the crisis. All of these historical single-shock tries lead to underestimation of the historical shocks. The single-shock was not accurately propagated to the other assets; It means that either the co-movements between assets, prior to the crisis, are not fully captured.

<u>Next Step</u>:

All other things being equal, a post-crisis historical covariance may incorporate some covariance updates, reflecting in a better the new relationships between the three equities during the crisis.

01/08/2008 - 30/10/2008		Analysis Date 01/12/2008 test_US_EQ				
Shock on US_EQUITY	-25%	Asset Name	Initial Market Value	Final Market Value	P&L	Asset Id
Portfolio	**-26.06%**	Total	**100.00**	**82.31**	**-17.69%**	
MSCI EMU Historical Return	**-23.41%**	*MSCI Europe*	33.33	26.66	*-20.02%*	MSEUR
MSCI JP Historical Return	***-29.92%***	*MSCI Japan*	33.33	30.45	*-8.64%*	MSCIJPN
MSCI US Historical Return	***-24.85%***	MSCI USA	33.33	25.20	-24.41%	MSCIUSA

01/08/2008 - 30/10/2008		Analysis Date 01/12/2008 test_JP_EQ				
Shock on JP_EQUITY	-25%	Asset Name	Initial Market Value	Final Market Value	P&L	Asset Id
Portfolio	**-26.06%**	Total	**100.00**	**87.48**	**-12.52%**	
MSCI EMU Historical Return	**-23.41%**	*MSCI Europe*	33.33	31.54	*-5.39%*	MSEUR
MSCI JP Historical Return	***-29.92%***	MSCI Japan	33.33	24.71	-25.87%	MSCIJPN
MSCI US Historical Return	***-24.85%***	*MSCI USA*	33.33	31.23	*-6.31%*	MSCIUSA

01/08/2008 - 30/10/2008		Analysis Date 01/12/2008 test_EU_EQ				
Shock on EMU_EQUITY	-25%	Asset Name	Initial Market Value	Final Market Value	P&L	Asset Id
Portfolio	**-26.06%**	Total	**100.00**	**83.35**	**-16.65%**	
MSCI EMU Historical Return	**-23.41%**	MSCI Europe	33.33	25.67	-22.98%	MSEUR
MSCI JP Historical Return	***-29.92%***	*MSCI Japan*	33.33	30.76	*-7.72%*	MSCIJPN
MSCI US Historical Return	***-24.85%***	*MSCI USA*	33.33	26.92	*-19.24%*	MSCIUSA

Table 2.2: Shock Replication, using Multivariate Normal framework (48-Month Half-Life Covariance Matrix after to the Crisis)

First try (top table), we apply a -25% on MSCI US (est. universe). We obtain the following results:

- MSCI EMU historical shock is slightly under-estimated
- MSCI Japan historical shock is still under-estimated (-20% difference between historical and estimated shock)

Second try (middle table), we apply a -25% shock on MSCI Japan (est. universe).

- MSCI US historical shock remains under-estimated (~-20% difference between historical and estimated shock)
- MSCI EMU historical shock remains under-estimated (~ -20% difference between historical and estimated shock)

Last try (bottom table), we apply a -25% shock on MSCI EM U (est. universe).

- MSCI US historical shock is better replicated than with previous covariance matrix, but still under-estimated (~-5% difference between historical and estimated shock)
- MSCI Japan historical shock is still under-estimated (~ -20% difference between historical and estimated shock)

<u>Observations (Table 2.2)</u>:

Even with a partially updated covariance matrix, the historical shocks are still not fully replicated. This observation has been our motivation to improve correlated shock methodology and attempt to reverse-engineer factor covariance matrix to accurelaty replicate historical returns through the Latent Factor approach (Bender, Lee, Stefek). The objective is to find the best exposures to latent drivers of the correlation matrix, so the co-movements between assets are such that the replicated asset shocks, from a sample of applied asset shocks, are extremily close to their historical returns

III. Shocking Correlation Matrix to Replicate Historical Extreme Losses, Using Latent Factor Approach

E. Latent Factor Approach

Most of the investors wonder about the consequences on their portfolio if the correlations between two assets (or asset classes) increase dramatically. They would clearly lose the benefit from diversification, but what would happen? How big would the portfolio losses be given such a change?

Manipulating the correlation matrix provides one answer to these questions. The problem is that manipulating correlation is not straightforward: it has its own mathematical challenges (positive semi-definiteness). Plus, the approach has to be flexible: manipulating some correlation pairs, preserving some others, exposing the remaining correlations to these collateral changes of correlations.

Bender, Lee, Stefek ("Manipulating Correlations through Latent Drivers", Research Insight Paper, MSCI) present a framework to consistently shock any matrix correlation (asset, factor, or asset class).

A first assumption is the following: each asset is exposed to a (hidden/unobservable) driver. We can group assets in sub-groups within the correlation matrix, given that they are exposed to the same latent driver/factor, with a level of exposure defined as $\boldsymbol{\upsilon}$.

The proposed framework can not only change correlation within sub-blocks, it can also change cross block correlations.

$$\rho_{ij}^{new} = \upsilon_i \upsilon_j + \sqrt{1-\upsilon_i^2}\sqrt{1-\upsilon_j^2}\,\rho_{ij}^{orig} \quad if\ i\ and\ j\ are\ in\ the\ same\ block$$

$$\rho_{ij}^{new} = \upsilon_i \upsilon_j \rho_{r_1 r_2} + \sqrt{1-\upsilon_i^2}\sqrt{1-\upsilon_j^2}\,\rho_{ij}^{orig} \quad if\ i\ and\ j\ are\ in\ different\ block$$

With zero-value exposures, the correlations remain the same.

Therefore, by manipulating exposures using latent factors, the method offers a consistent way to "shock" within-block and cross-block correlations.

One interest is to identify which exposure/set of exposures led to such changes in correlation across factors/asset classes. These exposures could then be used to replicate these specific market conditions to assess the impact of these correlations increases on the portfolio and its volatility.

As we ignore the correlations between exposures ($\rho_{r_1 r_2}$), we will admit that these drivers are uncorrelated (as suggested in the research paper) by letting $\rho_{r_1 r_2} = 0$.

Original correlation matrix	US equity	Canada equity	UK equity	EMU equity
US equity	1.00			
Canada equity	0.76	1.00		
UK equity	0.71	0.64	1.00	
EMU equity	0.73	0.67	0.79	1.00

$\boldsymbol{\upsilon = 0}$

New correlation matrix	US equity	Canada equity	UK equity	EMU equity
US equity	1.00			
Canada equity	0.82	1.00		
UK equity	0.78	0.73	1.00	
EMU equity	0.80	0.75	0.84	1.00

$\boldsymbol{\upsilon = 0.5}$

$$\rho^{new}_{US,CN} = \underbrace{\upsilon_{US}\upsilon_{CN}}_{0.25} + \underbrace{\sqrt{1-\upsilon_{US}^2}}_{0.87}\,\underbrace{\sqrt{1-\upsilon_{CN}^2}}_{0.87}\,\rho^{orig}_{US,CN}$$

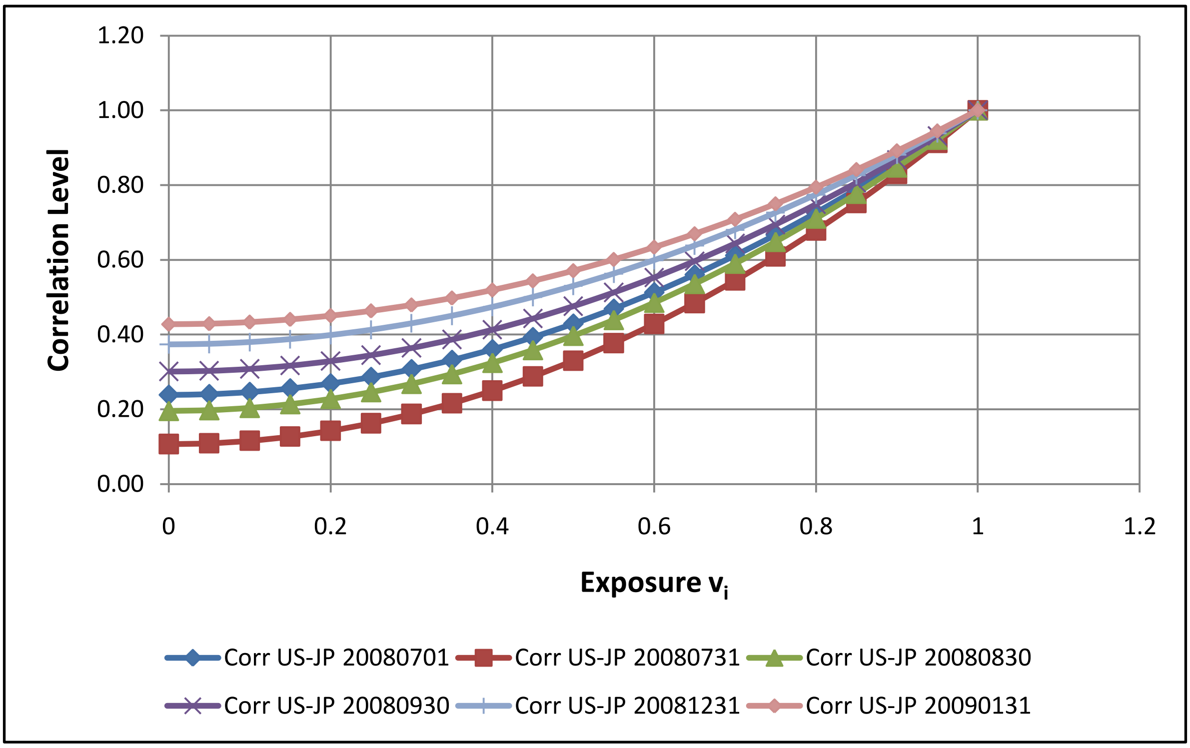


Figure 3.1: Evolution of correlation (using different monthly updated Barra Integrated Model covariance matrices)

Observation (Figure 3.1):

This chart presents the evolution of the correlation between MSCI US and MSCI Japan in function of its exposure. The trend of this evolution is totally linked to the equation driving the "shocked" correlation. Unsurprisingly, if the exposure is 0, then the new correlation remains equal to the original one; if the exposure is 1, then the new correlation is maximum and equal to 1.

Our objective is to shock the correlation matrix, so the derived covariance matrix replicated the historical shocks (using equation **(1)**), from a set of historical shock

$$\boxed{E(Y|S)} = E(Y) + \underbrace{\Sigma_{SY}^{T}\Sigma_{SS}^{-1}}_{\text{Shock}}[\underbrace{S}_{\text{Input}} - E(S)]$$

F. First attempt: Aug – Oct 2008

In this section, we will try to replicate the historical shocks from a single-shock, using the Latent Factor approach and adjusting the correlation matrix so this "shocked" correlation leads to the best replicated shocks.

Example:

Period: August-October 2008.

Why: Assess the accuracy of a correlated back test over a historical scenario, listed in BarraOne, using Latent Factor exposure approach so we can shock correlation matrix.

Specificity of the scenario: the scenario is not a single-day crisis, and goes over two months (consistency with the theoretical conditional expectation value in BarraOne). Lehman Brothers collapsed in September 2008.

Applied shocks: historical equity returns (MSCI USA, MSCI Japan, MSCI EMU), respectively -23.41%, -29.92 and -24.85%.

$$\boldsymbol{E(Y|S) = E(Y) + \Sigma_{SY}^{T}\Sigma_{SS}^{-1}[S - E(S)]} \qquad (\mathbf{1})$$

$$For\ our\ purpose, we\ assume\ E(S) = 0\ and\ E(Y) = 0$$

	US	**JP**	**EMU**
US	100.00%	7.54%	43.92%
JP	7.54%	100.00%	28.00%
EMU	43.92%	28.00%	100.00%

Table 3.1: EWMA Correlation matrix (80-day half-life) as of July 31st, 2008

1/ Shock on MSCI USA: -25%

- *Common exposure = 0* *(Original correlation matrix)*

	Replicated Shocks	Historical Shocks
US	**-25.000%**	-24.85%
JP	-2.407%	-29.92%
EMU	-11.816%	-23.40%

Exposure 0	US	JP	EMU
US	100.00%	7.54%	43.92%
JP	7.54%	100.00%	28.00%
EMU	43.92%	28.00%	100.00%

Table 3.2: Shock Replication, starting from MSCI US historical shock and using a zero-value exposure and covariance matrix (80-day half-life) as of July 31st, 2008

- *Common exposure = 0.4* *(new correlation matrix)*

	Replicated Shocks	Historical Shocks
US	**-25.000%**	-24.85%
JP	-7.131%	-29.92%
EMU	-14.230%	-23.40%

Exposure 0.4	US	JP	EMU
US	100.00%	22.33%	52.89%
JP	22.33%	100.00%	39.52%
EMU	52.89%	39.52%	100.00%

Table 3.3: Shock Replication, starting from MSCI US historical shock and using a 0.4-value exposure and covariance matrix (80-day half-life) as of July 31st, 2008

- *Common exposure = 0.8* *(new correlation matrix)*

	Replicated Shocks	Historical Shocks
US	**-25.000%**	-24.85%
JP	-21.301%	-29.92%
EMU	-21.473%	-23.40%

Exposure 0.8	US	JP	EMU
US	100.00%	66.71%	79.81%
JP	66.71%	100.00%	74.08%
EMU	79.81%	74.08%	100.00%

Table 3.4: Shock Replication, starting from MSCI US historical shock and using a 0.8-value exposure and covariance matrix (80-day half-life) as of July 31st, 2008

- *Common exposure = 0.95* *(new correlation matrix)*

	Replicated Shocks	Historical Shocks
US	**-25.000%**	-24.85%
JP	-29.051%	-29.92%
EMU	-25.433%	-23.40%

Exposure 0.95	US	JP	EMU
US	100.00%	90.99%	94.53%
JP	90.99%	100.00%	92.98%
EMU	94.53%	92.98%	100.00%

Table 3.5: Shock Replication, starting from MSCI US historical shock and using a 0.95-value exposure and covariance matrix (80-day half-life) as of July 31st, 2008

Observations (Table 3.2-3-4-5):

The introduction of a common latent factor exposure <u>does</u> improve the shock replications from the MSCI USA historical return, and for a certain exposure-value the expected shocks almost match the historical shocks (~0.95 seems the best common exposure).

<u>2/ Shock on MSCI Japan: -25%</u>

- *Common exposure = 0* *(Original correlation matrix)*

	Replicated Shocks	Historical Shocks
US	-1.476%	-24.85%
JP	**-25.000%**	-29.92%
EMU	-5.899%	-23.40%

Exposure	**0**	**US**	**JP**	**EMU**
	US	100.00%	7.54%	43.92%
	JP	7.54%	100.00%	28.00%
	EMU	43.92%	28.00%	100.00%

Table 3.6: Shock Replication, starting from MSCI JP historical shock using a zero-value exposure and covariance matrix (80-day half-life) as of July 31st, 2008

- *Common exposure = 0.95* *(New correlation matrix)*

	Replicated Shocks	Historical Shocks
US	-17.810%	-24.85%
JP	**-25.000%**	-29.92%
EMU	-19.587%	-23.40%

Exposure	**0.95**	**US**	**JP**	**EMU**
	US	100.00%	90.99%	94.53%
	JP	90.99%	100.00%	92.98%
	EMU	94.53%	92.98%	100.00%

Table 3.7: Shock Replication, starting from MSCI JP historical shock using a 0.95-value exposure and covariance matrix (80-day half-life) as of July 31st, 2008

<u>Observations (3.6-7):</u>

Using the MSCI JP historical shock, the reverse-engineer covariance method (using the Latent Factor Approach) results in better replication than the "traditional" approach, but the historical shocks are only partially replicated.

We will try to refine this case by defining sub-blocks within this covariance matrix, or introducing a shorter half-life covariance matrix.

<u>3/ Shock on MSCI EMU: -25%</u>

- *Common exposure = 0* *(Original correlation matrix)*

	Replicated Shocks	Historical Shocks
US	-10.203%	-24.85%
JP	-8.309%	-29.92%
EMU	**-25.000%**	-23.40%

Exposure	**0**	**US**	**JP**	**EMU**
	US	100.00%	7.54%	43.92%
	JP	7.54%	100.00%	28.00%
	EMU	43.92%	28.00%	100.00%

Table 3.8: Shock Replication, starting from MSCI EMU historical shock using a zero-value exposure and covariance matrix (80-day half-life) as of July 31st, 2008

- *Common exposure = 0.8* *(New correlation matrix)*

	Replicated Shocks	Historical Shocks
US	-18.540%	-24.85%
JP	-21.979%	-29.92%
EMU	**-25.000%**	-23.40%

Exposure	**0.8**	**US**	**JP**	**EMU**
	US	100.00%	66.71%	79.81%
	JP	66.71%	100.00%	74.08%
	EMU	79.81%	74.08%	100.00%

Table 3.9: Shock Replication, starting from MSCI EMU historical shock using a 0.8-value exposure and covariance matrix (80-day half-life) as of July 31st, 2008

- *Common exposure = 0.95* *(New correlation matrix)*

	Replicated Shocks	Historical Shocks
US	-21.960%	-24.85%
JP	-27.586%	-29.92%
EMU	**-25.000%**	-23.40%

Exposure	**0.95**	**US**	**JP**	**EMU**
	US	100.00%	90.99%	94.53%
	JP	90.99%	100.00%	92.98%
	EMU	94.53%	92.98%	100.00%

Table 3.10: Shock Replication, starting from MSCI EMU historical shock using a 0.8-value exposure and covariance matrix (80-day half-life) as of July 31st, 2008

Observations (Table 3.8-9-10):

The introduction of a common latent factor exposure does improve the shock replications from the MSCI EMU historical return, and for a certain exposure-value the expected shocks almost match the historical shocks (~0.9 seems to be the best common exposure).

Defining sub-blocks and shorter half-life covariance matrix would probably result in better replications.

IV. Back-Testing Reverse-Engineering GEM2 Covariance matrix with Latent Factor Method on Historical Scenarios

We propose to back test reverse-engineering GEM2 Covariance matrix using optimized Latent Factor Exposures on a sample of historical extreme scenarios (pre-listed in BarraOne).

We defined optimized Latent Factor Exposure(s) as the exposure(s) that minimize the sum of absolute deviations between expected and historical returns. The minization problem to solve is the following problem:

$\min_{\mathbf{v}} \sum_{i=1}^{n} (|E(Y|S) - H(Y)|^{\alpha})$ Y is the vector of targets, S the vector of shocks

We will assume $\alpha = 1$.

With $\boldsymbol{E}(\boldsymbol{Y}|\boldsymbol{S}) = \boldsymbol{E}(\boldsymbol{Y}) + \boldsymbol{\Sigma}_{\mathbf{SY}}^{\mathbf{T}} \boldsymbol{\Sigma}_{\mathbf{SS}}^{-1} [\mathbf{S} - \mathbf{E}(\mathbf{S})]$

And $\boldsymbol{\Sigma}_{\mathbf{SY}} = \mathbf{P}_{\mathbf{SY}} (\boldsymbol{\Sigma}_{\mathbf{SS}} \boldsymbol{\Sigma}_{\mathbf{YY}})^{\mathbf{1/2}}$,

P_{SY}being the "new" correlation matrix between S and Y with:

$$\rho_{ij}^{new} = \boldsymbol{v}_i \boldsymbol{v}_j + \sqrt{1 - \boldsymbol{v}_i^2} \sqrt{1 - \boldsymbol{v}_j^2} \rho_{ij}^{orig} \; if \; i \; and \; j \; are \; in \; the \; same \; block$$

$$\rho_{ij}^{new} = \boldsymbol{v}_i \boldsymbol{v}_j \rho_{r_1 r_2} + \sqrt{1 - \boldsymbol{v}_i^2} \sqrt{1 - \boldsymbol{v}_j^2} \rho_{ij}^{orig} \; if \; i \; and \; j \; are \; in \; different \; block$$

$\rho_{r_1 r_2}$ *will be assumed equal to* 0

The sample of extreme scenarios we back test is the following: 1997-Financial Crisis; 1998 Russian Financial Crisis; 2000 Emerging Market Decline; 2000 Tech Bubble; 2000-02 Argentina Economic Crisis; 2001 DotCom Slowdown; 2001 Sep 11 (Week); 2003 Iraq War; 2006 Emerging Market Crash; 2007-08 Oil Price Rise; 2007-09 Subprime Mortgage Meltdown; 2008 Bear Stearns Collapse; 2008 January; 2008 Nov-Mar; 2008 Sep-Nov; 2009 Jan-Mar.

As it seems unlikely to be able to replicate the 152 factor historical returns from a single-shock, we will test different set of shocks for the listed scenarios.

- The largest negative return over the period
- The largest negative factor return AND the largest positive factor return over the period
- The largest negative factor return, the median factor return and the largest positive factor return over the period

A. With a common exposure (Correlation Matrix fully Exposed to a Latent Factor)

	1 Shock: Neg		2 Shocks: Negative / Positive		3 Shocks: Negative / Median / Positive	
SCENARIOS	Optimized Common Exposure	Sum Errors	Optimized Common Exposure	Sum Errors	Optimized Common Exposure	Sum Errors
1997-98 Asian Financial Crisis	0.00	58.51	0.00	56.58	0.00	56.66
1998 Russian Financial Crisis	0.00	26.07	0.00	24.17	0.00	20.55
2000 Emerging Market Decline	0.00	3.08	0.13	2.93	0.14	2.93
2000 Tech Bubble	0.00	3.22	0.00	3.03	0.00	3.04
2000-02 Argentina Economic Crisis]	0.16	27.27	0.27	27.05	0.21	27.01
2001 DotCom Slowdown	0.00	246.78	0.00	239.71	0.00	248.70
2001 Sep 11	0.38	4.63	0.47	4.23	0.55	4.16
2003 Iraq War	0.41	3.27	0.99	3.33	0.99	3.33
2006 Emerging Market Crash	0.00	1.17	0.99	1.15	0.59	1.14
2007-08 Oil Price Rise	0.00	21.43	0.00	20.07	0.67	19.43
2007-08 Subprime Mortgage Meltdown	0.00	13.76	0.99	11.98	0.99	11.78
2007-09 Subprime Mortgage Meltdown	0.53	18.50	0.00	21.41	0.99	18.28
2008 Bear Stearns Collapse	0.00	2.38	0.00	2.22	0.00	2.23
2008 January	0.00	4.74	0.00	4.32	0.32	4.23
2008 Nov-Mar	0.00	10.01	0.00	10.09	0.00	10.21
2008 Sep-Nov	0.00	10.01	0.00	10.09	0.00	10.21
2009 Jan-Mar	0.00	10.87	0.00	9.72	0.00	9.69

Table 3.11: Scenarios and Optimized Common Exposure

Observations (Table 3.11):

Almost all the scenarios have an optimized common latent factor exposure equals to 0. When applying other shocks (largest negative/positive) and (largest negative/media/positive), the optimized exposure is less often 0, and more often close to 1.

Applying both the negative and positive shocks helps to replicate the upside and downside of the return distribution (cf. Figure3.2). When applying only the negative shock, we cannot replicate the upside of the return distribution.

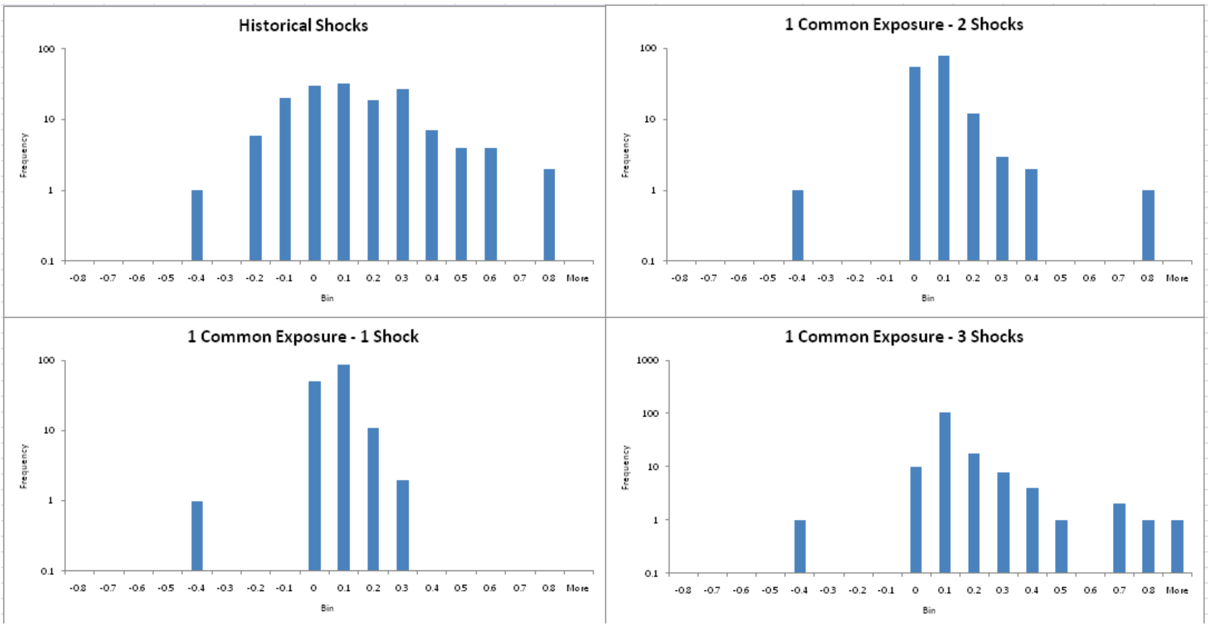


Figure 3.2: Historical Shock (top-right hand chart) vs. Replicated Shock distribution for the "2007-08 Oil Price Rise" scenario

One arising comment about reverse-engineering the covariance matrix using Latent Factor Approach with a common exposure that we cannot decrease correlations between factor returns; for example, during Asian Financial Crisis, Asian currencies get un-correlated (or even anti-correlated) with other currencies. This problem can be mitigated by introducing relevant sub-blocks and introducing negative across-block correlations. However, for the rest of our experiments, we will keep the across-block correlations (i.e. the correlations across latent drivers) equal to 0.

B. With two Sub-Blocks (Largest Negative Return vs. Other Return Block)

Now, we define two sub-blocks.The first sub-block will be composed of the factors that experienced negative returns that are 3-standard-deviation away from the average return. If there is no return over this threshold, this sub-block will be composed of the factors that experienced negative returns that are 2-standard-deviation away from the average. If no return matches this criterion, we will consider this sub-block as a single factor block: the largest negative return. The second sub-block is made of the remaining factors.

	1 Shock: Neg			2 Shock: Negative/Positive			3 Shock: Negative/Median/Positive		
SCENARIOS	Optimized Exposures		Sum Errors	Optimized Exposures		Sum Errors	Optimized Exposures		Sum Errors
1997-98 Asian Financial Crisis	0.95	0.00	24.68	0.95	0.00	26.25	0.95	0.00	24.00
1998 Russian Financial Crisis	0.95	0.95	13.08	0.55	0.95	13.14	0.40	0.95	12.86
2000 Emerging Market Decline	0.85	0.80	2.50	0.00	0.90	2.47	0.00	0.90	2.47
2000 Tech Bubble	0.00	0.00	3.22	0.40	0.00	2.95	0.25	0.00	3.00
2000-02 Argentina Economic Crisis	0.00	0.00	27.37	0.00	0.00	27.25	0.00	0.60	27.09
2001 DotCom Slowdown	0.95	0.95	35.23	0.95	0.95	49.91	0.95	0.95	33.23
2001 Sep 11	0.95	0.75	3.63	0.00	0.95	3.68	0.00	0.95	3.67
2003 Iraq War	0.95	0.95	3.31	0.00	0.95	3.49	0.00	0.80	3.42
2006 Emerging Market Crash	0.00	0.95	1.12	0.00	0.80	1.15	0.00	0.80	1.14
2007-08 Oil Price Rise	0.00	0.00	21.43	0.55	0.00	18.93	0.40	0.00	19.66
2007-08 Subprime Mortgage Meltdown	0.00	0.35	13.76	0.40	0.00	11.62	0.35	0.00	11.69
2007-09 Subprime Mortgage Meltdown	0.00	0.00	19.98	0.00	0.00	21.41	0.00	0.00	18.62
2008 Bear Stearns Collapse	0.95	0.45	1.52	0.35	0.50	1.76	0.35	0.50	1.76
2008 January	0.95	0.95	4.55	0.25	0.00	4.29	0.10	0.00	4.24
2008 Nov-Mar	0.55	0.90	9.82	0.00	0.00	10.09	0.00	0.00	10.21
2008 Sep-Nov	0.00	0.70	15.06	0.00	0.55	14.54	0.75	0.70	12.97
2009 Jan-Mar	0.00	0.55	10.79	0.50	0.00	9.19	0.55	0.00	9.35

Table 3.12: Scenarios and Optimized 2-Block Exposures

Observations (Table 3.12):

Defining two sub-blocks within the correlation matrix minimizes better, in most of the cases, the sum of errors. The benefit from applying mixed shocks rather than only large negative shock can not be established.

C. With three Sub-Blocks (Largest Negative Return / In Between / Largest Positive Return)

The first block, the "large negative return"-block remains defined as previously explained. We compose, on the same methodology, a "large positive return"-block; the remaining factors are grouped in a third sub-block.

	1 Shock: Neg				2 Shocks: Negative / Positive			
SCENARIOS	Optimized Exposures			Sum Errors	Optimized Exposures			Sum Errors
1997-98 Asian Financial Crisis	0.95	0.00	0.95	24.68	0.95	0.00	0.00	23.67
1998 Russian Financial Crisis	0.95	0.95	0.95	13.08	0.95	0.95	0.95	11.91
2000 Emerging Market Decline	0.85	0.80	0.95	2.49	0.85	0.80	0.95	2.34
2000 Tech Bubble	0.00	0.00	0.95	3.22	0.00	0.00	0.90	3.01
2000-02 Argentina Economic Crisis]	0.00	0.00	0.00	27.37	0.45	0.00	0.95	26.76
2001 DotCom Slowdown	0.95	0.95	0.95	35.23	0.95	0.95	0.00	33.13
2001 Sep 11	0.95	0.75	0.00	3.62	0.95	0.75	0.00	3.48
2003 Iraq War	0.95	0.95	0.95	3.31	0.95	0.95	0.95	3.16
2006 Emerging Market Crash	0.00	0.95	0.95	1.12	0.00	0.85	0.95	1.09
2007-08 Oil Price Rise	0.00	0.00	0.00	21.43	0.00	0.00	0.00	20.07
2007-08 Subprime Mortgage Meltdown	0.00	0.00	0.00	21.43	0.00	0.00	0.60	21.05
2007-09 Subprime Mortgage Meltdown	0.00	0.00	0.95	19.89	0.00	0.00	0.95	19.66
2008 Bear Stearns Collapse	0.95	0.45	0.95	1.52	0.95	0.50	0.65	1.39
2008 January	0.95	0.95	0.95	4.55	0.35	0.90	0.90	3.99
2008 Nov-Mar	0.50	0.90	0.95	9.79	0.35	0.00	0.90	8.50
2008 Sep-Nov	0.00	0.70	0.95	15.03	0.00	0.55	0.00	14.54
2009 Jan-Mar	0.00	0.50	0.95	10.74	0.00	0.45	0.60	9.70

	3 Shocks: Negative / Median / Positive			
SCENARIOS	Optimized Exposures			Sum Errors
1997-98 Asian Financial Crisis	0.95	0.00	0.00	23.64
1998 Russian Financial Crisis	0.95	0.95	0.95	11.37
2000 Emerging Market Decline	0.85	0.80	0.95	2.37
2000 Tech Bubble	0.30	0.00	0.65	3.00
2000-02 Argentina Economic Crisis]	0.75	0.00	0.95	26.49
2001 DotCom Slowdown	0.95	0.95	0.00	36.42
2001 Sep 11	0.95	0.95	0.00	3.39
2003 Iraq War	0.95	0.00	0.95	2.98
2006 Emerging Market Crash	0.75	0.90	0.95	1.08
2007-08 Oil Price Rise	0.45	0.00	0.95	19.36
2007-08 Subprime Mortgage Meltdown	0.70	0.00	0.70	20.22
2007-09 Subprime Mortgage Meltdown	0.60	0.00	0.95	16.89
2008 Bear Stearns Collapse	0.95	0.50	0.70	1.36
2008 January	0.35	0.00	0.90	3.73
2008 Nov-Mar	0.00	0.00	0.95	8.59
2008 Sep-Nov	0.60	0.85	0.00	12.71
2009 Jan-Mar	0.00	0.45	0.05	9.64

Table 3.13: Scenarios and Optimized 3-Block Exposures

Let's have a closer look for comparing the One-block version with the 2 and 3 sub-blocks.

	1 Shock: Neg		2 Shocks: Negative / Positive		3 Shocks: Negative / Median / Positive	
SCENARIOS	Optimized Common Exposure	Sum Errors	Optimized Common Exposure	Sum Errors	Optimized Common Exposure	Sum Errors
1997-98 Asian Financial Crisis	0.00	58.51	0.00	56.58	0.00	56.66
1998 Russian Financial Crisis	0.00	26.07	0.00	24.17	0.00	20.55
2000 Emerging Market Decline	0.00	3.08	0.13	2.93	0.14	2.93
2000 Tech Bubble	0.00	3.22	0.00	3.03	0.00	3.04
2000-02 Argentina Economic Crisis]	0.16	27.27	0.27	27.05	0.21	27.01
2001 DotCom Slowdown	0.00	246.78	0.00	239.71	0.00	248.70
2001 Sep 11	0.38	4.63	0.47	4.23	0.55	4.16
2003 Iraq War	0.41	3.27	0.99	3.33	0.99	3.33
2006 Emerging Market Crash	0.00	1.17	0.99	1.15	0.59	1.14
2007-08 Oil Price Rise	0.00	21.43	0.00	20.07	0.67	19.43
2007-08 Subprime Mortgage Meltdown	0.00	13.76	0.99	11.98	0.99	11.78
2007-09 Subprime Mortgage Meltdown	0.53	18.50	0.00	21.41	0.99	18.28
2008 Bear Stearns Collapse	0.00	2.38	0.00	2.22	0.00	2.23
2008 January	0.00	4.74	0.00	4.32	0.32	4.23
2008 Nov-Mar	0.00	10.01	0.00	10.09	0.00	10.21
2008 Sep-Nov	0.00	10.01	0.00	10.09	0.00	10.21
2009 Jan-Mar	0.00	10.87	0.00	9.72	0.00	9.69

3 Shock: Negative/Median/Positive			3 Shocks: Negative / Median / Positive			
Optimized Exposures		Sum Errors	Optimized Exposures			Sum Errors
0.95	0.00	24.00	0.95	0.00	0.00	23.64
0.40	0.95	12.86	0.95	0.95	0.95	11.37
0.00	0.90	2.47	0.85	0.80	0.95	2.37
0.25	0.00	3.00	0.30	0.00	0.65	3.00
0.00	0.60	27.09	0.75	0.00	0.95	26.49
0.95	0.95	33.23	0.95	0.95	0.00	36.42
0.00	0.95	3.67	0.95	0.95	0.00	3.39
0.00	0.80	3.42	0.95	0.00	0.95	2.98
0.00	0.80	1.14	0.75	0.90	0.95	1.08
0.40	0.00	19.66	0.45	0.00	0.95	19.36
0.35	0.00	11.69	0.70	0.00	0.70	20.22
0.00	0.00	18.62	0.60	0.00	0.95	16.89
0.35	0.50	1.76	0.95	0.50	0.70	1.36
0.10	0.00	4.24	0.35	0.00	0.90	3.73
0.00	0.00	10.21	0.00	0.00	0.95	8.59
0.75	0.70	12.97	0.60	0.85	0.00	12.71
0.55	0.00	9.35	0.00	0.45	0.05	9.64

Table 3.14: Comparing results for the One-block version

<u>Observations (Table 3.13-14):</u>

Comparing with a common exposure approach (which was already an improvement itself for better replicating historical shocks), 2 and 3 sub-block version minimizes the sum of absolute deviation for every scenario (but one: "2008 Nov-Mar")

D. With seven Sub-Blocks (World, Styles, Industries, Dev Countries, Others, Dev Currencies, Others)

The aim with a seven sub-block version is to a block division that would make sense economically speaking. Indeed, we can group the factors by their own inherent characteristics – following MSCI classification – : World factor, Styles factors, Industries, Developed Countries, Emerging and Frontier Countries, Developed country currencies, Emerging and Frontier country currencies.

	1 Shock: Neg							
SCENARIOS	Optimized Exposures							Sum Errors
1997-98 Asian Financial Crisis	1.00	1.00	1.00	1.00	1.00	1.00	0.00	21.56
1998 Russian Financial Crisis	1.00	1.00	1.00	1.00	1.00	1.00	0.00	13.30
2000 Emerging Market Decline	0.80	0.00	0.00	0.60	0.80	1.00	1.00	2.77
2000 Tech Bubble	0.00	0.00	0.00	1.00	0.00	1.00	1.00	3.05
2000-02 Argentina Economic Crisis]	0.00	0.00	0.00	1.00	1.00	0.00	0.20	26.52
2001 DotCom Slowdown	1.00	1.00	1.00	1.00	1.00	1.00	0.00	48.73
2001 Sep 11	1.00	1.00	0.00	1.00	1.00	1.00	1.00	3.87
2003 Iraq War	0.00	0.80	1.00	1.00	0.40	1.00	1.00	3.11
2006 Emerging Market Crash	0.00	1.00	1.00	1.00	0.00	0.00	0.00	1.14
2007-08 Oil Price Rise	0.00	0.00	0.00	1.00	0.00	0.00	0.00	21.16
2007-08 Subprime Mortgage Meltdown	1.00	0.00	0.00	0.40	1.00	0.00	0.00	13.05
2007-09 Subprime Mortgage Meltdown	1.00	0.00	0.00	0.00	0.20	1.00	0.00	19.83
2008 Bear Stearns Collapse	0.20	0.00	1.00	1.00	1.00	1.00	0.00	1.57
2008 January	1.00	0.00	1.00	0.00	0.00	1.00	1.00	4.48
2008 Nov-Mar	1.00	0.00	1.00	0.00	0.00	1.00	0.60	9.57
2008 Sep-Nov	1.00	1.00	0.00	1.00	0.40	1.00	1.00	5.04
2009 Jan-Mar	0.00	0.00	1.00	1.00	1.00	0.00	0.00	9.95

	2 Shocks: Neg / Pos							
SCENARIOS	Optimized Exposures							Sum Errors
1997-98 Asian Financial Crisis	1.00	1.00	1.00	1.00	1.00	1.00	0.00	25.64
1998 Russian Financial Crisis	1.00	1.00	1.00	1.00	0.80	1.00	0.00	13.49
2000 Emerging Market Decline	0.80	0.00	0.00	0.60	0.00	1.00	1.00	2.67
2000 Tech Bubble	0.00	0.00	0.00	0.80	0.40	1.00	1.00	2.84
2000-02 Argentina Economic Crisis]	0.00	0.00	0.00	1.00	0.00	0.00	0.80	26.50
2001 DotCom Slowdown	1.00	1.00	1.00	1.00	1.00	1.00	0.00	49.37
2001 Sep 11	0.00	1.00	0.00	1.00	1.00	1.00	1.00	3.70
2003 Iraq War	1.00	1.00	1.00	1.00	0.80	1.00	1.00	3.06
2006 Emerging Market Crash	1.00	1.00	1.00	0.00	0.80	0.00	1.00	1.12
2007-08 Oil Price Rise	0.00	0.00	0.00	1.00	0.40	0.00	0.00	19.29
2007-08 Subprime Mortgage Meltdown	0.80	0.60	0.00	0.40	0.20	1.00	1.00	11.43
2007-09 Subprime Mortgage Meltdown	1.00	0.00	0.00	0.00	0.20	1.00	1.00	20.18
2008 Bear Stearns Collapse	0.80	0.40	1.00	1.00	1.00	0.60	0.00	1.34
2008 January	0.00	0.00	0.00	0.00	0.00	0.00	1.00	4.32
2008 Nov-Mar	0.80	1.00	0.40	0.40	0.00	1.00	1.00	8.47
2008 Sep-Nov	0.00	0.40	0.80	0.00	0.80	1.00	0.00	6.60
2009 Jan-Mar	0.00	0.00	1.00	0.00	0.80	0.00	0.00	8.20

	3 Shocks: Neg / Median / Pos							
SCENARIOS	Optimized Exposures							Sum Errors
1997-98 Asian Financial Crisis	0.00	0.00	1.00	0.00	0.00	0.00	1.00	20.63
1998 Russian Financial Crisis	1.00	1.00	1.00	1.00	0.80	0.00	0.00	11.48
2000 Emerging Market Decline	0.80	0.00	0.00	0.60	0.00	1.00	1.00	2.68
2000 Tech Bubble	0.00	1.00	0.00	0.80	0.40	1.00	1.00	2.86
2000-02 Argentina Economic Crisis]	0.00	0.60	0.00	1.00	0.00	0.40	0.80	26.39
2001 DotCom Slowdown	1.00	1.00	1.00	1.00	1.00	1.00	0.00	49.34
2001 Sep 11	0.00	0.00	0.20	1.00	1.00	1.00	1.00	3.60
2003 Iraq War	1.00	1.00	1.00	1.00	0.80	1.00	1.00	3.06
2006 Emerging Market Crash	1.00	1.00	1.00	0.00	0.80	0.00	0.00	1.11
2007-08 Oil Price Rise	0.00	1.00	0.00	1.00	0.40	0.00	0.00	19.38
2007-08 Subprime Mortgage Meltdown	0.80	0.60	0.00	0.40	0.20	1.00	1.00	10.81
2007-09 Subprime Mortgage Meltdown	1.00	0.00	0.00	0.00	0.20	0.00	0.00	18.56
2008 Bear Stearns Collapse	0.80	0.40	1.00	1.00	1.00	0.60	0.00	1.33
2008 January	1.00	1.00	0.00	0.00	0.00	0.60	0.40	4.11
2008 Nov-Mar	0.80	1.00	0.40	0.60	0.20	1.00	0.00	8.89
2008 Sep-Nov	1.00	1.00	0.00	1.00	1.00	0.00	0.00	5.25
2009 Jan-Mar	0.00	0.00	1.00	0.00	0.80	0.00	0.00	8.20

Table 3.15: Scenarios and Optimized 7-Block Exposures

Observations (Table 3.15):

Comparing these results with the previous, we don't find any evidence of motivation to prefer using the 7-Block version rather than 3-Block one. However, it seems very likely that applying Latent Factor technique improves the accuracy of the historical shock replication (in our multivariate normal framework).

Moreover, it seems unlikely to be able to replicate, using a long half-life covariance matrix, every historical factor shock from a single shock. The 3-shock vector is likely to help providing better results in most of the listed historical scenarios. Another result from these tests is the following: the accuracy of the replication is improved by actually obtaining exposures with "extreme" value (0 or 1, in many cases), resulting in more 0-value replicated shocks. Therefore, over a specified scenario period, the distribution of the factor returns can't be fully replicated. It may be explained by the 0-correlation across blocks that we made (as we have no clear views about the correlations between the latent factor exposures).

E. Revisiting the Aug-Nov 08 Scenario

In this section, we will re-visit the Aug-Nov 08 scenario, by using EWMA shorter half-life covariance matrix to assess risk contagion and capture the latest co-movements between assets, prior to the crisis.

1. *With a Shorter Half-Life Covariance Matrix*

	US	**JP**	**EMU**
US	100.00%	1.02%	42.88%
JP	1.02%	100.00%	20.96%
EMU	42.88%	20.96%	100.00%

Table 4.1: EWMA Correlation matrix (21-day half-life) as of July 31st, 2008

1/ Shock on MSCI USA: -25%

- *Common exposure = 0* *(Original correlation matrix)*

	Replicated Shocks	Historical Shocks
US	-25.000%	-24.85%
JP	-0.285%	-29.92%
EMU	-11.200%	-23.40%

Exposure	**0**	US	JP	EMU
	US	100.00%	1.02%	42.88%
	JP	1.02%	100.00%	20.96%
	EMU	42.88%	20.96%	100.00%

Table 4.2: Shock Replication, starting from MSCI USA historical shock using a zero-value exposure and the covariance matrix (21-day half-life) as of July 31st, 2008

- *Common exposure = 0.4* *(new correlation matrix)*

	Replicated Shocks	Historical Shocks
US	**-25.000%**	-24.85%
JP	-0.285%	-29.92%
EMU	-11.200%	-23.40%

Exposure	**0.4**	**US**	**JP**	**EMU**
	US	100.00%	1.02%	42.88%
	JP	1.02%	100.00%	20.96%
	EMU	42.88%	20.96%	100.00%

Table 4.3: Shock Replication, starting from MSCI USA historical shock using a 0.4-value exposure and the covariance matrix (21-day half-life) as of July 31st, 2008

- *Common exposure = 0.8* *(new correlation matrix)*

	Replicated Shocks	Historical Shocks
US	**-25.000%**	-24.85%
JP	-18.076%	-29.92%
EMU	-20.747%	-23.40%

Exposure	**0.8**	**US**	**JP**	**EMU**
	US	100.00%	64.37%	79.44%
	JP	64.37%	100.00%	71.55%
	EMU	79.44%	71.55%	100.00%

Table 4.3: Shock Replication, starting from MSCI USA historical shock using a 0.8-value exposure and the covariance matrix (21-day half-life) as of July 31st, 2008

- *Common exposure = 0.95* *(new correlation matrix)*

	Replicated Shocks	Historical Shocks
US	**-25.000%**	-24.85%
JP	-18.076%	-29.92%
EMU	-20.747%	-23.40%

Exposure	**0.95**	**US**	**JP**	**EMU**
	US	100.00%	64.37%	79.44%
	JP	64.37%	100.00%	71.55%
	EMU	79.44%	71.55%	100.00%

Table 4.4: Shock Replication, starting from MSCI USA historical shock using a 0.95-value exposure and the covariance matrix (21-day half-life) as of July 31st, 2008

<u>Observations (Table 4.2-3-4)</u>:

Using a shorter half-life doesn't result in more accurate replications. The shock on MSCI Japan is under-replicated by almost 5%.

2/ Shock on MSCI JP: -25%

- *Common exposure = 0* *(Original correlation matrix)*

	Replicated Shocks	Historical Shocks
US	-0.226%	-24.85%
JP	**-25.000%**	-29.92%
EMU	-4.874%	-23.40%

Exposure	**0**	**US**	**JP**	**EMU**
	US	100.00%	1.02%	42.88%
	JP	1.02%	100.00%	20.96%
	EMU	42.88%	20.96%	100.00%

Table 4.5: Shock Replication, starting from MSCI Japan historical shock using a 0-value exposure and the covariance matrix (21-day half-life) as of July 31st, 2008

- *Common exposure = 1* *(new correlation matrix)*

	Replicated Shocks	Historical Shocks
US	-22.255%	-24.85%
JP	**-25.000%**	-29.92%
EMU	-23.251%	-23.40%

Exposure	**1**	**US**	**JP**	**EMU**
	US	100.00%	100.00%	100.00%
	JP	100.00%	100.00%	100.00%
	EMU	100.00%	100.00%	100.00%

Table 4.6: Shock Replication, starting from MSCI Japan historical shock using a 1-value exposure and the covariance matrix (21-day half-life) as of July 31st, 2008

Observations (Table 5-6):

Using a short half-life covariance matrix results in better replication than with a long half-life one. However, it must be pointed out that the actual historical shock for MSCI Japan is -30%. Then, using the 21-day-half life covariance matrix provides an acceptable replication of the historical shocks. EMU shock is over-replicated by 2%.

	Replicated Shocks	Historical Shocks
US	-24.129%	-24.85%
JP	**-30.000%**	-29.92%
EMU	-25.751%	-23.40%

Exposure	**0.95**	**US**	**JP**	**EMU**
	US	100.00%	90.35%	94.43%
	JP	90.35%	100.00%	92.29%
	EMU	94.43%	92.29%	100.00%

Table 4.7: Shock Replication, starting from MSCI Japan historical shock using a 0.95-value exposure and the covariance matrix (21-day half-life) as of July 31st, 2008

3/ Shock on MSCI EMU: -25%

- *Common exposure = 0* *(Original correlation matrix)*

	Replicated Shocks	Historical Shocks
US	-10.262%	-24.85%
JP	-5.635%	-29.92%
EMU	**-25.000%**	-23.40%

Exposure 0	**US**	**JP**	**EMU**
US	100.00%	1.02%	42.88%
JP	1.02%	100.00%	20.96%
EMU	42.88%	20.96%	100.00%

Table 4.8: Shock Replication, starting from MSCI EMU historical shock using a 0-value exposure and the covariance matrix (21-day half-life) as of July 31st, 2008

- *Common exposure = 1* *(New correlation matrix)*

	Replicated Shocks	Historical Shocks
US	-23.930%	-24.85%
JP	-26.881%	-29.92%
EMU	**-25.000%**	-23.40%

Exposure 1	**US**	**JP**	**EMU**
US	100.00%	100.00%	100.00%
JP	100.00%	100.00%	100.00%
EMU	100.00%	100.00%	100.00%

Table 4.9: Shock Replication, starting from MSCI EMU historical shock using a 1-value exposure and the covariance matrix (21-day half-life) as of July 31st, 2008

Observations (Table 8-9):

This good replication, starting from MSCI EMU historical shock, leads to an extreme exposure. The MSCI Japan replicated is under-estimated by 4%.

With the common exposure to a latent driver, and using a short-half covariance matrix that allows to capture the latest co-movements across assets, we obtain acceptable shock replications. However, sub-block version should lead to better results.

2. *With a 2-Block Version and a Correlation between Latent Drivers*

In the following test, we will focus on the correlation between latent drivers. This parameter $\boldsymbol{\rho_{r_1 r_2}}$ is mandatory to preserve consistency across blocks (cf. equation (1)). $\boldsymbol{\rho_{r_1 r_2}}$ was supposed equal to 0, as we don't have clear views about on the correlation pairs. With 2-block versions and $\boldsymbol{\rho_{r_1 r_2}} = 0$, we present below the best replications:

A) [USA] vs. [Japan;EMU]

	Replicated Shocks	Historical Shocks	# Block
US	**-25.00%**	-24.85%	1
JP	-0.29%	-29.92%	2
EMU	-11.20%	-23.40%	2
Block 1 exposure	0		
Block 2 exposure	0		

Table 4.9: Shock Replication, starting from MSCI US historical shock using a 2-Block version and the covariance matrix (21-day half-life) as of July 31st, 2008

B) [USA;EMU] vs. [Japan]

	Replicated Shocks	Historical Shocks	# Block
US	**-25.00%**	-24.85%	1
JP	-0.12%	-29.92%	2
EMU	-23.28%	-23.40%	1
Block 1 exposure	0.9		
Block 2 exposure	0		

Table 4.0: Shock Replication, starting from MSCI US historical shock using a 2-Block version and the covariance matrix (21-day half-life) as of July 31st, 2008

C) [USA;Japan] vs. [EMU]

	Replicated Shocks	Historical Shocks	# Block
US	**-25.00%**	-24.85%	1
JP	-28.08%	-29.92%	2
EMU	0.00%	-23.40%	2

Block 1 exposure 1

Block 2 exposure 0

Table 4.11: Shock Replication, starting from MSCI US historical shock using a 2-Block version and the covariance matrix (21-day half-life) as of July 31st, 2008

<u>Observations (Table 4.9-10-11)</u>:

With $\boldsymbol{\rho_{r_1 r_2}} = 0$, it seems unlikely to be able to spread the shock from one block to another one. Applying the shock on MSCI Japan or EMU won't change this as long as we keep $\boldsymbol{\rho_{r_1 r_2}} = 0$.

With a 2-block version and $\boldsymbol{\rho_{r_1 r_2}} \neq 0$, we obtain the best results with both exposures and correlation parameter close to 1 (therefore, we are close to the common exposure framework). Nevertheless, the benefit is to adjust specific replicated shock(s) by changing the block exposures. Below are the parameters for the best replications:

<u>MSCI US Shock: -25%</u> $\boldsymbol{\rho_{r_1 r_2} = 1}$

	Replicated Shocks	Historical Shocks	# Block
US	**-25.00%**	-24.85%	1
JP	-28.08%	-29.92%	1
EMU	-23.51%	-23.40%	2

Block 1 exposure 1

Block 2 exposure 0.9

Table 4.12 Shock Replication, starting from MSCI US historical shock using a 2-Block version and the covariance matrix (21-day half-life) as of July 31st, 2008

MSCI JP Shock: -30% $\rho_{r_1 r_2} = 0.9$

	Replicated Shocks	Historical Shocks	# Block
US	-24.63%	-24.85%	1
JP	**-30.00%**	-29.92%	1
EMU	-23.41%	-23.40%	2

Block 1 exposure	0.96
Block 2 exposure	0.96

Table 4.13: Optimized Shock Replication, starting from MSCI EMU historical shock using a 2-Block version and the covariance matrix (21-day half-life) as of July 31st, 2008

MSCI EMU Shock: -25% $\rho_{r_1 r_2} = 1$

	Replicated Shocks	Historical Shocks	# Block
US	-23.93%	-24.85%	1
JP	-26.88%	-29.92%	1
EMU	**-25.00%**	-23.40%	2

Block 1 exposure	1
Block 2 exposure	1

Table 4.14: Optimized Shock Replication, starting from MSCI EMU historical shock using a 2-Block version and the covariance matrix (21-day half-life) as of July 31st, 2008

This simple analysis highlights that, from a single shock, an acceptable replication of an historical scenario can lead to different parameters (one block vs. sub-blocks, 0-correlation between blocks vs. non-null correlation between latent drivers, etc.).

Our instinctive sub-block organization would have been the following: [USA; EMU] vs. [Japan] (cf. our remark on the different economies). However, [USA; Japan] vs. [EMU] has provided the best replications. It seems likely that the block division would change, given the historical scenario.

This last analysis opens up new prospects for stress testing techniques.

V. Next Steps

A. Refining Examples

We want to refine the previous examples in order to:

- Find a consistent and simple way to define sub-blocks to replicate accurately historical shocks.
- Improve the replication of the shocks
- Validate our approach
- Building an optimizer (GPU coding technique?) to find the best exposures and across-block correlations.

B. Motivating examples for practitioners

One next step is to demonstrate the usefulness of the shocking correlations using specific sub-blocks and the accuracy of the replicated shocks. Practitioners should be interested in seeing this technique applied to very recent/current concerns (European Sovereign Debt …).

C. Questions

- A limitation of the accuracy of the replication might be because of the numerous risk factors of GEM2 multi-factor risk model.
- By definition, using GEM2, the specific risks are assumed to be uncorrelated. How do their correlations actually evolved during major crises? Could we make these specific risks be correlated, using Latent Factor approach?

References

Alexander, C., Sheedy, E., 2008. Developing a Stress Testing Framework Based on Market Risk Models.

J. Bender, J.-H. Lee, D. Stefek, 2010, Manipulation Correlations through Latent Drivers, MSCI Barra Research Insights

Berkowitz, J., 2000. A Coherent Framework for Stress Testing.

Campbell, R., Koedijk, K., Kofman P., 2002. Increased correlation in bear markets: a downside risk perspective.

Kupiec, Paul, 1998. Stress Testing in a Value at Risk Framework. Journal of Derivatives, Fall 1998: 7-24.

Longin, F., Solnik, B., 2001. Extreme Correlation of International Equity Markets.

MSCI Inc., 2010 Barra Extreme Risk Analytics Guide version 1.0

J. Menchero, A. Morozov, P. Shepard, 2008, The Barra Global Equity Model (GEM2) - Research Notes –