RiskMetrics

An MSCI Brand

Research Technical Note

Swaptions

November 26, 2012

Keywords: European Swaption, Bermudan Swaption, SABR, Hull-White, BDT.

Contents

1 Introduction

This document is an overview of swaption pricing in RiskManager. The simplest and most-widely used pricing methodology (Black model) is covered in detail, while more advanced pricing models are introduced and references are provided to more detailed documents.

A swaption is an option to enter into an interest rate swap at a certain time in the future. In a *Payer Swaption* the option holder would pay a fixed rate and receive a floating rate. In a *Receiver Swaption* the option holder would pay a floating rate and receive a fixed rate.

A basic (vanilla) swaption would exhibit the following terms and conditions:

- A single exercise date (the option expiry date)
- Option expiration date coinciding with the swap start date
- Floating payments based on a simple term rate (LIBOR), reset in-advance
- Uniform fixed payments occuring at regular intervals

Additional features can include

- Swap start date later than option expiration date (option on forward swap)
- Floating payments based on LIBOR plus a constant spread, or a spread that varies across payment dates (spread schedule)
- Fixed payments of the underlying swap occuring at arbitrary dates and varying by coupon date
- Interest payments based on a variable notional amount (option on amortizing swap)
- Multiple exercise dates that occur throughout the life of the swap (Bermudan swaption).
- Multiple exercise dates, each corresponding to a swap with a fixed term and thus different maturity dates (Constant-maturity Bermudan Swaption).

Swaptions in RiskManager can be priced using a number of different models, including:

- Black price-based (PB)
- Black yield-based (YB)
- Hull-White One-Factor (HW)
- SABR
- Black-Derman-Toy (BDT)

Supported swaption types depend on the model chosen, and are summarized in the following table:

Supported Swaption Features by Pricing Model:

	PB	YB	SABR	BDT	HW
Vanilla	✓	✓	✓	✓	✓
Option on forward swap	✓	✓	✓	✓	✓
Constant spread	✓	✓	✓	✓	✓
Spread schedule				✓	✓
Fixed coupon schedule	✓			✓	
Bermudan (fixed swap maturity)				✓	✓
Constant-maturity Bermudan				✓	
Amortizing				✓	✓
Multi-curve pricing	✓				

2 Definitions

RiskManager Fields:

	Name	Spec Variable	Required?	Default	Example
1.	Notional	N	Yes	-	-2000000
2.	Currency	-	Yes	-	EUR
3.	Fixed Leg				
(a)	Coupon Frequency	-	No	semiannual	monthly
(b)	First Coupon Date	-	No	-	1/5/2007
(c)	Coupon Schedule	-	No	-	-
4.	Float Leg				
(a)	Coupon Frequency	-	No	semiannual	quarterly
(b)	Reference Curve	-	Yes	-	EUR Swap
(c)	Reset Spread	-	No	-	2.1
(d)	Reset Spread Schedule	-	No	-	-
5.	useMultiCurvePricing	-	No	-	-
6.	Discount Curve	-	No	-	USD Govt
7.	Swaption Type	-	No	Payer	Receiver
8.	Exercise Type	-	No	European	Bermudan
9.	Option Expiry Date	t_e	Yes	-	9/5/2006
10.	Swap Start Date	t_s	No	Option Expiry	10/5/2006
11.	Swap Maturity Date	t_m	Yes	-	10/5/2016
12.	Underlying Swap Rate	S	No	0	4.5
13.	Strike Rate	X	Yes	-	4
14.	Market Price	PV	No	-	2.17
15.	Volatility	σ_s	No	-	20
16.	Volatility Series	-	No	-	EU.EUR.ISO
17.	First Exercise Date	-	-	-	10/5/2006
18.	Exercise Frequency	-	-	-	quarterly
19.	Exercise Term	-	-	-	3 months
20.	Swap Type	-	No	variableMaturity	constantMaturity
21.	Swap Term	T	No	-	10 Years
22.	Nominal Schedule List	-	No	-	-

1. **Notional** - Notional amount of underlying swap, in units of specified Currency, upon which interest rates and principal return are based. A positive value indicates a long option position while a negative value indicates a short option position. Must be a non-zero number.

2. **Currency** - Currency of underlying swap. Must be a currency code that matches one currently defined in the Market Data database. The default discount curve is the government curve associated with the Currency. This can be overridden with the discountCurveIsReferenceCurve tag.

3. **Fixed leg**

   (a) **Coupon Frequency** - The frequency at which swap payments are made. As with swaps, the floating and fixed legs can have different coupon frequencies. Defaults to semiannual.

   (b) **First Coupon Date** - The date on which the first fixed coupon payment is made.

   (c) **Coupon Schedule** - List of date/coupon pairs specifying fixed coupon payment dates and amounts. This input takes precedence over Coupon Frequency and First Coupon Date, and is only allowed for Black price-based model.

4. **Float leg**

   (a) **Coupon Frequency** - The frequency at which swap payments are made. As with swaps, the floating and fixed legs can have different coupon frequencies. Defaults to semiannual.

   (b) **Reference Curve** - Yield curve upon which the floating leg coupons are based. Must be a yield curve name that matches one currently defined in the Market Data database.

   (c) **Reset Spread** - Additional coupon interest to be paid, in basis points, above (positive) or below (negative) reference rate on fixing dates for underlying swap. If left blank, default is zero.

   (d) **Reset Spread Schedule** - List of dates and spreads paid above floating rate. This feature is supported under the BDT pricing model and the Hull-White pricing model. The interpretation of this input differs depending on which pricing model is used. For BDT, this specifies spreads that apply only to the specified dates; other coupon dates are taken to have no spread applied. In contrast, for HW, the specified spreads are filled forward to cover all coupon dates, and the earliest spread is filled backward. For example, if only a single spread is specified, this spread is taken to apply to all coupon payments. If two spreads are specified, the first spread applies to all coupon payments on or before the first date, and all coupon payents up to but not including the second date; the second spread applies to all remaining coupon payments.

5. **Swaption Type** - Payer or Receiver.

6. **Exercise Type** - European or Bermudan.

7. **Option Expiry Date** The exercise date for European swaptions. For Bermudan swaptions, exercise dates start at the First Exercise Date, and occur at the specified Exercise Frequency or Exercise Term up to and including the Option Expiry Date. (If the Option Expiry Date falls between regular exercise dates, the Option Expiry Date is not an exercise date.)

8. **Swap Start Date** - The date on which the swap underlying this option starts. i.e. when interest starts accruing for the first coupon payment. Must be equal to or later than the expiry date for a European swaption. Defaults to the option expiry date.

9. **Swap Maturity Date** - Last payment date of underlying swap. Standard coupon schedules are backed out from the maturity date using coupon frequencies.

10. **Underlying Swap Rate** - Latest closing swap rate for underlying swap, as an annualized percentage. If specified, RM calibrates its risk model to match this rate by applying a parallel shift to the reference curve. If left blank, current swap rate is computed from appropriate historical time series. This is always interpreted as a swap rate, even if the Black price-based model is used. If the swaption exercise type is Bermudan, this input is ignored. See Section 4.1 for details.

11. **Strike Rate** - Strike rate of swaption, as an annual percentage, that represents the fixed leg coupon of underlying swap.

12. **Market Price** - Latest closing market price (premium) of the option, as a percentage of Notional. If specified, RiskServer calibrates its risk model to match this price as described in Section 4. If left blank, current market price is computed from appropriate historical time series.

13. **Volatility** - Implied volatility of the underlying swap rate, as an annualized percentage. Only applies to Black model. In the Black priced-based model, this volatility is converted to a price volatility. If left blank, volatility is computed from appropriate historical time series.

14. **Volatility Series** - Implied volatility time-series for Black model (swaption implied volatility, e.g. US.-USD.ISO), or cap implied volatility time-series for BDT model (e.g. US.USD.ISC). See [9] for details on how the swaption implied volatility is calculated from risk factors. Volatility series for other pricing models appear under the pricingModel tag.

15. **Pricing Model Type** - specifies pricing model and associated inputs. See Section 2.1 for details.

16. **First Exercise Date** - Only applies to Bermudan swaptions, in which case it is required.

17. **Exercise Frequency** - Frequency of exercise opportunities; only applies to Bermudan swaptions.

18. **Exercise Term** - Number of months between exercise opportunities; only applies to Bermudan exercise, in which case it overrides Exercise Frequency.

19. **Swap Type** - Variable maturity or constant maturity swap. Defaults to variable maturity.

20. **Swap Term** - The swap term in a constant maturity swap, in months, quarters, or years. Not used for variable maturity swap.

21. **Nominal Schedule List** - List of dates and notional amounts for an amortizing swap.

2.1 Pricing Model Specification

The Pricing Model input is a complex tag allowing the choice of one of the following model selections:

	Name	Spec Variable	Required	Default	Example
1.	Hull-White One-Factor Model		No	-	
(a)	Mean-Reversion Rate	κ	No	-	0.01
(b)	Short-Rate Volatility	σ_{HW}	No	-	1.2
(c)	Volatility Series		No	-	US.USD.ISC
2.	Price-based	-	No	-	-
3.	Yield-based	-	No	-	-
4.	SABR Model				
(a)	Beta	β	No	0.5	0.2
(b)	SABR formula	-	No	Hagan	Obłój
(c)	Interest rate interpolation	-	No	Cubic spline	Linear
(d)	Skew data	-	Yes	-	USD Swaption Grid
(e)	ATM data	-	No	Skew data	US.USD.ISO
(f)	Rho	ρ	No	-	0.1
(g)	VolVol	ν	No	-	0.2
5.	BDT				

1. **Hull-White One-Factor Model** - Specifies that Hull-White one factor model should be used for pricing and specifies its parameters as follows.
   (a) **Mean-Reversion Rate** - Hull-White One-Factor model mean-reversion rate parameter. If not specified, it is calibrated from market data.
   (b) **Short-rate Volatility** - Hull-White One-Factor model annualized volatility of the short-rate parameter, specified as a percentage. If not specified, it is calibrated from market data.
   (c) **Volatility Series** - The name of the volatility time series to be used to calibrate the Hull-White model parameters. This can be either a Cap or a Swaption volatility time-series and must be a name that matches one currently defined in the Market Data database. If not specified, model parameters are calibrated by using the historical time-series of the Reference Curve as described in [7].
2. **Price-Based** - Specifies that a price-based Black model should be used for pricing.
3. **Yield-Based** - Specifies that yield-based Black model should be used for pricing.

4. **SABR Model**

   (a) **Beta** - SABR model parameter that specifies the type of underlying process that is used in the model; acceptable values are between 0 and 1. $\beta = 0$ corresponds to a normal process, $\beta = 1$ corresponds to a lognormal process, and $\beta = 1/2$ corresponds to a square-root process.

   (b) **SABR formula** - The analytical approximation used to price European options under the SABR model. This formula is used for both calibration of the SABR parameters and for pricing. The approximation is expressed as a Black implied volatility. Choices are *Hagan*, *Obłój*, and *Normal*. *Hagan* and *Obłój* give lognormal Black implied volatility that is used in the Black lognormal pricing formula. *Normal* gives a normal Black implied volatility that is used in the Black normal pricing formula.[1] The default is *Hagan*.

   (c) **Interest rate interpolation** - The technique used to interpolate between discrete nodes on the yield curve; default is cubic spline, and the alternative is linear.

   (d) **Skew data** - Identifies the volatility skew data that the SABR model should be calibrated to. Should correspond to the underlying rate or be a reasonable proxy. The format of the identifier is *USD Interest Rate Cap*, *EUR Swaption Grid*, where USD and EUR can be replaced by other currency codes.

   (e) **ATM Volatility Series For Calibration** - Identifies the ATM volatility data the SABR model should be calibrated to. Users can specify the ATM component of the skew data, or a separate pure ATM volatility data source. If not specified, the ATM component of the skew data is used. The format of the identifier for pure ATM volatility is *EU.EUR.ISC* for caps/floors, *US.USD.ISO* for swaptions, where USD and EUR can be replaced by other currency codes. Note that the pure ATM volatility data typically has a much longer history than the skew data, so pure ATM volatility data is more appropriate when a long historical window is required.

   (f) **Rho** - SABR parameter defining the correlation between the volatility and the underlying rate. Must be between -1 and 1. If *rho* is specified, *volVol* must also be specified. If not specified, these parameters are calibrated to skew data. This value of *rho* is used for each point on the swaption grid.

   (g) **VolVol** - SABR parameter defining the annualized volatility of volatility, specified as an absolute number (not a percentage). Must be greater than 0. If *volVol* is specified, *rho* must also be specified. If not specified, these parameters are calibrated to skew data. This value of *volVol* is used for each point on the swaption grid.

5. **BDT** - Specifies that Black-Derman-Toy model should be used for pricing.

[1] The skew data that is available for calibration is given in terms of Black lognormal implied volatility, which cannot directly be used with the *Normal* pricing formula. Therefore, even when *Normal* is chosen, the calibration is still done using lognormal volatilities and the *Hagan* formula.

3 Pricing Methodology

A European Swaption gives the holder the right to enter into an interest rate swap at a certain time in the future. The swaption holder will choose to exercise the option only if the value of the swap is greater than 0.

The payoff function *PO* of the European swaption at option expiry is:

$$\begin{aligned} PO &= \max\left[\text{Swap Value, } 0\right] \\ &= \max\left[B_{float} - B_{fixed}, 0\right] \end{aligned} \tag{1}$$

where B_{float} is the price of a floating coupon bond, and B_{fixed} is the price of a fixed coupon bond. The terms and conditions of these two bonds are part of the swap contract specification.

In the remainder of this section we describe the different pricing models that can be used to price swaptions.

3.1 Black Model

The pricing methodology is based on Black's approach [1]. The *price-based* approach assumes that the fixed coupon bond price follows Geometric Brownian Motion (GBM). The *yield-based* approach assumes that forward swap rate follows GBM. The Black price-based approach is the default for European swaptions. These two approaches are described in the remainder of this section.

3.1.1 Price-Based Model

A feature of a standard FRN is that its present value is always equal to its notional value, regardless of interest rate levels, as long as the floating reference rate is the same as the discount rate.[2] This means that the payoff of the swaption in Eq. 1 can be rewritten in this case as

$$PO = N \cdot \max\left[1 - B_{fixed}, 0\right] \tag{2}$$

where N is the swaption notional and B_{fixed} is the value of the fixed coupon bond with unit notional. This payoff is equivalent to the payoff of a put option on the fixed-coupon bond, with strike equal to 1.

If we assume that the price of the fixed-coupon bond in both cases follows Geometric Brownian Motion, then the value of the swaption is given by the Black equation.

Let t be the analysis date and t_e the option expiry date. Further, let $D(t,t_e)$ be the discount factor from t to t_e, and $B_{fixed}(t_e)$ be the expected value of the (unit notional) fixed-coupon bond on the option expiry date.

[2] If the discount rate is equal to the reference rate plus a constant spread, the PV is equal to the notional times 1 plus the discounted spread payments. We neglect this correction here, although it can be applied manually by the user by specifying a Reset Spread equal to the discount curve spread.

The present value PV of the swaption given by the Black equation is then:

$$PV = N \cdot D(t,t_e) \cdot \left[\Phi(-d_2) - B_{fixed}(t_e) \cdot \Phi(-d_1)\right] \tag{3}$$

where $\Phi(\cdot)$ is the normal cumulative distribution function, and

$$\begin{aligned} d_1 &= \frac{\ln\left(B_{fixed}(t_e)\right) + \frac{\sigma_P^2}{2}(t_e - t)}{\sigma_P \sqrt{t_e - t}} \\ d_2 &= d_1 - \sigma_P \sqrt{t_e - t} \end{aligned} \tag{4}$$

Here σ_P is the volatility of the bond price. This price volatility is computed from yield volatility σ_y using the following approximation:

$$\sigma_P \approx D \times y \times \sigma_y \tag{5}$$

where y is the yield-to-maturity of the fixed-coupon bond, and D is the Macaulay duration.[3] The conversion factor $D \times y$ is computed in the base scenario and held constant in each simulated scenario.

The price-based approach allows for great flexibility in the terms and conditions of the fixed coupon leg of the underlying swap. Furthermore, any spread above the floating rate can be treated as negative coupon payments on the fixed-coupon leg. See Appendix A for details on how the coupon schedule is computed.

For an option on a forward swap, the only modification is to discount the expected payoff from option expiry to swap start:

$$PO = N \cdot D(t_e, t_s) \cdot \max\left[1 - B_{fixed}, 0\right] \tag{6}$$

3.1.2 Yield-Based Model

If the fixed-coupon bond pays a coupon rate X on each fixed-coupon payment date, we can write the value of the underlying swap at time t as follows:

$$\begin{aligned} \text{Swap Value} &= B_{float} - B_{fixed} \\ &= 1 - \left(X \sum_{i=1}^{N} \tau_i \cdot D(t, T_i) + D(t, T_N) \right) \end{aligned} \tag{7}$$

where $D(t,T)$ is the discount factor from t to T, T_i are fixed-coupon payment dates, and $\tau_i = T_i - T_{i-1}$. Define the *par swap rate*

$$S = \frac{1 - D(t, T_m)}{\sum_{i=1}^{N} \tau_i \cdot D(t, T_i)} \tag{8}$$

[3] This is the standard transformation from yield volatility to price volatility described in e.g. [4]. In this case our starting point is swap rate volatility, so this formula involves an additional assumption that the swap rate is proportional to yield.

as the fixed-coupon rate that gives the swap zero value. Then we can rewrite the swap value as

$$\text{Swap Value} = (S-X)\sum_{i=1}^{N}\tau_i \cdot D(t,T_i) \tag{9}$$

$$\equiv A\cdot(S-X) \tag{10}$$

where we have defined the *annuity factor A* as the sum of discount factors. This give the payoff function

$$PO = A\cdot\max\left[S-X,0\right] \tag{11}$$

which is proportional to the payoff of a call option on the swap rate S, with strike X. If we assume that the swap rate follows Geometric Brownian Motion, then the value of the swaption is given by the Black equation.

Let t be the analysis date and t_e the option expiry date. The present value PV of the swaption given by the Black equation is then:

$$PV = A\cdot N\left[S\cdot\Phi(d_1) - X\cdot\Phi(d_2)\right] \tag{12}$$

where S and X are expressed as annual percentages, $\Phi(\cdot)$ is the normal cumulative distribution function, and

$$\begin{aligned} d_1 &= \frac{\ln\left(\frac{S}{X}\right)+\frac{\sigma_s^2}{2}(t_e-t)}{\sigma_s\sqrt{t_e-t}} \\ d_2 &= d_1 - \sigma_s\sqrt{t_e-t} \end{aligned} \tag{13}$$

The volatility σ_s is the implied volatility of the swap rate, which is directly observable from the market price of swaptions.

If there is a constant spread s above the floating rate, the swap value becomes

$$\text{Swap Value} = 1 + s\sum_{j=1}^{M}\tau_j\cdot D(t,T_j) - \left(X\sum_{i=1}^{N}D(t,T_i) + D(t,T_N)\right) \tag{14}$$

where T_j are the floating payment dates. The par swap rate becomes

$$S = \frac{1 + s\sum_{j=1}^{M}\tau_j\cdot D(t,T_j) - D(t,T_m)}{\sum_{i=1}^{N}D(t,T_i)} \tag{15}$$

3.2 Black-Derman-Toy (BDT) Model

The Black-Derman-Toy model is a stochastic short rate model with deterministic volatility, which also allows for mean-reversion. It is especially straightforward to calibrate the BDT model and price some interest rate derivatives using a binomial tree. The BDT model is the default pricing model for Bermudan swaptions.

Details on the pricing of swaptions using the BDT model are provided in [6].

3.3 SABR Model

SABR model can be considered a generalization of the Black yield-based model which specifies a more general stochastic process on the underlying rates [3]. Depending on the parameter values, the rate process can be normal, lognormal, a square-root (CIR) process, or others. In addition, the volatility is allowed to be stochastic and correlated with the underlying.

An advantage of the SABR model is that it accounts for different Black implied volatilities across different strikes (the *volatility skew*). In addition, SABR provides a deterministic relationship between the underlying and Black implied volatility that is consistent with the chosen stochastic process. Details on SABR pricing in RiskManager are provided in [8].

3.4 Hull-White One-Factor Model

The Hull-White (HW) One-Factor model is a mean-reverting short rate model, meaning that the dynamics of the entire term structure of interest rates is modeled using a single stochastic short-rate (see [5] for more details on the Hull-White model). The HW model is particularly useful during low interest-rate environments, because it is a model of absolute interest rate changes (a *normal* process), so it can accommodate negative interest rate scenarios. In contrast, many other models[4] are models of relative interest rate changes (a *lognormal* process), so do not naturally handle negative interest rates.

The Hull-White (HW) model is specified by two parameters: the mean-reversion rate κ (the rate at which the underlying process reverts to its long-term mean), and the volatility of the short-rate σ_{HW}. The HW parameters are generally calibrated to one of three types of market data:

- Swaption implied volatility
- Cap implied volatility
- Historical time-series of the reference curve

The implied volatility is specified through the Volatility Series under the Hull-White model input. If no Volatility Series is specified, the Reference Curve is used for historical calibration. These calibration procedures are detailed in [7]. In addition, either of these parameters can be user-specified under the Hull-White model input.

[4]For example, the Black yield-based model, the Black-Derman-Toy (BDT) model, and many common variations on the SABR model

The Hull-White mean-reversion parameter (κ) is always static (not a risk factor). If κ is specified, this κ is used and only σ_{HW} is calibrated to market data. Otherwise, κ is calibrated in the base scenario and set to this value in each simulated scenario. On the other hand, when a Volatility Series under the Hull-White model input is specified, volatility is a risk factor. The simulated implied volatilities are not used directly as in the Black case, but trigger re-calibration of the Hull-White short-rate volatility in each scenario. Unlike in the Black case, the entire market implied volatility grid contributes to the vega risk of instruments priced using Hull-White model.

When using Hull-White model, a volatility model parameter stress corresponds to shifting the Hull-White short-rate volatility σ_{HW} directly. Likewise, the Greek Sensitivity Vega is the sensitivity to σ_{HW}. On the other hand, a volatility risk factor stress corresponds to shifting the HW calibration data, as specified by the Volatility Series under the Hull-White model input, and triggers a re-calibration of σ_{HW}. If there is no volatility series specified, then there is no sensitivity to a volatility risk factor stress.

When using the Hull-White model, we value European swaptions using a semi-analytical formula [5] and Bermudan swaptions using a trinomial tree.

4 Price Calibration

In this section we describe price calibration procedures when using Black and Hull-White models. For price calibration when using the BDT or SABR models see [6] and [8] respectively.

In Risk Manager price calibration is the process by which valuation and risk models are adjusted to match the option price or volatility information provided by the user. To describe the behavior under all possible input combinations, we make the following definitions:

- **User Volatility (UV)** Volatility supplied by the user, either Black or Hull-White as appropriate.
- **Market Volatility (MV)** Volatility computed from market data. The MV is computed from the Volatility Series if specified; otherwise it is computed from historical underlying time-series.

Volatility is a Risk Factor

When a Volatility Series is specified, then volatility is a risk factor, i.e. MV can change in each simulated scenario. In order to reconcile the MV with user-supplied data we apply the following adjustments:

- If a Market Price is specified, the MV is scaled multiplicatively so that the model price matches the market price in the base scenario. The same multiplicative factor is then applied in each simulated scenario.

[5] The semi-analytical formula used for Hull-White pricing of swaptions is pretty involved and we will not present it here. The formula can be found in [2].

- If UV is specified, we compute the ratio between the UV and the MV in the base scenario. The MV is multiplied by this ratio in each scenario.

If both the Market Price and UV are specified, the Market Price input is ignored.

Volatility is Static

If no Volatility Series is specified, then volatility is static, i.e. it is the same in base and simulated scenarios. If no Market Price or UV is specified, the MV is computed from historical underlying time-series. If either Market Price or UV are specified, the model volatility is fully determined by the user-inputs, so no MV is needed:

- If a Market Price is specified, the volatility is chosen such that the model price matches the Market Price in the base scenario.
- If UV is specified, the volatility is set to the UV.

If both the Market Price and UV are specified, the Market Price input is ignored.

Notice that the only difference between Black and Hull-White model calibration procedures described above is the interpretation of the volatility parameter (Black-implied volatility or Hull-White short rate volatility resp.), and how this volatility parameter is computed from implied volatility time-series or underlying time-series. In the Black case, an instrument implied volatility is computed directly from the set of implied volatility time-series that define the volatility term structure or surface. In the Hull-White case, the instrument implied volatility is computed through calibration of swaption or cap prices that are given implicitly through swaption or cap volatilities. Similarly, when computing volatility from underlying time series: in the Black case, the volatility is given by an average over historical underlying levels; in the Hull-White case, the volatility is given by calibrating the short-rate volatility to historical time-series volatility of underlying levels.

4.1 Underlying Price Calibration

If the underlying swap rate is specified, we calibrate the reference curve so that the computed swap rate matches the user-specified value. This is done by applying a constant additive shift to the reference curve. This same constant is applied in each simulated scenario.

Bermudan swaptions do not have a well-defined swap term (or swap dates in the case of constant maturity swaps), so this input is ignored if the exercise type is Bermudan. Similarly, when a fixed coupon schedule is provided, there is not a single underlying swap rate, so this input is irrelevant.

A Coupon Schedule and Day-Count Conventions

The fixed coupon schedule is determined from five inputs: *optionExpiryDate*, *swapStartDate*, *firstCouponDate*, *maturityDate*, and *couponFrequency*. The coupon payment dates are computed by advancing or decreasing a reference date by intervals corresponding to the *couponFrequency*. All choices of *couponFrequency* correspond to an integer number of months, with the exception of *lunar-monthly*, which corresponds to 28 days. No business day rule is applied, meaning that any calendar date can be a coupon payment date.

- If *firstCouponDate* is not specified, the reference date is the *maturityDate*, and coupon payment dates are computed by counting backward from *maturityDate* until the first coupon payment date is reached.
- If *firstPaymentDate* is specified, the reference date is the *firstPaymentDate*, and coupon payment dates are computed by counting forward from *firstPaymentDate* until the *maturityDate* is reached.

If this procedure does not lead exactly to the first coupon payment date (or *maturityDate*), the first (or last) coupon period is taken to be shorter than a regular coupon period.

The first coupon period starts on the *startDate* if specified, otherwise it starts on the *optionExpiryDate*.

If the reference date falls on the last day of the month, and the *couponFrequency* defines an integer number of months, all coupon payments are adjusted to fall on the last day of the month.

Coupon period lengths (τ_k) are computed using an actual/365 day-count convention.

References

[1] F. Black. The pricing of commodity contracts. *Journal of Financial Economics*, 3:167–179, 1976.

[2] D. Brigo and F. Mercurio. *Interest Rate Models—Theory and Practice: With Smile, Inflation and Credit*. Springer, 2 edition, 2007.

[3] P. S. Hagan, D. Kumar, A. S. Lesniewski, and D. E. Woodward. Managing smile risk. *Wilmott Magazine*, pages 84–108, July 2002.

[4] J. C. Hull. *Options, Futures and Other Derivatives*. Prentice Hall, 5th edition, 2003.

[5] J. C. Hull and A. White. One-factor interest-rate models and the valuation of interest-rate derivatives securities. *Journal of Financial and Quantitative Analysis*, 28(2):235–254, 1993.

[6] RiskMetrics Research Department. Bdt swaption pricing. *Research Technical Note*. `http://research.rmgapps.com/documents/instruments/Swaption/BDT_swaption_pricing_spec-external.pdf`.

[7] RiskMetrics Research Department. Interest rates MC simulation with the Hull-White model. *Research Technical Note*. http://research.rmgapps.com/documents/methodologies/HullWhiteInterestRateSimulation/IR_Hull_White_spec-internal.pdf.

[8] RiskMetrics Research Department. Pricing Swaptions and Caps/Floors under the SABR model. *Research Technical Note*. http://research.rmgapps.com/external_docs/methodologies/Pricing/SABR.pdf.

[9] RiskMetrics Research Department. Swaption implied volatility interpolation. *Research Technical Note*. http://research.rmgapps.com/external_docs/methodologies/Volatilities/SwaptionVolInterpolation.pdf.

Client Service Information is Available 24 Hours a Day

clientservice@msci.com

Americas

Americas	+ 1.888.588.4567 (toll free)
Atlanta	+ 1.404.551.3212
Boston	+ 1.617.532.0920
Chicago	+ 1.312.675.0545
Montreal	+ 1.514.847.7506
Monterrey	+ 52.81.1253.4020
New York	+ 1.212.804.3901
San Francisco	+ 1.415.836.8800
Sao Paulo	+ 55.11.3706.1360
Stamford	+1.203.325.5630
Toronto	+ 1.416.628.1007

Europe, Middle East & Africa

Amsterdam	+ 31.20.462.1382
Cape Town	+ 27.21.673.0100
Frankfurt	+ 49.69.133.859.00
Geneva	+ 41.22.817.9777
London	+ 44.20.7618.2222
Madrid	+ 34.91.700.7275
Milan	+ 39.02.5849.0415
Paris	0800.91.59.17 (toll free)
Zurich	+ 41.44.220.9300

Asia Pacific

China North	10800.852.1032 (toll free)
China South	10800.152.1032 (toll free)
Hong Kong	+ 852.2844.9333
Seoul	+827.0768.88984
Singapore	800.852.3749 (toll free)
Sydney	+ 61.2.9033.9333
Tokyo	+ 81.3.5226.8222

Notice and Disclaimer