

# Math 573

## Lecture Notes



Department of Mathematics  
530 Church Street  
Ann Arbor, MI 48109-1043, USA

# Chapter 1

## Introduction

### 1.1 What the course covers

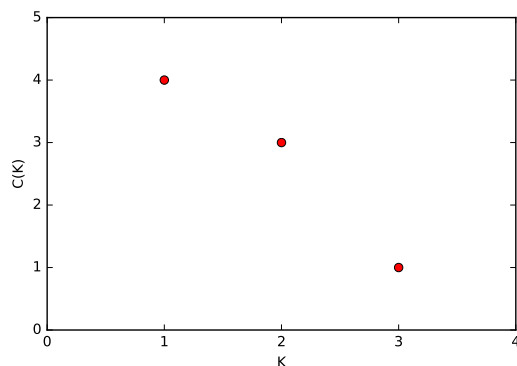
The course is divided broadly into three sections:

1. Arbitrage pricing and hedging
2. Optimal investment
3. Risk measurement

Examples of each follow. Don't worry if the examples are unfamiliar at this time. They serve to indicate where we are headed.

#### 1.1.1 Arbitrage Pricing and Hedging

Let  $C(K) \equiv$  the price of a European call option struck at  $K$ . Suppose that there are call options with strike prices of 1, 2, and 3 as depicted.



Is arbitrage possible? We note that if, for example, we buy 1 call struck at 1, sell 2 calls struck at 2, and buy 2 struck at 3, we will have spent  $4 - 2(3) +$

$2(1) = 0$ . But as we will see, this portfolio will never have negative value, and can have positive value at expiry. So there is arbitrage.

### 1.1.2 Optimal Investment

Consider stocks that have been selected from an index. We wish to construct a portfolio from these stocks. A commonly used approach is to weight each stock  $i$  according to its relative market cap:

$$w_i = \frac{\text{market cap of } i}{\text{combined market cap of stocks}}$$

Why do portfolio managers use this weighting scheme? The answer is that according to CAPM (Capital Asset Pricing Model) theory, this portfolio gives the best return relative to the risk of the portfolio.

### 1.1.3 Risk Measurement

How can we measure the risk of the portfolio? ...

## 1.2 Markets, Assets, Trades, and Derivatives

### 1.2.1 Markets

We distinguish two types of markets:

1. exchanges, where prices are well-known
2. OTC (over-the-counter), where prices are not necessarily observed

How do we price an OTC security?

### 1.2.2 Assets

We will consider the following types of assets:

1. stocks (equities)
2. bonds (fixed income securities). For our purposes, we will assume these are risk-free (i.e. no credit risk, meaning there is no chance of default).
3. commodities and currencies
4. derivatives of the above

### 1.2.3 Trades

A *contract* is an agreement to exchange asset flows.

A *trade* is the act of entering into a contract.