1 Geometric measure

The geometric measure of entanglement [1, 2, 3, 4] is another way to quantify the entanglement of a quantum state. It is defined for both pure and mixed states. It is based on the geometric approach of quantifying how "far" a given quantum state is from the set of all separable (or unentangled) states. This "distance" is used as a measure of the entanglement of the state.

For a given pure state $|\psi\rangle$, the geometric measure of entanglement is defined as the minimum squared Euclidean distance between $|\psi\rangle$ and the set of all separable states. Mathematically, it is given by

$$E_G(\psi) = 1 - \max_{|\phi\rangle} |\langle\phi|\psi\rangle|^2$$

where

- $|\phi\rangle$ is any separable state.
- The maximum is taken over all separable states $|\phi\rangle$.

Interpreting this, $|\langle\phi|\psi\rangle|^2$ is the overlap between the state $|\psi\rangle$ and a separable state $|\phi\rangle$. The geometric measure of entanglement is then $1 - \max_{|\phi\rangle} |\langle\phi|\psi\rangle|^2$, which means it is $1-$ the maximum overlap of $|\psi\rangle$ with any separable state.

This measure takes values between 0 and 1, where 0 indicates no entanglement and 1 indicates maximum entanglement.

For mixed states, the geometric measure of entanglement is defined as the minimum geometric measure of entanglement over all pure-state decompositions of the mixed state. Mathematically, it is given by

$$E_G(\rho) = \min_{\{p_k, |\psi_k\rangle\}} \sum_k p_k E_G(\psi_k)$$

where

- p_k are probabilities such that $0 \leq p_k \leq 1$ and $\sum_k p_k = 1$.
- $|\psi_k\rangle$ are pure states.
- The minimum is taken over all possible decompositions of ρ as a mixture of pure states $\{p_k, |\psi_k\rangle\}$.

The geometric measure of entanglement has the property that if $E_G(\rho) = 0$, then ρ is a separable state (i.e., no entanglement), and if $E_G(\rho) = 1$, then ρ is a maximally entangled state. It is a useful measure of entanglement as it provides a geometric perspective on entanglement, and can be used to quantify the entanglement of any quantum state, pure or mixed.

This measure has several desirable properties, such as being invariant under local unitary transformations, and being continuous and monotonous under local operations and classical communication (LOCC). This makes it a useful and widely accepted measure of entanglement.

References

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