1 Single-qubit unitary decomposition

An arbitrary one-qubit operator can be decomposed as a sequence of at most three R_y e R_z [1, 2] gates using the ZYZ decomposition[1] [3]. Given any unitary matrix $U_{2\times 2}$, there exist angles ϕ, α, β, and γ that satisfy the following equation:

$$U = e^{i\phi} R_z(\alpha) R_y(\beta) R_z(\gamma). \tag{1}$$

Such a decomposition is possible because the rows and columns of unitary matrices are orthonormal vectors, so every unitary matrix 2×2 can be written in the form [1, 2]

$$\begin{bmatrix} e^{i(\phi+\frac{\alpha}{2}+\frac{\gamma}{2})}\cos\frac{\beta}{2} & e^{i(\phi+\frac{\alpha}{2}-\frac{\gamma}{2})}\sin\frac{\beta}{2} \\ -e^{i(\phi-\frac{\alpha}{2}+\frac{\gamma}{2})}\sin\frac{\beta}{2} & e^{i(\phi-\frac{\alpha}{2}-\frac{\gamma}{2})}\cos\frac{\beta}{2} \end{bmatrix}. \tag{2}$$

From the matrix (2), factoring into the form of the expression (1) is straightforward.

For the case where $U_{2\times 2}$ belongs to the special unitary group $SU(2)$, the property $\det(U) = 1$ implies that $e^{i\phi} = \pm 1$. Thus, the expression (1) can be simplified to:

$$U = R_z(\alpha) R_y(\beta) R_z(\gamma). \tag{3}$$

Listing 1: Decomposition of single-qubit operators using qclib.

```
from qiskit import transpile
from scipy.stats import unitary_group
from qclib.unitary import unitary as decompose
n_qubits = 1
U = unitary_group.rvs(2 ** n_qubits)
circuit = decompose(U)
t_circuit = transpile(circuit, basis_gates=['u', 'cx'],
                      optimization_level=0)
n_depth = t_circuit.depth()
n_cx = t_circuit.count_ops().get('cx', 0)
print(f'cnots={n_cx}\t␣depth={n_depth}')
# cnots=0        depth=1
```

References

[1] Adriano Barenco, Charles H. Bennett, Richard Cleve, David P. DiVincenzo, Norman Margolus, Peter Shor, Tycho Sleator, John A. Smolin, and Harald Weinfurter. Elementary gates for quantum computation. Phys. Rev. A, 52:3457–3467, 1995.

[2] Michael A. Nielsen and Isaac L. Chuang. Quantum computation and quantum information. Cambridge University Press, Cambridge ; New York, 10th anniversary ed edition, 2010.

[3] Vivek V. Shende, Igor L. Markov, and Stephen S. Bullock. Minimal universal two-qubit controlled-NOT-based circuits. Physical Review A, 69(6), 2004a.

[1] The choice of y and z axes is arbitrary. Any pair of orthogonal vectors can be chosen.