1 Bottom-up (divide-and-conquer)

In the following, I explain how the input vector $\mathbf{x} = (ac, ad, be, bf)$ is encoded into a quantum state using the divide-and-conquer strategy (as the goal is to highlight how the states are combined, I will assume that the angles have already been calculated).

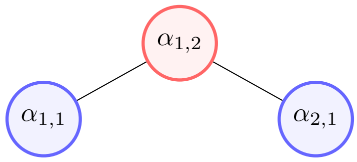

Figure 1: Angle tree constructed from a 4-dim input vector.

Rotations applied to the circuit will initialize the following single-qubit states (the sub-indice(s) indicate the qubit(s) associated with the state):

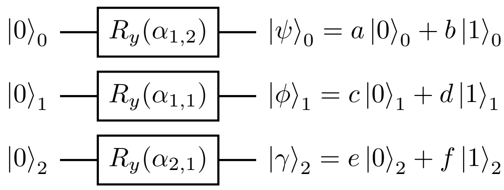

Figure 2: Initialization of the single-qubit states.

Next, the single-qubit states are combined using a CSWAP gate. Since qubit 2 is ancilla, any state associated with it is considered auxiliary. Note the swap of qubits between states $|\phi\rangle$ and $|\gamma\rangle$.

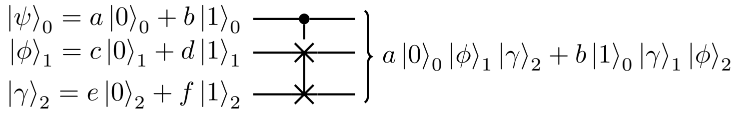

Figure 3: Combination of states that were created by rotations using the Angle Tree parameters.

Now we can calculate the final state, which encodes the input vector into the quantum state amplitudes (states associated with the ancilla qubit are not expanded for readability).

$$\begin{aligned}
|\Psi\rangle =& a\,|0\rangle_0\,|\phi\rangle_1\,|\gamma\rangle_2 + b\,|1\rangle_0\,|\gamma\rangle_1\,|\phi\rangle_2 \\
& a\,|0\rangle_0\,(c\,|0\rangle_1 + d\,|1\rangle_1)\,|\gamma\rangle_2 + b\,|1\rangle_0\,(e\,|0\rangle_1 + f\,|1\rangle_1)\,|\phi\rangle_2 \\
& ac\,|00\rangle_{0,1}\,|\gamma\rangle_2 + ad\,|01\rangle_{0,1}\,|\gamma\rangle_2 + be\,|10\rangle_{0,1}\,|\phi\rangle_2 + bf\,|11\rangle_{0,1}\,|\phi\rangle_2
\end{aligned}$$

As expected, the state amplitudes are equal to the coordinates of the input vector.