1 Meyer-Wallach measure

The Meyer-Wallach (MW) measure [1, 2, 3] is a global measure of entanglement that quantifies the entanglement of a state of a multi-qubit system by averaging the single-qubit ”purity” over all qubits.

Purity, in this context, refers to how mixed a state is, and is given by the trace of the square of the density matrix, $\mathrm{Tr}(\rho^2)$. For a single qubit, the maximum purity is 1, which corresponds to a pure state, and the minimum purity is 1/2, which corresponds to a maximally mixed state.

The measure is defined as the average entanglement of a single qubit with the rest of the system. Mathematically, it is given by

$$Q(\rho) = \frac{1}{N} \sum_{i=1}^{N} Q_i(\rho)$$

where

- N is the total number of qubits in the system.
- ρ is the density matrix of the state.
- $Q_i(\rho)$ is the entanglement of the i-th qubit with the rest of the system, given by:

$$Q_i(\rho) = 2(1 - \mathrm{Tr}(\rho_i^2))$$

- ρ_i is the reduced density matrix of the i-th qubit, obtained by taking the partial trace of ρ over all qubits except the i-th one.

The MW measure $Q(\rho)$ takes values between 0 and 1. A value of 0 indicates no entanglement, and a value of 1 indicates maximum entanglement. Specifically

- If $Q(\rho) = 0$, then the state ρ is a product state (i.e., no entanglement).
- If $Q(\rho) = 1$, then the state ρ is maximally entangled.

While $Q(\rho) = 0$ clearly indicates that the state ρ is a product state (i.e., no entanglement), and $Q(\rho) = 1$ indicates that the state ρ is maximally entangled, the interpretation of values of $Q(\rho)$ between 0 and 1 is not straightforward.

The MW measure provides a simple and computationally efficient way to estimate the entanglement of a quantum state, especially for large systems where other entanglement measures may be computationally intractable. However, it is worth noting that the MW measure does not capture all aspects of entanglement, and other measures, such as the entanglement entropy or the geometric measure of entanglement [4, 5, 6], may provide different insights into the entanglement structure of a quantum state.

References

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