Multiplexor rotation gate

A multiplexor rotation gate is equivalent to a uniformly controlled rotation, as shown in the figure below.

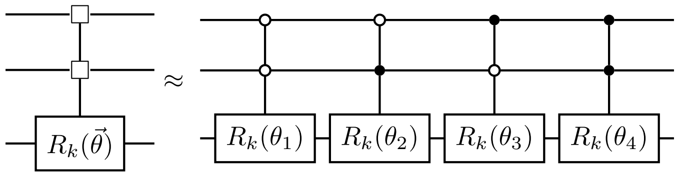

Figure 1: Example diagram of a multiplexor rotation gate with three qubits (two controls and one target). The square symbols indicate that control occurs for every bitstring appearing on the select qubits.

There are two main methods for decomposing this type of gate, one by Mottonen et al.[1] and another by Shende et al.[2] The former involves large matrices and Gray codes. Shende's method is more efficient and is therefore the choice here. The following figure describes how the decomposition can be done recursively.

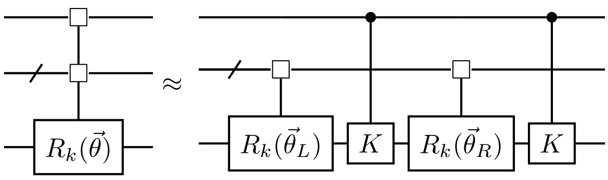

Figure 2: Recursive decomposition of a multiplexed R_k gate (k={y,z} and K={X, Z}).

The set of angles $\vec{\theta}_L$ and $\vec{\theta}_R$ are calculated as follows:

$$
\begin{aligned}
A &= \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & -0.5 \end{bmatrix} \otimes \mathbb{I}_{2^{\text{num\_ctrls}-1} \times 2^{\text{num\_ctrls}-1}} \\
\vec{\theta}_{L,R} &= (A\vec{\theta}^T)^T \\
\vec{\theta}_L &= \vec{\theta}_{L,R}[1 \ldots \text{size}/2] \\
\vec{\theta}_R &= \vec{\theta}_{L,R}[\text{size}/2 \ldots \text{size}]
\end{aligned}
$$

The matrix A is called *angle multiplexor*, and the vector $\vec{\theta}_{L,R}$ is the set of *multiplexed angles*. The vectors $\vec{\theta}_L$ and $\vec{\theta}_R$ are the first and second half of the multiplexed angles.

References:

[1] Mikko Möttönen, Juha J. Vartiainen, Ville Bergholm, and Martti M. Salomaa. Quantum Circuits for General Multiqubit Gates. Phys. Rev. Lett. 93, 130502

[2] Shende, V.; Bullock, S.; Markov, I. Synthesis of quantum-logic circuits. IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems, v. 25, n. 6, p. 1000–1010, 2006