Autocorrelation Function

TIME SERIES ANALYSIS IN PYTHON



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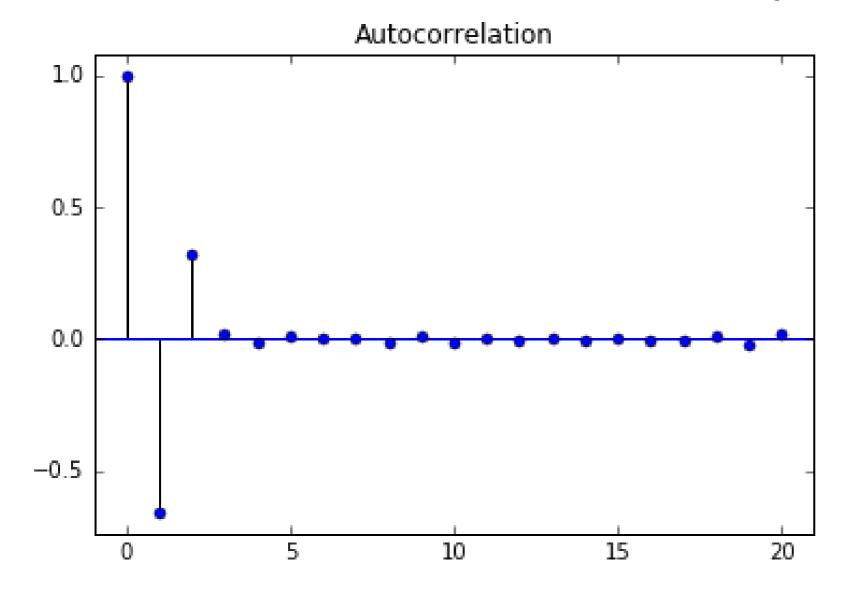


Autocorrelation Function

- Autocorrelation Function (ACF): The autocorrelation as a function of the lag
- Equals one at lag-zero
- Interesting information beyond lag-one

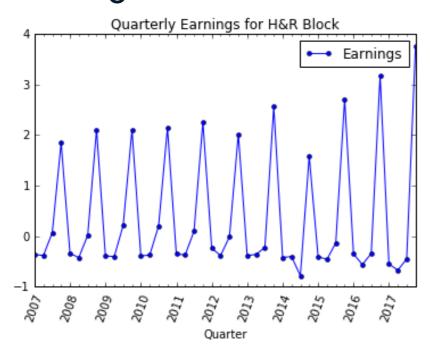
ACF Example 1: Simple Autocorrelation Function

Can use last two values in series for forecasting

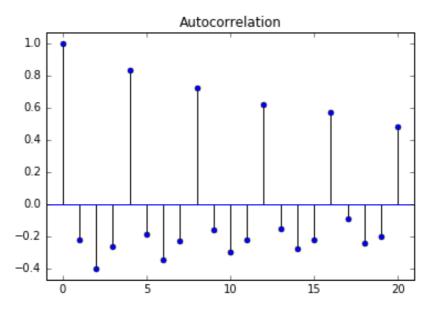


ACF Example 2: Seasonal Earnings

Earnings for H&R Block

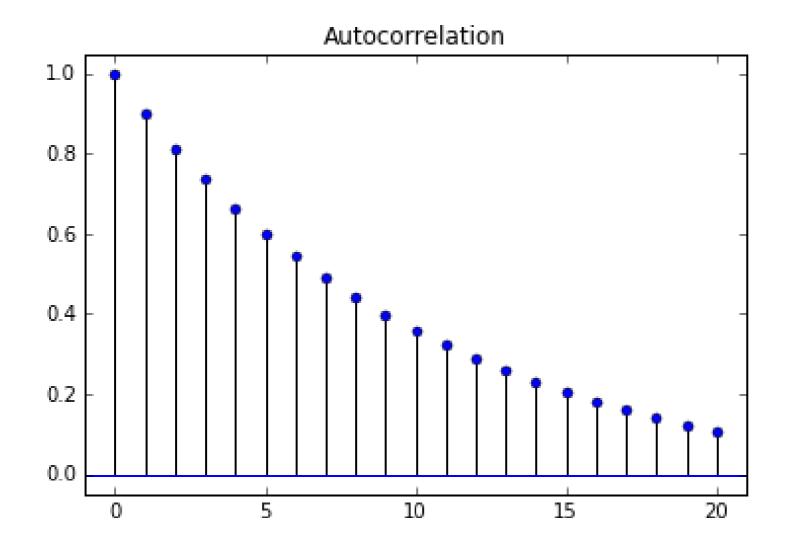


• ACF for H&R Block



ACF Example 3: Useful for Model Selection

Model selection



Plot ACF in Python

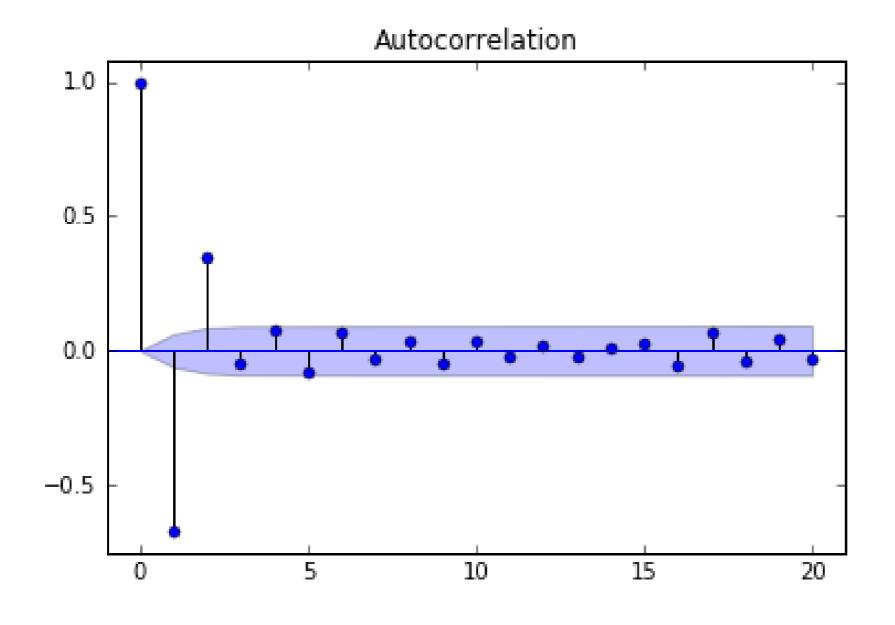
• Import module:

```
from statsmodels.graphics.tsaplots import plot_acf
```

Plot the ACF:

```
plot_acf(x, lags= 20, alpha=0.05)
```

Confidence Interval of ACF





Confidence Interval of ACF

- Argument alpha sets the width of confidence interval
- Example: alpha=0.05
 - 5% chance that if true autocorrelation is zero, it will fall outside blue band
- Confidence bands are wider if:
 - Alpha lower
 - Fewer observations
- Under some simplifying assumptions, 95% confidence bands are $\pm 2/\sqrt{N}$
- If you want no bands on plot, set alpha=1

ACF Values Instead of Plot

```
from statsmodels.tsa.stattools import acf
print(acf(x))
```

```
      [ 1.
      -0.6765505
      0.34989905
      -0.01629415
      -0.02507

      -0.03186545
      0.01399904
      -0.03518128
      0.02063168
      -0.02626

      ...
      0.07191516
      -0.12211912
      0.14514481
      -0.09644228
      0.05215
```

Let's practice!

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White Noise

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What is White Noise?

- White Noise is a series with:
 - Constant mean
 - Constant variance
 - Zero autocorrelations at all lags
- Special Case: if data has normal distribution, then Gaussian White Noise

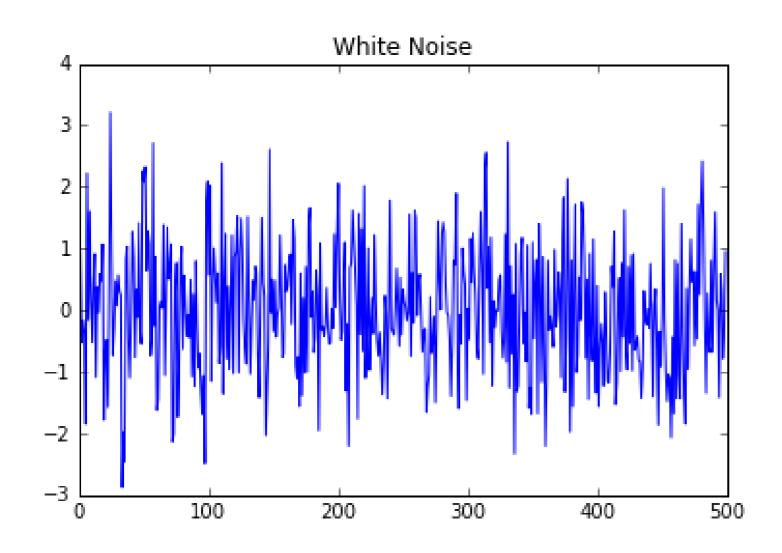
Simulating White Noise

• It's very easy to generate white noise

```
import numpy as np
noise = np.random.normal(loc=0, scale=1, size=500)
```

What Does White Noise Look Like?

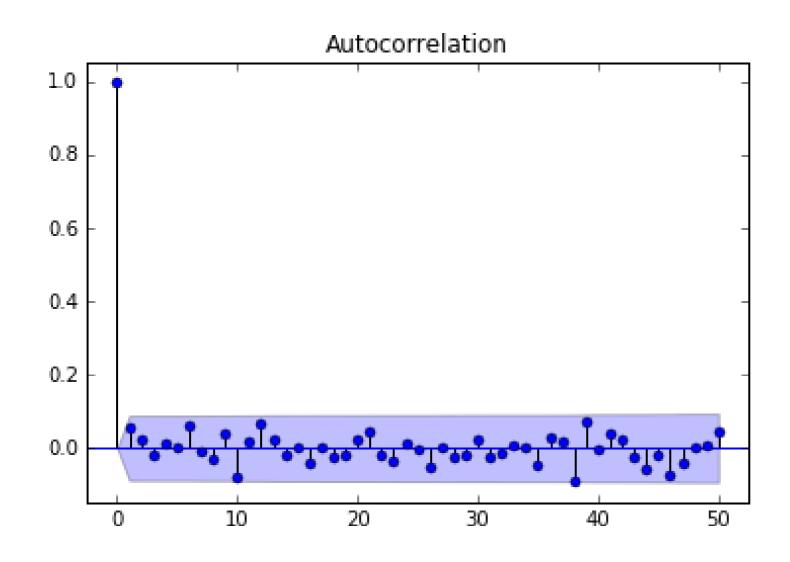
plt.plot(noise)





Autocorrelation of White Noise

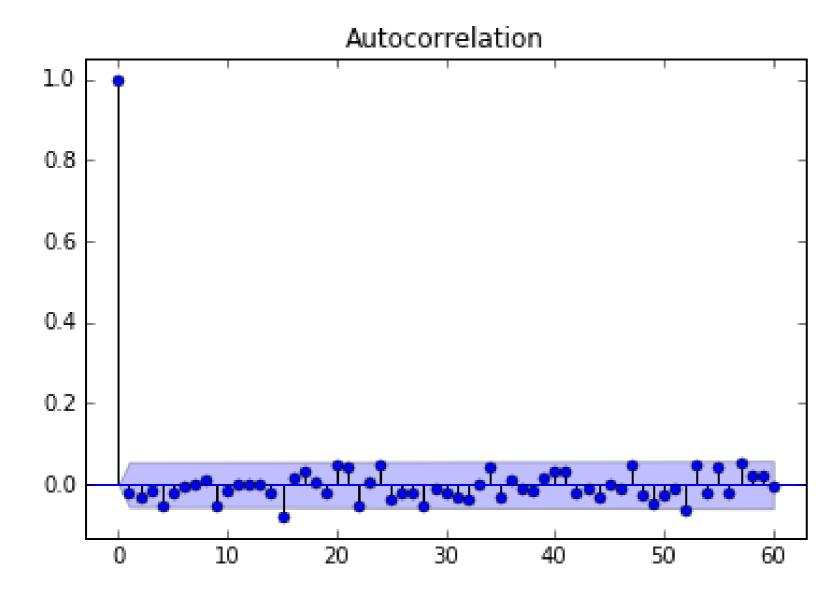
plot_acf(noise, lags=50)





Stock Market Returns: Close to White Noise

Autocorrelation Function for the S&P500



Let's practice!

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Random Walk

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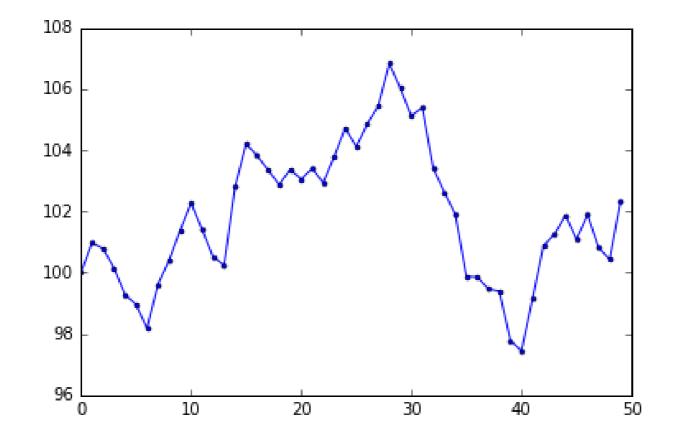


What is a Random Walk?

Today's Price = Yesterday's Price + Noise

$$P_t = P_{t-1} + \epsilon_t$$

Plot of simulated data



What is a Random Walk?

• Today's Price = Yesterday's Price + Noise

$$P_t = P_{t-1} + \epsilon_t$$

Change in price is white noise

$$P_t - P_{t-1} = \epsilon_t$$

- Can't forecast a random walk
- Best forecast for tomorrow's price is today's price

What is a Random Walk?

• Today's Price = Yesterday's Price + Noise

$$P_t = P_{t-1} + \epsilon_t$$

Random walk with drift:

$$P_t = \mu + P_{t-1} + \epsilon_t$$

• Change in price is white noise with non-zero mean:

$$P_t - P_{t-1} = \mu + \epsilon_t$$

Statistical Test for Random Walk

Random walk with drift

$$P_t = \mu + P_{t-1} + \epsilon_t$$

Regression test for random walk

$$P_t = \alpha + \beta P_{t-1} + \epsilon_t$$

• Test:

$$H_0:eta=1$$
 (random walk)

$$H_1:eta<1$$
 (not random walk)

Statistical Test for Random Walk

Regression test for random walk

$$P_t = \alpha + \beta P_{t-1} + \epsilon_t$$

Equivalent to

$$P_t - P_{t-1} = \alpha + \beta P_{t-1} + \epsilon_t$$

• Test:

$$H_0:eta=0$$
 (random walk)

$$H_1:eta<0$$
 (not random walk)

Statistical Test for Random Walk

Regression test for random walk

$$P_t - P_{t-1} = \alpha + \beta P_{t-1} + \epsilon_t$$

• Test:

$$H_0:eta=0$$
 (random walk)

$$H_1:eta<0$$
 (not random walk)

- This test is called the Dickey-Fuller test
- If you add more lagged changes on the right hand side, it's the **Augmented Dickey-Fuller** test

ADF Test in Python

Import module from statsmodels

```
from statsmodels.tsa.stattools import adfuller
```

Run Augmented Dickey-Test

```
adfuller(x)
```

Example: Is the S&P500 a Random Walk?

```
# Run Augmented Dickey-Fuller Test on SPX data
results = adfuller(df['SPX'])
# Print p-value
print(results[1])
0.782253808587
# Print full results
print(results)
(-0.91720490331127869,
 0.78225380858668414,
 0,
 1257,
 \{'1\%': -3.4355629707955395,
  '10%': -2.567995644141416,
  '5%': -2.8638420633876671},
 10161.888789598503)
```



Let's practice!

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Stationarity

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What is Stationarity?

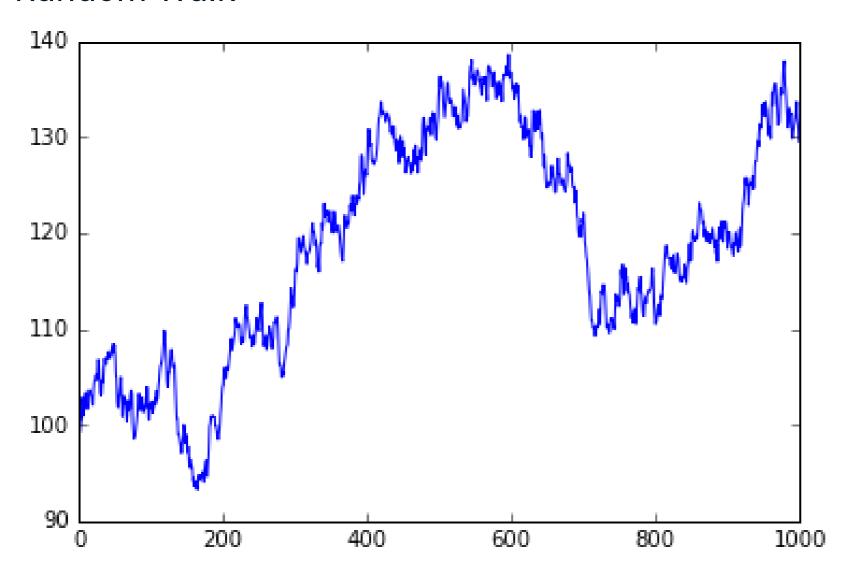
- Strong stationarity: entire distribution of data is timeinvariant
- Weak stationarity: mean, variance and autocorrelation are time-invariant (i.e., for autocorrelation, $\mathrm{corr}(X_t,X_{t- au})$ is only a function of au)

Why Do We Care?

- If parameters vary with time, too many parameters to estimate
- Can only estimate a parsimonious model with a few parameters

Examples of Nonstationary Series

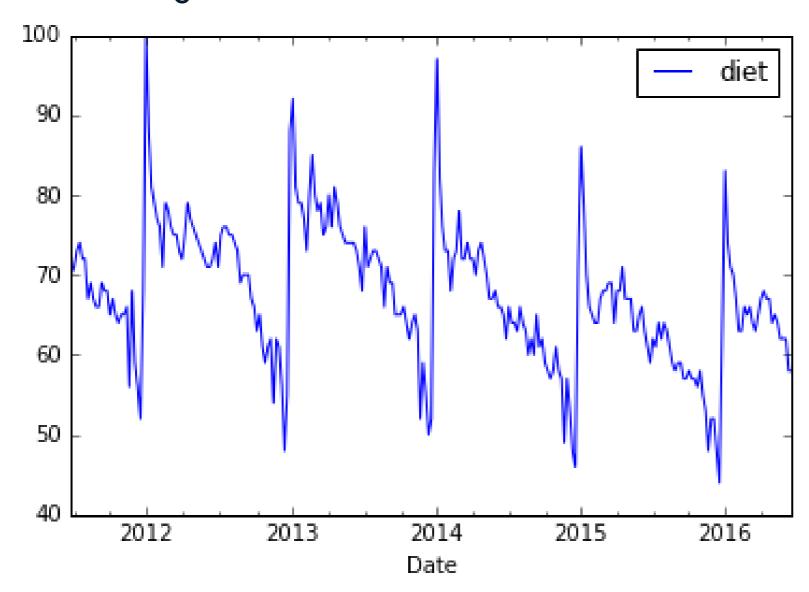
Random Walk





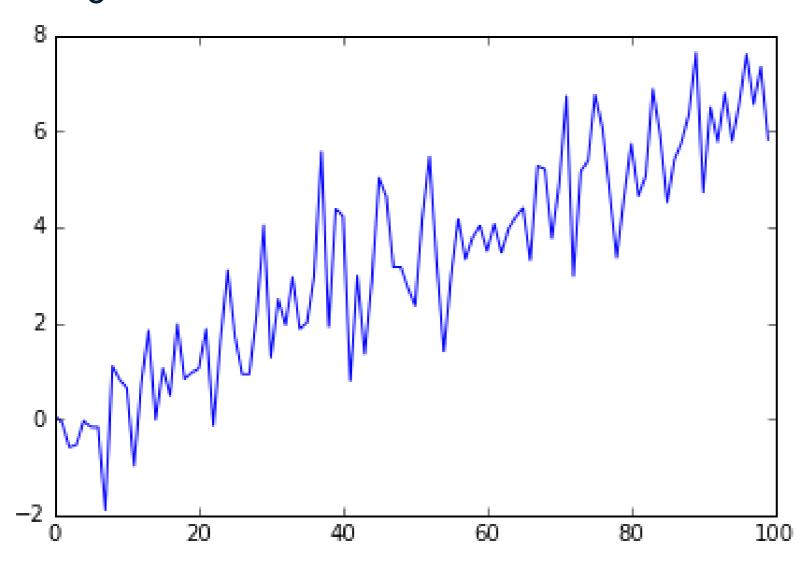
Examples of Nonstationary Series

• Seasonality in series



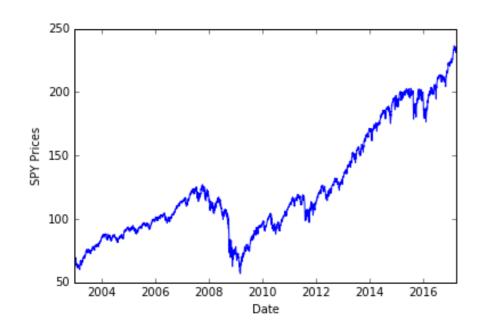
Examples of Nonstationary Series

Change in Mean or Standard Deviation over time



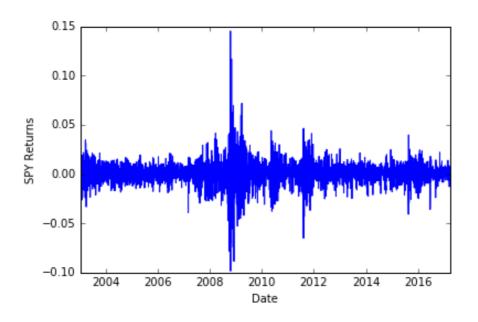
Transforming Nonstationary Series Into Stationary Series

Random Walk



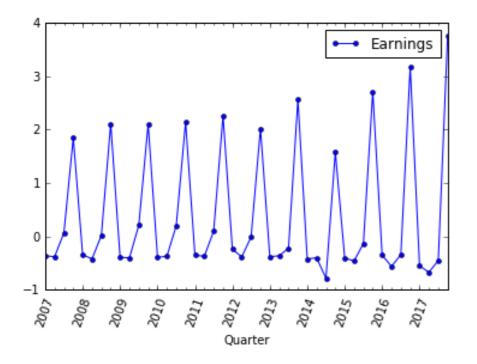
First difference

```
plot.plot(SPY.diff())
```



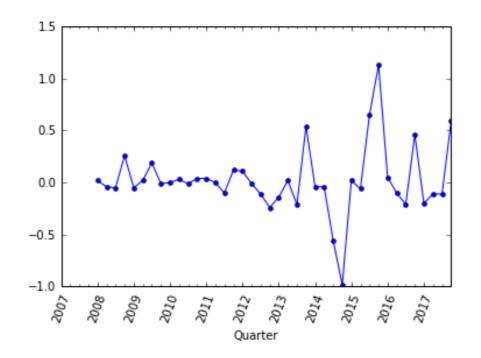
Transforming Nonstationary Series Into Stationary Series

Seasonality



Seasonal difference

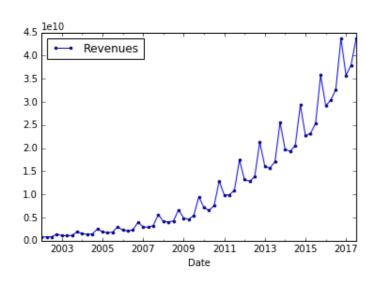
```
plot.plot(HRB.diff(4))
```



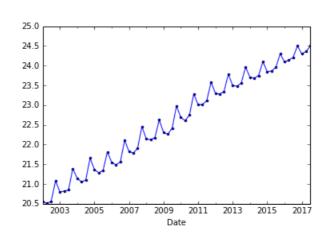
Transforming Nonstationary Series Into Stationary Series

AMZN Quarterly Revenues

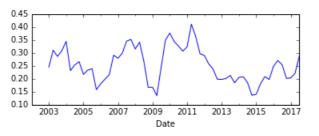
plt.plot(AMZN)



Log of AMZN Revenues
plt.plot(np.log(AMZN))



Log, then seasonal difference
plt.plot(np.log(AMZN).diff(4))



Let's practice!

TIME SERIES ANALYSIS IN PYTHON

