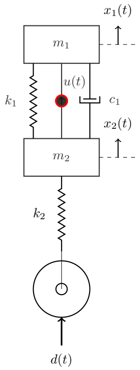
Design Problems

Raktim Bhattacharya
Aerospace Engineering, Texas A&M University

Active Suspension

Robust $\mathcal{H}_\infty$ Disturbance Rejection

Disturbance Rejection with Active Suspension



Equations of motion

$$m_1\ddot{x}_1 = -k_1(x_1 - x_2) - c_1(\dot{x}_1 - \dot{x}_2) + u,$$
$$m_2\ddot{x}_2 = k_1(x_1 - x_2) + c_1(\dot{x}_1 - \dot{x}_2) - k_2(x_2 - d) - u.$$

State Variables

$$q_1 := x_1, \qquad q_2 := x_2,$$
$$q_3 := \dot{x}_1, \qquad q_4 := \dot{x}_2.$$

Robust $\mathcal{H}_\infty$ Disturbance Rejection

Disturbance Rejection with Active Suspension

Linear System

$$\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\\ \dot{q}_3\\ \dot{q}_4\end{bmatrix} = \begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -k_1/m_1 & k_1/m_1 & -c_1/m_1 & c_1/m_1\\ k_1/m_2 & -(k_1+k_2)/m_2 & c_1/m_2 & -c_1/m_2\end{bmatrix}\begin{bmatrix}q_1\\ q_2\\ q_3\\ q_4\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 1/m_1\\ -1/m_2\end{bmatrix}u + \begin{bmatrix}0\\ 0\\ 0\\ k_2/m_2\end{bmatrix}d.$$

Output $z(t)$

$$z = \begin{bmatrix}q_1\\ u\end{bmatrix}.$$

Robust $\mathcal{H}_\infty$ Disturbance Rejection

System Parameters and Definition

```
clear; clc;
% System Parameters
m1 = 290; % kg -- Body mass
m2 = 60;  % kg -- suspension mass

k1 = 16200; % N/m
k2 = 191000; % N/m
c1 = 1000; % Ns/m

A = [0 0 1 0;
    0 0 0 1;
    -k1/m1 k1/m1 -c1/m1 c1/m1;
    k1/m1 -(k1+k2)/m2 c1/m2 -c1/m2];
Bu = [0;0;1/m1;-1/m2];
Bw = [0;0;0;k2/m2];

nx = 4; nu = 1;
nz = 1; nw = 1;

Cz = [1,0,0,0];
Du = 1*ones(nz,nu);
```

Robust $\mathcal{H}_\infty$ Disturbance Rejection

System Uncertainty

- There is 20% uncertainty in the body mass m_1 – variation in passenger mass and luggage
- There is 10% uncertainty in tire springiness k_2
- There is 10% uncertainty in c_1 – uncertainty in shock absorbers

System Analysis

- Make a grid over parameters (m_1, k_2, c_1) with 10 points along each parameter
- Plot bode plot for the 1000 systems
- Assess variation in system response due to parametric uncertainty

Robust $\mathcal{H}_\infty$ Disturbance Rejection

Robustness of Nominal Controller

- Make a grid over parameters (m_1, k_2, c_1) with 10 points along each parameter.
- Run 100 simulations for the closed-loop system with nominal controller and assess the quality of disturbance rejection
  - *Plot state and control trajectories for the 1000 realizations*
  - *Compute the average and standard deviation of the $\mathcal{H}_2$ norm of $G_{w\rightarrow z}$ from the 1000 systems*

Robust $\mathcal{H}_\infty$ Disturbance Rejection

Design of Robust Controller

- Design $\mathcal{H}_\infty$ optimal controller including uncertainty in the system – using `hinfsyn()` MATLAB command.
- Make a grid over parameters (m_1, k_2, c_1) with 10 points along each parameter.
- Run 1000 simulations for the closed-loop system with robust controller and assess the quality of disturbance rejection
  - *Plot state and control trajectories for the 1000 realizations*
  - *Compare the average and standard deviation of the $\mathcal{H}_\infty$ norm obtained using nominal controller with the $\mathcal{H}_\infty$ norm obtained from the synthesis of the robust controller.*

Robust $\mathcal{H}_\infty$ Disturbance Rejection

Design of Robust Controller (contd).

- To assess the conservativeness of the design, compute $\|G_{w\to z}\|_\infty$ for the 1000 systems with the robust controller. How does the $\mathcal{H}_\infty$ norm from robust synthesis compare with these 1000 values. What can you say about the conservativeness of the controller?
- Compare the performance of the nominal and robust controller based on trajectory plots and $\mathcal{H}_2$ norms

Robust $\mathcal{H}_\infty$ Regulator

Design of Robust Controller – Extra Credit

The above design specifications do not include rate limits on control u and does not penalize magnitude of x_2.

How can you modify the control problem to impose limits on these variables?

It is unclear what the limits on $u, \dot{u}$, and x_2 should be and what is their effect on $\|G_{w\to z}\|_\infty$. You are to vary limits on $u, \dot{u}$, and x_2 and plot the corresponding $\|G_{w\to z}\|_\infty$ to help decide them. The limit on $\dot{u}$ can be expressed as a low pass filter with a cut off frequency ω_c.

Robust Flight Control

Tracking Controller for Nonlinear System

Trimming F-16 Dynamics

- The longitudinal nonlinear F16 dynamics is given by

$$\dot{x} = f(x, u),$$

where $x = [V, \alpha, \theta, q]$ and $u = [T, \delta_e]$.

- Trim states and control are $(\bar{x}, \bar{u})$, such that

$$\dot{x} = f(\bar{x}, \bar{u}) = 0.$$

- Trim conditions for steady-level flight implies

$$\gamma = \theta - \alpha = 0, \text{ and } q = 0.$$

Tracking Controller for Nonlinear System

Trimming F-16 Dynamics (contd.)

- Trimming is done as a constrained (nonlinear) optimization problem.

$$\min_{x,u} \dot{x}^T \dot{x}$$

subject to

$$LB \leq \begin{bmatrix} x \\ u \end{bmatrix} \leq UB, \quad A_{\text{eq}} \begin{bmatrix} x \\ u \end{bmatrix} = b_{\text{eq}}.$$

- This is implemented in `trimF16.m`. Study the code and understand how it works.
- Using this code, you are to trim the vehicle at velocities

$$V = [900, 1000, 1100, 1200, 1300, 1400, 1500, 1600, 1700].$$

- Linearize the system about $(\bar{x}, \bar{u})$ to obtain a family of A, B, C, D. This is done in `trimF16.m` using MATLAB command `linmod(...)`.

Tracking Controller for Nonlinear System

Robust Control Design

1. Model the variation in A, B, C, D as parametric uncertainty using LFTs.
2. Include disturbance in α dynamics as uniform white noise $\in [-1, 1]$ rad.
3. Design output feedback $\mathcal{H}_\infty$ controller to track V, γ reference signals in the presence of model uncertainty and disturbance.
4. Show performance of the robust controller for a doublet in $\gamma_r = \pm 30^\circ$ and $V_r = \pm 50 ft/s$, on the linear model obtained at each of the trim velocities.
5. Compare with the performance of the nominal controller designed at $V = 1200$ ft/s trim, on the linear model obtained at each of the trim velocities.
6. Repeat steps 4,5 for the nonlinear model, i.e. test your robust controller and nominal controller on the nonlinear F16 model.