AERO 422: Active Controls for Aerospace Vehicles
## Introduction

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Laboratory For Uncertainty Quantification
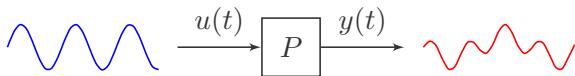Aerospace Engineering, Texas A&M University.

Preliminaries

- Signals & Systems
- Laplace transforms
- Transfer functions – from ordinary linear differential equations
- System interconnections
- Block diagram algebra – simplification of interconnections
- General feedback control system interconnection.
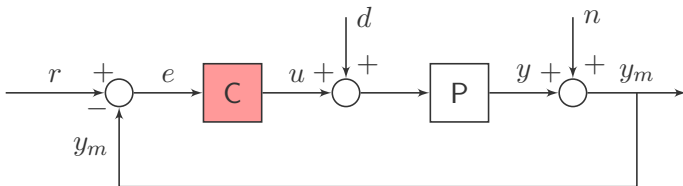
Signals & Systems

Signals & Systems

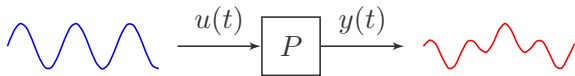

$$u(t) \quad \boxed{P} \quad y(t)$$

- Actuator applies $u(t)$
- Sensor provides $y(t)$

- Feedback controller takes $y(t)$ and determines $u(t)$ to achieve desired behavior
- The controller is typically implemented as software, running in a micro controller

- Imperfections exist in real world
    - *sensors have noise*
    - *actuators have irregularities*
    - *plant P is not fully known*

System Response to $u(t)$

Given plant P and input $u(t)$, what is $y(t)$?

- P is defined in terms of ordinary differential equations
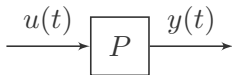- $y(t)$ is the forced $+$ initial condition response.

Linear Dynamics

$m\ddot{x} + c\dot{x} + kx = u(t)$ dynamics

$y(t) = x(t)$ measurement

Nonlinear Dynamics

$\ddot{x} - \mu(1 - x^2)\dot{x} + x = u(t)$ dynamics

$y(t) = x(t)$ measurement

In this class we focus on linear systems

Linear Systems

$$\xrightarrow{\;u(t)\;}\boxed{P}\xrightarrow{\;y(t)\;}$$

- Dynamics is defined by linear ordinary differential equation
- Super position principle applies

$$\begin{aligned}u_1(t) \mapsto y_1(t)\\u_2(t) \mapsto y_2(t)\end{aligned} \implies (u_1(t) + u_2(t)) \mapsto (y_1(t) + y_2(t))$$

Laplace Transforms

Laplace Transforms

Given signal $u(t)$, Laplace transform is defined as

$$\mathcal{L}\{u(t)\} := \int_0^\infty u(t)e^{-st}dt$$

Exists when

$$\lim_{t \to \infty} |u(t)e^{-\sigma t}| = 0, \text{ for some } \sigma > 0$$

Very useful in studying linear dynamical systems and designing controllers

Properties Laplace Transforms

Linear operator

- Additive

$$\mathcal{L}\{u_1(t) + u_2(t)\} = \int_0^\infty (u_1(t) + u_2(t))\, e^{-st} dt$$
$$= \int_0^\infty u_1(t) e^{-st} dt + \int_0^\infty u_2(t) e^{-st} dt$$
$$= \mathcal{L}\{u_1(t)\} + \mathcal{L}\{u_2(t)\}$$

- Superposition

$$\mathcal{L}\{au(t)\} = a\mathcal{L}\{u(t)\}, \quad a \text{ is a constant}$$
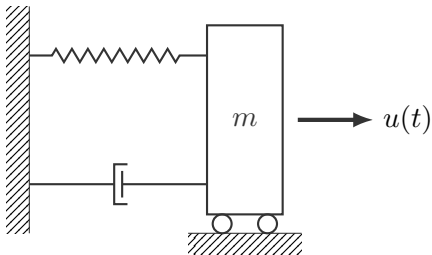
Properties (contd.)

1. $U(s) := \mathcal{L}\{u(t)\}$

2. $\mathcal{L}\{au_1(t) + bu_2(t)\} = a\mathcal{L}\{u_1(t)\} + b\mathcal{L}\{u_2(t)\} = aU_1(s) + bU_2(s)$

3. $\dfrac{1}{s}U(s) \Longleftrightarrow \displaystyle\int_0^t u(\tau)d\tau$

4. $U_1(s)U_2(s) \Longleftrightarrow u_1(t) * u_2(t)$ Convolution

5. $\displaystyle\lim_{s\to 0} sU(s) \Longleftrightarrow \lim_{t\to\infty} u(t)$ Final value theorem

6. $\displaystyle\lim_{s\to\infty} sU(s) \Longleftrightarrow u(0^+)$ Initial value theorem

7. $-\dfrac{dU(s)}{ds} \Longleftrightarrow tu(t)$

8. $\mathcal{L}\left\{\dfrac{du}{dt}\right\} \Longleftrightarrow sU(s) - u(0)$

9. $\mathcal{L}\{\ddot{u}\} \Longleftrightarrow s^2U(s) - su(0) - \dot{u}(0)$

Important Signals

1. $\mathcal{L}\left\{\delta(t)\right\} = 1$ $\delta(t)$ is impulse function

2. $\mathcal{L}\left\{1(t)\right\} = \dfrac{1}{s}$ $1(t)$ is unit step function at $t = 0$

3. $\mathcal{L}\left\{t\right\} = \dfrac{1}{s^2}$

4. $\mathcal{L}\left\{\sin(\omega t)\right. = \dfrac{\omega}{s^2 + \omega^2}$

5. $\mathcal{L}\left\{\cos(\omega t)\right. = \dfrac{s}{s^2 + \omega^2}$

Transfer Functions

Spring Mass Damper System

Equation of Motion

$$m\ddot{x} + c\dot{x} + kx = u(t)$$

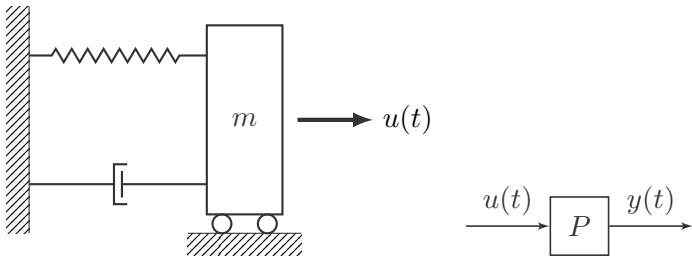Take $\mathcal{L}\{\cdot\}$ on both sides

$$\mathcal{L}\{m\ddot{x} + c\dot{x} + kx\} = \mathcal{L}\{u(t)\}$$

$$m\mathcal{L}\{\ddot{x}\} + c\mathcal{L}\{\dot{x}\} + k\mathcal{L}\{x\} = \mathcal{L}\{u(t)\}$$

$$m\left(s^2 X(s) - sx(0) - \dot{x}(0)\right) + c\left(sX(s) - x(0)\right) + kX(s) = U(s)$$

$$(ms^2 + cs + k)X(s) = U(s) \quad \text{$\dot{x}(0)$ and $x(0)$ are assumed to be zero}$$

Transfer Function

$$(ms^2 + cs + k)X(s) = U(s) \implies \frac{X(s)}{U(s)} = \frac{1}{ms^2 + cs + k}$$

Choose output $y(t) = x(t) \implies Y(s) = X(s)$.
Therefore

$$P(s) := \frac{Y(s)}{U(s)} = \frac{1}{ms^2 + cs + k} \quad \text{Transfer function}$$

Transfer Function (contd.)

In general

$$P(s) = \frac{N(s)}{D(s)}$$

where $N(s)$ and $D(s)$ are polynomials in s

- Roots of $N(s)$ are the zeros
- Roots of $D(s)$ are the poles – determine stability
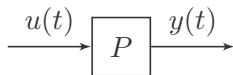
Response to $u(t)$

Given

– input signal $u(t)$ and transfer function $P(s)$.

Determine

– output response $y(t)$

1. **Laplace transform**
   $U(s) := \mathcal{L}\{u(t)\}$

2. **Determine** $Y(s) := P(s)U(s)$

3. **Laplace inverse**

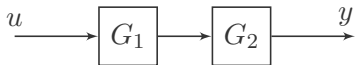 $y(t) := \mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\{P(s)U(s)\}$

$$\xrightarrow{\quad u(t) \quad} \boxed{P} \xrightarrow{\quad y(t) \quad}$$
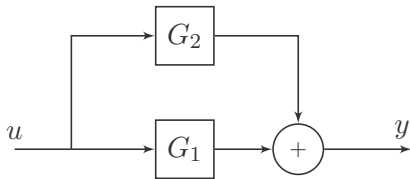
System Interconnection

Block Diagram

Representation of System Interconnections

- Series
- Parallel
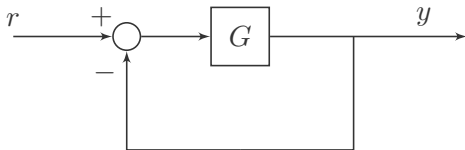- Feedback
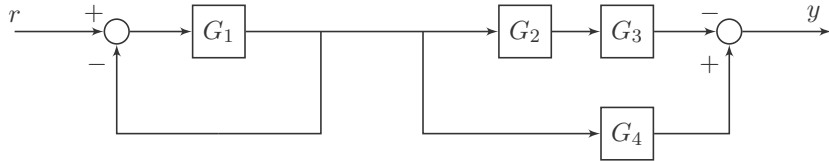- A simple example
- A complex example

Series Connection

Parallel Connection

Feedback Connection

Simple Example

Complex Example