AERO 422: Active Controls for Aerospace Vehicles

Introduction

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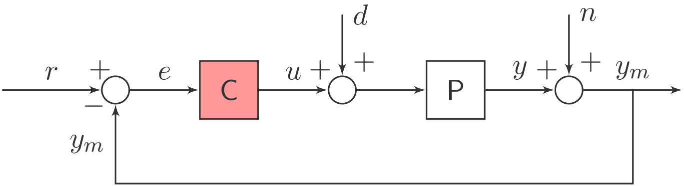
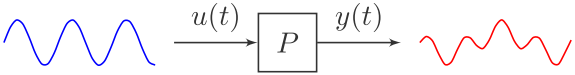
Preliminaries

- Signals & Systems
- Laplace transforms
- Transfer functions – from ordinary linear differential equations
- System interconnections
- Block diagram algebra – simplification of interconnections
- General feedback control system interconnection.



Signals & Systems

Signals & Systems

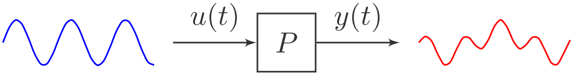


- Actuator applies $u(t)$
- Sensor provides $y(t)$

- Feedback controller takes $y(t)$ and determines $u(t)$ to achieve desired behavior
- The controller is typically implemented as software, running in a micro controller

- Imperfections exist in real world
  - *sensors have noise*
  - *actuators have irregularities*
  - *plant P is not fully known*

System Response to $u(t)$



Given plant P and input $u(t)$, what is $y(t)$?

- P is defined in terms of ordinary differential equations
- $y(t)$ is the forced + initial condition response.

Linear Dynamics

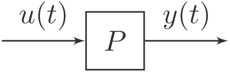
$$m\ddot{x} + c\dot{x} + kx = u(t) \quad \text{dynamics}$$
$$y(t) = x(t) \quad \text{measurement}$$

Nonlinear Dynamics

$$\ddot{x} - \mu(1 - x^2)\dot{x} + x = u(t) \quad \text{dynamics}$$
$$y(t) = x(t) \quad \text{measurement}$$

In this class we focus on linear systems

Linear Systems



- Dynamics is defined by linear ordinary differential equation
- Super position principle applies

$$\begin{array}{l} u_1(t) \mapsto y_1(t) \\ u_2(t) \mapsto y_2(t) \end{array} \implies (u_1(t) + u_2(t)) \mapsto (y_1(t) + y_2(t))$$

Laplace Transforms

Laplace Transforms

Given signal $u(t)$, Laplace transform is defined as

$$\mathcal{L}\{u(t)\} := \int_0^\infty u(t)e^{-st}dt$$

Exists when

$$\lim_{t\to\infty} |u(t)e^{-\sigma t}| = 0, \text{ for some } \sigma > 0$$

Very useful in studying linear dynamical systems and designing controllers

Properties Laplace Transforms

Linear operator

- Additive

$$\begin{aligned}\mathcal{L}\left\{u_1(t)+u_2(t)\right\} &= \int_0^\infty \left(u_1(t)+u_2(t)\right)e^{-st}dt \\ &= \int_0^\infty u_1(t)e^{-st}dt + \int_0^\infty u_2(t)e^{-st}dt \\ &= \mathcal{L}\left\{u_1(t)\right\} + \mathcal{L}\left\{u_2(t)\right\}\end{aligned}$$

- Superposition

$$\mathcal{L}\left\{au(t)\right\} = a\mathcal{L}\left\{u(t)\right\}, \quad a \text{ is a constant}$$

Properties (contd.)

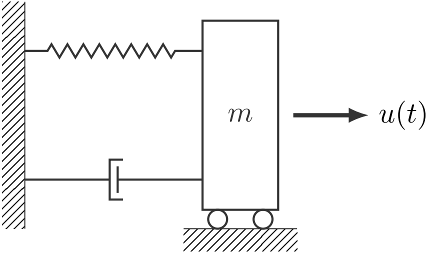
1. $U(s) := \mathcal{L}\{u(t)\}$
2. $\mathcal{L}\{au_1(t) + bu_2(t)\} = a\mathcal{L}\{u_1(t)\} + b\mathcal{L}\{u_2(t)\} = aU_1(s) + bU_2(s)$
3. $\frac{1}{s}U(s) \iff \int_0^t u(\tau)d\tau$
4. $U_1(s)U_2(s) \iff u_1(t) * u_2(t)$ Convolution
5. $\lim_{s\to 0} sU(s) \iff \lim_{t\to\infty} u(t)$ Final value theorem
6. $\lim_{s\to\infty} sU(s) \iff u(0^+)$ Initial value theorem
7. $-\frac{dU(s)}{ds} \iff tu(t)$
8. $\mathcal{L}\left\{\frac{du}{dt}\right\} \iff sU(s) - u(0)$
9. $\mathcal{L}\{\ddot{u}\} \iff s^2U(s) - su(0) - \dot{u}(0)$

Important Signals

1. $\mathcal{L}\{\delta(t)\} = 1$ $\delta(t)$ is impulse function
2. $\mathcal{L}\{1(t)\} = \frac{1}{s}$ $1(t)$ is unit step function at $t = 0$
3. $\mathcal{L}\{t\} = \frac{1}{s^2}$
4. $\mathcal{L}\{\sin(\omega t\} = \frac{\omega}{s^2 + \omega^2}$
5. $\mathcal{L}\{\cos(\omega t\} = \frac{s}{s^2 + \omega^2}$

Transfer Functions

Spring Mass Damper System



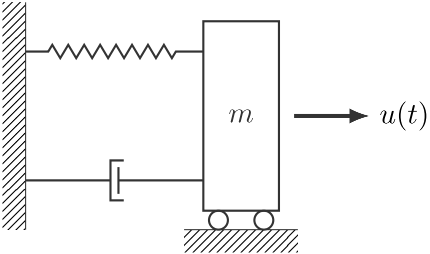
Equation of Motion

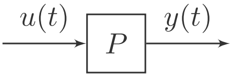
$$m\ddot{x} + c\dot{x} + kx = u(t)$$

Take $\mathcal{L}\left\{\cdot\right\}$ on both sides

$$\mathcal{L}\left\{m\ddot{x} + c\dot{x} + kx\right\} = \mathcal{L}\left\{u(t)\right\}$$
$$m\mathcal{L}\left\{\ddot{x}\right\} + c\mathcal{L}\left\{\dot{x}\right\} + k\mathcal{L}\left\{x\right\} = \mathcal{L}\left\{u(t)\right\}$$
$$m\left(s^2X(s) - sx(0) - \dot{x}(0)\right) + c\left(sX(s) - x(0)\right) + kX(s) = U(s)$$
$$(ms^2 + cs + k)X(s) = U(s)$$ $\dot{x}(0)$ and $x(0)$ are assumed to be zero

Transfer Function





$$(ms^2 + cs + k)X(s) = U(s) \implies \frac{X(s)}{U(s)} = \frac{1}{ms^2 + cs + k}$$

Choose output $y(t) = x(t) \implies Y(s) = X(s)$.
Therefore

$$P(s) := \frac{Y(s)}{U(s)} = \frac{1}{ms^2 + cs + k}$$ Transfer function

Transfer Function (contd.)

In general

$$P(s) = \frac{N(s)}{D(s)}$$

where $N(s)$ and $D(s)$ are polynomials in s

- Roots of $N(s)$ are the zeros
- Roots of $D(s)$ are the poles – determine stability

Response to $u(t)$

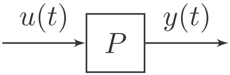
Given

– input signal $u(t)$ and transfer function $P(s)$.

Determine

– output response $y(t)$

1. **Laplace transform**
   $U(s) := \mathcal{L}\{u(t)\}$



2. **Determine** $Y(s) := P(s)U(s)$

3. **Laplace inverse**

$$y(t) := \mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\{P(s)U(s)\}$$

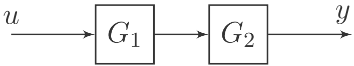
System Interconnection

Block Diagram

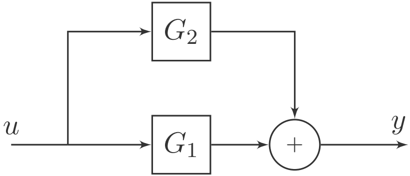
Representation of System Interconnections

- Series
- Parallel
- Feedback
- A simple example
- A complex example

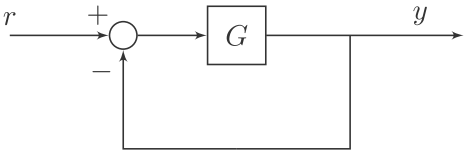
Series Connection



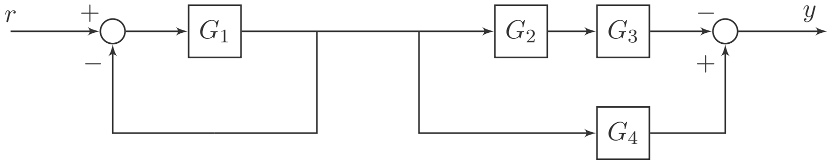
Parallel Connection



Feedback Connection



Simple Example



Complex Example