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Realized GARCH Model in Volatility Forecasting and Option Pricing

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Abstract

We have developed a novel option pricing model that relies on forecasting realized volatility. By incorporating past conditional volatility from the underlying asset based on the GARCH model, we address heteroscedasticity in time-varying realized volatility. To overcome the GARCH model's inability to capture the long-range persistence of volatility in financial time series, our model leverages the additive cascade model for estimating realized volatility components across various frequencies. Easily estimated from historical data, our model's parameters yield forecasts with reduced measurement error and accurately capture the time series pattern of volatility in financial data. Additionally, our model can be adapted as a new option pricing method based on discrete-time stochastic volatility. We obtain martingale measures and option prices through Monte Carlo simulations. In our empirical analysis, we applied this model to the S & P 500 equity index, Nasdaq, and Dow Jones Industrial Average market indices. We also explored the model's application in pricing European options for the S & P 500 market index.

Keywords Realized volatility · Volatility forecasting · High frequency data · Long memory · Option pricing

1 Introduction

To the best of our knowledge, minimal work has been dedicated to option pricing that combines realized volatility and the GARCH model family introduced by Bollerslev (1986). The most renowned and Nobel Prize-winning option pricing model is the Black-Scholes-Merton (BSM) model, but this traditional model Black

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and Scholes (1973); Merton (1973); Black (1975) falls short in accurately measuring real volatility in financial time series data or market prices of underlying assets (Lim et al., 2005). Implied volatility considers the expectation of future volatility, and the option pricing model with implied volatility functions as the risk premiums in the underlying asset. For instance, the volatility parameter in the BSM model is not directly observable, and Fischer Black and Myron Scholes (Black & Scholes, 1973; Merton, 1973) presume that implied volatility is the best estimate of future variability in the underlying asset and that the price of the underlying asset follows geometric Brownian motion, particularly in European options. On the other hand, the BSM model also assumes constant volatility, overlooking liquidity risk in the options market and disregarding situations with significant price fluctuations that lead to volatility clustering. Duan (1995) addresses these challenges by linking the powerful econometric model-generalized autoregressive conditional heteroskedastic (GARCH) model-to option pricing, assuming that asset volatility follows the GARCH process and incorporating implied volatility to tackle the difficulty of observing the variance rate in stochastic volatility option models (Lehar et al., 2002; Badescu & Kulperger, 2008; Hsieh & Ritchken, 2005; Zhang & Zhang, 2020; Fromkorth & Kohler, 2015; Fang et al., 2021; Hua et al., 2018).

Duan's GARCH option pricing model is based on non-Markovian nature, as it uses the GARCH(1,1) process (Engle & Mustafa, 1992); our proposed model in this paper also follows this nature. We extend the GARCH model to realized volatility instead of employing implied volatility estimates. Generally, option pricing in the BSM model is a special case of the GARCH model, with the main difference being homoskedastic asset return processes in BSM but heteroskedastic in GARCH. Härdle and Hafner extended the GARCH option pricing model to the MTGARCH option model, which measures the pattern of volatility closer to the real volatility in financial time series data. Heston and Nandi (2000) improved the GARCH(1,1) process in option pricing to the GARCH(p,q) process and developed a closed-form option valuation formula, which considers the correlation with spot asset returns. Chorro et al. (2012) further extended the GARCH option pricing model to the American option based on generalized hyperbolic innovations. Christoffersen and Jacobs enhanced Heston's model and proposed a two-factor Component GARCH in discrete-time, which improved the performance in terms of Root Mean Squared Error (RMSE) by 26% Christoffersen et al. (2008). The GARCH family's option pricing model performance has also been significantly improved by Christoffersen et al. (2011), as they proposed a new GARCH model with quadratic pricing and presented semi-parametric evidence of a U-shape in volatility, improving performance in terms of RMSE by 14% from Heston's 2000 model (Corsi et al., 2013). However, implied volatility has some drawbacks related to volatility smiles and illiquidity.

In this paper, we employ realized volatility in conjunction with the GARCH volatility forecasting model for option pricing. As Corsi et al. (2013) notes, realized volatility is a non-parametric measure of the variability for the underlying asset and is frequently used in high-frequency data due to its ease of computation compared to implied volatility. The intraday price movement offers valuable information about realized volatility, enabling the model to quickly respond to changing market conditions. This is because it depends on measuring historical volatility for the underlying asset, and filtering procedures are not required for realized volatility option pricing. In recent years, models linking options and realized volatility have gained popularity. Barndorff-Nielsen and Shephard (2002) investigated the relationship between traditional econometric models, such as the Autoregressive-Moving-Average (ARMA) model, and realized volatility in intraday data. With advancements in computer and communication technology, high-frequency trading has emerged as a profitable opportunity for large financial institutions. Numerous researchers have proposed volatility models for high-frequency data. Bandi et al. (2008) introduced high-frequency data in option pricing, demonstrating that realized volatility more reliably captures the microstructure within financial time series data. Christensen and Prabhala (1998) provided substantial evidence supporting the application of realized volatility in option pricing across various time series models.

Corsi (2009) proposed an effective realized volatility model called the Heterogeneous Autoregressive model of Realized Volatility (HAR-RV), which comprises three volatility components: daily, weekly, and monthly realized volatility calculated by summing intraday data returns, such as one-minute frequency financial data. Corsi modeled the relationship between latent partial volatility-volatility generated by a specific market-and realized volatility, exhibiting strong performance in volatility forecasting. Inspired by the HAR-RV and Hansen's RealGARCH model Hansen et al. (2012), we propose a high-frequency realized volatility model combined with GARCH for forecasting and option pricing. We gathered extensive high-frequency data for the S &P 500, Nasdaq 100, and Dow Jones, each containing more than ten years of trading history. Latent volatility is calculated using one-minute frequency realized volatility, with the construction process projecting one-minute volatility into daily, weekly, and monthly time horizons. The standard GARCH model's lagged volatility serves as the latent volatility for conducting forecasting, and we use Monte Carlo simulation to construct the option pricing model.

The remainder of this paper is organized as follows: Sect. 2 constructs the GARCH-realized volatility model and demonstrates its application in option pricing. 3 presents the empirical analysis and discussion. Section 4 summarizes the new findings and offers conclusions.

2 Methodology

In this section, we introduce our volatility forecasting model that combines the GARCH and HAR models. Our model captures the information on variations of underlying asset prices across different frequencies and extracts all fluctuation information from the time series data. Moreover, we construct a hybrid GARCH and HAR model for option pricing and derive an equivalent martingale measure for various maturities using Monte Carlo simulations.

2.1 Assumption of Observable Volatility

In this initial subsection, we lay the groundwork for our model by addressing the assumption of observable volatility in financial markets. This assumption is crucial as it directly influences the subsequent development of our volatility forecasting and option pricing models.

Volatility, while inherently latent and unobservable, can be approximated through various proxies. In our framework, we consider the realized volatility (RV) as a measurable proxy for latent market volatility. This approach is supported by extensive empirical research indicating that RV, derived from high-frequency financial data, offers a robust approximation of true market volatility.

The rationale behind using RV as a proxy stems from its ability to capture the market's consensus on volatility, as reflected in the pricing of financial instruments. Furthermore, RV has been empirically validated to provide a consistent measure of latent integrated variance, particularly in markets with minimal microstructure noise and price jumps. This consistency makes RV an effective tool for volatility estimation in our model.

The assumption of observable volatility through RV is not only theoretically sound but also practically applicable. Financial markets generate a wealth of high-frequency trading data, allowing for the accurate computation of RV. This data availability, coupled with advanced computational techniques, enables the practical implementation of RV-based volatility measurement in real-world financial scenarios.

In the following sections, we will demonstrate how this assumption of observable volatility via RV plays a pivotal role in constructing our hybrid GARCH and HAR model for option pricing. The model leverages RV to extract comprehensive fluctuation information from time series data, facilitating effective volatility forecasting and option valuation.

2.2 Volatility Model

Firstly, based on Barndorff-Nielsen and Shephard (2002) and Andersen et al. (2003), the realized volatility (RV) is calculated as sum of the squared returns over the trading day:

$$\mathrm{RV}_t = \sqrt{\sum_{j=1}^{M} (r_{t,j})^2} \tag{1}$$

where M is the total number of intraday returns for the day and $r_{t,j}$ is j-th intraday return for the day t and RV_t is daily volatility. Under the ideal conditions where the microstructure noise is not exist or without price jumps, the RV provides an asymptotically consistent measure of the latent integrated variance (Sharma, 2016). Another advantage of realized volatility is that the multi-period volatilities are average of the one-period realized volatility which is given as:

$$\mathrm{RV}_t^n = \frac{1}{N}\left(\mathrm{RV}_{t1} + \mathrm{RV}_{t2} + \cdots + \mathrm{RV}_{tn}\right) \tag{2}$$

where N is the number of the trading days in one period, such as the weekly volatility is easy to be calculated by average of the daily volatility which $N = 5$. The latent volatility-in GARCH theories, the volatility is always changing and that it is therefore not directly observable, the variableσ_t can be expressed as:

$$\sigma_t = \mathrm{RV}_t + \omega_t \tag{3}$$

where ω_t is zero mean and independent volatility innovations $\omega_t \sim$ i.i.d $(0, \sigma_{\omega_t})$ and an alternative way to ensure positiveness of partial volatilities would be to write the model in terms of the log of RV.

To illustrate our framework, consider a most simple GARCH-HAR(1, 1) model, this model is based on the three-factor stochastic volatility model, which is given by the following two equations:

$$\begin{aligned} h_{t+1} &= c + \alpha_1 h_t + \gamma \sigma_t \\ \mathrm{RV}_t &= c + \beta^d \mathrm{RV}_t^d + \beta^w \mathrm{RV}_t^w + \beta^m \mathrm{RV}_t^m + \omega_t \end{aligned} \tag{4}$$

In this equation, RV_t^d, RV_t^w, and RV_t^m are the multi-period realized volatility for daily, weekly, and monthly frequencies, respectively. In the GARCH-HAR model, these variables are related to the realized volatility of the previous day, week, and month. Since the target prediction h_{t+1} is daily, the GARCH-HAR model shares similarities with the autoregressive (AR) model but requires far fewer parameters to be estimated. This is because the GARCH-HAR model calculates the rolling sum of volatility for the previous 22 days, summarizing the information about changes in asset prices over the last month. The GARCH-HAR model is more flexible in applying volatility analysis to financial time series data and performs better for realized measures than the original HAR model. In terms of multi-period volatility forecasting, the prediction result of h_{t+1} is used to update the realized volatility terms RV and continues to conduct rolling basis forecasting. By simplifying Eq. (4), we obtain the following equation, which is much easier to estimate.

$$h_{t+1} = c + \alpha_1 h_t + \beta^d \mathrm{RV}_t^d + \beta^w \mathrm{RV}_t^w + \beta^m \mathrm{RV}_t^m + \omega_t \tag{5}$$

The return process based on σ_t in financial data is given as:

$$r_t = \mu + \sigma_t e_t \tag{6}$$

where $e \sim$ i.i.d $(0, 1)$.

2.3 Deriving Option Price with Estimated Overall Volatility

This subsection delves into the process of option pricing in a context where overall market volatility is unobservable. While historical data allows us to calculate realized volatility (RV), we acknowledge that the total volatility, which includes unobservable components, needs to be estimated for effective option pricing.

Our approach combines the Monte Carlo simulation with the cascade model to estimate this overall volatility. The cascade model, utilizing RV derived from observable historical data, helps in capturing the underlying volatility dynamics across different time horizons. However, recognizing that RV does not encompass all aspects of market volatility, especially the unobservable components, we employ Monte Carlo simulations to bridge this gap.

The Monte Carlo method simulates a multitude of potential future market scenarios, incorporating both the observed RV and additional stochastic elements that represent the unobservable aspects of volatility. This comprehensive approach allows us to estimate the fair price of options more accurately, taking into account the entire spectrum of market volatility.

In the following subsection, by integrating both observable RV and estimations for the unobservable components, our model aims to provide a more robust and realistic framework for option pricing, reflecting the complex nature of financial markets.

2.4 Realized GARCH Option Pricing Model

In this subsection, we build a new option pricing model based discrete time series and the realized volatility GARCH model. The GARCH option pricing model firstly proposed by Duan (1995). Denote the S_t is price of the underlying asset-which is stock price in our paper at time t, the continuously compounded returns y_t is given as following:

$$y_t = \ln \frac{S_t}{S_{t-1}} \tag{7}$$

Also, denote the h_t is the conditional variance of the logarithmic return over the period between t and $t-1$, which is the period of one day in reality. We adapt the GARCH option pricing by Duan (1995) with the combination of return process modeling by realized volatility in different frequencies, and the y_t are modeled by:

$$\begin{aligned} y_t &= r_f + \lambda\sqrt{h_t} - \frac{1}{2}h_t + \beta^d \mathrm{RV}^d_{t-1} + \beta^w \mathrm{RV}^w_{t-1} + \beta^m \mathrm{RV}^m_{t-1} + h_t e_t \\ e &\sim N(0,\ 1) \\ h_t &= c + \alpha_1 h_{t-1} + \gamma \epsilon_{t-1} \end{aligned} \tag{8}$$

where r_f is the risk-free rate, $e \sim$ i.i.d with a standard normal distribution, and λ can be interpreted as the unit risk premium. Denote the innovation $\epsilon_t = h_t e_t$ with zero mean and conditional variance h_t. This model assumes that the returns are drawn from a normal distribution with time-dependent volatility h_t. In financial time series data, the return of the underlying asset is more sensitive to factors such as monetary policies, geopolitics, or economic cycles. Therefore, using realized volatility with frequencies longer than one day is reasonable for modeling conditional variance. Unlike the standard GARCH, which considers short-term volatility clustering, our model uncovers systematic risk from various patterns, ranging from long-term (monthly) to short-term (daily). Furthermore, our model can capture heterogeneity, especially the heterogeneity that arises from various reasons and different time horizons. This is because Corsi (2009) pointed out that heterogeneity originates from factors such as agents' endowments, institutional constraints, temporal horizons, and geographical locations. Our model encourages the community to analyze the inter-day volatility change for financial asset returns in greater detail and price options by considering fluctuating features across different time horizons.

Duan (1995) provided the theory of locally risk-neutral valuation relationship (LRNVR), which has proven that measure Q is mutually absolutely continuous with respect to measure P. That paper demonstrated that the conventional risk-neutral valuation relationship must be generalized to accommodate the heteroskedasticity of the asset return process. The LRNVR is given by the following equations:

$$\begin{aligned} &E^Q\left(S_t/S_{t-1} \mid \mathcal{F}_{t-1}\right) = e^r \qquad E^Q\left(y_t \mid \mathcal{F}_{t-1}\right) = r \\ &\mathrm{Var}^Q\left(y_t \mid \mathcal{F}_{t-1}\right) = \mathrm{Var}^P\left(y_t \mid \mathcal{F}_{t-1}\right) \end{aligned} \tag{9}$$

where S_t/S_{t-1} distributes log-normally under measure Q, it is equal to the conditional variances under measure P. The option pricing model based on measure Q under LRNVR is given as following:

$$\begin{aligned} &y_t = r_f - \frac{1}{2}h_t + \xi_t \\ &\xi_t \mid \mathcal{F}_{t-1} \sim N(0, h_t) \\ &h_t = c + \alpha_1\left(\xi_{t-1} - \beta^d \mathrm{RV}^d_{t-1} - \beta^w \mathrm{RV}^w_{t-1} - \beta^m \mathrm{RV}^m_{t-1} - \lambda\sqrt{h_{t-1}}\right)^2 + \gamma \in_{t-1} \end{aligned} \tag{10}$$

Option pricing based on the Eq. (10) is conducted by the Monte Carlo simulation. Because it is really difficult to derive the distribution for our option pricing model analytically, the Monte Carlo simulation used in this paper is similar to Duan (1995). We simulated the parameter h_t and ξ_t in in the period length of option maturity. Denote the exercise price for European call option is K in the maturity of time $T-t$ and the stock price in the maturity denoted as S_T, the price of underlying asset under GARCH specification equal to:

$$S_T = S_t \exp\left[(T-t)r_f - \frac{1}{2}\sum_{i=t+1}^{T} h_i + \sum_{i=t+1}^{T} \xi_i\right] \tag{11}$$

Assuming the simulation m as large as possible, then the European call option price is obtained by:

$$C_t = e^{-(T-t)r_f}\frac{1}{m}\sum_{i=1}^{m} \max\left(S_T - K, 0 \mid \beta\right) \tag{12}$$

3 Empirical Analysis

In this section, we present the empirical analysis and discussion for volatility forecasting and option pricing based on the model we proposed in this paper. We utilize the predicted results with loss functions to evaluate the daily volatility forecasting performance of our model. Patton (2011) noted that only the mean squared error (MSE) and quasi-likelihood (QLIKE) loss functions are robust to imperfections in the volatility proxy. To evaluate the performance of the volatility model, we employ the MSE to assess the in-sample fits.

$$\mathrm{MSE}\left(\mathrm{RV}_{t+1}, h_{t+1}\right) = \left(\mathrm{RV}_{t+1} - h_{t+1}\right)^2 \tag{13}$$

where the h_t refers to the fits from our model in Eq. (5), and compared to the standard GARCH(1, 1) model. We also calculate the one-day-ahead out of sample forecast, we compared our model with GARCH(1, 1) in MSE and QLIKE loss:

$$\mathrm{QLIKE}\left(\mathrm{RV}_{t+1}, h_{t+1}\right) = \frac{\mathrm{RV}_{t+1}}{h_{t+1}} - \ln \frac{\mathrm{RV}_{t+1}}{h_{t+1}} - 1 \tag{14}$$

The volatility forecasting model, as shown in Eq. (5), can be estimated iteratively, provided an initial normal random value for h, denoted as h_0. The parameters c, α_1, β^d, β^w, and β^m can be easily estimated through maximum likelihood estimation. We used over 200,000 rows of high-frequency trading history for three market indexes and 17 individual stocks listed in the United States, including S &P 500, Nasdaq 100, and Dow Jones, each utilizing 1-min frequency data.

The option pricing model is based on European call options, and only call option prices are calculated. The price of the European put option can be determined using the Put-Call parity. We also employed the Monte Carlo method to simulate price paths, with each simulation including $m = 10,000$ sample paths. We computed option prices with different maturities and compared the results with the standard Black-Scholes model, including the price-bias.

3.1 Volatility Forecasting

Figure 1 displays the daily, weekly, and monthly realized volatility for the S &P 500, Nasdaq, and Dow Jones indices. The Dow Jones has been the most volatile over the past year, making it particularly challenging to model without a long-memory model. In this subsection, we present the in-sample fit and out-of-sample forecasting, including one-period-ahead forecasts and multi-period rolling forecasts with a window size of $s = 5$. We compared our realized GARCH model with different parameters (p, q) in the GARCH model and HAR model using the loss functions of MSE and QLIKE. To conserve space, we only report the mean values for the five windows in the rolling forecast.

Table 1 presents the summary statistics for the daily realized volatility in the three market indices and 17 individual stocks. All of the returns exhibit excess kurtosis, indicating a fat-tailed distribution. Figure 2 illustrates the in-sample fit of MSE for the three market indices and 17 individual stocks. The realized GARCH model has the lowest MSE error in most cases compared to other volatility forecasting models, as the realized volatility is more reliable for modeling high-frequency data. As expected, the realized GARCH model enhances the HAR model, since the realized GARCH model partially relies on the autoregressive component of h_{t+1} through the

Fig. 1 The realized volatility for three market indexes in the United States

coefficient of α_1. Realized GARCH also outperforms the standard GARCH because it incorporates the coefficients β^d, β^w, β^m of realized volatility components across different time horizons.

Also, the improvements afforded by the realized GARCH models are depending on the choice of time horizon, it gives the flexibility to the user to determine how many β coefficients are required to model high frequency data. In general, more high-frequency observations help reduce the magnitude of the measurement errors.

Table 3 provides the MSE and QLIKE comparison in terms of one period ahead out-sample forecasting, the demonstrated figures are present as percentages. The results suggested that the realized GARCH model is powerful in the high-frequency volatility analysis and able to capture the short-term autocorrelation structure, this model also captures the structure over a much longer horizon.

Table 4 also highlights the average of MSE and QLIKE in between one period to five periods out-sample forecasting. The realized GARCH also outperforms the standard GARCH and the HAR model in the out sample prediction, as lower values of both loss functions. The results confirm that at the high-frequency stock price data, the

Table 1 Summary statistics for the realized volatility of selected financial assets

Assets	Obs	#RV[d]	Mean	Std	Skew	Kurt	Median
SP500	200000	515	0.0773	0.0486	2.1638	6.074	0.0621
Nasdaq100	200000	515	0.3198	0.1641	1.9997	6.1482	0.2703
Dow Jones	200000	496	0.592	0.4198	8.7093	126.2334	0.4922
NVDA	300484	500	0.0191	0.0085	0.5679	0.4113	0.0186
TMUS	205860	500	0.0056	0.0027	2.0561	6.7517	0.0049
ZM	249784	500	0.0218	0.0111	2.4524	15.7901	0.0198
TSLA	369839	500	0.042	0.0293	2.3588	7.7273	0.0318
AAPL	203192	247	0.0027	0.001	2.0701	5.8508	0.0024
AMZN	242552	500	0.114	0.0462	1.6967	4.7355	0.1048
GS	216814	500	0.0132	0.0047	1.1215	2.1786	0.0125
JPM	252702	500	0.005	0.0022	3.9962	29.8384	0.0045
FB	296024	500	0.0101	0.0032	1.5729	4.5629	0.0095
EBAY	210432	500	0.0031	0.0013	2.136	8.1207	0.0029
INTC	288747	500	0.0024	0.0013	2.2943	7.0533	0.0019
SBUX	224006	500	0.004	0.0017	1.92	4.381	0.0036
QCOM	235095	500	0.0063	0.0025	1.8617	6.9384	0.0056
PYPL	244668	500	0.011	0.0042	1.7213	8.0704	0.0102
NFLX	234281	500	0.0244	0.0091	2.3227	9.8878	0.0229
REGN	181890	500	0.036	0.0159	1.8018	4.4936	0.0325
TXN	204928	500	0.0073	0.0033	2.3116	7.934	0.0065

realized GARCH is far more persistent in realized volatility measurement and inter-day volatility forecast than the standard HAR model and traditional GARCH model. The accuracy of daily volatility prediction is the very first step to constructing the option pricing model, the methodology will show in the following section.

3.2 Option Pricing

In this section, we apply our realized GARCH model to price European options using Eqs. (10) and (12). The Monte Carlo simulation method is highly convenient for derivative pricing, as it simulates the sample path of the underlying asset and the payoff of the derivative contract. It works well for complex option pricing without an analytical solution. We price European call options and use Put-Call parity to calculate European put option prices. We denote the exercise price as K and the maturity of a European call option as T. We assume the volatility is based on the realized GARCH proposed in this paper, and the price of the underlying asset follows:

$$S_{T,i} = S_t \exp \sum_{j=t+1}^{T} y_{j,i} \tag{15}$$

Table 2 In sample fit (MSE). 'R'GARCH' for 'Realized GARCH', 'G(1, 1)' for 'GARCH(1, 1)', 'G(1, 0)' for 'GARCH(1, 0)', and 'G(2, 1)' for 'GARCH(2, 1)'

Assets	R'GARCH	G(1, 1)	G(1, 0)	G(2, 1)	HAR
SP500	**0.003**	0.0415	0.0418	0.0414	0.0241
Nasdaq100	**0.0423**	0.1543	0.1527	0.1543	0.0941
Dow Jones	0.3225	0.2435	0.2407	0.2435	**0.1636**
AAPL	**0.0002**	0.001	0.001	0.001	0.0008
AMZN	**0.0034**	0.0453	0.0453	0.0453	0.038
GS	**0.0009**	0.0041	0.0041	0.0041	0.0031
JPM	**0.0007**	0.0013	0.0013	0.0562	0.0011
FB	**0.0008**	0.003	0.003	0.003	0.0028
EBAY	**0.0002**	0.0013	0.0013	0.0013	0.0011
INTC	**0.0003**	0.0008	0.0008	0.0008	0.0006
NVDA	**0.0013**	0.0094	0.0093	0.0094	0.0051
TMUS	**0.0002**	0.0026	0.1035	0.0075	0.0021
ZM	0.011	0.0109	**0.0108**	0.0109	0.0181
TSLA	0.0107	0.0174	0.016	0.0174	**0.009**
SBUX	**0.0006**	0.0012	0.0012	0.0012	0.0011
QCOM	**0.0003**	0.0026	0.1453	0.0026	0.0019
PYPL	**0.0005**	0.004	0.0041	0.004	0.0034
NFLX	**0.0008**	0.0085	0.0086	0.0085	0.0073
REGN	0.0128	0.0164	0.0161	0.0164	**0.0113**
TXN	**0.0003**	0.0033	0.0034	0.0033	0.0026

The definition of bold is smallest MSE in the row

where $S_{T,i}$ is the price of the underlying asset at maturity time T and the ith simulation by the Monte Carlo method, and $y_{j,i}$ is the return at time t. The data used in this section are the one-min high-frequency data of the S &P 500 and the European call options on the market indices. Moreover, we present the option price of the realized GARCH and compare it to the BS model and the standard GARCH(1, 1). We compare performances across different maturities and for in-the-money and out-of-the-money options using different exercise to current price ratios (K/S). The price bias or error criterion Υ is calculated as follows:

$$\begin{aligned} \upsilon_r &= \left| \frac{\hat{V}_r - V_r}{V_r} \right| \\ \Upsilon &= \sum_{r=1}^{R} \upsilon_r^2 \end{aligned} \tag{16}$$

where $\hat{V}_r$ is the option price given by the models and the V_r is the real market value, Υ is the overall price biased in different maturity. The S &P 500 option is represented as the S &P 500 Index ($SPX) in the market. Figure 2 demonstrated the daily returns and the one-minute frequency returns for the underlying asset S &P 500.

Table 5 test the Monte Carlo Simulation and provided the summary statistics for the results with sample $m = 50,000$ and initial price of \$100, we conduct the

Table 3 Comparison of Mean Squared Error (MSE) and Quasi-Likelihood (QLIKE) for one-period forecasting across various financial assets (%). 'R'GARCH' for 'Realized GARCH', 'G(m, n)' for 'GARCH(m, n)' as above

	Assets	R'GARCH	G(1, 1)	G(1, 0)	G(2, 1)	HAR
	SP500	0.01	6.6811	6.7925	6.661	1.2073
	Nasdaq100	0.6208	23.2803	22.796	23.2803	3.3699
	Dow Jones	33.035	41.8978	40.7692	41.8978	8.0257
	AAPL	0.0101	0.007	0.0202	0.008	0.1519
	AMZN	0.0115	2.7284	2.7283	2.7287	5.0053
	GS	0.0101	0.2384	0.2514	0.2417	0.4185
	JPM	0.1397	0.1397	0.1652	0.1555	0.1611
	FB	0.01	0.0289	0.0193	0.0181	0.0604
	EBAY	0.0101	0.065	0.0714	0.0662	0.069
	INTC	0.0301	0.0639	0.0639	0.0639	0.0412
MSE	NVDA	0.0104	1.2666	1.2387	1.269	0.6075
	TMUS	0.0101	0.134	0.1374	0.1404	0.1041
	ZM	0.0103	0.7561	0.7908	0.7342	0.2604
	TSLA	1.1748	1.8534	1.4913	1.854	0.1113
	SBUX	0.0932	0.0909	0.1037	0.0989	0.1105
	QCOM	0.0101	0.0204	31.1517	0.0176	0.3105
	PYPL	0.0101	0.5385	0.5245	0.5436	0.3635
	NFLX	0.0102	0.1614	0.1333	0.1614	0.3795
	REGN	1.8566	0.3583	0.5589	0.3643	0.7087
	TXN	0.0108	0.1397	0.1364	0.1436	0.1667
	SP500	0.01	36.2092	38.3117	35.8415	0.5148
	Nasdaq100	0.008	26.8475	25.1316	26.8476	0.2652
	Dow Jones	4.4289	24.591	22.6076	24.5909	0.4519
	AAPL	0.0817	0.0401	0.3102	0.0518	10.058
	AMZN	0.0001	3.7815	3.7813	3.7821	9.8011
	GS	0.0047	1.9834	2.1753	2.0318	5.1083
	JPM	6.2802	6.2814	8.1268	7.4154	7.8287
	FB	0.0044	0.0386	0.017	0.0149	0.1514
	EBAY	0.0755	2.4226	2.8469	2.4969	2.682
	INTC	2.065	7.1444	7.1444	7.1444	3.5273
QLIKE	NVDA	0.0074	31.5548	30.761	31.6217	12.3236
	TMUS	0.0325	3.9593	4.1262	4.2739	2.5861
	ZM	0.0022	6.9691	7.4681	6.6586	1.1509
	TSLA	2.6957	19.3652	10.5078	19.3829	0.0324
	SBUX	3.8096	3.6562	4.5392	4.2033	5.0365
	QCOM	0.014	0.0609	NaN	0.0453	7.6643
	PYPL	0.014	16.8443	16.2604	17.0579	9.6952
	NFLX	0.0011	0.2436	0.1689	0.2436	1.1962
	REGN	10.5251	0.6588	1.4784	0.6796	2.2442
	TXN	0.0184	2.3184	2.2257	2.43	3.1369

This table provides a concise overview of the performance metrics for each model and asset

Table 4 MSE and QLIKE comparison-average of five periods forecasting (%). 'R'GARCH' for 'Realized GARCH', 'G(m, n)' for 'GARCH(m, n)' as above

	Stocks	R'GARCH	G(1, 1)	G(1, 0)	G(2, 1)	HAR
	SP500	0.01	7.0948	7.1988	7.058	0.9529
	Nasdaq100	2.5941	24.6617	24.1809	24.6618	2.0906
	Dow Jones	33.9054	46.069	44.9247	46.0687	8.311
	AAPL	0.0101	0.2175	0.2091	0.1439	0.1234
	AMZN	0.0105	4.2396	3.9156	3.9158	2.9001
	GS	0.0101	0.5655	0.5581	0.5638	0.4465
	JPM	0.092	0.49	0.1093	0.1079	0.1122
	FB	0.01	0.3259	0.3243	0.3239	0.2356
	EBAY	0.0101	0.1455	0.0816	0.0832	0.0707
	INTC	0.0233	0.0451	0.0451	0.0451	0.0293
MSE	NVDA	0.0103	0.8133	0.7782	0.8152	0.4085
	TMUS	0.0101	0.1241	17.7596	0.1248	0.1248
	ZM	0.0101	0.6514	0.6846	0.6296	0.2311
	TSLA	1.2216	2.1774	1.9536	2.2008	0.6461
	SBUX	0.061	0.0889	0.0879	0.0886	0.0896
	QCOM	0.0101	0.3291	2.3708	0.3299	0.2017
	PYPL	0.01	0.3122	0.2588	0.316	0.2374
	NFLX	0.0101	0.5128	0.5416	0.5252	0.3873
	REGN	1.7201	0.3177	0.4361	0.3196	1.4723
	TXN	0.0105	0.1461	0.1491	0.1494	0.1439
	SP500	0.001	39.9408	42.0338	39.2246	0.0287
	Nasdaq100	0.1218	29.55	27.7366	29.5504	0.0095
	Dow Jones	4.3538	28.7157	26.4925	28.7152	0.0428
	AAPL	0.0367	22.4323	19.6527	6.6926	0.469
	AMZN	0.0	6.6938	5.4302	5.4307	0.2796
	GS	0.0027	7.463	7.2101	7.4035	0.4345
	JPM	2.4255	23.968	3.192	3.1211	0.3309
	FB	0.0036	3.6536	3.6058	3.5932	0.162
	EBAY	0.0479	13.7525	2.7791	2.8994	0.205
	INTC	1.1169	3.4216	3.4216	3.4216	0.1681
QLIKE	NVDA	0.0034	11.2621	10.6411	11.2953	0.4319
	TMUS	0.0227	3.0606	263.0304	3.0921	0.305
	ZM	0.0019	5.0159	5.4242	4.7531	0.0861
	TSLA	2.7915	24.8638	17.7609	25.7322	0.0874
	SBUX	1.3204	2.5203	2.4605	2.5056	0.2558
	QCOM	0.0079	11.1228	11.1131	11.1984	0.2775
	PYPL	0.0072	5.2485	3.9875	5.3399	0.3476
	NFLX	0.0008	1.9097	2.1823	2.024	0.1046
	REGN	8.5389	0.4808	0.8758	0.4867	0.596
	TXN	0.0118	1.992	2.0803	2.088	0.1908

simulation in different maturities including 30, 60, 90, 180 and 270 days. The column path is the sample path for the price and h_t and ξ, the row of price is S_T in each simulation, we calculated the range, mean, and quartiles for the 50,000 simulated underlying asset price in time T. The row of Sum Sq and Sum ξ is the sum of h_t and ξ from Eq. (10), because the results of Monte Carlo Simulation for h_t and ξ is a matrix format, the matrix size is maturity $(T - t) \times 50,000$. We take the column sum of those matrix, and calculate the statistics for the h_t and ξ.

Table 5 clearly presents that the mean of ξ is really close to 0. The results suggest that the parameters estimation and Monte Carlo simulation method are reliable to be further used for option pricing, we will upload the entire results separately. To compare the real market value of the European option, we used the midpoint of the bid-ask spread to be the real value and calculated the price bias with different models. Tables 6 and 7 show the European call option prices calculated by the realized GARCH model denoted as RG and provide the comparison with the standard GARCH (SG) and the standard Black Scholes model (BS). The option price biases calculated by the Eq. (16), denoted as the RG_υ, SG_υ and BS_υ for different models. The results demonstrated that the option price by the realized GARCH model outperforms the standard GARCH and the BS model with lower price biases. Those results also suggest that the daily, weekly, and monthly realized volatility are better explains the short-term and long-term series pattern for the time-varying volatility rather than the simple GARCH(1, 1) model or the BS model of homoscedasticity of asset return. Realized GARCH option pricing model is expected to generate more flexibility, particularly for the market of frequently changing volatility. Similar to the GARCH option pricing model from Duan (1995), our model also provides the measurement of initial conditional variance on the price of the option, it is also estimated through

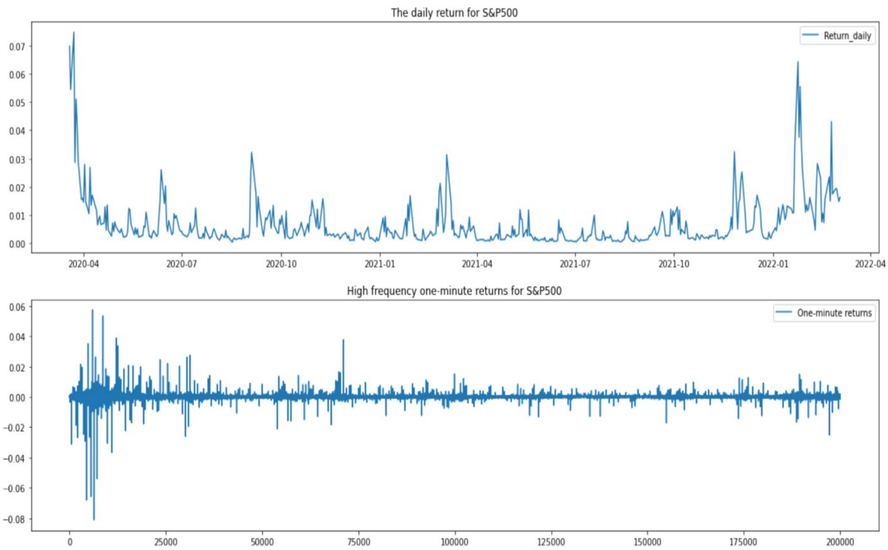

Fig. 2 The daily return and one-minute frequency returns for S & P 500

Table 5 Summary statistics for the results from Monte Carlo Simulation in different maturities

T	Path	Min	Max	Mean	Q1	Median	Q3
	Price	72.1656	132.5361	100.4133	95.9019	100.327	104.8254
30	Sum Sq	0.0036	0.0095	0.0044	0.0042	0.0044	0.0046
	Sum ξ	−0.326	0.2823	0.0	−0.0437	0.0014	0.0452
	Price	65.4107	146.9249	100.8933	94.6129	100.5367	106.8669
60	Sum Sq	0.007	0.0126	0.0081	0.0078	0.0081	0.0084
	Sum ξ	−0.4282	0.3807	0.0006	−0.0594	0.0012	0.0622
	Price	52.3332	163.8658	101.3259	93.6564	100.8422	108.4157
90	Sum Sq	0.0105	0.0187	0.0119	0.0115	0.0118	0.0122
	Sum ξ	−0.6505	0.4872	0.0009	−0.0717	0.002	0.0744
	Price	52.5964	189.2265	102.5514	91.6235	101.4916	112.3679
180	Sum Sq	0.021	0.0293	0.0231	0.0227	0.0231	0.0235
	Sum ξ	−0.6548	0.6247	0.0006	−0.1005	0.0017	0.1035
	Price	47.1819	215.2826	103.7612	90.053	102.1284	115.5743
270	Sum Sq	0.0317	0.0402	0.0343	0.0338	0.0342	0.0348
	Sum ξ	−0.7701	0.7467	−0.0002	−0.1245	0.0012	0.1249
	Price	41.3553	241.9282	105.2047	89.0883	102.9147	118.725
360	Sum Sq	0.0427	0.0502	0.0454	0.0448	0.0453	0.0459
	Sum ξ	−0.908	0.8566	0.0012	−0.1421	0.0021	0.1449

the MLE or algorithm implementation. Intuitively, if the initial volatility of the underlying asset at the time of option valuation is high (low), the prices of the option will be written relatively more (less).

Table 8 provided the error criterion Υ for the realized GARCH, and standard GARCH and the Black Scholes, it is obvious that our model has the lower error than other models.

Rubinstein (1985) presented that the implied volatility of traded options has the systematic pattern in different strike prices and maturities, which is also called the volatility smile. We utilized the Black Scholes model to invert the option prices calculated by our realized GARCH model, and Fig. 3 shows the annualized implied volatility for different maturities. When T = 30, 60, 180 days, it demonstrates U-shape implied volatility phenomenon, volatility smile is less obviously in the maturity of 360 days.

Based on all the aforementioned theorems, calculations, and tables, it is evident that incorporating realized volatility with the standard GARCH model enhances volatility modeling. Indeed, it offers an options pricing model with fewer price biases compared to the traditional Black Scholes model and the standard GARCH model. This is because it efficiently extracts different time horizon volatilities from high-frequency data, which reflect changes in the underlying asset price. The innovation and effectiveness of the realized GARCH option pricing model are technically sound and validated by all the presented results.

Table 6 The option price biases and the option prices by realized GARCH (RG), standard GARCH (SG) and Black-Scholes model (BS)—the maturity of 30, 60, and 90 days

T	(K/S)	RG	RG_{υ}	SG	SG_{υ}	BS	BS_{υ}
	0.9889	130.1986	0.0725	136.2511	0.1223	139.3533	0.1479
	0.99	127.2042	0.0785	133.3027	0.1302	136.4085	0.1565
	0.9911	124.2521	0.0847	130.3919	0.1383	133.502	0.1654
	0.9922	121.3422	0.0917	127.5206	0.1473	130.6337	0.1753
	0.9933	118.4744	0.099	124.6908	0.1567	127.8039	0.1856
	0.9944	115.6465	0.1061	121.9014	0.166	125.0126	0.1957
	0.9955	112.8584	0.1147	119.1501	0.1768	122.2598	0.2075
	0.9966	110.112	0.1224	116.4386	0.1869	119.5457	0.2186
	0.9978	107.4069	0.1312	113.7676	0.1982	116.8703	0.2309
	0.9989	104.744	0.1416	111.1355	0.2113	114.2336	0.2451
30	1.0	102.1229	0.1513	108.5442	0.2237	111.6356	0.2586
	1.0011	99.5453	0.1622	105.9933	0.2375	109.0763	0.2735
	1.0022	97.0096	0.1737	103.4822	0.2521	106.5557	0.2892
	1.0033	94.5179	0.1852	101.0122	0.2666	104.0738	0.305
	1.0044	92.0706	0.1981	98.5809	0.2828	101.6306	0.3225
	1.0055	89.6649	0.2117	96.1874	0.2998	99.2259	0.3409
	1.0066	87.3006	0.2261	93.8327	0.3179	96.8596	0.3604
	1.0077	84.9766	0.2405	91.5189	0.336	94.5318	0.38
	1.0088	82.6978	0.2568	89.244	0.3563	92.2423	0.4019
	1.0099	80.4591	0.2731	87.0077	0.3767	89.991	0.4239
	1.011	78.2625	0.2915	84.8131	0.3996	87.7777	0.4485
	0.9889	181.0675	0.118	187.6515	0.1587	190.6959	0.1775
	0.99	178.1542	0.1236	184.7982	0.1656	187.8338	0.1847
	0.9911	175.2716	0.129	181.97	0.1721	184.9987	0.1916
	0.9922	172.4171	0.1351	179.1695	0.1795	182.1908	0.1994
	0.9933	169.5903	0.1416	176.3957	0.1875	179.41	0.2077
	0.9944	166.7917	0.1479	173.65	0.1951	176.6564	0.2158
	0.9955	164.0203	0.1547	170.9328	0.2033	173.93	0.2244
	0.9966	161.2768	0.1615	168.2412	0.2117	171.2309	0.2332
	0.9978	158.562	0.1689	165.5767	0.2206	168.559	0.2426
	0.9989	155.8781	0.1764	162.9383	0.2297	165.9143	0.2522
60	1.0	153.2215	0.185	160.3257	0.24	163.2969	0.2629
	1.0011	150.5966	0.1933	157.7426	0.2499	160.7067	0.2734
	1.0022	148.0005	0.2018	155.1879	0.2602	158.1437	0.2842
	1.0033	145.434	0.2109	152.6615	0.2711	155.6079	0.2957
	1.0044	142.8937	0.2203	150.1604	0.2823	153.0994	0.3074
	1.0055	140.3847	0.2298	147.686	0.2938	150.618	0.3195
	1.0066	137.9061	0.2396	145.238	0.3055	148.1638	0.3318
	1.0077	135.4571	0.2508	142.8171	0.3187	145.7367	0.3457
	1.0088	133.0386	0.2622	140.4232	0.3323	143.3366	0.3599
	1.0099	130.6509	0.2734	138.0595	0.3456	140.9636	0.3739

Table 6 (continued)

T	(K/S)	RG	RG_v	SG	SG_v	BS	BS_v
	1.011	128.2938	0.2855	135.7259	0.36	138.6175	0.389
	0.9889	219.9365	0.0564	229.6021	0.1028	232.4006	0.1162
	0.99	217.0336	0.0597	226.7763	0.1073	229.57	0.1209
	0.9911	214.1536	0.0631	223.9736	0.1118	226.7615	0.1256
	0.9922	211.2948	0.0666	221.1931	0.1166	223.975	0.1306
	0.9933	208.4611	0.0701	218.4341	0.1213	221.2107	0.1356
	0.9944	205.6512	0.0739	215.6964	0.1264	218.4684	0.1408
	0.9955	202.8651	0.0779	212.9823	0.1317	215.7483	0.1464
	0.9966	200.1039	0.0819	210.2922	0.137	213.0502	0.1519
	0.9978	197.3686	0.0862	207.6262	0.1427	210.3744	0.1578
	0.9989	194.6585	0.0899	204.9839	0.1477	207.7206	0.163
90	1.0	191.9733	0.0945	202.3635	0.1537	205.089	0.1693
	1.0011	189.3139	0.0994	199.7671	0.1601	202.4795	0.1758
	1.0022	186.6799	0.1043	197.1944	0.1665	199.8922	0.1824
	1.0033	184.0705	0.1092	194.6432	0.1729	197.3269	0.1891
	1.0044	181.4841	0.1148	192.1131	0.1801	194.7838	0.1965
	1.0055	178.9206	0.12	189.6049	0.1869	192.2627	0.2035
	1.0066	176.3829	0.126	187.1214	0.1945	189.7637	0.2114
	1.0077	173.87	0.1316	184.6582	0.2018	187.2867	0.2189
	1.0088	171.3825	0.138	182.2176	0.2099	184.8318	0.2273
	1.0099	168.9196	0.1441	179.7992	0.2177	182.3989	0.2353
	1.011	166.4806	0.1509	177.4035	0.2264	179.9879	0.2443

4 Conclusion

This paper developed a realized GARCH model for volatility modeling and option pricing. The model achieved higher accuracy in volatility forecasting, with lower MSE and QLIKE compared to the GARCH and HAR models. We utilized one-minute high-frequency data to compute daily, weekly, and monthly realized volatilities, allowing for a systematic analysis of the time series patterns in financial assets. Accurate volatility modeling is a crucial first step in developing a better option pricing model. The realized GARCH model employs the rolling sum of realized volatility, which is more effective in addressing real market data. Our results indicate that the realized GARCH model can generate option prices with lower price deviations compared to the BS model and Standard GARCH. In future research, we will explore the use of more advanced computing technologies to handle large volumes of financial data and high-frequency data. We aim to improve the accuracy of option pricing by leveraging the benefits of neural networks and other computer science algorithms for optimizing forecasting.

Table 7 The option price biases and the option prices by realized GARCH (RG), standard GARCH (SG) and Black-Scholes model (BS)—the maturity of 180, 270, and 360 days-continued

T	(K/S)	RG	RG_v	SG	SG_v	BS	BS_v
180	0.9513	422.3124	0.0227	437.1815	0.0587	437.5945	0.0597
	0.9568	405.4473	0.0266	420.754	0.0653	421.1924	0.0664
	0.9624	388.9845	0.0312	404.6932	0.0729	405.16	0.0741
	0.9679	372.9126	0.0369	389.0171	0.0817	389.5017	0.083
	0.9734	357.2318	0.0433	373.7127	0.0915	374.2207	0.0929
	0.9789	341.9519	0.051	358.8083	0.1028	359.3201	0.1044
	0.9845	327.0931	0.0601	344.3137	0.1159	344.802	0.1175
	0.99	312.6498	0.0705	330.2035	0.1306	330.668	0.1322
	0.9955	298.6392	0.0826	316.4785	0.1473	316.919	0.1489
	1.0011	285.0516	0.0968	303.1347	0.1664	303.5554	0.168
	1.0066	271.8641	0.1126	290.1756	0.1875	290.577	0.1892
	1.0121	259.0919	0.1307	277.5947	0.2114	277.9827	0.2131
	1.0177	246.7283	0.1513	265.3851	0.2384	265.7713	0.2402
	1.0232	234.7595	0.1741	253.5493	0.2681	253.9405	0.27
	1.0287	223.2026	0.2003	242.0654	0.3018	242.4877	0.304
	1.0343	212.0582	0.23	230.9645	0.3397	231.4099	0.3423
	1.0398	201.321	0.2634	220.2432	0.3821	220.7034	0.385
	1.0453	190.9893	0.3006	209.8779	0.4292	210.3641	0.4325
	1.0508	181.0552	0.3421	199.878	0.4817	200.3872	0.4854
	1.0564	171.5072	0.3882	190.2476	0.5398	190.7678	0.5441
	1.0619	162.3333	0.4398	180.975	0.6051	181.5005	0.6098
270	0.9513	499.368	0.043	521.8915	0.09	519.1484	0.0843
	0.9568	483.0788	0.047	505.9349	0.0965	503.279	0.0908
	0.9624	467.1192	0.0516	490.3037	0.1038	487.7121	0.098
	0.9679	451.4849	0.057	474.962	0.1119	472.4498	0.1061
	0.9734	436.1803	0.0628	459.9306	0.1207	457.4938	0.1148
	0.9789	421.2016	0.0696	445.2118	0.1306	442.8456	0.1245
	0.9845	406.5394	0.0772	430.791	0.1415	428.5062	0.1354
	0.99	392.2252	0.0859	416.6615	0.1535	414.4763	0.1475
	0.9955	378.2494	0.0956	402.8633	0.1669	400.7563	0.1608
	1.0011	364.5984	0.1062	389.3736	0.1814	387.3463	0.1752
	1.0066	351.2689	0.1182	376.1842	0.1975	374.246	0.1913
	1.0121	338.2724	0.1315	363.3177	0.2153	361.4548	0.2091
	1.0177	325.6	0.1463	350.7619	0.2349	348.9718	0.2286
	1.0232	313.252	0.1626	338.5162	0.2563	336.7958	0.2499
	1.0287	301.2324	0.1808	326.5792	0.2802	324.9251	0.2737
	1.0343	289.5251	0.2009	314.9485	0.3063	313.3581	0.2997
	1.0398	278.1552	0.2229	303.6349	0.335	302.0926	0.3282
	1.0453	267.1015	0.2461	292.6085	0.3651	291.1262	0.3582
	1.0508	256.3716	0.2742	281.8847	0.401	280.4563	0.3939
	1.0564	245.9745	0.3035	271.4756	0.4387	270.0801	0.4313

Table 7 (continued)

T	(K/S)	RG	RG_{υ}	SG	SG_{υ}	BS	BS_{υ}
	1.0619	235.8938	0.3384	261.3419	0.4832	259.9945	0.4756
360	0.9513	574.5921	0.0904	595.0834	0.1293	592.3749	0.1242
	0.9568	558.5779	0.0955	579.4863	0.1365	576.8451	0.1313
	0.9624	542.8282	0.1	564.1366	0.1431	561.5754	0.1379
	0.9679	527.3568	0.1075	549.0355	0.1531	546.5671	0.1479
	0.9734	512.164	0.1144	534.1881	0.1623	531.8214	0.1571
	0.9789	497.2553	0.122	519.6099	0.1724	517.3389	0.1673
	0.9845	482.6333	0.1303	505.3057	0.1834	503.1204	0.1783
	0.99	468.2974	0.139	491.2685	0.1949	489.1661	0.1898
	0.9955	454.253	0.1494	477.509	0.2083	475.4763	0.2031
	1.0011	440.4813	0.1601	464.0266	0.2221	462.051	0.2169
	1.0066	426.9977	0.1719	450.8129	0.2373	448.8898	0.232
	1.0121	413.8158	0.185	437.8676	0.2539	435.9923	0.2485
	1.0177	400.9112	0.1989	425.1912	0.2715	423.3578	0.266
	1.0232	388.2849	0.2143	412.7808	0.2909	410.9854	0.2853
	1.0287	375.9444	0.2312	400.6247	0.312	398.8742	0.3063
	1.0343	363.8932	0.2494	388.7139	0.3346	387.0228	0.3288
	1.0398	352.1242	0.2694	377.0651	0.3593	375.4298	0.3534
	1.0453	340.619	0.2907	365.6626	0.3856	364.0937	0.3797
	1.0508	329.3768	0.3136	354.5218	0.4138	353.0127	0.4078
	1.0564	318.411	0.3384	343.6411	0.4445	342.1847	0.4384
	1.0619	307.7195	0.3652	333.0066	0.4774	331.6079	0.4712

Table 8 The error criterion Υ - sum of price biases squared

Maturity - T	Υ_{RG}	Υ_{SG}	Υ_{BS}
30	0.6456	1.3283	1.7365
60	0.8192	1.3568	1.6179
90	0.2188	0.5527	0.6632
180	0.8296	1.7533	1.7817
270	0.5969	1.3874	1.3246
360	0.9193	1.6626	1.6016

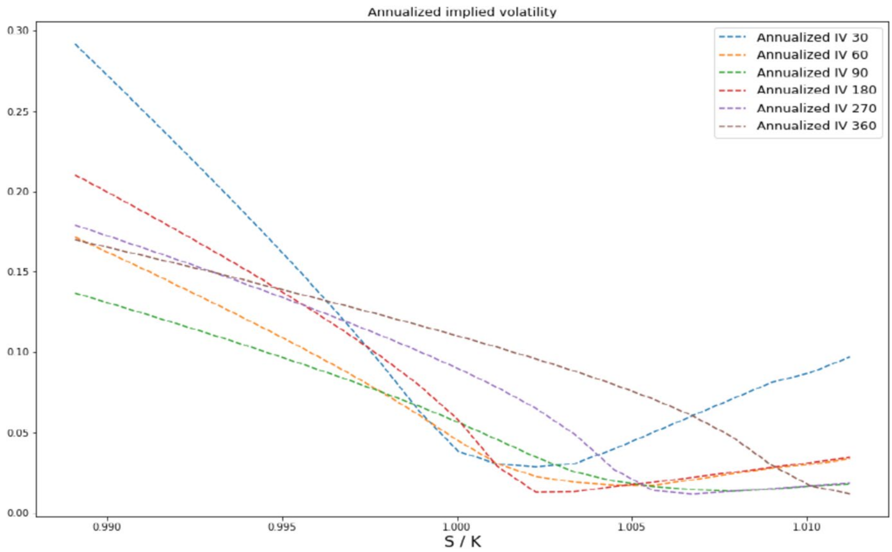

Fig. 3 Annualized implied volatility for the different maturities and S/K ratio

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Declarations

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