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Performance of GARCH Models in Forecasting Stock Market Volatility

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ABSTRACT

This paper studies the performance of GARCH model and its modifications, using the rate of returns from the daily stock market indices of the Kuala Lumpur Stock Exchange (KLSE) including Composite Index, Tins Index, Plantations Index, Properties Index, and Finance Index. The models are stationary GARCH, unconstrained GARCH, non-negative GARCH, GARCH-M, exponential GARCH and integrated GARCH. The parameters of these models and variance processes are estimated jointly using the maximum likelihood method. The performance of the within-sample estimation is diagnosed using several goodness-of-fit statistics. We observed that, among the models, even though exponential GARCH is not the best model in the goodness-of-fit statistics, it performs best in describing the often-observed skewness in stock market indices and in out-of-sample (one-step-ahead) forecasting. The integrated GARCH, on the other hand, is the poorest model in both respects. Copyright © 1999 John Wiley & Sons, Ltd.

KEY WORDS forecasting volatility; GARCH; time-series; rate of returns

INTRODUCTION

Following the seminal work of Mandelbrot (1963) and Fama (1965), many researchers have found that the empirical distribution of stock returns is significantly non-normal, such as Hsu *et al*. (1974), Hagerman (1978), Lau *et al*. (1990), Kim and Kon (1994), etc. They found that (1) the kurtosis of the stock returns time series is obviously larger than the kurtosis of the normal distribution, in order words, the time series of stock returns are leptokurtic; (2) the distribution of stock returns is skewed, either to the right (positive skewness) or to the left (negative skewness); (3) the variance of the stock returns is not constant over time or the volatility is clustering. Some researchers regarded this as the persistency of the stock market volatility and the financial analyst called this uncertainty or risk. This uncertainty is crucially important in modern finance theory. Before the seminal paper by Engle (1982), the uncertainty of speculative prices, changing

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over time (Mandelbrot, 1963; Fama, 1965) measured by the variances and covariance has been accepted for decades.

Many conventional time series and econometric models work only if the variance is constant. Until lately, the financial and economic researchers have started modelling time variation in second- or higher-order moments. Engle (1982) has characterized the changing variances using the Autoregressive Conditional Heteroscedasticity (ARCH) model and its extensions and modifications. Since then, hundreds of researchers have applied these models to financial time series data.

In many of the applications, the linear ARCH(q) model requires a long lag length of q. The alternative and more flexible lag structure is the generalized ARCH (GARCH) introduced by Bollerslev (1986). It is proven that a small lag such as GARCH(1,1) is sufficient to model the variance changing over long sample periods (French *et al.*, 1987; Franses and Van Dijk, 1996).

Even though GARCH model can effectively remove the excess kurtosis in returns, the GARCH model cannot cope with the skewness of the distribution of returns, especially the stock market indices which are commonly skewed. Hence, the forecasts and forecast error variances from a GARCH model can be expected to be biased for a skewed time series. Recently, a few modifications to the GARCH model have been proposed which explicitly take skewed distributions into account. One of the alternatives of non-linear models that can cope with skewness is the Exponential GARCH or EGARCH model introduced by Nelson (1990). For stock indices, Nelson's exponential GARCH is proven to be the best model of the conditional heteroscedasticity. Franses and Van Dijk (1996) found that there are difficulties when comparing this EGARCH model to out-of-sample forecasting performance. In order to know the out-of-sample forecasting performance of EGARCH, we compare the performance of EGARCH and the other five modifications of the GARCH model to the simple random walk forecasting scheme.

The models are presented in the following section. The third section gives the background to stock market indices data and the methodology used in this study. All the results will be discussed in the fourth section. Conclusions are presented in the final section.

MODELS

Consider a stock market index I_t and its rate of return r_t, which we construct as $r_t = I_t - I_{t-1}/(I_{t-1})$. The index t denotes daily closing observations. Instead of return, rate of return was used to make the data dimension less. The conditional distribution of the series of disturbances which follows the GARCH process can be written as

$$\varepsilon_t|\Psi_{t-1} \sim \mathrm{N}(0, h_t)$$

where Ψ_{t-1} denotes all available information at time $t-1$. The conditional variance h_t is

$$h_t = w + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-1}^2 + \sum_{j=1}^{p} \beta_j h_{t-j}$$

where $p \geqslant 0$, $q > 0$ and $w > 0$, $a_i \geqslant 0$, $\beta_j \geqslant 0$ for non-negative GARCH (NG(p,q)) process. The GARCH(p,q) model reduces to the ARCH(q) process when $p = 0$ and at least one of the ARCH parameters must be non-zero ($q > 0$). The GARCH regression model for the series of r_t can be written as

$$\begin{aligned}
\phi_s(B)r_t &= \mu + \varepsilon_t, \text{ with } \phi_s(B) = 1 - \phi_1 B - \cdots - \phi_s B^s \\
\varepsilon_t &= \sqrt{h_t} e_t \\
e_t &\sim \mathrm{N}(0, 1) \\
h_t &= w + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-1}^2 + \sum_{j=1}^{p} \beta_j h_{t-j}
\end{aligned}$$

where B is the backward shift operator defined by $B^k y_t = y_{t-k}$. The parameter μ reflects a constant term, which in practice is typically estimated to be close or equal to zero. The order of s is usually 0 or small, indicating that there are usually no opportunities to forecast r_t from its own past. In other words, there is never an auto-regressive process in r_t. The parameter of w, α_i and β_j can be unconstrained, thus yielding the unconstrained GARCH (UG(p,q)) model. If the parameters are constrained such that

$$\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \nu_j < 1 \quad ,$$

they imply the weakly stationary GARCH (SG(p,q)) process since the mean, variance, and autocovariance are finite and constant over time. Sometimes the multistep forecasts of the variance do not approach the unconditional variance when the model is integrated in variance; that is,

$$\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j = 1$$

The unconditional variance for the IGARCH model does not exist. However, it is interesting that the integrated GARCH or IGARCH (IG(p,q)) model can be strongly stationary even though it is not weakly stationary (Nelson, 1990a,b). The exponential GARCH or EGARCH (EG(p,q)) model was proposed by Nelson (1991). Nelson and Cao (1992) argue that the non-negativity constraints in the linear GARCH model are too restrictive. The GARCH model imposes the non-negative constraints on the parameters, α_i and β_j, while there is no restriction on these parameters in the EGARCH model. In the EGARCH model, the conditional variance, h_t, is an asymmetric function of lagged disturbances, ε_{t-i}:

$$\ln(h_t) = w + \sum_{i=1}^{q} \alpha_i g(Z_{t-i}) + \sum_{j=1}^{p} \beta_j \ln(h_{t-j})$$

where

$$\begin{aligned}
g(Z_t) &= \theta Z_t + \gamma[|Z_t| - E|Z_t|] \\
Z_t &= \varepsilon_t | \sqrt{h_t}
\end{aligned}$$

The coefficient of the second term in $g(Z_t)$ is set to be 1 ($\gamma = 1$) in this formulation. Note that $E|Z_t| = (2/\pi)^{1/2}$ if $Z_t \sim N(0, 1)$. The GARCH-M ((G(p,q)-M) model has the added regressor that is the conditional standard deviation

$$r_t = \mu + \delta\sqrt{h_t} + \varepsilon_t$$
$$\varepsilon_t = \sqrt{h_t}e_t$$

where h_t follows the GARCH process.

In the case of stock market indices, the GARCH models are often preferred for analysis. Furthermore, since a small lag of the GARCH model is sufficient to model the long-memory processes of changing variance (French *et al.*, 1987; Franses and Van Dijk, 1996), the performance of GARCH models is evaluated by using SG(1,1), UG(1,1), NG(1,1), G(1,1)-M, EG(1,1) and IG(1,1) in this study.

DATA AND METHODOLOGY

The data used in this paper is daily observed stock market indices in the Kuala Lumpur Stock Exchanges (KLSE), including Composite Index, Tins Index, Plantations Index, Properties Index and Finance Index. This data was collected from 1 January 1989 to 31 December 1990. In this study, we consider the daily closing prices as the daily observations.

Some characteristics of the rate of returns, r_t, are given in Table I. The number of observations are 483 for all the five indices. The means and variances are quite small. The high kurtosis indicates the necessity of fat-tailed distributions to describe these variables. The estimated measure of skewness is either positive or negative and is large in an absolute sense.

The family of GARCH models are estimated using the maximum likelihood method. This enables the rate of return and variance processes to be estimated jointly. The log-likelihood function is computed from the product of all conditional densities of the prediction error:

$$l = \sum_{t=1}^{N} \frac{1}{2}\left[-\ln(2\pi) - \ln(h_t) - \frac{\varepsilon_t^2}{h_t}\right]$$

where $\varepsilon_t = r_t - \mu$ and h_t is the conditional variance. When the GARCH(p,q)-M model is estimated, $\varepsilon_t = r_t - \mu - \delta\sqrt{h_t}$. When there are no regressors (trend or constant, μ), the residuals ε_t are denoted as r_t or $r_t - \delta\sqrt{h_t}$. The likelihood function is maximized via the dual quasi-Newton

Table I. Summary statistics of data on rate of returns from 1989 to 1990

Indices	n	Mean ($\times 10^{-4}$)	Variance ($\times 10^{-4}$)	Skewness	Kurtosis
Composite	483	7.613	1.772	−1.533	18.772
Tins	483	−1.733	3.278	−0.070	8.630
Plantations	483	−0.698	1.660	−0.019	16.593
Properties	483	7.903	5.367	1.783	26.246
Finance	483	6.868	2.988	0.303	13.142

Source of data: Investors Digest, KLSE.

and trust region algorithm. The starting values for the regression parameters μ are obtained from the OLS estimates. When there are autoregressive parameters in the model, the initial values are obtained from the Yule–Walker estimates.

In order to test for independence of the indices series, the portmanteau test statistic based on squared residual is used (McLeod and Li, 1983). This Q-statistic is used to test the non-linear effects, such as GARCH effects, present in the residuals. The GARCH(p,q) process can be considered as an ARMA(max(p,q),p) process. Therefore, the Q-statistic calculated from the squared residuals can be used to identify the order of the GARCH process. The Lagrange multiplier (LM) test, proposed by Engle (1982), is used to test for the ARCH disturbances.

The LM and Q-statistics are computed from the OLS residuals, assuming that disturbances is white noise. The Q and LM statistics have an approximate ($\chi^2_{(q)}$) distribution under the white-noise null hypotheses.

Various goodness-of-fit statistics are used to compare the six models in this study. The diagnostics are the mean of square error (MSE), the loglikelihood (Log L), Schwarz's Bayesian information criterion (SBC) by Schwarz (1978) and Akaike's information criterion (AIC) (Judge *et al.*, 1985).

The 'true volatility' is measured to evaluate the performance of the six GARCH models in forecasting the volatility in stock returns. As in the studies by Pagan and Schwert (1990) and Day and Lewis (1992), the volatility is measured by

$$v_t = (r_t - \bar{r})^2$$

where $\bar{r}$ is the average return. The measure of the one-step-ahead forecast error is

$$e_{t+1} = v_{t+1} - \hat{h}_{t+1}$$

where $\hat{h}_{t+1}$ is generated using the h_t equations of the GARCH models being studied. The estimated parameters of the GARCH models such as w, α, β, θ and δ are substituted during the generation of $\hat{h}_{t+1}$. In order to show the performance of GARCH models over a naïve no-change forecast, the forecast errors of the random walk (RW) is calculated as follows:

$$e_{t+1} = v_{t+1} - v_t$$

This is a very important naïve benchmark in comparison of the forecasts from the GARCH models (Brooks, 1997).

RESULTS AND DISCUSSION

The parameter estimates for the six variation GARCH models used in this study is presented in Table II(a). The additional parameter estimates of EG(1,1) and G(1,1)-M are reported in Table II(b). These within-sample estimation results enable us to know the possible usefulness of the GARCH models in modelling the volatility of stock returns. It is clear from Tables II(a) and II(b) that almost all the parameter estimates including μ, ω, α, β, θ and δ are significant at the 5% level. Hence, the constant variance model can be rejected, at least within sample. For linear GARCH models such as SG(1,1), the sum of the α and β parameters is close to unity for the Finance Index. However, for the Properties Index, we observed that $\alpha + \beta = 1$ for a stationary

Table II(a). Estimation results of rate of returns for five stock market indices

		Parameter estimates							
Index	Model	μ ($\times 10^{-4}$)	t-ratio	ω ($\times 10^{-5}$)	t-ratio	α	t-ratio	β	t-ratio
Composite	SG(1,1)	3.229	0.523	8.11768	6.500	0.420	2.938	0.179	1.722
	UG(1,1)	3.228	0.512	8.11766	6.525	0.420	2.911	0.179	1.727
	NG(1,1)	3.229	0.514	8.11763	6.519	0.420	2.893	0.179	1.718
	G(1,1)-M	−53.301	−2.384	7.058	6.686	0.488	3.066	0.212	2.249
	EG(1,1)	−3.7492	−2.815	—	−5.237	0.721	4.691	0.481	4.935
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	IG(1,1)	−4.593	−0.888	7.299	7.599	0.922	15.67	0.078	—
Tins	SG(1,1)	−8.543	−1.101	1.330	5.620	0.317	3.485	0.310	3.367
	UG(1,1)	−8.5474	−1.108	1.330	5.888	0.317	3.605	0.310	3.457
	NG(1,1)	−8.543	−1.101	1.330	5.620	0.317	3.486	0.310	3.370
	G(1,1)-M	−86.971	−2.129	1.344	5.604	0.322	3.704	0.296	3.289
	EG(1,1)	−16.459	−7.381	−32,767.92	−4.971	0.485	5.117	0.591	7.317
	IG(1,1)	−15.220	−2.159	1.129	6.157	0.755	10.75	0.245	—
Plantations	SG(1,1)	−1.434	−2.578	0.476	6.086	0.458	2.782	0.324	3.682
	UG(1,1)	−1.434	−2.520	0.476	5.900	0.458	2.828	0.324	3.741
	NG(1,1)	−1.434	−2.541	0.476	6.213	0.458	3.123	0.324	3.999
	G(1,1)-M	−5.574	−3.575	0.458	5.417	0.548	3.222	0.290	3.763
	EG(1,1)	−1.648	−6.516	−28,679.48	−5.378	0.731	6.098	0.671	11.16
	IG(1,1)	−1.836	−4.100	0.424	6.223	0.730	11.13	0.270	—
Properties	SG(1,1)	5.404	0.884	0.264	4.717	0.435	10.39	0.565	—
	UG(1,1)	2.427	0.413	0.170	3.099	0.652	5.834	0.545	12.38
	NG(1,1)	2.429	0.417	0.170	2.977	0.652	6.155	0.545	12.25
	G(1,1)-M	−3.503	−3.258	0.171	3.006	0.663	6.006	0.527	11.52
	EG(1,1)	−1.003	−0.169	−8034.68	−4.180	0.785	8.839	0.892	38.81
	IG(1,1)	5.403	0.884	0.264	4.717	0.435	10.39	0.565	—
Finance	SG(1,1)	9.229	1.385	0.402	2.937	0.262	3.733	0.631	10.15
	UG(1,1)	9.229	1.347	0.402	2.936	0.262	3.755	0.631	10.15
	NG(1,1)	9.229	1.332	0.402	2.914	0.262	3.974	0.631	9.974
	G(1,1)-M	1.502	0.060	0.408	3.021	0.262	3.915	0.628	11.03
	EG(1,1)	−7.064	−0.988	−14,013.14	−4.402	0.407	5.665	0.826	21.30
	IG(1,1)	8.529	1.288	0.290	3.586	0.359	7.101	0.641	—

Table II(b). Estimation results of rate of returns for five stock market indices

		Parameter estimates			
Index	Model	δ	t-ratio	θ	t-ratio
Composite	G(1,1)-M	0.482	2.446		
	EG(1,1)			0.060	0.530
Tins	G(1,1)-M	0.489	1.951		
	EG(1,1)			−0.128	−1.038
Plantations	G(1,1)-M	0.402	2.605		
	EG(1,1)			0.175	2.327
Properties	G(1,1)-M	0.334	3.910		
	EG(1,1)			−0.120	−1.765
Finance	G(1,1)-M	0.055	0.323		
	EG(1,1)			−0.311	−2.996

GARCH(1,1) model and $\alpha + \beta > 1$ for an unconstrained GARCH(1,1) model and a non-negative GARCH(1,1) model. This indicates the weaknesses of imposing the parameter estimates of a GARCH model to certain constraints such as stationarity or non-negativity. As argued by Nelson and Cao (1992), the non-negativity constraint in the linear GARCH models is too restrictive, while there is no restriction on the parameters in the EGARCH model. From Table II(b), the parameter estimates of EG(1,1) are significant at the 10% level for the Properties Index, the Plantations Index and the Finance Index. The approximated probability values (p-value) for θ in EG(1,1) are 0.5958 (Composite Index), 0.2933 (Tins), 0.0200 (Plantations Index), 0.0776 (Properties Index) and 0.0027 (Finance Index). The approximated probability values (p-value) for δ in G(1,1)-M are 0.0144 (Composite Index), 0.0510 (Tins), 0.0092 (Plantations Index), 0.0001 (Properties Index) and 0.7471 (Finance Index).

The results for Q-statistics and the Lagrange multiplier (LM) test shown in Table III can help to determine the order of the ARCH model appropriates for the data. The tests are significant ($p < 0.0001$) though order 12, which indicate that a very high-order ARCH model is needed to model the heteroscedasticity. The basic ARCH(q) model is a short-memory process in that only the most recent q-squared residuals are used to estimate the changing variance. The GARCH model allows long-memory processes, which use all the past squared residuals to estimate the current variance. The Q-statistic and the LM test in Table III suggest the use of the GARCH model instead of the ARCH model.

Table III. Diagnostics for five stock market indices using the Q-statistic and the Lagrange Multiplier test

	Diagnostics			
Index	$Q(12)$	Prob > $Q(12)$	LM(12)	Prob > LM(12)
Composite	48.0672	0.0001	38.3500	0.0001
Tins	30.5062	0.0023	26.7916	0.0083
Plantations	88.8191	0.0001	58.3975	0.0001
Properties	73.9152	0.0001	50.8146	0.0001
Finance	44.3089	0.0001	42.8426	0.0001

In Table IV, the results of goodness-of-fit statistics for GARCH models are presented. Table V shows the rankings of the models averaged across five indices based on the performance of the goodness-of-fit statistics. The AIC and Log L values suggest that the G(1,1)-M is the best model following by EG(1,1). However, the values of MSE and SBC suggest the reverse.

In any case, the good performance in parameter estimates and goodness-of-fit statistics do not guarantee good performance in forecasting. Hence, the performance of the GARCH models is further evaluated through the one-step-ahead forecasting. Fifty one-step-ahead forecasts are generated and the mean square error (MSE) is used in the forecast evaluation. The results of forecasting for the GARCH models and the random walk model are shown in Table VI. The rankings of the models averaged across five indices based on the performance of one-step-ahead forecasting are presented in Table VII.

The results indicate that EG(1,1) is the best model in forecasting volatility for all the five stock market indices, following by the SG(1,1) and then NG(1,1). The UG(1,1) and G(1,1)-M are ranked 4 which means that both perform equally well. The random walk (RW) model is ranked second last in forecasting volatility while the integrated GARCH, IG(1,1) model is ranked last.

Table IV. Goodness-of-fit statistics on rate of returns for five stock market indices

		Goodness-of-Fit Statistics			
Index	Model	MSE ($\times 10^{-3}$)	Log L	SBC	AIC
Composite	SG(1,1)	0.18	1299.262	−2574.24	−2590.52
	UG(1,1)	0.18	1299.262	−2574.24	−2590.52
	NG(1,1)	0.18	1299.262	−2574.24	−2590.52
	G(1,1)-M	0.179	1302.11	−2573.87	−2594.22
	EG(1,1)	0.181	1300.832	−2571.31	−2591.66
	IG(1,1)	0.182	1296.86	−2575.51	−2587.72
Tins	SG(1,1)	0.333	1147.125	−2269.97	−2286.25
	UG(1,1)	0.333	1147.125	−2269.97	−2286.25
	NG(1,1)	0.333	1147.125	−2269.97	−2286.25
	G(1,1)-M	0.333	1149.153	−2267.95	−2288.31
	EG(1,1)	0.335	1147.49	−2264.63	−2284.98
	IG(1,1)	0.334	1140.807	−2263.4	−2275.61
Plantations	SG(1,1)	0.167	1334.912	−2645.54	−2661.82
	UG(1,1)	0.167	1334.912	−2645.54	−2661.82
	NG(1,1)	0.167	1334.912	−2645.54	−2661.82
	G(1,1)-M	0.17	1338.044	−2645.73	−2666.09
	EG(1,1)	0.167	1336.142	−2641.93	−2662.28
	IG(1,1)	0.168	1334.087	−2649.96	−2662.17
Properties	SG(1,1)	0.559	1176.929	−2335.64	−2347.86
	UG(1,1)	0.56	1180.872	−2337.46	−2353.74
	NG(1,1)	0.56	1180.872	−2337.46	−2353.74
	G(1,1)-M	0.642	1188.171	−2345.99	−2366.34
	EG(1,1)	0.56	1184.448	−2338.54	−2358.9
	IG(1,1)	0.559	1176.929	−2335.64	−2347.86
Finance	SG(1,1)	0.299	1181.016	−2337.75	−2354.03
	UG(1,1)	0.299	1181.016	−2337.75	−2354.03
	NG(1,1)	0.299	1181.016	−2337.75	−2354.03
	G(1,1)-M	0.3	1181.068	−2331.78	−2352.14
	EG(1,1)	0.301	1185.7	−2341.05	−2361.4
	IG(1,1)	0.299	1179.852	−2341.49	−2353.7

Using the random walk model as the naïve benchmark, all models are useful tools for forecasting volatility except IG(1,1).

CONCLUSION

Using five daily observed stock market indices in the Kuala Lumpur Stock Exchange (KLSE), this paper has examined the performance of six variations of GARCH models in explaining the commonly observed characteristics of the unconditional distribution of daily stock rate of returns, including leptokurtosis, skewness and the changing of variance over time.

It has been seen that the hypotheses of constant variance models could be rejected, at least within sample, since almost all the parameter estimates are significant at the 5% level.

Table V. Rankings of the models averaged across five indices based on the performance of various goodness-of-fit statistics

Model	Composite Index				Tins Index				Plantations Index				Properties Index				Finance Index				Average			
	MSE	LogL	SBC	AIC	MSE	LogL	SBC	AIC	MSE	LogL	SBC	AIC	MSE	LogL	SBC	AIC	MSE	LogL	SBC	AIC	MSE	LogL	SBC	AIC
SG(1,1)	2	3	2	3	1	3	1	2	1	3	3	4	1	5	5	5	1	3	3	2	1	5	3	5
UG(1,1)	2	3	2	3	1	3	1	2	1	3	3	4	3	3	3	3	1	3	3	2	2	3	1	3
NG(1,1)	2	3	2	3	1	3	1	2	1	3	3	4	3	3	3	3	1	3	3	2	2	3	1	3
G(1,1)-M	1	1	5	1	1	1	4	1	6	1	2	1	6	1	1	1	5	2	6	6	5	1	5	1
EG(1,1)	5	2	6	2	6	2	5	5	1	2	6	2	3	2	2	2	6	1	2	1	6	2	6	2
IG(1,1)	6	6	1	6	5	6	6	6	5	6	1	3	1	5	5	5	1	6	1	5	4	6	3	6

Table VI. Out-of-sample forecasting performance of SG(1,1), UG(1,1), NG(1,1), G(1,1)-M, EG(1,1), IG(1,1) and random walk models for the volatility of stock market indices

Model	MSE ($\times 10^{-7}$) of one-step-ahead forecast (forecast period = 50)				
	Composite Index	Tins Index	Plantations Index	Properties Index	Finance Index
SG(1,1)	0.712	3.240	0.899	18.344	2.637
UG(1,1)	0.712	3.240	0.899	41.912	2.638
NG(1,1)	0.712	3.240	0.899	41.755	2.637
G(1,1)-M	0.756	3.225	1.018	23.232	2.631
EG(1,1)	0.693	3.183	0.840	5.359	2.510
IG(1,1)	44.980	120.510	15.863	18.345	15.704
RW	1.055	5.785	1.801	7.147	5.199

Table VII. Rankings of the models averaged across five indices based on the performance of one-step-ahead forecasting

Model	MSE of one-step-ahead forecast					
	Composite Index	Tins Index	Plantations Index	Properties Index	Finance Index	Average
SG(1,1)	2	3	2	3	3	2
UG(1,1)	2	3	2	7	5	4
NG(1,1)	2	3	2	6	3	3
G(1,1)-M	5	2	5	5	2	4
EG(1,1)	1	1	1	1	1	1
IG(1,1)	7	7	7	4	7	7
RW	6	6	6	2	6	6

From the aspect of restriction, we observed the weaknesses of imposing the parameter estimates of the GARCH model to certain constraints such as stationary or non-negativity. In contrast, the EGARCH model has no restriction on the parameters.

The Q-statistics and Lagrange multiplier test reveal the use of the long-memory GARCH model instead of the short-memory and high-order ARCH model.

The results from various goodness-of-fit statistics are not consistent for the six variations of GARCH models. However, the EGARCH model outperforms the random walk and other variations of GARCH models in one-step-ahead forecasting. On the other hand, the integrated GARCH model cannot be recommended for forecasting volatility.

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