The Official
Calculus 1A/1B Survival Guide

1. Background

Over the five year period ending in 2009, half the students who enrolled in Calculus I either withdrew from the course or failed outright. One third of the rest earned less than the "C" needed to go on to Calculus II. Clearly, many students lack an adequate grounding in the precalculus fundamentals needed to succeed in Calculus I.

The Calculus 1A/1B sequence is designed to provide those students whose pre-calculus skills are weak an extra semester to master the full scope of material previously provided over one semester.

Calculus 1A/1B is <u>NOT</u> a one semester precalculus course followed by a traditional one semester calculus course. With the exception of the first few weeks, which are devoted to a review of basic principles, this course is focused on calculus – both its concepts and related skills – both semesters.

The major difference between this sequence and the single semester calculus alternative (Calculus I) is pacing. In Calculus 1A/1B, extra time is allotted to review and strengthen precalculus skills as they are needed when the various calculus topics are presented.

Do not mistake this change of pace with a watering down of the rigor of the material that you will be expected to master. The standards of accomplishment for this course are identical to those imposed in Calculus I.

Do not conclude from this change of pace that your level of effort can be reduced. In fact, the effort needed for you to be successful in this course is probably greater than the effort required of your fellow students enrolled in Calculus I. You are enrolled in 1A/1B because your mastery of mathematical fundamentals is lacking. It is likely that your missing skills constitute a deficit that is greater than the extra semester afforded in this sequence.

You need to commit to filling this deficit once and for all. This will require focused/concentrated effort on your part. Your instructors and the resources of the university are here to help, BUT YOU MUST DO IT. This survival guide provides proven guidance based on years of experience teaching calculus at the university level. Those who take this advice seriously <u>and act on it</u> have a substantially better chance of succeeding than those that don't. Ignore this advice at your peril.

2. Course Resources – Use Them To Their Fullest

- **Your textbook (whether physical or on-line) has "text" – not just homework problems!**
  - **Preview upcoming sections of material in your textbook before class. Even 15 minutes spent doing this will help you absorb the concepts more fully when they are presented.**
  - **Within 24 hours of each lecture and BEFORE you do your homework problems, read the associated sections of the text. If you read the text carefully, the homework will be much easier AND you will be better prepared for upcoming quizzes and tests.**
  - **Reading your math text will be difficult at first, but it is vitally important to your success. You cannot read a math textbook like you would a novel. It is much slower and requires substantially greater concentration. You must dwell on and understand each point as it is made. This process can be compared to weightlifting for the mind. It**

forces your mind to think about the concepts, understand how they are used, and how to write mathematics (more on this later). Over time it will get easier.
  - **IF you do not understand everything you read. That is OK. Mark your questions and then *ASK THEM IN CLASS!***
- **Participate in class by asking questions**
  - **Your questions are almost surely shared by many other students in the class. By asking them you ensure that your gaps in understanding are filled AND you help the instructor understand what areas need greater attention.**
  - **Do not hesitate to ask. Break the ice early … it gets easier every time you do it!**
- **Sample Exams will be made available. They are a GREAT guide to the actual exams**
  - **The problems on the actual tests are based on the same concepts as the problems on the sample exams. Focus on understanding each of these concepts <u>and</u> how they are applied; not just the specific problems!**
  - **A week or more (not less) before each test, take the relevant sample test and then compare your answers to the solutions provided.**
  - **If you get a problem wrong, determine why. Did you not understand the concept? Was your understanding of how to apply it flawed? If so, *ASK ABOUT IT IN CLASS!***
- **Homework Problems … Practice, Practice, Practice**
  - **Doing ALL the assigned homework in MyMathLab™ and a substantial portion of any other recommended textbook problems is critically important**
  - **If you are having difficulty with a concept, or a type of problem … DO MORE than the number of problems assigned and then *ASK ABOUT IT IN CLASS!***
- **Tutoring … a most underutilized resource**
  - **All instructors provide some opportunities for tutoring outside of class. *TAKE ADVANTAGE OF THIS ONE-ON-ONE TUTORING! IT IS FREE!***
  - **If you can't make these hours, there is free tutoring available on campus that stretches into the late evening … Get help early and often!! (Call 978 934 2947 or go to http://www.uml.edu/class/tutoring/tutor.html for up to date information)**
- **Calculators … Lurking in the shadows and waiting to destroy you**
  - **Well maybe that is a little strong … BUT ONLY A LITTLE!**
  - **There are no test problems in Calculus 1A/1B that require a calculator.**
  - **Calculators are viewed as a savior by students … something that will substitute for understanding the key concepts. Unfortunately, this <u>never</u> works. Relying on calculators hinders learning and understanding, so we have outlawed them on tests.**
  - **LESSON: FOCUS ON CONCEPTS AND METHODS and leave the calculators at home or sell them on E-bay.**

3. Good Habits and Basic Mechanics … and our expectations of students

<u>Writing Mathematical Expressions</u>   Mathematics has evolved a language and style that makes expression of mathematical statements <u>easier</u> and ensures that those ideas are <u>expressed precisely</u> while minimizing the potential for confusion.

The "easier" part means less work for you. The "precise" part is the part that will raise your grade because it helps you think clearly about what you are doing and makes it clearer to you whether or not what you are writing is correct. If it is not, you can work on fixing or improving it.

Nobody passes calculus by scrawling random mathematical expressions on an exam page in the hope that the grader will give partial credit. Graders know what the correct answer is and know the mathematical steps needed to get there.

A short summary of the basics of mathematical expression is provided at the end of this survival guide. This summary provides a list of approximately fifteen mathematical symbols. You probably know half of them already. That leaves around seven or eight for you to learn. Do it now and start using them.

Orderly Development/Demonstration of Mathematical Facts **Beyond learning mathematical notation and being able to properly state mathematical facts (the equivalent of English sentences), you need to learn to express a logical and orderly progression of mathematical reasoning to arrive at a solution (the equivalent of an English paragraph, chapter, or book). Writing scattered mathematical phrases in random order on the page with no logic or order, or using incomplete sentences, will earn little or no credit … even if something resembling the correct answer emerges.**

Figure 1 illustrates what we're referring to. Hopefully no further elaboration is needed.

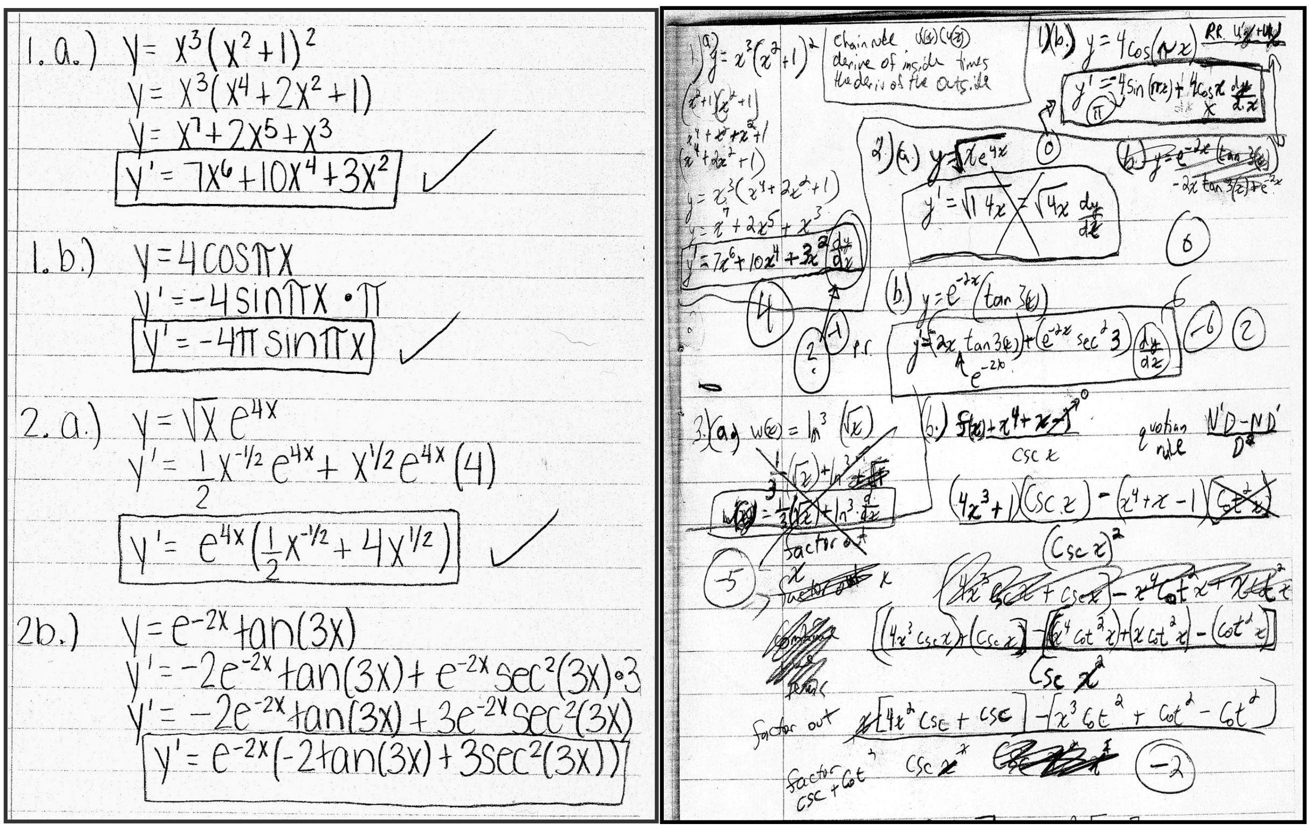

Figure 1 Which example does your mathematical writing most resemble?

How do you learn to write mathematics? There are two necessary ingredients:

1. **READ. Your textbook provides a great model. Just as you couldn't write an English essay if you never read the writings of accomplished authors, you can't expect to write mathematics if you don't read and digest well-written mathematics.**
2. **PRACTICE. Discipline yourself to write full mathematical sentences in a neat, logical and orderly fashion all the time. Start with your homework, and the skills will carry over to quizzes and exams. Significantly higher grades are nearly certain.**

Tips on word problems – Many students have difficulty with word problems. Word problems require translating English sentences into mathematical equations that are subsequently used to find a solution. The translation process is often the biggest stumbling block to solving these problems.

If you have difficulty with this translation process, consider performing it in two steps.

The first step is to create a very neat, clear and well-labeled diagram that encompasses ALL of the facts stated in the English version of the problem. The diagram should show the various elements of the problem with the correct geometry, the needed variables/dimensions clearly labeled, and the desired unknowns explicitly indicated. Complete this BEFORE going on to step two.

The second step is to translate this diagram into the needed equations. If the diagram is drawn neatly and accurately, it will contain everything you need to create the needed equations.

This process takes practice, but most students find this approach much easier than directly translating from English to the required mathematical equations.

Finally, watch when your Instructor solves word problems – He/she will inevitably follow this process.

4. Avoiding These Errors Is Critical to Your Success

This short guide is not designed to be a replacement for your math textbook, but it is difficult to resist listing the most common errors seen when grading quizzes and exams. These are elementary errors that should never be committed at the college level. A high percentage of the points deducted when grading exams are as a result of one or more of these errors.

Before listing these elementary errors, a short comment on PARENTHESES is in order: Better to overuse parentheses than under use them. It never hurts to add a redundant set of parentheses, but leaving out a needed set of parentheses invariably leads to an incorrect result.

For example, suppose y(x) is the product of 3x + 7 and 2x – 3. If you were to be sloppy and write: y(x) = (3x + 7) 2x – 3, you are likely to follow this on the next line with the false claim that y(x) = $6x^2$ + 14x – 3. Always make the order of operations explicit: in this case, y(x) = (3x + 7) (2x – 3).

A short list of common algebraic errors are listed below with very brief elaboration. If there is ever a chance that you might slip and be tempted to make any of these errors, NOW is the time to permanently imprint a flashing WARNING in your brain: BEWARE – BEWARE – BEWARE!!

$\frac{a+b}{a+c} \neq \frac{1+b}{1+c}$ or $1+\frac{b}{c}$ **The a's do not cancel! Only common FACTORS cancel.**

$(a^n)^m \neq a^{n+m}$ **It equals** a^{nm}**. Try a=2, n=3 & m=2. Review exponent laws.**

$f(x+y) \neq f(x) + f(y)$ and $f(ax) \neq a\,f(x)$ **UNLESS f(x) is a linear function[1].**

For example: $(a+b)^n \neq a^n + b^n$ **Taking powers ($n \neq 1$) is a nonlinear process.**

For example, for n=3, $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$. **Review the binomial theorem.**

$\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$ **Square root is a nonlinear function. Try a=4 & b=5.**

---

[1] Note: Even f(x) = ax + b is a nonlinear function because $f(x+y) = a(x + y) + b \neq f(x) + f(y) = ax + ay + 2b$. The only truly linear function is f(x) = ax. Linear functions when plotted must be straight lines that pass through the origin, i.e., (0,0).

$\log(x + y) \neq \log(x) + \log(y)$ **Log is a nonlinear function. See the pattern?**

$\mathrm{Sin}(2x) \neq 2\,\mathrm{Sin}(x)$ **and** $\sin(x+y) \neq \sin(x) + \sin(y)$ **Same reason!**

$\sin(x) \neq \sin$ **Trig functions always require arguments. Always, always, always.**

While there are many other common errors, these mistakes seem to be made most frequently. Spend a few moments thinking about each one. Try some sample numbers to make it clear that these expressions are not equal.

These errors are your enemy. Your mind needs to immediately reject any thought of committing them.

The Basics of Mathematical Expression …

It's just English with some very specialized notation

If you write an essay for your English class and your essay consists of a number of disconnected phrases with no verbs or complete thoughts, you should realistically expect a grade of F. The same is true in mathematics. Students routinely scribble a series of mathematical expressions with no verbs and, therefore, no complete statements. Often the grader tries to infer or guess what the student meant to claim as he scrawled these expressions. Sometimes we try too hard.

Your job is to make your assertions clear. If you do not write complete sentences, you make no claims. If you make no claims, there is nothing to grade.

Mathematics is written using both English grammar and mathematical symbols. Look at your textbook. You see paragraphs of English with mathematical statements interspersed. In all cases, if you read the book out loud, you will notice that it consists of fully-formed English sentences.

Sometimes, mathematical expressions form complete English sentences by themselves. For example:

$$\mathbf{y^2 = 3\sin(\pi x) + 7}$$

Notice that there is a subject (y^2), a verb (=) and a predicate ("$3\sin(\pi x) + 7$"). If you read the equation out loud, it forms a complete sentence. A claim is being made that y^2 is equal to the expression on the right hand side of the equation. The statement may be true or false, but it makes a claim nevertheless. If you write such an expression, we can evaluate it and give you credit (or not). If you just write "$3\sin(\pi x) + 7$" you have made no claim. It is just an expression sitting by itself with little value.

Sometimes a mathematical expression or equation is merely part of a larger sentence that also contains English prose. For example, you might see:

"**The distance traveled by a rocket in t seconds is given by $\mathbf{3t^3 + t^2 + 4}$**."

While, in this case, the mathematical expression does not contain a verb, it is used in the context of a complete sentence. A claim is made. It can be evaluated as true or false and it can be graded. Just writing "$\mathbf{3t^3 + t^2 + 4}$" by itself is meaningless. It makes no claim so it can't be judged true or false. It can't be graded and no credit is deserved.

In other cases, mathematical "sentences" are embedded in English text such as:

“Note that f(x) = e^{x+5} > 0 for all x and that f(x) = $e^5 e^x$.”

This is no different than a compound English sentence such as, “Note that Sally went to the store and she bought some makeup.”

So, always write mathematics in complete sentences**.** If you don’t, it is unlikely that you will be given credit.

Learn how to write mathematics as you read your textbooks. Your textbooks provide you a consistent model of proper mathematical writing. You will find that your textbook consistently uses complete sentences, including those portions written in mathematical notation. Why? Because the textbook is always making observations or claims and these are always written in complete, grammatically correct, English sentences. The use of mathematical notation is not an excuse for illiteracy.

To make a claim in mathematics, you must use verbs. You know the verbs when they show up in English prose. Here are some common verbs expressed in mathematical notation: =, >, <, $\neq$, $\cong$, $\le$, $\ge$, $\Leftrightarrow$, $\propto$, $\perp$, $\exists$, $\Rightarrow$, $\in$, and $\notin$. You should be familiar with their English translation:

=	is equal to
>	is greater than
<	is less than
$\neq$	is not equal to
$\cong$	is approximately equal to
$\le$	is less than or equal to
$\ge$	is greater than or equal to
$\Leftrightarrow$	is true if and only if or is equivalent to
$\propto$	is proportional to
$\perp$	is perpendicular to
$\exists$	There exists a (an)
$\Rightarrow$	implies that
$\in$	belongs to or is an element of
$\notin$	does not belong to or is not an element of

The “slash” is often used to negate the meaning of a symbol as is illustrated above by the $\neq$ and the $\notin$ symbols. Several other useful mathematical symbols are: $\ni$, which translates to “such that”, $\forall$, which translates to “for all”, and the set operators $\cup$ and $\cap$ meaning “union with” and “intersection with” respectively. And finally, (a,b] refers to the set of real numbers greater than a and less than or equal to b. The rounded parentheses – either “(” or “)” – implies the end value is not included in the set while the square bracket – “[“ or “]” – implies the end value is included in the set. You will be exposed to more symbols as your exposure to mathematics progresses.

Based on this brief introduction, you can write, “For all x greater than or equal to zero, there exists a y such that y squared equals x.” as follows:

$\forall$ **X** $\ge$ **0,** $\exists$ **Y** $\ni$ **Y^2 = X ... or as ...** $\forall$ **X** $\in [0, \infty)$, $\exists$ **Y** $\ni$ **Y^2 = X** **Cool!!**

Try reading this: $\exists$ **y** $\in (4,9]$ $\ni$ **y**$^{1/2} = 3$ …. But just barely! Thank goodness for the “]” !!

Try writing this in mathematical notation: For all y having values between –1 to 1 there exists a number x between $-\pi/2$ and $+\pi/2$ such that y = sin(x). This might be more fun than text messaging!!