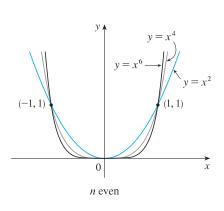
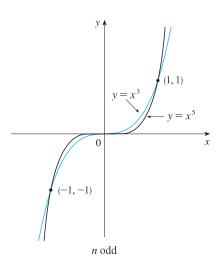
SPECIAL FUNCTIONS

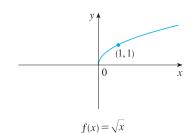
Power Functions $f(x) = x^a$

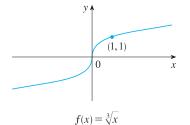
(i) $f(x) = x^n$, n a positive integer



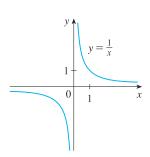


(ii) $f(x) = x^{1/n} = \sqrt[n]{x}$, n a positive integer





(iii)
$$f(x) = x^{-1} = \frac{1}{x}$$

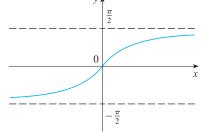


Inverse Trigonometric Functions

$$\arcsin x = \sin^{-1} x = y \iff \sin y = x \text{ and } -\frac{\pi}{2} \le y \le \frac{\pi}{2}$$

$$\arccos x = \cos^{-1} x = y \iff \cos y = x \text{ and } 0 \le y \le \pi$$

$$\arctan x = \tan^{-1} x = y \iff \tan y = x \text{ and } -\frac{\pi}{2} < y < \frac{\pi}{2}$$



$$\lim_{x \to -\infty} \tan^{-1} x = -\frac{\pi}{2}$$

$$\lim_{x \to \infty} \tan^{-1} x = \frac{\pi}{2}$$

$$y = \tan^{-1} x = \arctan x$$

SPECIAL FUNCTIONS

Exponential and Logarithmic Functions

$$\log_a x = y \iff a^y = x$$

$$\ln x = \log_e x$$
, where $\ln e = 1$

$$\ln x = y \iff e^y = x$$

Cancellation Equations

$$\log_a(a^x) = x \qquad a^{\log_a x} = x$$

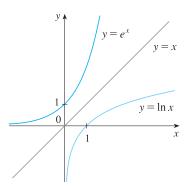
$$\ln(e^x) = x \qquad e^{\ln x} = x$$

Laws of Logarithms

$$1. \log_a(xy) = \log_a x + \log_a y$$

$$2. \log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$3. \log_a(x^r) = r \log_a x$$

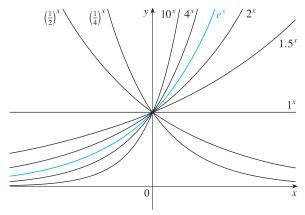


$$\lim_{x \to -\infty} e^x = 0$$

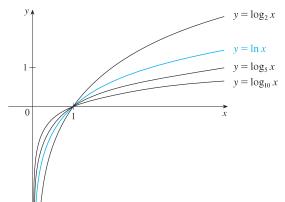
$$\lim_{x\to\infty}e^x=\infty$$

$$\lim_{x \to 0^+} \ln x = -\infty \qquad \qquad \lim_{x \to \infty} \ln x = \infty$$





Exponential functions



Logarithmic functions

Hyperbolic Functions

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

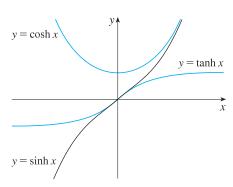
$$\operatorname{csch} x = \frac{1}{\sinh x}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\operatorname{sech} x = \frac{1}{\cosh x}$$

$$\tanh x = \frac{\sinh x}{\cosh x}$$

$$\coth x = \frac{\cosh x}{\sinh x}$$



Inverse Hyperbolic Functions

$$y = \sinh^{-1}x \iff \sinh y = x$$

$$y = \cosh^{-1}x \iff \cosh y = x \text{ and } y \ge 0$$

$$y = \tanh^{-1} x \iff \tanh y = x$$

$$\sinh^{-1}x = \ln(x + \sqrt{x^2 + 1})$$

$$\cosh^{-1}x = \ln(x + \sqrt{x^2 - 1})$$

$$\tanh^{-1}x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$

DIFFERENTIATION RULES

General Formulas

$$1. \ \frac{d}{dx}(c) = 0$$

3.
$$\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$$

5.
$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$$
 (Product Rule)

7.
$$\frac{d}{dx} f(g(x)) = f'(g(x))g'(x)$$
 (Chain Rule)

$$2. \ \frac{d}{dx} [cf(x)] = cf'(x)$$

4.
$$\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$$

6.
$$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$
 (Quotient Rule)

8.
$$\frac{d}{dx}(x^n) = nx^{n-1}$$
 (Power Rule)

Exponential and Logarithmic Functions

$$9. \ \frac{d}{dx}(e^x) = e^x$$

11.
$$\frac{d}{dx} \ln |x| = \frac{1}{x}$$

$$\mathbf{10.} \ \frac{d}{dx}(a^x) = a^x \ln a$$

Trigonometric Functions

13.
$$\frac{d}{dx}(\sin x) = \cos x$$

16.
$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

14.
$$\frac{d}{dx}(\cos x) = -\sin x$$

17.
$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

15.
$$\frac{d}{dx}(\tan x) = \sec^2 x$$

18.
$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

Inverse Trigonometric Functions

19.
$$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$$

22.
$$\frac{d}{dx}(\csc^{-1}x) = -\frac{1}{x\sqrt{x^2-1}}$$

20.
$$\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}}$$

23.
$$\frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2 - 1}}$$

21.
$$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$$

24.
$$\frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+x^2}$$

Hyperbolic Functions

25.
$$\frac{d}{dx}(\sinh x) = \cosh x$$

28.
$$\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \operatorname{coth} x$$

26.
$$\frac{d}{dx}(\cosh x) = \sinh x$$

29.
$$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$$

27.
$$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$$

30.
$$\frac{d}{dx} \left(\coth x \right) = -\operatorname{csch}^2 x$$

Inverse Hyperbolic Functions

31.
$$\frac{d}{dx} \left(\sinh^{-1} x \right) = \frac{1}{\sqrt{1 + x^2}}$$

34.
$$\frac{d}{dx} (\operatorname{csch}^{-1} x) = -\frac{1}{|x|\sqrt{x^2 + 1}}$$

32.
$$\frac{d}{dx} (\cosh^{-1}x) = \frac{1}{\sqrt{x^2 - 1}}$$

35.
$$\frac{d}{dx} (\operatorname{sech}^{-1} x) = -\frac{1}{x\sqrt{1-x^2}}$$

33.
$$\frac{d}{dx} (\tanh^{-1} x) = \frac{1}{1 - x^2}$$

36.
$$\frac{d}{dx} \left(\coth^{-1} x \right) = \frac{1}{1 - x^2}$$

REFERENCE PAGES

TABLE OF INTEGRALS

Basic Forms

$$1. \int u \, dv = uv - \int v \, du$$

2.
$$\int u^n du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$$

$$3. \int \frac{du}{u} = \ln|u| + C$$

$$4. \int e^u du = e^u + C$$

$$5. \int a^u du = \frac{a^u}{\ln a} + C$$

$$\mathbf{6.} \int \sin u \, du = -\cos u + C$$

$$7. \int \cos u \, du = \sin u + C$$

$$8. \int \sec^2 u \, du = \tan u + C$$

$$\mathbf{9.} \int \csc^2 u \ du = -\cot u + C$$

$$10. \int \sec u \, \tan u \, du = \sec u + C$$

$$11. \int \csc u \cot u \, du = -\csc u + C$$

$$12. \int \tan u \, du = \ln |\sec u| + C$$

$$\mathbf{13.} \int \cot u \, du = \ln |\sin u| + C$$

$$14. \int \sec u \, du = \ln |\sec u + \tan u| + C$$

$$15. \int \csc u \, du = \ln |\csc u - \cot u| + C$$

16.
$$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} \frac{u}{a} + C, \quad a > 0$$

17.
$$\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$$

18.
$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1} \frac{u}{a} + C$$

19.
$$\int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \left| \frac{u + a}{u - a} \right| + C$$

20.
$$\int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln \left| \frac{u - a}{u + a} \right| + C$$

Forms Involving $\sqrt{a^2 + u^2}$, a > 0

21.
$$\int \sqrt{a^2 + u^2} \, du = \frac{u}{2} \sqrt{a^2 + u^2} + \frac{a^2}{2} \ln(u + \sqrt{a^2 + u^2}) + C$$

22.
$$\int u^2 \sqrt{a^2 + u^2} \, du = \frac{u}{8} \left(a^2 + 2u^2 \right) \sqrt{a^2 + u^2} - \frac{a^4}{8} \ln \left(u + \sqrt{a^2 + u^2} \right) + C$$

23.
$$\int \frac{\sqrt{a^2 + u^2}}{u} du = \sqrt{a^2 + u^2} - a \ln \left| \frac{a + \sqrt{a^2 + u^2}}{u} \right| + C$$

24.
$$\int \frac{\sqrt{a^2 + u^2}}{u^2} du = -\frac{\sqrt{a^2 + u^2}}{u} + \ln(u + \sqrt{a^2 + u^2}) + C$$

25.
$$\int \frac{du}{\sqrt{a^2 + u^2}} = \ln(u + \sqrt{a^2 + u^2}) + C$$

26.
$$\int \frac{u^2 du}{\sqrt{a^2 + u^2}} = \frac{u}{2} \sqrt{a^2 + u^2} - \frac{a^2}{2} \ln(u + \sqrt{a^2 + u^2}) + C$$

27.
$$\int \frac{du}{u\sqrt{a^2 + u^2}} = -\frac{1}{a} \ln \left| \frac{\sqrt{a^2 + u^2} + a}{u} \right| + C$$

28.
$$\int \frac{du}{u^2 \sqrt{a^2 + u^2}} = -\frac{\sqrt{a^2 + u^2}}{a^2 u} + C$$

29.
$$\int \frac{du}{(a^2 + u^2)^{3/2}} = \frac{u}{a^2 \sqrt{a^2 + u^2}} + C$$

TABLE OF INTEGRALS

Forms Involving $\sqrt{a^2-u^2},\ a>0$

30.
$$\int \sqrt{a^2 - u^2} \, du = \frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \sin^{-1} \frac{u}{a} + C$$

31.
$$\int u^2 \sqrt{a^2 - u^2} \, du = \frac{u}{8} (2u^2 - a^2) \sqrt{a^2 - u^2} + \frac{a^4}{8} \sin^{-1} \frac{u}{a} + C$$

32.
$$\int \frac{\sqrt{a^2 - u^2}}{u} du = \sqrt{a^2 - u^2} - a \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$$

33.
$$\int \frac{\sqrt{a^2 - u^2}}{u^2} du = -\frac{1}{u} \sqrt{a^2 - u^2} - \sin^{-1} \frac{u}{a} + C$$

34.
$$\int \frac{u^2 du}{\sqrt{a^2 - u^2}} = -\frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \sin^{-1} \frac{u}{a} + C$$

35.
$$\int \frac{du}{u\sqrt{a^2 - u^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$$

36.
$$\int \frac{du}{u^2 \sqrt{a^2 - u^2}} = -\frac{1}{a^2 u} \sqrt{a^2 - u^2} + C$$

37.
$$\int (a^2 - u^2)^{3/2} du = -\frac{u}{8} (2u^2 - 5a^2) \sqrt{a^2 - u^2} + \frac{3a^4}{8} \sin^{-1} \frac{u}{a} + C$$

38.
$$\int \frac{du}{(a^2 - u^2)^{3/2}} = \frac{u}{a^2 \sqrt{a^2 - u^2}} + C$$

Forms Involving $\sqrt{u^2-a^2}$, a>0

39.
$$\int \sqrt{u^2 - a^2} \, du = \frac{u}{2} \sqrt{u^2 - a^2} - \frac{a^2}{2} \ln \left| u + \sqrt{u^2 - a^2} \right| + C$$

40.
$$\int u^2 \sqrt{u^2 - a^2} \, du = \frac{u}{8} \left(2u^2 - a^2 \right) \sqrt{u^2 - a^2} - \frac{a^4}{8} \ln \left| u + \sqrt{u^2 - a^2} \right| + C$$

41.
$$\int \frac{\sqrt{u^2 - a^2}}{u} du = \sqrt{u^2 - a^2} - a \cos^{-1} \frac{a}{|u|} + C$$

42.
$$\int \frac{\sqrt{u^2 - a^2}}{u^2} du = -\frac{\sqrt{u^2 - a^2}}{u} + \ln |u + \sqrt{u^2 - a^2}| + C$$

43.
$$\int \frac{du}{\sqrt{u^2 - a^2}} = \ln \left| u + \sqrt{u^2 - a^2} \right| + C$$

44.
$$\int \frac{u^2 du}{\sqrt{u^2 - a^2}} = \frac{u}{2} \sqrt{u^2 - a^2} + \frac{a^2}{2} \ln \left| u + \sqrt{u^2 - a^2} \right| + C$$

45.
$$\int \frac{du}{u^2 \sqrt{u^2 - a^2}} = \frac{\sqrt{u^2 - a^2}}{a^2 u} + C$$

46.
$$\int \frac{du}{(u^2 - a^2)^{3/2}} = -\frac{u}{a^2 \sqrt{u^2 - a^2}} + C$$

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TABLE OF INTEGRALS

Forms Involving a + bu

47.
$$\int \frac{u \, du}{a + bu} = \frac{1}{b^2} (a + bu - a \ln |a + bu|) + C$$

48.
$$\int \frac{u^2 du}{a + bu} = \frac{1}{2b^3} \left[(a + bu)^2 - 4a(a + bu) + 2a^2 \ln|a + bu| \right] + C$$

49.
$$\int \frac{du}{u(a+bu)} = \frac{1}{a} \ln \left| \frac{u}{a+bu} \right| + C$$

50.
$$\int \frac{du}{u^2(a+bu)} = -\frac{1}{au} + \frac{b}{a^2} \ln \left| \frac{a+bu}{u} \right| + C$$

51.
$$\int \frac{u \, du}{(a + bu)^2} = \frac{a}{b^2 (a + bu)} + \frac{1}{b^2} \ln|a + bu| + C$$

52.
$$\int \frac{du}{u(a+bu)^2} = \frac{1}{a(a+bu)} - \frac{1}{a^2} \ln \left| \frac{a+bu}{u} \right| + C$$

53.
$$\int \frac{u^2 du}{(a+bu)^2} = \frac{1}{b^3} \left(a + bu - \frac{a^2}{a+bu} - 2a \ln|a+bu| \right) + C$$

54.
$$\int u\sqrt{a+bu}\,du = \frac{2}{15b^2}(3bu-2a)(a+bu)^{3/2} + C$$

55.
$$\int \frac{u \, du}{\sqrt{a + bu}} = \frac{2}{3b^2} (bu - 2a) \sqrt{a + bu} + C$$

56.
$$\int \frac{u^2 du}{\sqrt{a + bu}} = \frac{2}{15b^3} (8a^2 + 3b^2u^2 - 4abu) \sqrt{a + bu} + C$$

57.
$$\int \frac{du}{u\sqrt{a+bu}} = \frac{1}{\sqrt{a}} \ln \left| \frac{\sqrt{a+bu} - \sqrt{a}}{\sqrt{a+bu} + \sqrt{a}} \right| + C, \text{ if } a > 0$$
$$= \frac{2}{\sqrt{-a}} \tan^{-1} \sqrt{\frac{a+bu}{-a}} + C, \text{ if } a < 0$$

58.
$$\int \frac{\sqrt{a+bu}}{u} du = 2\sqrt{a+bu} + a \int \frac{du}{u\sqrt{a+bu}}$$

59.
$$\int \frac{\sqrt{a+bu}}{u^2} du = -\frac{\sqrt{a+bu}}{u} + \frac{b}{2} \int \frac{du}{u\sqrt{a+bu}}$$

60.
$$\int u^n \sqrt{a + bu} \, du = \frac{2}{b(2n+3)} \left[u^n (a + bu)^{3/2} - na \int u^{n-1} \sqrt{a + bu} \, du \right]$$

61.
$$\int \frac{u^n du}{\sqrt{a + bu}} = \frac{2u^n \sqrt{a + bu}}{b(2n + 1)} - \frac{2na}{b(2n + 1)} \int \frac{u^{n-1} du}{\sqrt{a + bu}}$$

62.
$$\int \frac{du}{u^n \sqrt{a + bu}} = -\frac{\sqrt{a + bu}}{a(n - 1)u^{n-1}} - \frac{b(2n - 3)}{2a(n - 1)} \int \frac{du}{u^{n-1} \sqrt{a + bu}}$$

TABLE OF INTEGRALS

Trigonometric Forms

63.
$$\int \sin^2 u \ du = \frac{1}{2}u - \frac{1}{4}\sin 2u + C$$

64.
$$\int \cos^2 u \ du = \frac{1}{2}u + \frac{1}{4}\sin 2u + C$$

$$\mathbf{65.} \int \tan^2 u \ du = \tan u - u + C$$

$$\mathbf{66.} \int \cot^2 u \, du = -\cot u - u + C$$

67.
$$\int \sin^3 u \ du = -\frac{1}{3}(2 + \sin^2 u) \cos u + C$$

68.
$$\int \cos^3 u \ du = \frac{1}{3}(2 + \cos^2 u) \sin u + C$$

69.
$$\int \tan^3 u \ du = \frac{1}{2} \tan^2 u + \ln|\cos u| + C$$

70.
$$\int \cot^3 u \ du = -\frac{1}{2} \cot^2 u - \ln|\sin u| + C$$

71.
$$\int \sec^3 u \, du = \frac{1}{2} \sec u \, \tan u + \frac{1}{2} \ln |\sec u + \tan u| + C$$

72.
$$\int \csc^3 u \, du = -\frac{1}{2} \csc u \, \cot u + \frac{1}{2} \ln |\csc u - \cot u| + C$$

73.
$$\int \sin^n u \, du = -\frac{1}{n} \sin^{n-1} u \, \cos u + \frac{n-1}{n} \int \sin^{n-2} u \, du$$

74.
$$\int \cos^n u \, du = \frac{1}{n} \cos^{n-1} u \, \sin u + \frac{n-1}{n} \int \cos^{n-2} u \, du$$

75.
$$\int \tan^n u \ du = \frac{1}{n-1} \tan^{n-1} u - \int \tan^{n-2} u \ du$$

Inverse Trigonometric Forms

87.
$$\int \sin^{-1} u \, du = u \sin^{-1} u + \sqrt{1 - u^2} + C$$

88.
$$\int \cos^{-1} u \, du = u \cos^{-1} u - \sqrt{1 - u^2} + C$$

89.
$$\int \tan^{-1} u \, du = u \tan^{-1} u - \frac{1}{2} \ln(1 + u^2) + C$$

90.
$$\int u \sin^{-1} u \, du = \frac{2u^2 - 1}{4} \sin^{-1} u + \frac{u\sqrt{1 - u^2}}{4} + C$$

91.
$$\int u \cos^{-1} u \, du = \frac{2u^2 - 1}{4} \cos^{-1} u - \frac{u\sqrt{1 - u^2}}{4} + C$$

76.
$$\int \cot^n u \ du = \frac{-1}{n-1} \cot^{n-1} u - \int \cot^{n-2} u \ du$$

77.
$$\int \sec^n u \, du = \frac{1}{n-1} \tan u \sec^{n-2} u + \frac{n-2}{n-1} \int \sec^{n-2} u \, du$$

78.
$$\int \csc^n u \, du = \frac{-1}{n-1} \cot u \csc^{n-2} u + \frac{n-2}{n-1} \int \csc^{n-2} u \, du$$

79.
$$\int \sin au \, \sin bu \, du = \frac{\sin(a-b)u}{2(a-b)} - \frac{\sin(a+b)u}{2(a+b)} + C$$

80.
$$\int \cos au \cos bu \, du = \frac{\sin(a-b)u}{2(a-b)} + \frac{\sin(a+b)u}{2(a+b)} + C$$

81.
$$\int \sin au \cos bu \, du = -\frac{\cos(a-b)u}{2(a-b)} - \frac{\cos(a+b)u}{2(a+b)} + C$$

82.
$$\int u \sin u \, du = \sin u - u \cos u + C$$

83.
$$\int u \cos u \, du = \cos u + u \sin u + C$$

84.
$$\int u^n \sin u \, du = -u^n \cos u + n \int u^{n-1} \cos u \, du$$

85.
$$\int u^n \cos u \, du = u^n \sin u - n \int u^{n-1} \sin u \, du$$

86.
$$\int \sin^n u \, \cos^m u \, du = -\frac{\sin^{n-1} u \, \cos^{m+1} u}{n+m} + \frac{n-1}{n+m} \int \sin^{n-2} u \, \cos^m u \, du$$
$$= \frac{\sin^{n+1} u \, \cos^{m-1} u}{n+m} + \frac{m-1}{n+m} \int \sin^n u \, \cos^{m-2} u \, du$$

92.
$$\int u \tan^{-1} u \, du = \frac{u^2 + 1}{2} \tan^{-1} u - \frac{u}{2} + C$$

93.
$$\int u^n \sin^{-1} u \, du = \frac{1}{n+1} \left[u^{n+1} \sin^{-1} u - \int \frac{u^{n+1} \, du}{\sqrt{1-u^2}} \right], \quad n \neq -1$$

94.
$$\int u^n \cos^{-1} u \, du = \frac{1}{n+1} \left[u^{n+1} \cos^{-1} u + \int \frac{u^{n+1} \, du}{\sqrt{1-u^2}} \right], \quad n \neq -1$$

95.
$$\int u^n \tan^{-1} u \, du = \frac{1}{n+1} \left[u^{n+1} \tan^{-1} u - \int \frac{u^{n+1} \, du}{1+u^2} \right], \quad n \neq -1$$

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Exponential and Logarithmic Forms

96.
$$\int ue^{au} du = \frac{1}{a^2} (au - 1)e^{au} + C$$

97.
$$\int u^n e^{au} du = \frac{1}{a} u^n e^{au} - \frac{n}{a} \int u^{n-1} e^{au} du$$

98.
$$\int e^{au} \sin bu \, du = \frac{e^{au}}{a^2 + b^2} (a \sin bu - b \cos bu) + C$$

99.
$$\int e^{au} \cos bu \ du = \frac{e^{au}}{a^2 + b^2} (a \cos bu + b \sin bu) + C$$

100.
$$\int \ln u \, du = u \ln u - u + C$$

101.
$$\int u^n \ln u \, du = \frac{u^{n+1}}{(n+1)^2} [(n+1) \ln u - 1] + C$$

102.
$$\int \frac{1}{u \ln u} du = \ln |\ln u| + C$$

Hyperbolic Forms

$$\mathbf{103.} \int \sinh u \, du = \cosh u + C$$

$$\mathbf{104.} \int \cosh u \, du = \sinh u + C$$

105.
$$\int \tanh u \, du = \ln \cosh u + C$$

$$\mathbf{106.} \int \coth u \, du = \ln |\sinh u| + C$$

107.
$$\int \mathrm{sech} \, u \, du = \tan^{-1} | \sinh u | + C$$

$$108. \int \operatorname{csch} u \, du = \ln \left| \tanh \frac{1}{2} u \right| + C$$

$$109. \int \operatorname{sech}^2 u \, du = \tanh u + C$$

$$\mathbf{110.} \int \operatorname{csch}^2 u \, du = -\coth u + C$$

111.
$$\int \operatorname{sech} u \, \tanh u \, du = -\operatorname{sech} u + C$$

112.
$$\int \operatorname{csch} u \operatorname{coth} u \, du = -\operatorname{csch} u + C$$

Forms Involving $\sqrt{2au-u^2}$, a>0

113.
$$\int \sqrt{2au - u^2} \, du = \frac{u - a}{2} \sqrt{2au - u^2} + \frac{a^2}{2} \cos^{-1} \left(\frac{a - u}{a} \right) + C$$

114.
$$\int u\sqrt{2au - u^2} \, du = \frac{2u^2 - au - 3a^2}{6} \sqrt{2au - u^2} + \frac{a^3}{2} \cos^{-1} \left(\frac{a - u}{a}\right) + C$$

115.
$$\int \frac{\sqrt{2au - u^2}}{u} du = \sqrt{2au - u^2} + a \cos^{-1} \left(\frac{a - u}{a}\right) + C$$

116.
$$\int \frac{\sqrt{2au - u^2}}{u^2} du = -\frac{2\sqrt{2au - u^2}}{u} - \cos^{-1} \left(\frac{a - u}{a}\right) + C$$

$$117. \int \frac{du}{\sqrt{2au - u^2}} = \cos^{-1}\left(\frac{a - u}{a}\right) + C$$

118.
$$\int \frac{u \, du}{\sqrt{2au - u^2}} = -\sqrt{2au - u^2} + a \cos^{-1} \left(\frac{a - u}{a}\right) + C$$

119.
$$\int \frac{u^2 du}{\sqrt{2au - u^2}} = -\frac{(u + 3a)}{2} \sqrt{2au - u^2} + \frac{3a^2}{2} \cos^{-1} \left(\frac{a - u}{a}\right) + C$$

120.
$$\int \frac{du}{u\sqrt{2au - u^2}} = -\frac{\sqrt{2au - u^2}}{au} + C$$