Circuit Theory and Electronics Fundamentals

Masters of Aeroespace Engineer, Técnico, University of Lisbon

Laboratory Report

Group 37

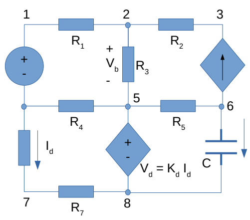
Afonso Magalhães, nº95765
Fábio Monteiro, nº95786
Leonardo Encarnação, nº95816

March 25, 2021

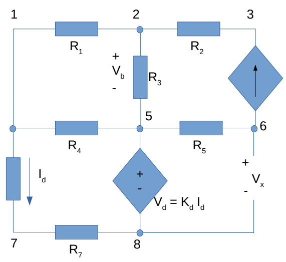
Contents

1 Introduction

In this laboratory assignment we study a circuit containing various elements, amongst them a capacitor, resistances and dependent and independent sources of voltage and current in order to accomplish various objectives. At first we analyse the circuit when t¡0, using the nodal method to determine the voltages in all nodes and currents in all branches. Then, in order to find the R_eq, we change the circuit into what is displayed in Figure 2, and with this new circuit, we'll find the total solution of the voltage V_s. To finalize, we are going to determine the frequency responses of the voltage in the capacitor and in the node 6.



(a) First Circuit



(b) Second Circuit

In Section 2, a theoretical analysis of the circuit is presented. In Section 3, the circuit is analysed by simulation, and the results are compared to the theoretical results obtained in Section 2. The conclusions of this study are outlined in Section 4.

2 Theoretical Analysis

In this section we will explain with some detail how do we analyse the circuit theoretically. To do so, and because there were several things to be analyse, the following subsections are going to detail the analysis of each of the six powerpoint points related to the theoretical analysis. Just as a final note, the results obtained theoretically will only be shown in the symulation section, to compare the both.

2.1 Point 1: Determine the initial state of the circuit

Initially, the circuits's independent voltage source inputs in the circuit a constant voltage. Because of this, and considering that the circuit is working for a really long time (started working for t = $-\infty$), we can assume that the capacitor is already fully charged and, consequentaly, it behaves like an open circuit. So to analyse the circuit's initial state we just need to do a node analysis just like we did in *laboratory nº1*.

After performing the node analysis we've obtained the following matrix:

$$\begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ -G1 & G1+G2+G3 & -G2 & 0 & -G3 & 0 & 0 & 0 \\ 0 & -G2-Kb & G2 & 0 & Kb & 0 & 0 & 0 \\ G1 & -G1 & 0 & G4+G6 & -G4 & 0 & -G6 & 0 \\ 0 & 0 & 0 & -Kd*G6 & 1 & 0 & Kd*G6 & 0 \\ 0 & Kb & 0 & 0 & -G5-Kb & G5 & 0 & 0 \\ 0 & 0 & 0 & -G6 & 0 & 0 & G6+G7 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \\ V4 \\ V5 \\ V6 \\ V7 \\ V8 \end{pmatrix} = \begin{pmatrix} V5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

2.2 Point 2: Determine the equivalent resistance seen by the capacitor

To determine the equivalent resistance, we've defined a voltage source Vx between the nodes 6 and 8 and we've powered off the independent voltage source Vs. After that, we use the node analysis to determine the volatge in all nodes, which resulted in the following matrix:

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ -G1 & G1+G2+G3 & -G2 & 0 & -G3 & 0 & 0 & 0 \\ 0 & -G2-Kb & G2 & 0 & Kb & 0 & 0 & 0 \\ G1 & -G1 & 0 & G4+G6 & -G4 & 0 & -G6 & 0 \\ 0 & 0 & 0 & -Kd*G6 & 1 & 0 & Kd*G6 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -G6 & 0 & 0 & G6+G7 & -G7 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \\ V4 \\ V5 \\ V6 \\ V7 \\ V8 \end{pmatrix} = \begin{pmatrix} Vx \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

After knowing these voltages, it's easy to determine the currents in all branches. To determine the equavalent resistance we need to determine the current in the branch 6-8, Ix, just because $R_{eq} = V_x/I_x$. We've obtained Ix by analysing the node 8 and we've obtained the following expression for Ix:

After knowing Ix, we now can obtain Req, using the previous shown formula.

2.3 Point 3: Obtaining the natural solution for the capacitor

Using the results in point 2, we can determine the natural solution of the circuit by making a simple RC circuit, with the only components being the capacitor and the equivalent resistance, just as shown in the figure:

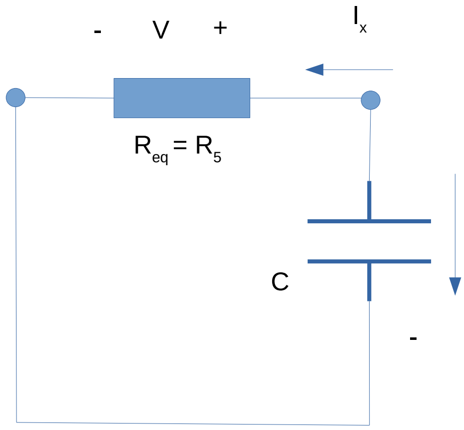


Figure 2: Circuit3

Using the Kirchhoff's voltage law for this circuit, we've obtain the folling diferential equation:

Knowing the that the $V_{capacitor_i} = V_6 - V_8$ and using the values for these voltages obtained in point 1, the natural solution for the capacitor is given by:

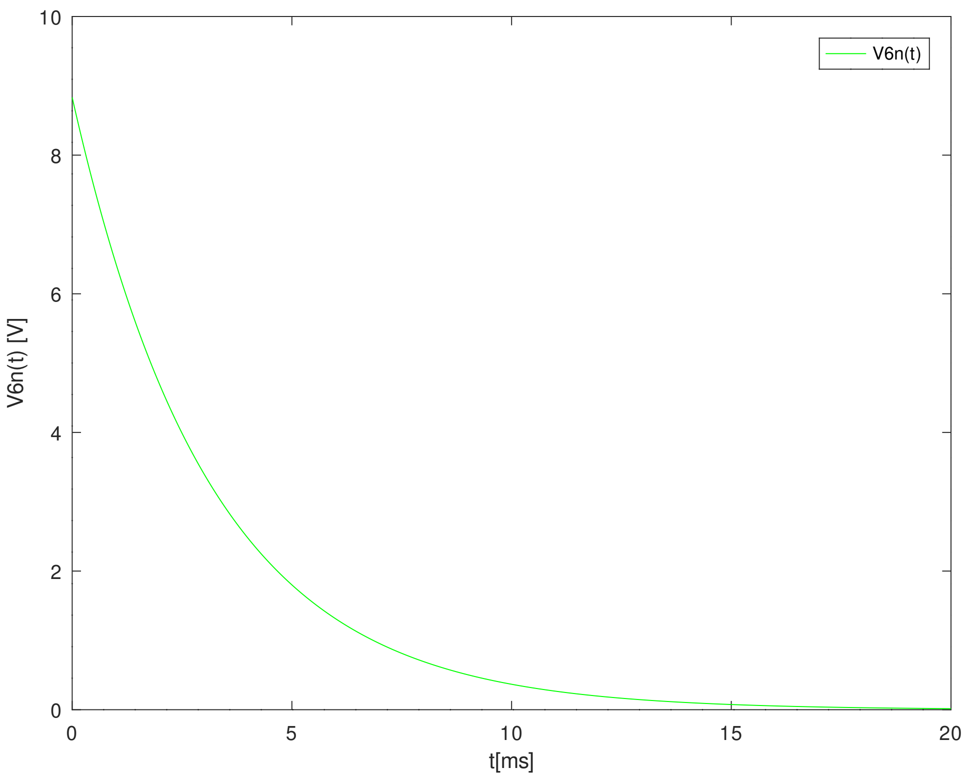


Figure 3: Natural Response

2.4 Point 4: Determine the forced solution for the circuit

We already know that in a forced circuit the voltages in all nodes will gain the same frequency as the source. Knowing this we know that the solution for the node i will be $V_i = V_{i_{max}} * cos(w * t + \phi_i)$. So, in order to determine the voltages for all nodes, we just need to determine the voltages amplitudes and phases, or in other words, their complex amplitude. This brings an interesting point: if we consider the complex world the solutions will be in the form of $V_i = V_{i_{max}} * e^{w*t} * e^{\phi_i}$, which makes easy to obtain both the amplitudes and the phases. Alongside this, we just need to solve the node analysis for all the circuit, which will result in the following matrix:

$$\begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ -G1 & G1+G2+G3 & -G2 & 0 & -G3 & 0 & 0 & 0 \\ 0 & -G2-Kb & G2 & 0 & Kb & 0 & 0 & 0 \\ G1 & -G1 & 0 & G4+G6 & -G4 & 0 & -G6 & 0 \\ 0 & 0 & 0 & -Kd*G6 & 1 & 0 & Kd*G6 & -1 \\ 0 & Kb & 0 & 0 & -G5-Kb & G5+\frac{1}{Z_c} & 0 & -\frac{1}{Z_c} \\ 0 & 0 & 0 & -G6 & 0 & 0 & G6+G7 & -G7 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \\ V4 \\ V5 \\ V6 \\ V7 \\ V8 \end{pmatrix} = \begin{pmatrix} V_s(t) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Note: For the node 6 we have the following equation:

And this is what we see in the matrix

Solving the matrix using the given values, the complex amplitudes generated are the following:

2.5 Point 5: Determine the solution for the voltage in node 6

Using the data obtained in the points 3 and 4 is relatively easy to find the solution for the voltage in the node 6. We just need to find the forced solution using the amplitude and phase for the node 6 obtained in point 4 e aplying it in this formula: $V_{6f} = V_{6_{max}} * cos(w * t + \phi_6)$. After that it's just add ir to the natural solution and we've obtained the following:

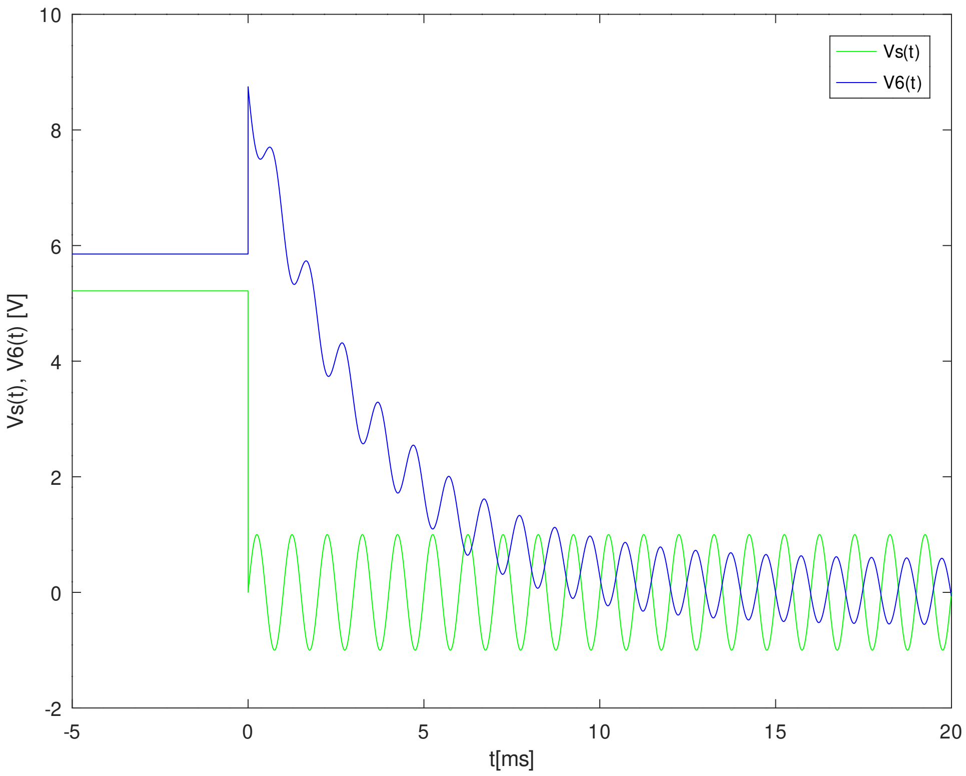


Figure 4: Natural and forced response

2.6 Point 6: Determine the frequency response for Vc(f), V6(f) and Vs(f)

In the final section of the work we just needed to repeat the point 4 multiples times modifying the fequencies of the source, to see how the amplitude and phase of Vc(f) (that is V6 - V8), V6(f) and Vs(f). The results can be seen below.

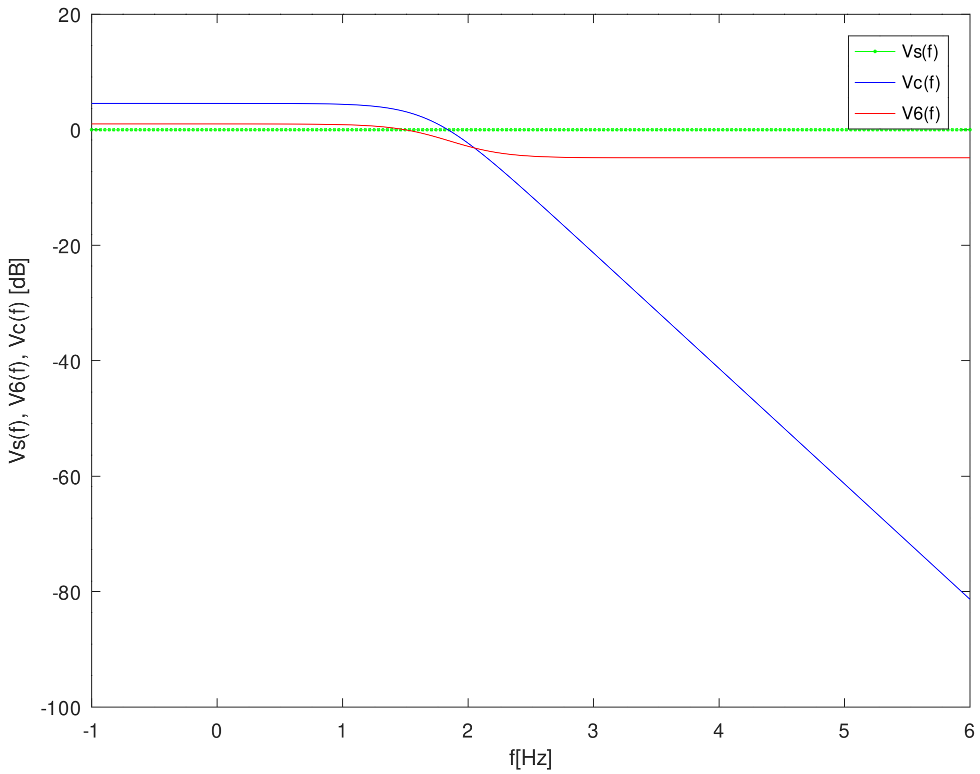


Figure 5: Amplitude

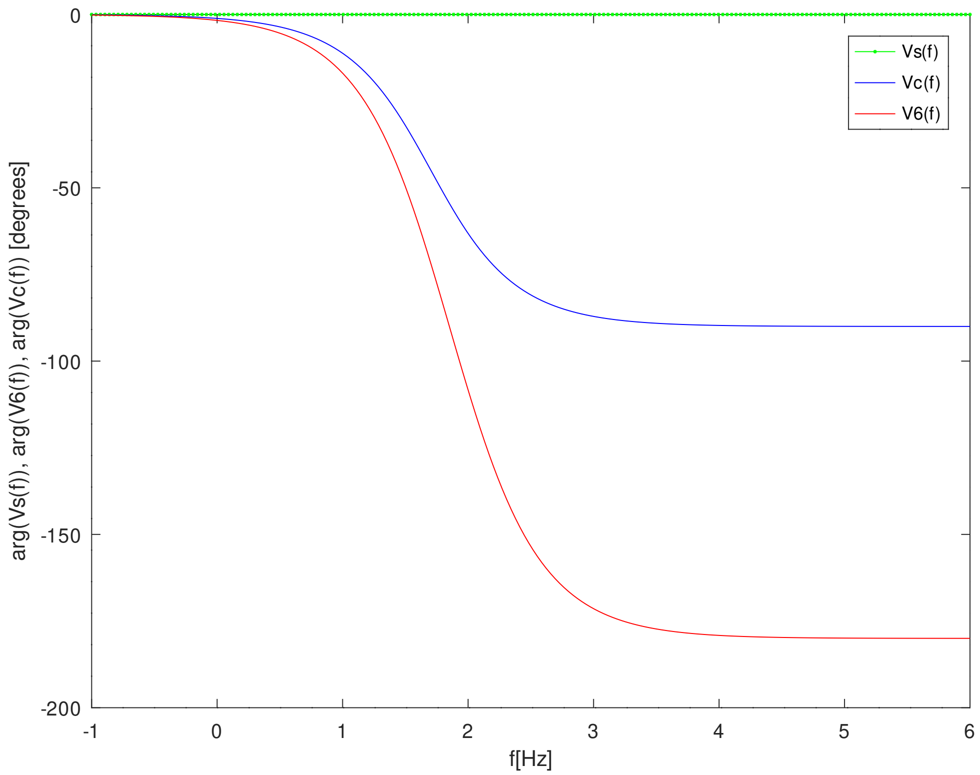


Figure 6: Arguments

3 Simulation Analysis

3.1 1

The results obtained from the simulation are shown in the table below.

Name	Value [A or V]	Name	Value [A or V]
@gb	-2.86455e-04	V1	5.216040e+00
@R1[i]	2.732478e-04	V2	4.939074e+00
@R2[i]	-2.86455e-04	V3	4.361415e+00
@R3[i]	-1.32073e-05	V5	4.978786e+00
@R4[i]	1.229564e-03	V6	5.853598e+00
@R5[i]	1.316042e-03	V7	-2.00109e+00
@R6[i]	9.563162e-04	V8	-2.97875e+00
@R7[i]	9.563162e-04	V9	0.000000e+00

Table 1: NgSpice simulation results 1

3.2 2

Name	Value [A or V]	Name	Value [A or V]
@gb	0.000000e+00	V1	0.000000e+00
@R1[i]	0.000000e+00	V2	0.000000e+00
@R2[i]	0.000000e+00	V3	0.000000e+00
@R3[i]	0.000000e+00	V5	0.000000e+00
@R4[i]	0.000000e+00	V6	5.000000e+01
@R5[i]	-1.63724e-02	V7	0.000000e+00
@R6[i]	0.000000e+00	V8	0.000000e+00
@R7[i]	0.000000e+00	V9	0.000000e+00

Table 2: NgSpice simulation results 2

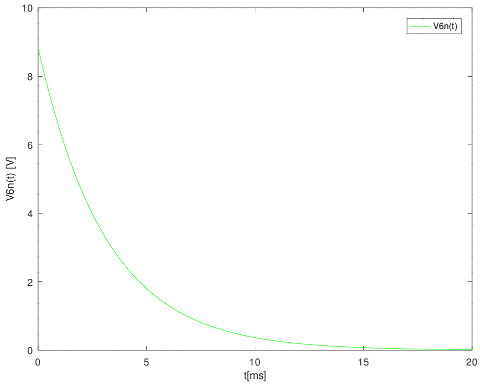
3.3 3

3.4 4

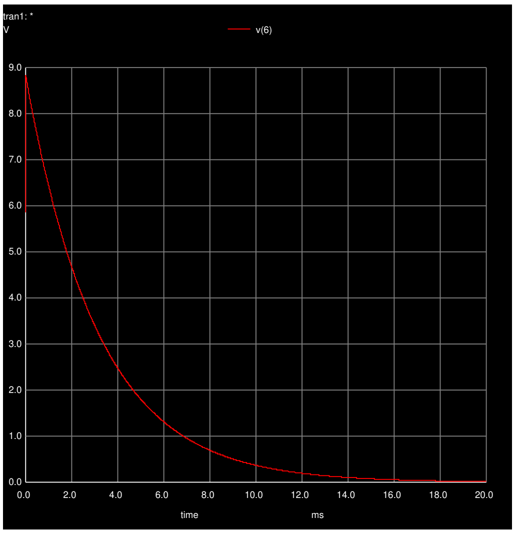
3.5 5

4 Conclusion

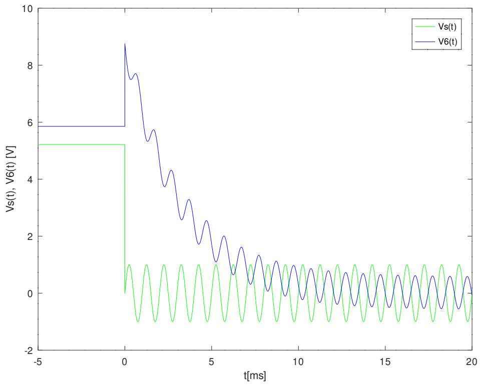
In this laboratory assignment the objective of analysing the circuit especified in the introduction has been achieved. All analyses have been performed both theoretically using the Octave maths tool and by circuit simulation using the Ngspice tool.



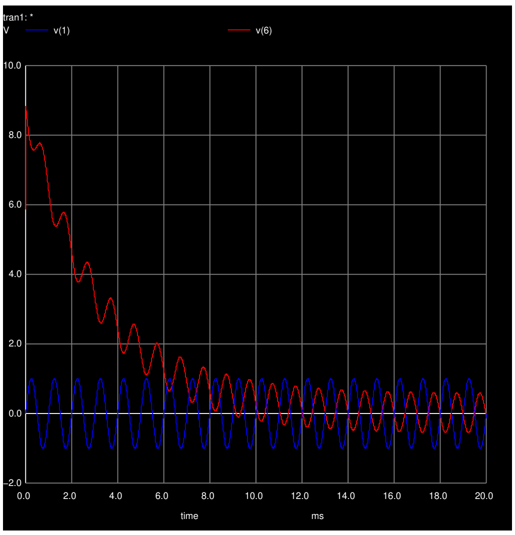
(a) Natural Response (Octave)



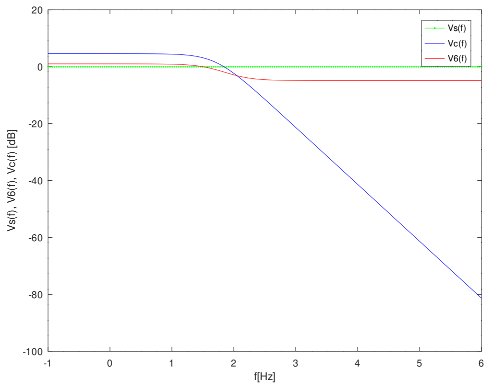
(b) Natural Response (NGSpice)



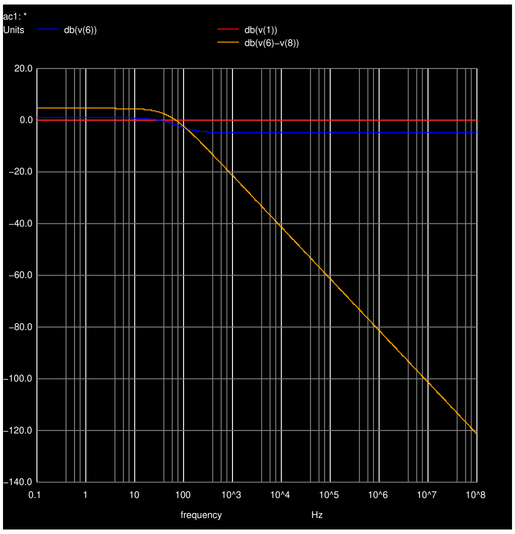
(a) Natural and forced response (Octave)



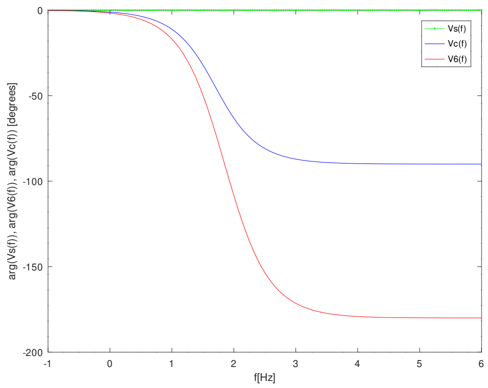
(b) Natural and forced response



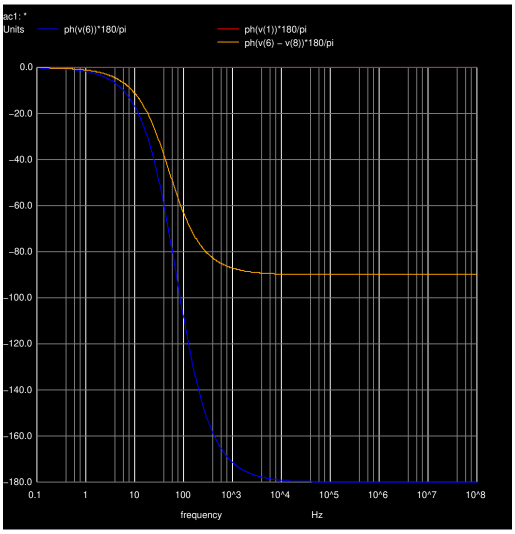
(a) First Circuit



(b) Second Circuit



(a) First Circuit



(b) Second Circuit