Circuit Theory and Electronics Fundamentals

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Laboratory Report: Lab Assignment T1, Group 26

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1 Introduction

The objective of this laboratory assignment is to study a circuit composed of four elementary meshes, where these can contain resistors, independent current or voltage sources and dependent current or voltage sources as seen in figure 1. In mesh B, we find a current source whose current is proportional to the voltage V_b (voltage controlled current source). In mesh C, on the other hand, there is a voltage source that varies proportionally with the current I_c (current controlled voltage source). The proportionality constants for these two cases are called, respectively, transconductance (SI unit: Siemens - S) and transresistance (SI unit: Ohm - Ω).

In Section 2, a theoretical analysis of the circuit is presented. In Section 3, the circuit is analysed by simulation, and the results are compared to the theoretical results obtained in Section 2. The conclusions of this study are outlined in Section 4. In both the theoretical and simulation analysis the current directions for each branch are defined as in figure 1. The potencial 0 was assigned to node 0 (the one where the voltage is V_0), so this was the node used as a reference.

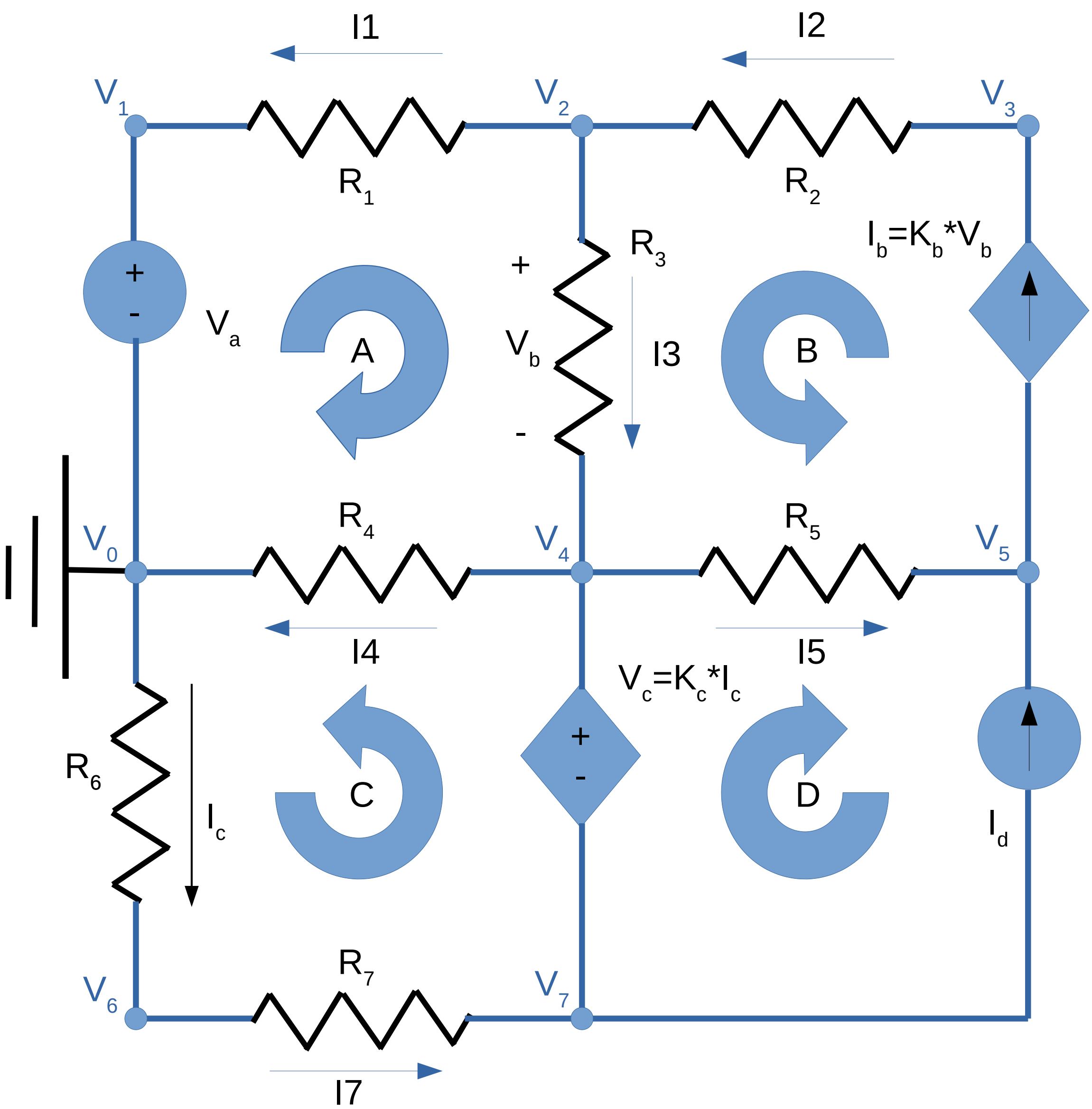

Figure 1: Circuit analysed.

2 Theoretical Analysis

In this section, the circuit shown in figure 1 is analysed theoretically, using both the mesh and the nodal methods.

2.1 Mesh method

The 4 meshes used for this method are identified in figure 1, as well as the direction chosen for the current assigned to each mesh. Using this method the following system was obtained:

$$\begin{cases} (A) : (R_1 + R_3 + R_4)\, I_A + R_3 I_B + R_4 I_C = V_a \\ (B) : -K_b R_3 I_A + (1 - K_b R_3)\, I_B = 0 \\ (C) : R_4 I_A + (R_4 + R_6 + R_7 - K_c)\, I_c = 0 \\ (D) : I_D = -I_d \end{cases} \tag{1}$$

For meshes A and C (meshes without current sources), we used Kirchhoff's Voltage Law (KVL) around each mesh (equations (A) and (C)). For meshes B and D (meshes with current sources), we used the relations between the current of the current sources and the mesh's current (equations (B) and (D)). Using the following equations, we can then find the currents through each resistor:

$$\begin{cases} I_1 = -I_A \\ I_2 = I_B \\ I_3 = I_A + I_B \\ I_4 = I_A + I_C \\ I_5 = I_B + I_D \\ I_c = I_C \\ I_7 = I_C \end{cases} \tag{2}$$

The voltages for each node can be directly obtained using Ohm's Law. The results obtained are written below.

Name	Value [mA]
IA	0.222012
IB	-0.232363
IC	0.929850
ID	-1.009821

Table 1: Mesh analysis: values obtained for mesh currents.

Name	Value [mA]
I1	-0.222012
I2	-0.232363
I3	-0.010351
I4	1.151862
I5	-1.242183
Ic	0.929850
I7	0.929850

Table 2: Mesh analysis: values obtained for the currents running through the resistors.

Name	Value [V]
V2-V1	-0.232598
V3-V2	-0.480374
V2-V4	-0.032271
V4-V0	4.821580
V4-V5	-3.819900
V0-V6	1.909867
V6-V7	0.934990

Table 3: Mesh analysis: values obtained for the voltage drops across resistors (in order, R_1, ..., R_7).

2.2 Nodal method

Applying this method the system obtained was the following:

$$\begin{cases} (1) : V_1 = V_a \\ (2) : -G_1V_1 + (G_1 + G_2 + G_3)\, V_2 - G_2V_3 - G_3V_4 = 0 \\ (3) : -(G_2 + K_b)\, V_2 + G_2V_3 + K_bV_4 = 0 \\ (4) : V_4 + K_cG_6V_6 - V7 = 0 \\ (5) : K_bV_2 - (G_5 + K_b)\, V_4 + G_5V_5 = I_d \\ (6) : (G_6 + G_7)\, V_6 - G_7V_7 = 0 \\ (7) : -G_3V_2 + (G_3 + G_4 + G_5)\, V_4 - G_5V_5 - G_7V_6 + G_7V_6 = 0 \end{cases} \tag{3}$$

For this method, Kirchhoff's Current Law (KCL) was applied directly for nodes 2, 3, 5 and 6 (there are no voltage sources connected to these nodes), giving us equations (2),(3),(5) and (6). Equations (1) and (4) appear directly from the voltage sources and the voltages of the nodes which they are connected to. Finally, since the dependent voltage source is connected to 2 non-reference nodes (nodes 4 and 7), we were required to apply the supernode construct (equation (7)). Using the voltages of every node, the currents and voltages for each branch can be calculated easily.

Name	Value [V]
V1	5.021907
V2	4.789309
V3	4.308935
V4	4.821580
V5	8.641480
V6	-1.909867
V7	-2.844856

Table 4: Node analysis: values obtained for node potentials.

Name	Value [mA]
I1	-0.222012
I2	-0.232363
I3	-0.010351
I4	1.151862
I5	-1.242183
Ic	0.929850
I7	0.929850

Table 5: Node analysis: values obtained for the currents running through the resistors.

Name	Value [V]
V2-V1	-0.232598
V3-V2	-0.480374
V2-V4	-0.032271
V4-V0	4.821580
V4-V5	-3.819900
V0-V6	1.909867
V6-V7	0.934990

Table 6: Node analysis: values obtained for the voltage drops across resistors (in order, R_1, ..., R_7).

As we can see, the results obtained are equivalent to the ones achieved through the mesh analysis method.

3 Simulation Analysis

We decided to, once again, put the circuit's scheme down below, so as to make the interpretation of the following results easier (fig 2).

3.1 Operating Point Analysis

Table 7 shows the simulated operating point results for the circuit under analysis. As can be seen, we obtained similar results to the ones calculated in the theoretical analysis section. This is proof that our theoretical analysis is, indeed, correct. We noticed, however, that *Octave* rounded the values of the circuit's parameters ($R_1, ..., R_7, V_a, I_d, K_b, K_c$), whereas *Ngspice* operated with the same precision as the one provided initially (no rounding). As such, one could expect to find slightly different results. However, this did not prove to be the case. Another important thing to note is that we had to insert a fictitional voltage source, with value $0\,V$, in series with R_6 and R_7, in order to properly introduce the current I_c in V_c's calculation, in the current dependent voltage source. This was due to *Ngspice*'s limitations, though. Thus, the value $V(6.5)$ in the table, which is equal to $V(6)$, like we required it to.

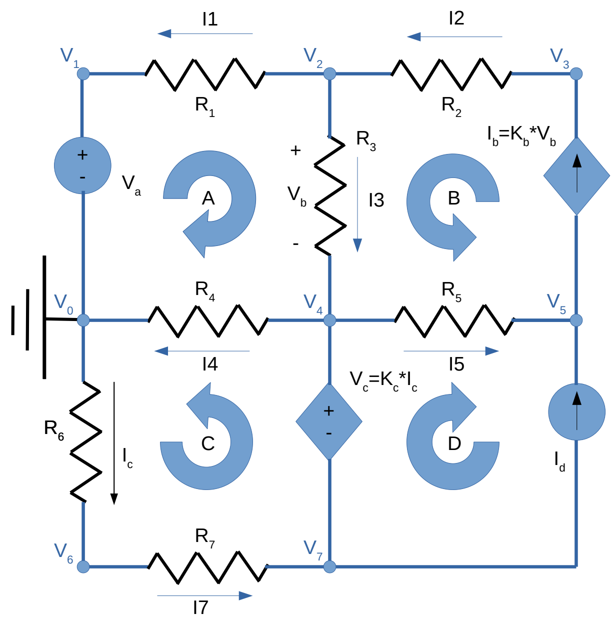

Figure 2: Circuit analysed.

Name	Value [A or V]
v(1)	5.021907e+00
v(2)	4.789309e+00
v(3)	4.308935e+00
v(4)	4.821580e+00
v(5)	8.641480e+00
v(6)	-1.90987e+00
v(6.5)	-1.90987e+00
v(7)	-2.84486e+00
@r1[i]	-2.22012e-04
@r2[i]	-2.32363e-04
@r3[i]	-1.03507e-05
@r4[i]	1.151862e-03
@r5[i]	-1.24218e-03
@r6[i]	9.298504e-04
@r7[i]	9.298504e-04
@hc[i]	7.997019e-05

Table 7: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

4 Conclusion

In this laboratory assignment, the objective of analysing the circuit presented in figure 1 has been achieved. The analysis was performed both theoretically using the Octave maths tool and by circuit simulation using the Ngspice tool. The simulation results matched the theoretical results precisely. The reason for this perfect match is the fact that this is a straightforward circuit containing only linear components, so the theoretical and simulation models cannot differ. For more complex components, the theoretical and simulation models could differ, but this is not the case in this work. Even though the calculations were successful, we realised that we could have done a smarter choice of the reference node. For instance, if we had chosen node 4 as the reference node, the equations would be a little simpler overall, since there would be more voltage values in the equations that would be equal to 0. In addition, this choice of reference node would make us apply the supernode construct for the independent voltage source instead of the dependent one, making the equations easier once again, since there would be less diverging and converging currents to take into account.

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