Open-Circuit Time Constant Analysis

General Form $$H(s) = K\frac{1 + a_1 s + a_2 s^2 + \bullet\bullet\bullet + a_m s^m}{1 + b_1 s + b_2 s^2 + \bullet\bullet\bullet + b_n s^n}$$

When the poles and zeros are easily found, then it is relatively easy to determine a dominant pole, if one exists. But sometimes it is not easy to determine the dominant pole.

The coefficient b_1 in the transfer function is especially important

$$b_1 = \frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} + \frac{1}{\omega_{p3}} + \ldots \frac{1}{\omega_{pn}} = \tau_{p1} + \tau_{p2} + \tau_{p3} + \ldots \tau_{pn}$$

How do we determine the ω_{pi} *or* τ_{pi} values? We next examine all of the capacitors in the overall circuit individually.

Open-Circuit Time Constant Analysis

We consider each capacitor in the overall circuit one at a time by setting every other small capacitor to an open circuit and letting independent voltage sources be short circuits.

The value of b_1 is computed by summing the individual time constants, called the "sum of the open-circuit time constants."

$$b_1 = \sum_{i=1}^{n} R_{io} C_i$$

RC is a time constant

And the pole frequency ω_H is given by

$$\omega_H = \frac{1}{b_1} = \frac{1}{\sum_{i=1}^{n} R_{io} C_i}$$

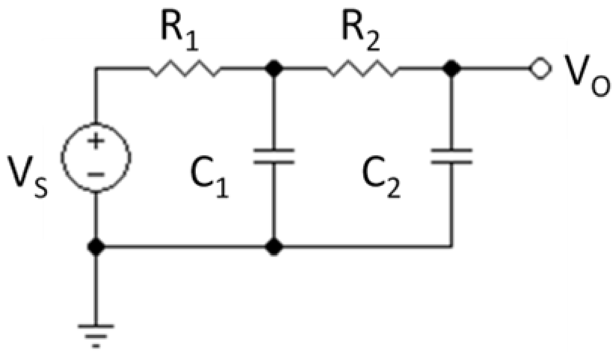
Open-Circuit Time Constant (OCTC) Description

The method of open-circuit time constants provides a simple and powerful way to obtain a reasonably good estimate of the upper 3-dB frequency, f_H. The capacitors that contribute to the high-frequency response are considered one at a time, with independent source V_S turned off (set to zero), and all other capacitances set to zero (that is, open-circuited). The Thévenin resistance presented to each capacitance is then determined, and the time constants (τ_{pi}) are summed to find the overall cutoff frequency f_H is found from $1/(2\pi\sum\tau_{pi})$.

Open-Circuit Time Constant (OCTC) Computation Rules

- For each "small" capacitor C_j in the circuit:
  - Open-circuit all other "small" capacitors
  - Short circuit all "big" capacitors (*e.g.*, coupling capacitors)
  - Turn off all independent sources (but not dependent sources)
  - Replace the capacitor under consideration (C_j) with a current or voltage source for resistance calculation (or determine by inspection)
  - Find the Thévenin equivalent input resistance R_j as seen by the capacitor C_j
  - R_jC_j is the open-circuit time constant for the j^{th} capacitor
- Procedure is best illustrated with an example . . .

Open-Circuit Time Constant (OCTC) Example 1

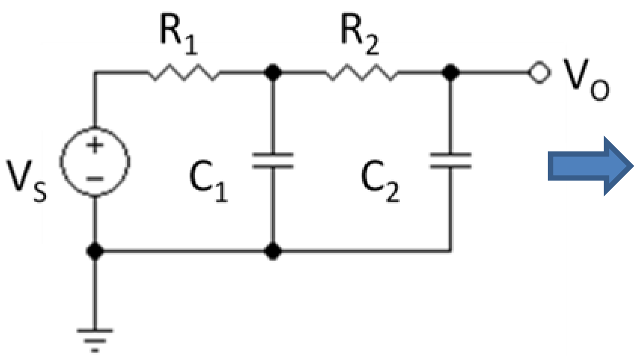


Doing the full analysis gives

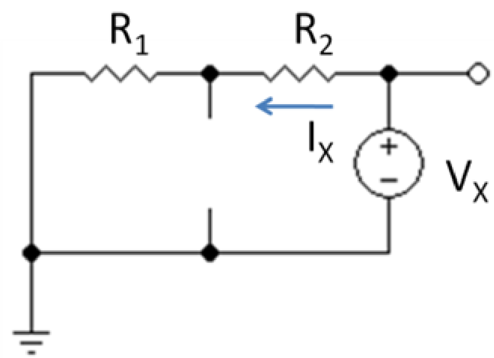
$$\frac{V_O}{V_S} = \frac{1}{1 + s[R_1C_1 + (R_1 + R_2)C_2] + s^2(R_1R_2C_1C_2)} = \underbrace{\frac{1}{1 + b_1 s + b_2 s^2}}_{\text{Standard format}}$$

Remember: $b_1 = \tau_1 + \tau_2$ and $b_2 = \tau_1\tau_2$

Open-Circuit Time Constant (OCTC) Example 1



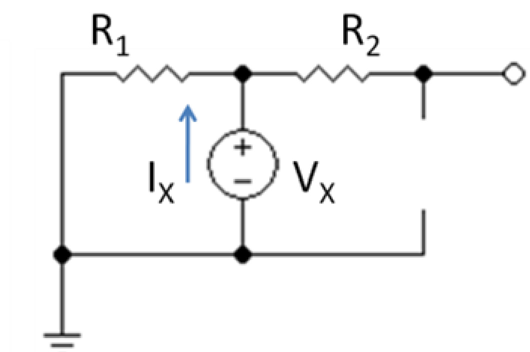
Circuit



Determining τ_2

Set C_1 to open, & replace capacitor C_2 with voltage source V_x & determine Thévenin resistance through which current I_x flows.

$V_x = I_x\,(R_1 + R_2)$, so
Then $\boldsymbol{\tau_2} = (R_1 + R_2)C_2$



Determining τ_1

Set C_3 to open, & replace capacitor C_1 with voltage source V_x & determine Thévenin resistance through which current I_x flows.

$V_x = I_x\,(R_1)$, so
Then $\boldsymbol{\tau_1} = R_1C_2$

Open-Circuit Time Constant (OCTC) Example 1

Recalling the expression for the transfer function,

$$\frac{V_O}{V_S} = \frac{1}{1 + s[\underbrace{R_1 C_1}_{\tau_1} + \underbrace{(R_1 + R_2) C_2}_{\tau_2}] + s^2 (R_1 R_2 C_1 C_2)} = \frac{1}{1 + b_1 s + b_2 s^2}$$

Let's put in some numbers:

Suppose $R_1 = R_2 = 10\ \text{k}\Omega$ and $C_1 = C_2 = 100$ pF.
What are the pole frequencies?

$$\tau_1 = R_1 C_1 = (10^4) 100 \times 10^{-12}\ \sec = 1\ \mu\sec \quad \Rightarrow \quad \omega_{p1} = 1\ \text{MHz}$$

$$\tau_2 = (R_1 + R_2) C_2 = (10^4 + 10^4) 100 \times 10^{-12}\ \sec = 2\ \mu\sec \quad \Rightarrow \quad \omega_{p2} = 0.5\ \text{MHz}$$

Open-Circuit Time Constant (OCTC)

Why does the Open-Circuit Time Constant method work?

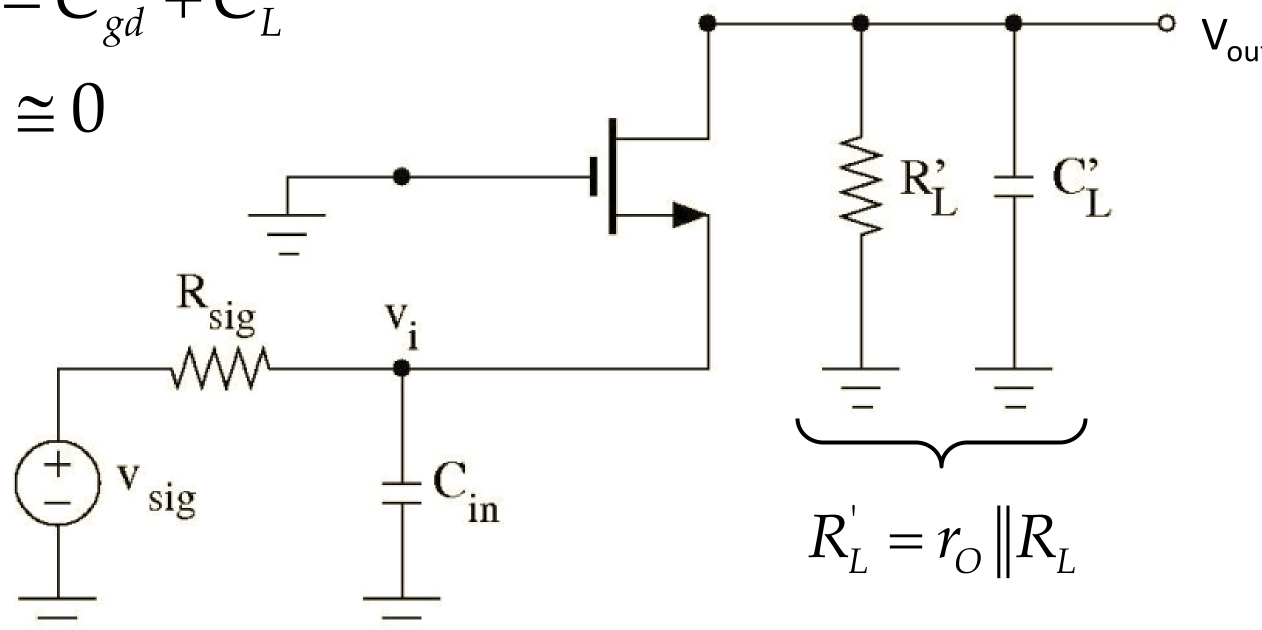
Answer:

Common-gate MOSFET Amplifier

$$C_{in} = C_{gs}$$

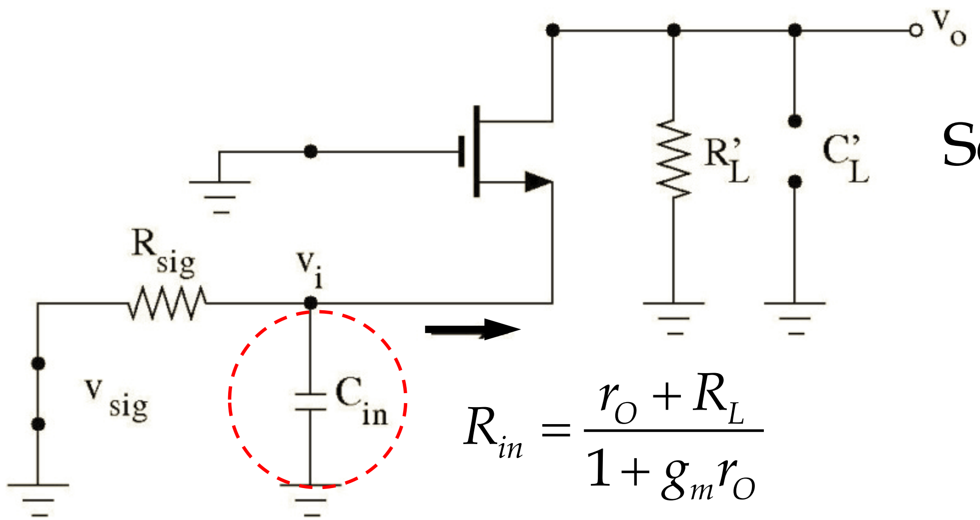
$$C_L^{'} = C_{gd} + C_L$$

$$C_{ds} \cong 0$$



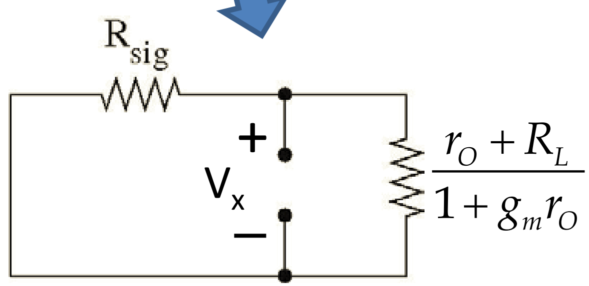
$$R_L^{'} = r_O \| R_L$$

Common-gate MOSFET Amplifier – Focus on C_{in}



Set C_L to open

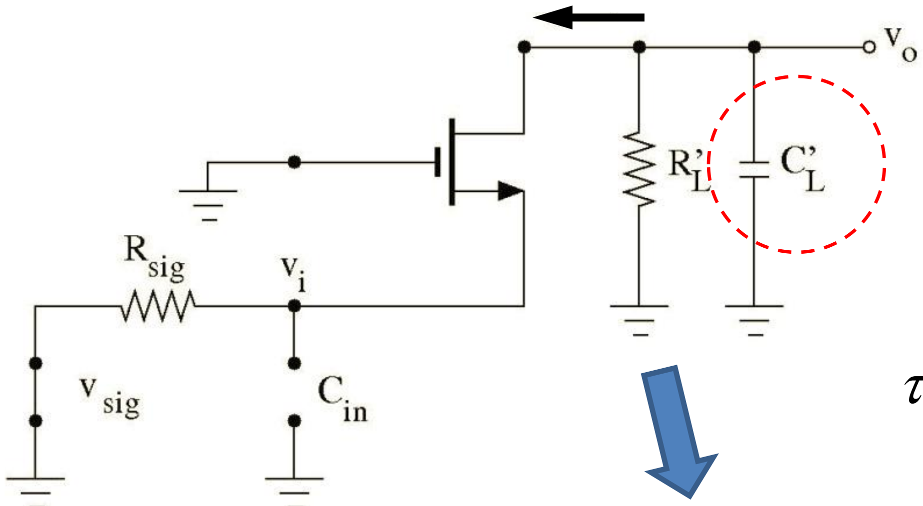
$$R_{in} = \frac{r_O + R_L}{1 + g_m r_O}$$



$$\tau_1 = C_{in}\left[R_{sig} \left\| \left(\frac{r_O + R_L}{1 + g_m r_O}\right)\right.\right]$$

Common-gate MOSFET Amplifier – Focus on C’$_L$

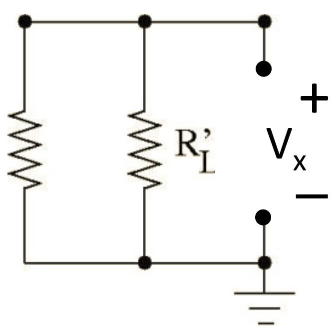
$$R_{out} = r_O\left(1 + g_m r_O\right)$$



$$\tau_2 = C_L^{'}\left[R_L^{'} \| r_O\left(1 + g_m R_{sig}\right)\right]$$

Set C_{in} to open

$r_O\left(1 + g_m r_O\right)$



Common-gate MOSFET Amplifier – Conclusion

The midband voltage gain of the CG stage is

$$A_V = +\frac{(r_O + R_L^{'})}{(r_O + R_L^{'}) + g_m r_O R_{sig}} \cdot [g_m (r_O \| R_L^{'})]$$

The two time constants are

$$\tau_1 = C_{in}\left[R_{sig} \left\| \left(\frac{r_O + R_L}{1 + g_m r_O} \right) \right. \right] \qquad \tau_2 = C_L^{'}\left[R_L^{'} \| r_O \left(1 + g_m R_{sig}\right) \right]$$

$$f_H = \frac{1}{2\pi b_1} = \frac{1}{2\pi(\tau_1 + \tau_2)}$$

Miller's Theorem vs. Miller's Approximation

For Miller Theorem to work, ratio of V_2/V_1 (amplifier gain) must be calculated in the presence of the impedance Z being transformed.

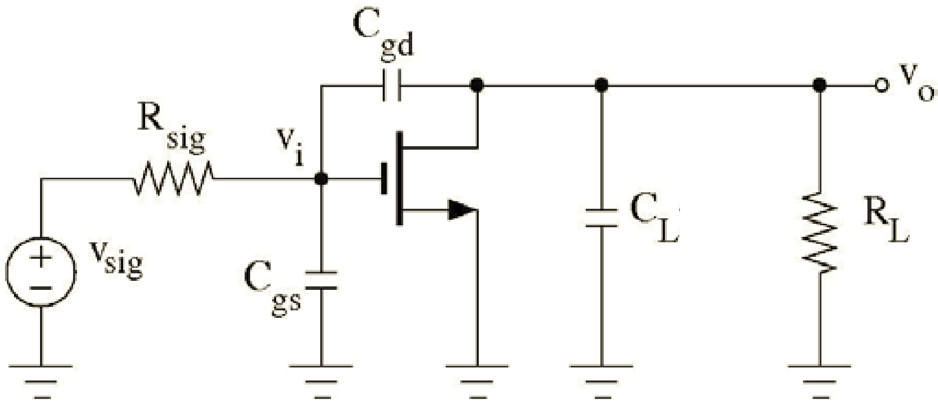
Most books use the mid-band gain of the amplifier and ignore changes in the gain due to the feedback capacitor, C_{gd}. This is called "Miller's Approximation."

The amplifier gain in the presence of C_{gd} is smaller than the mid-band gain (*i.e.*, high-frequency portion of the Bode gain plot), so Miller's approximation overestimates the $C_{gd,input}$ term and it underestimates the capacitor $C_{gd,output}$.

Note: But the OCTC method using b_1 and f_H does better.

Also, Miller's Approximation "misses" the zero introduced by the feedback capacitor C_{gd} or C_{μ} (important for analyzing stability of feedback amplifiers as it affects both gain and phase margins).

The origin of the zero in the CS MOSFET amplifier



1) Definition of a zero: $V_o(s = s_z) = 0$
2) Because $V_{out} = 0$, *zero current will flow in* r_o, C_L *and* R_L
3) Using KCL, a current of $g_m v_{gs}$ *flows in* C_{gd}.
4) Ohm's law for C_{gd} *gives:*

$$\left| sC_{gd} v_{gs} \right| = g_m v_{gs}$$

$$s_Z = \frac{g_m}{C_{gd}} \quad \text{and} \quad f_H = \frac{g_m}{2\pi C_{gd}}$$

Comparison of CS and CG MOSFET Amplifiers

1) Both CS and CG amplifiers have high gain $\left|g_m\left(r_O \| R_L\right)\right|$

2) CS amplifier has an ≈ infinite input resistance whereas CG amplifier has a low input resistance (≈ $1/g_m$).

- CG amplifier has a much better high-*frequency response.*
- *CS amplifier has a large capacitor at the input due to the Miller's effect:* $C_{in} = C_{gs} + C_{gd}[1 + g_m(r_O \| R_L)]$ *compared to that of a CG amplifier:* $C_{in} = C_{gs}$
- In addition, a CS amplifier has a zero.

Note: The Cascode amplifier combines the desirable properties of high input impedance with a reasonably high-frequency response. (It has a better high-frequency response than a two-stage CS amplifier.)

Caution: Miller's Approximation

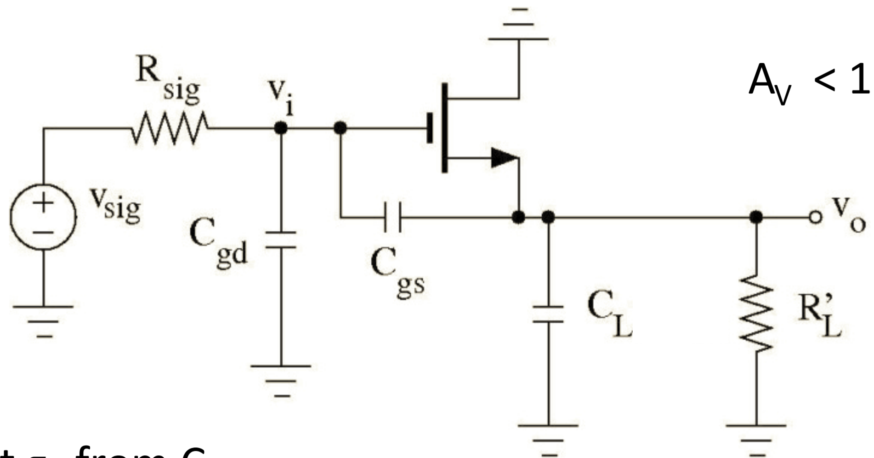
The main value of Miller's Theorem is to demonstrate that a large capacitance will appear at the input of a CS amplifier (Miller's capacitor).

Whereas, Miller's Approximation gives a reasonable approximation to f_H, it fails to provide accurate values for each pole and misses the zero in the transfer function.

- Miller's approximation should be used only as a first guess in analysis. Simulation can be used to more accurately find the amplifier response.
- Stability analysis (*i.e.*, gain and phase margins) should utilize simulations unless a dominant pole exists in the expression for f_H.

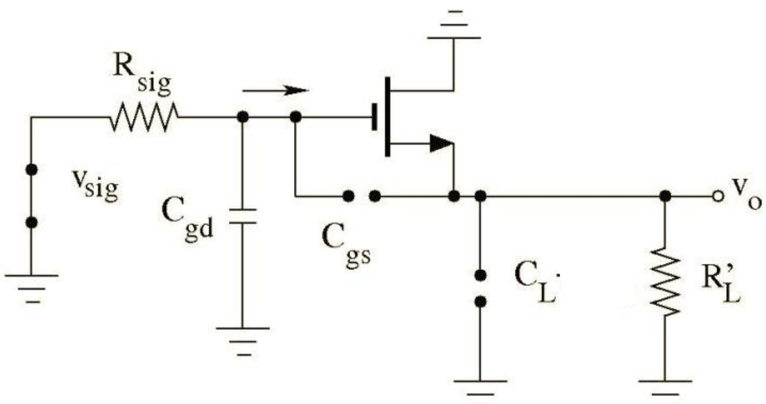
Miller's approximation breaks down when the gain is close to unity.

Common-Drain (Source Follower) Stage Example

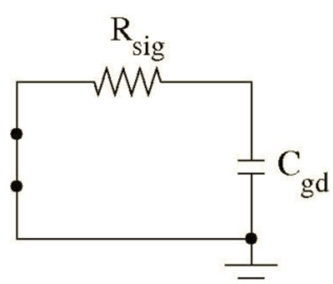


$A_V < 1$

Time constant τ_1 from C_{gs}

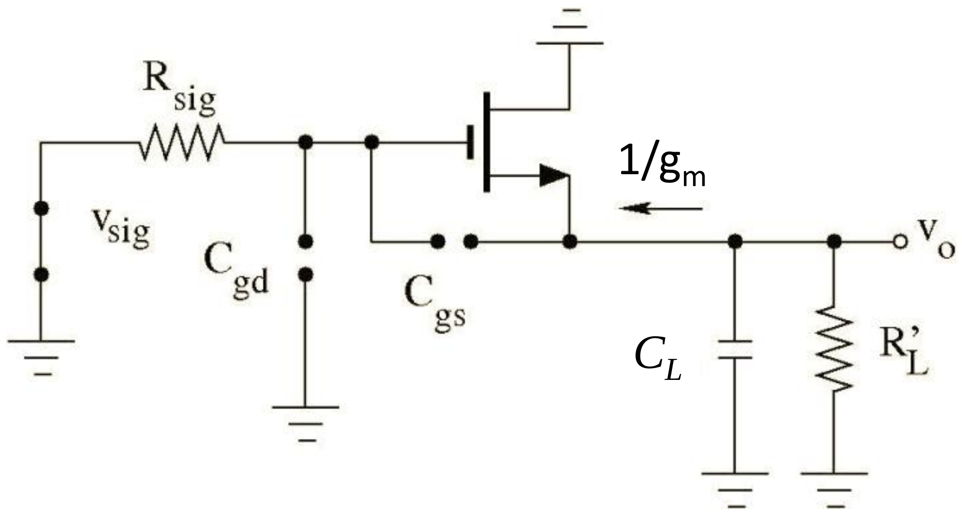


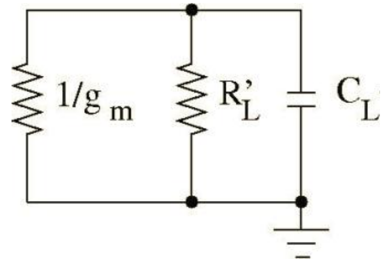
$$\tau_1 = R_{sig} C_{gd}$$



Common-Drain (Source Follower) Stage (2)

Time constant τ_2 from C_L

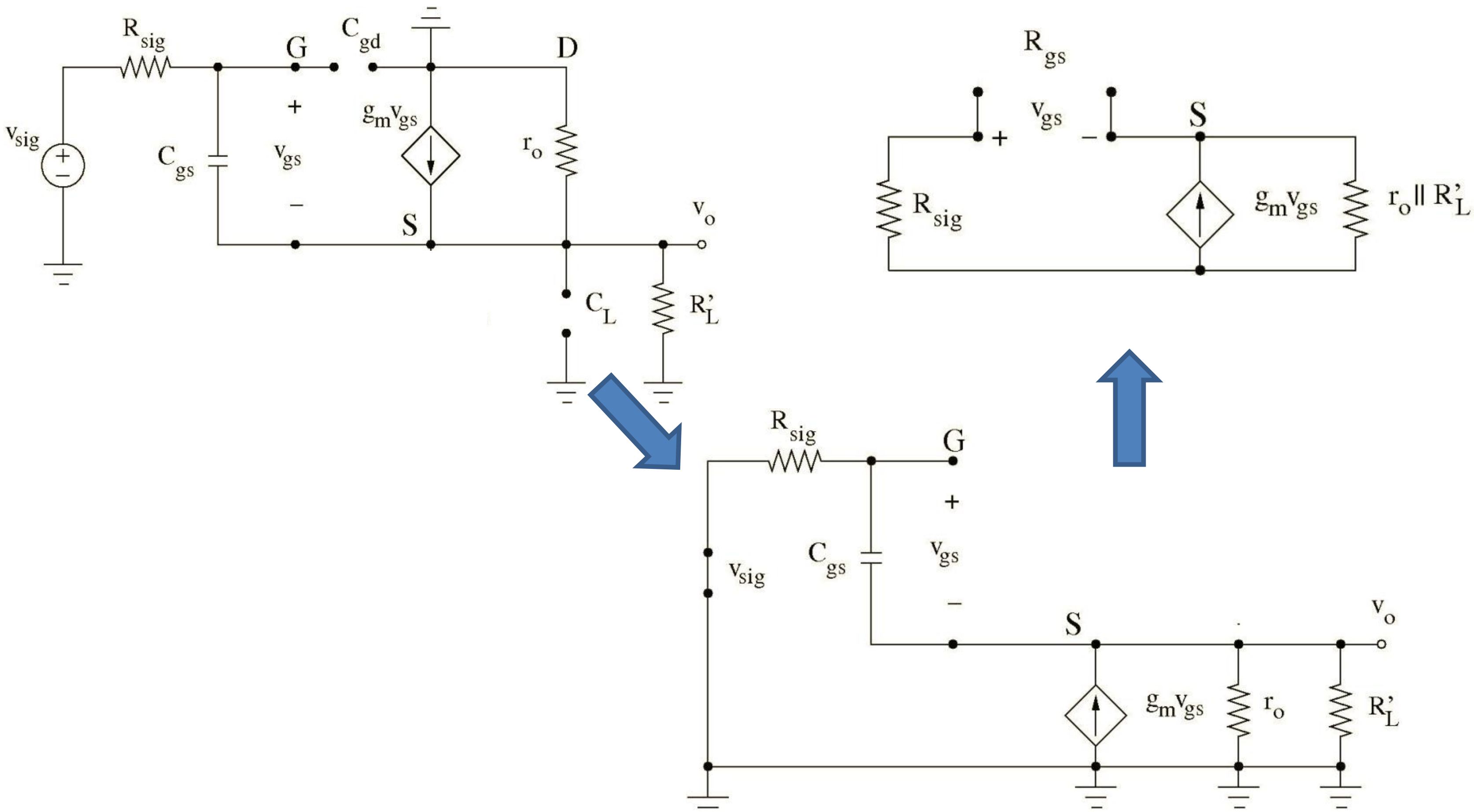




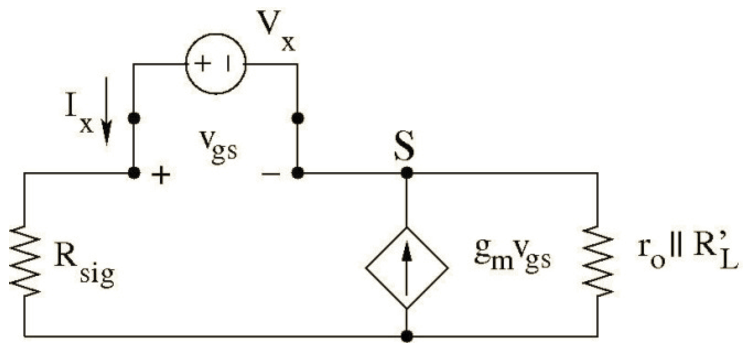
$$\tau_2 = \left(\frac{1}{g_m} \Big\| R_L^{'} \right) C_L$$

Common-Drain (Source Follower) Stage (3)

Time constant τ_3 from C_{gs}



Common-Drain (Source Follower) Stage (4)



Note: $v_x = v_{gs}$, using KVL gives

$$v_x = i_x R_{sig} + (r_O \| R_L^{'})(i_x - g_m v_{gs})$$

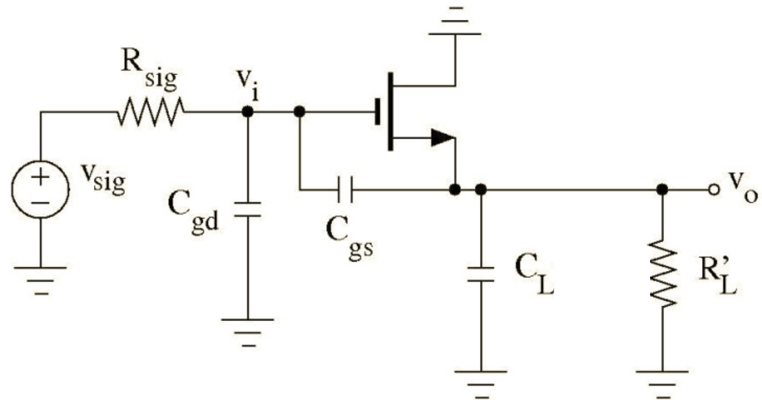
$$v_x = i_x R_{sig} + (r_O \| R_L^{'}) i_x - g_m (r_O \| R_L^{'}) v_x$$

$$v_x \left[1 + g_m (r_O \| R_L^{'}) \right] = [R_{sig} + (r_O \| R_L^{'})] i_x$$

$$\therefore \quad R_{gs} = \frac{v_x}{i_x} = \frac{R_{sig} + (r_O \| R_L^{'})}{\left[1 + g_m (r_O \| R_L^{'}) \right]}$$

$$\Rightarrow \quad \tau_3 = \left(\frac{R_{sig} + (r_O \| R_L^{'})}{1 + g_m (r_O \| R_L^{'})} \right) C_{gs}$$

Common-Drain (Source Follower) Stage (5)



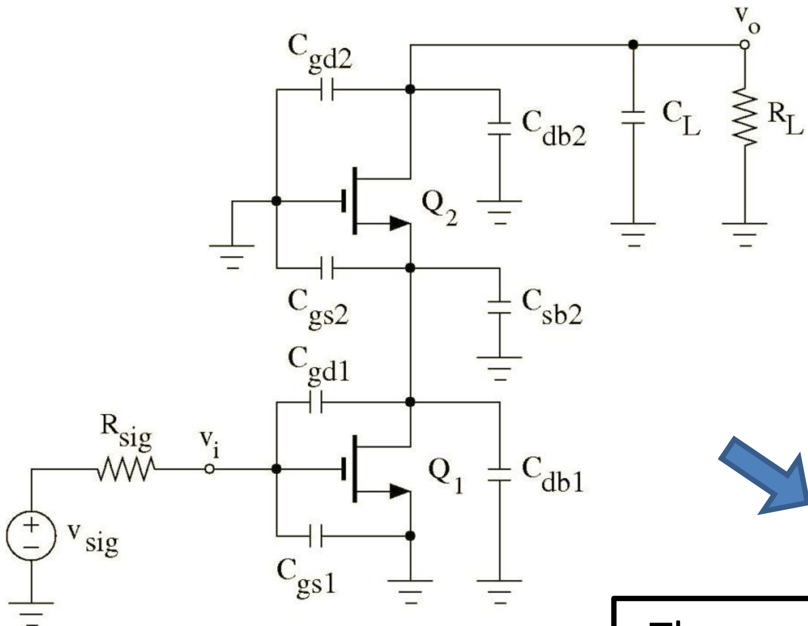
$$\frac{1}{2\pi f_H} = b_1 = \tau_1 + \tau_2 + \tau_3$$

$$\frac{1}{2\pi f_H} = R_{sig}C_{gd} + \left(\frac{1}{g_m}\middle\| R_L^{'}\right)C_L + \frac{R_{sig} + (r_O \| R_L^{'})}{\left[1 + g_m(r_O \| R_L^{'})\right]}C_{gs}$$

Selected comments on high-frequency response in MOSFET amplifiers

- Include internal-capacitances of MOSFETs and simplify the circuit as much as possible.
- Use Miller's approximation for Miller capacitance in configurations with a large (and negative) voltage gain A_V .
- Use the open-circuit time constant method to find f_H .
- Do not neglect zeros in the CS and CD configurations.

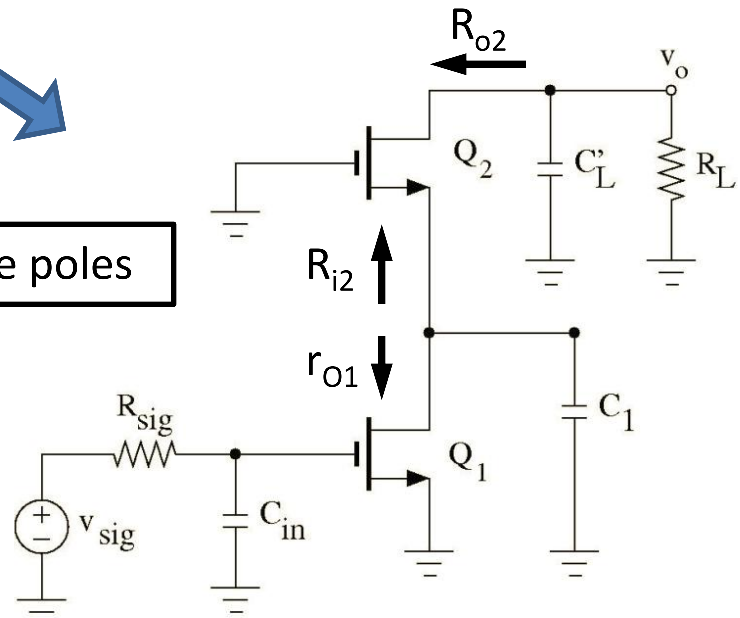
Common-source stage with active load example



$$\tau_1 = R_{sig} C_{in}$$

$$\tau_2 = \left(r_{O1} \| R_{i2} \right) C_1$$

$$\tau_3 = \left(r_{O1} \| R_L \right) C_L^{'}$$



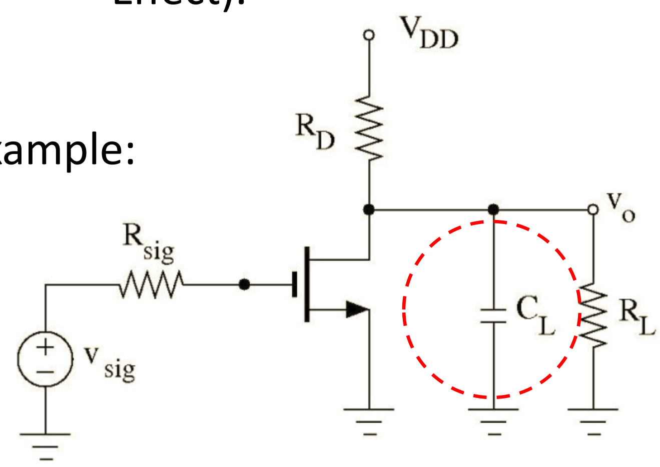
Three poles

$$\frac{1}{2\pi f_H} = b_1 = \tau_1 + \tau_2 + \tau_3$$

Dominant Pole Compensation

- Sometimes we must purposely introduce an additional "pole" in a circuit (such as to control gain or phase margin in feedback amplifiers for stability). This is called "dominant pole compensation. "
- This pole must be a "dominant pole" (that is, several octaves below any zero or other pole).
- In this case, we can ignore transistor internal capacitances in the analysis because the poles introduced by these capacitances are at higher frequencies and do not significantly impact the "dominant pole."
    1. Dominant pole is introduce by capacitor between output & ground
    2. Capacitor between input and output of a stage (*i.e.*, uses Miller Effect).

Example:



Dominant pole created by adding large capacitance C_L at the output .