Ejercicio: Obtenga el Jacobiano del robot Scara

- Partiendo de la cinemática directa

$$H_4^0(\vec{q}) = \left[\begin{array}{ccc|c} \multicolumn{3}{c|}{R} & \vec{o} \\ C_{124} & S_{124} & 0 & a_1C_1 + a_2C_{12} \\ S_{124} & -C_{124} & 0 & a_1S_1 + a_2S_{12} \\ 0 & 0 & -1 & d_1 - d_3 - d_4 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

1. Jacobiano tangencial (J_v)

- Derivando $\vec{o}$

$$\frac{d\vec{o}}{dt} = \begin{bmatrix} -a_1S_1\dot{\theta}_1 - a_2S_{12}(\dot{\theta}_1+\dot{\theta}_2) \\ a_1C_1\dot{\theta}_1 + a_2C_{12}(\dot{\theta}_1+\dot{\theta}_2) \\ -\dot{d}_3 \end{bmatrix}$$

- Factorizando $\dot{\vec{q}}$

$$\dot{\vec{o}} = \begin{bmatrix} \dot{o}_x \\ \dot{o}_y \\ \dot{o}_z \end{bmatrix} = \begin{bmatrix} -a_1S_1 - a_2S_{12} & -a_2S_{12} & 0 & 0 \\ a_1C_1 + a_2C_{12} & a_2C_{12} & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{d}_3 \\ \dot{\theta}_4 \end{bmatrix}$$

- Entonces J_v es:

$$J_v = \begin{bmatrix} -a_1S_1 - a_2S_{12} & -a_2S_{12} & 0 & 0 \\ a_1C_1 + a_2C_{12} & a_2C_{12} & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

2. Jacobiano angular:

$$S(\vec{\omega}) = \dot{R}R^T$$

$$\begin{bmatrix} C_{124} & S_{124} & 0 \\ S_{124} & -C_{124} & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$S(\vec{\omega}) = \begin{bmatrix} -S_{124}(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_4) & C_{124}(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_4) & 0 \\ C_{124}(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_4) & S_{124}(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_4) & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} C_{124} & S_{124} & 0 \\ S_{124} & -C_{124} & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$= (\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_4) \begin{bmatrix} \cancel{-S_{124}C_{124} + S_{124}C_{124}} & -1 & 0 \\ \underbrace{C_{124}^2 + S_{124}^2}_{1} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$S(\vec{\omega}) = (\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_4) \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{aligned} \omega_x &= 0 \\ \omega_y &= 0 \\ \omega_z &= \dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_4 \end{aligned}$$

$$\omega_x = 0$$
$$\omega_y = 0$$
$$\omega_z = \dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_4$$

En forma de matriz:

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \dot{\theta}_1 & \dot{\theta}_2 & 0 & \dot{\theta}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{d}_3 \\ \dot{\theta}_4 \end{bmatrix}$$

- Entonces el Jacobiano angular es:

$$J_\omega = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

3. Jacobiano completo (J)

$$J(\dot{q}) = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} & 0 & 0 \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$