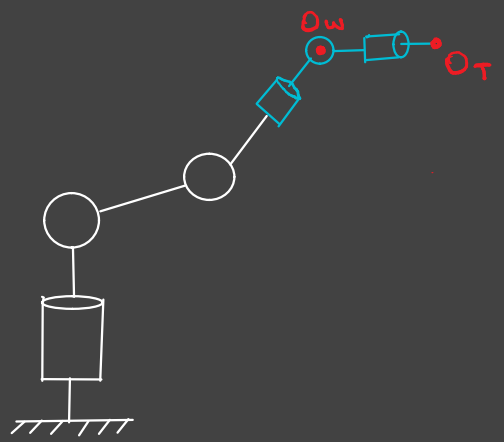
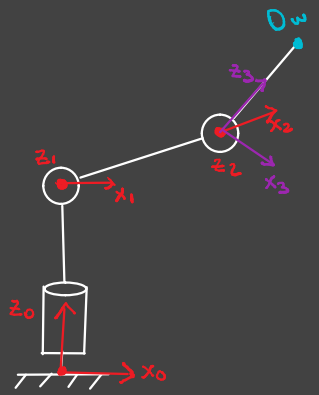
Forward Kinematics: Elbow - 6 DoF



1. Body



- D-H table

i	a_i	α_i	d_i	θ_i
1	0	$\pi/2$	d_1	θ_1^*
2	a_2	0	0	θ_2^*
3	0	$-\pi/2$	0	θ_3^*

- Homogeneous matrices

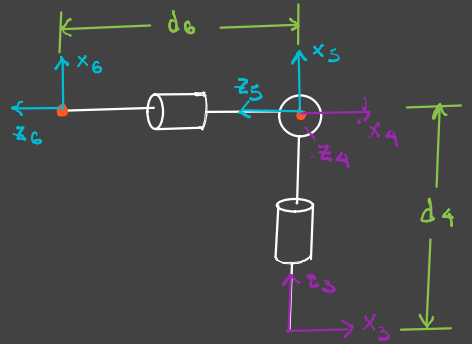
$$H_3^0 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & -s_2 & 0 & a_2 c_2 \\ s_2 & c_2 & 0 & a_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_3 & 0 & -s_3 & 0 \\ s_3 & 0 & c_3 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_1 c_2 & -c_1 s_2 & s_1 & a_2 c_1 c_2 \\ s_1 c_2 & -s_1 s_2 & -c_1 & a_2 s_1 c_2 \\ s_2 & c_2 & 0 & a_2 s_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_3 & 0 & -s_3 & 0 \\ s_3 & 0 & c_3 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_1(c_2 c_3 - s_2 s_3) & -s_1 & c_1(-c_2 s_3 - s_2 c_3) & a_2 c_1 c_2 \\ s_1(c_2 c_3 - s_2 s_3) & c_1 & s_1(-c_2 s_3 - s_2 c_3) & a_2 s_1 c_2 \\ s_2 c_3 + c_2 s_3 & 0 & -s_2 s_3 + c_2 c_3 & a_2 s_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3^0 = \begin{bmatrix} c_1 c_{23} & -s_1 & -c_1 s_{23} & a_2 c_1 c_2 \\ s_1 c_{23} & c_1 & -s_1 s_{23} & a_2 s_1 c_2 \\ s_{23} & 0 & c_{23} & d_1 + a_2 s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. Spherical wrist



D-H table

i	a_i	α_i	d_i	θ_i
4	0	$\pi/2$	d_4	θ_4
5	0	$-\pi/2$	0	θ_5
6	0	0	d_6	θ_6

- Homogeneous matrices

$$H_3^0 = \begin{bmatrix} c_4 & 0 & s_4 & 0 \\ s_4 & 0 & -c_4 & 0 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_5 & 0 & -s_5 & 0 \\ s_5 & 0 & c_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_4C_5 & -S_4 & -C_4S_5 & 0 \\ S_4C_5 & C_4 & -S_4S_5 & 0 \\ S_5 & 0 & C_5 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_6^3 = \begin{bmatrix} C_4C_5C_6 - S_4S_6 & -C_4C_5S_6 - S_4C_6 & -C_4S_5 & -d_6C_4S_5 \\ S_4C_5C_6 + C_4S_6 & -S_4C_5S_6 + C_4C_6 & -S_4S_5 & -d_6S_4S_5 \\ S_5C_6 & -S_5S_6 & C_5 & d_6C_5 + d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$