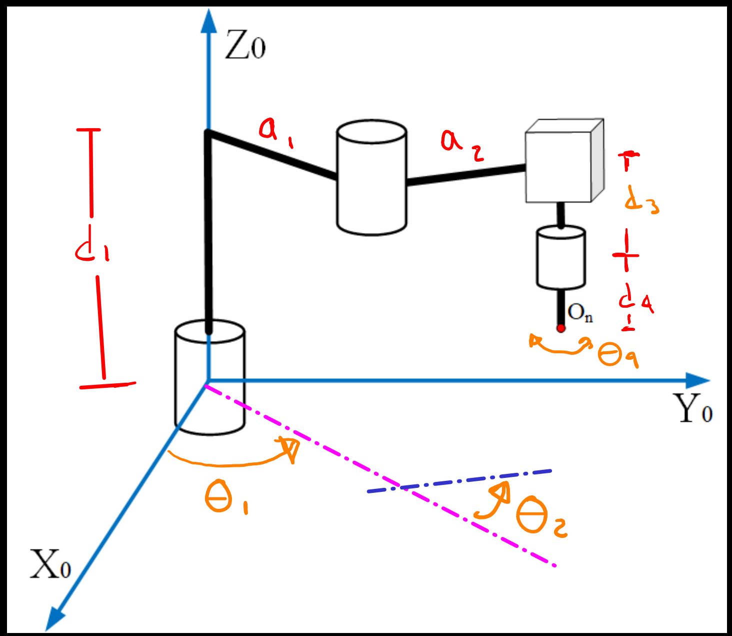
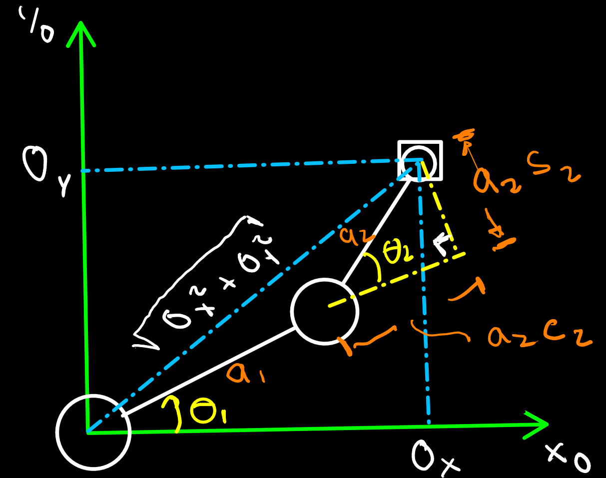
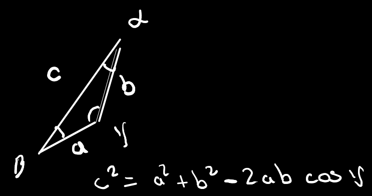
Inverse Kinematics : Scara robot

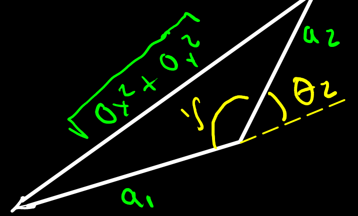


Upper view:





A. Obtaining θ_2



$$O_x^2+O_y^2 = a_1^2+a_2^2-2a_1a_2\cos\gamma$$

Si : $\theta_2+\gamma=\pi \rightarrow \gamma=\pi-\theta_2$

$$\cos(\pi-\theta_2) = -\cos(\theta_2)$$

$$O_x^2+O_y^2 = a_1^2+a_2^2+2a_1a_2\cos(\theta_2)$$

$$D = \cos(\theta_2) = \frac{O_x^2+O_y^2-a_1^2-a_2^2}{2a_1a_2}$$

- Expressing as a tangent

If:

$$\sin(\theta_2) = \sqrt{1-D^2}$$

So:

$$\tan\theta_2 = \frac{\sin\theta_2}{\cos\theta_2} = \frac{\sqrt{1-D^2}}{D}$$

$$\rightarrow \theta_2 = \operatorname{atan2}\left[\frac{\sqrt{1-D^2}}{D}\right]$$

- Obtaining θ_1

$$\rightarrow \theta_1 = \operatorname{atan2}\left[\frac{O_y}{O_x}\right] - \operatorname{atan2}\left[\frac{a_2 S_2}{a_1+a_2C_2}\right]$$

- Obtaining d_3 :

$$O_z = d_1 - d_3 - d_4$$

$$\rightarrow d_3 = d_1 - O_z - d_4$$

- Obtaining θ_4

- From the forward kinematics:

$$H_4^0(\vec{q}) = \begin{bmatrix} C_{124} & S_{124} & 0 & a_1C_1+a_2C_{12} \\ S_{124} & -C_{124} & 0 & a_1S_1+a_2S_{12} \\ 0 & 0 & -1 & d_1-d_3-d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_4^0 = \begin{bmatrix} C_{124} & S_{124} & 0 \\ S_{124} & -C_{124} & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

With:

$\alpha = \theta_1+\theta_2+\theta_4$

α: Proposed end-effector orientation

Then:

$$\rightarrow \theta_4 = \theta_1+\theta_2-\alpha$$