



$$\hat{\theta}_3 - \theta_3 = \frac{\pi}{2}$$

$$\hat{\theta}_3 = \frac{\pi}{2} + \theta_3$$

Angles have sign

1. Obtaining θ_3 (Cosine law)

$$O_x^2 + O_y^2 + (O_z - d_3)^2 = a_2^2 + d_4^2 - 2a_2d_4 \cos(\pi - \hat{\theta}_3)$$

$$O_x^2 + O_y^2 + (O_z - d_3)^2 = a_2^2 + d_4^2 - 2a_2d_4 \cos(\pi - \frac{\pi}{2} - \theta_3)$$

$$O_x^2 + O_y^2 + (O_z - d_3)^2 = a_2^2 - d_4^2 + 2a_2d_4 \sin(\theta_3)$$

$$D = \sin(\theta_3) = - \frac{O_x^2 + O_y^2 + (O_z - d_3)^2 - a_2^2 - d_4^2}{2a_2d_4}$$

$$\rightarrow \theta_3 = \text{atan2} \left[\frac{D}{\pm \sqrt{1 - D^2}} \right]$$

2. Obtaining θ_2

$$\theta_2 = \text{atan2} \left[\frac{O_z - d_1}{\sqrt{O_x^2 + O_y^2}} \right] - \text{atan2} \left[\frac{d_4 \sin \hat{\theta}_3}{a_2 + d_4 \cos \hat{\theta}_3} \right]$$

- If

$$\sin(\theta_3^*) = \sin\left(\frac{\pi}{2} + \theta_3\right) = \cos(\theta_3)$$

$$\cos(\theta_3^*) = \cos\left(\frac{\pi}{2} + \theta_3\right) = -\sin(\theta_3)$$

- Then:

$$\rightarrow \theta_2 = \arctan 2 \left[\frac{O_z - d_1}{\sqrt{O_x^2 + O_y^2}} \right] - \arctan 2 \left[\frac{d_4 c_3}{a_2 + s_3} \right]$$

3. obtaining θ_1

$$\rightarrow \theta_1 = \arctan 2 \left[\frac{O_y}{O_x} \right]$$