Toward FWI in Buildings
SEM modeling of the Timoshenko Model

[1] *Ricardo Caiza*, [2] *Romain Brossier*, [3] *Philippe Gueguen*

Institut des Sciences de la Terre

Resume

"The study of how buildings behave under earthquake arrivals is a trending area of research[1][2]. Beam-like building models provide reliable support in the comprehension of the dynamic evolution of these structures [1]. Timoshenko-Ehrenfest beam theory considers shear contributions manifested as deformation and rotation of beam's cross-section. The implementation of a code that describes Timoshenko's theory allows to emulate the dynamic of buildings and their different behaviors in shear and bending regimes [1]. It also allows computation of a forward model to solve an inverse problem. Therefore, it is possible to obtain the stiffness parameters of the building (E, G) such that we can estimate them. This affords the potential for localizing the internal damage in structures due to strong earthquakes as a non-invasive method. This work presents the implementation of Timoshenko's model with the Spectral Element Method [6] and provides the results of full wave form inversion (FWI) with synthetic data for standard real building parameters."

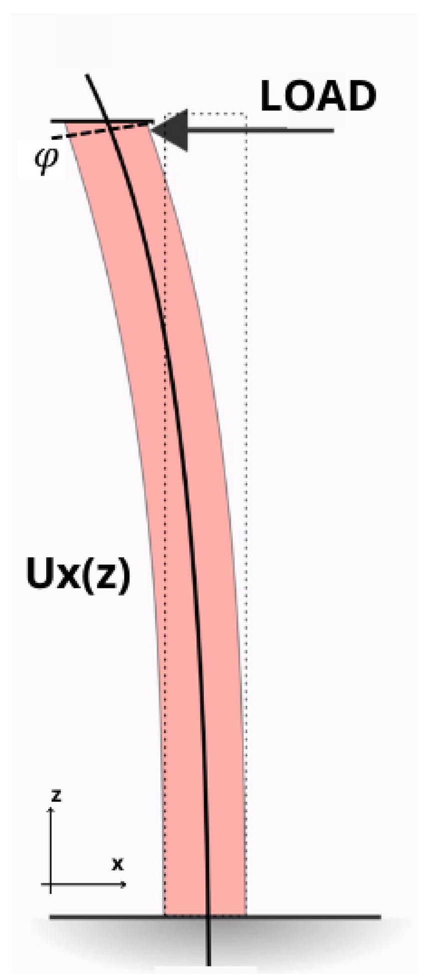
Key words: *Beam-like building, Timoshenko's model, FWI, Stiffness parameters*

Objectives

- Implementing a code that solves Timoshenko's model using the Spectral Element Method (SEM)[6]
- Solving the inverse problem applying the Full Wave Form Inversion (FWI) with a global optimization method [3][4]
- Computing the couple (G,E)

Theory

Timoshenko-Ehrenfest beam model considers shear contributions assuming that *the distribution of shear stress over the cross-section is a constant [2].*



$$\rho A\frac{\partial^2 u_x}{\partial t^2} = \frac{\partial}{\partial x}[\kappa AG(\frac{\partial u_x}{\partial x} - \varphi)]$$

$$\rho I\frac{\partial^2 \varphi}{\partial t^2} = \frac{\partial}{\partial x}(EI\frac{\partial \varphi}{\partial x}) + \kappa AG(\frac{\partial u_x}{\partial x} - \varphi)$$

- E Young's modulus
- G Shear modulus
- I Second moment of area
- A Surface cross section area
- κ Timoshenko's constant
- ρ Density
- u_x Displacement field
- φ Cross section rotation

Boundary conditions

Base:

No rotation $\varphi(z=0,t) = 0$

No displacement $u_x(z=0,t) = 0$

Top:

Free surface $\partial_z u_x\Big|_{z=H,t} = 0$

Implementation

SEM of Timoshenko's model. We followed the classical recipe [5]. As result for the weak form we obtain the next system:

$$\mathbf{M1}_g\ddot{u}_x = -\frac{\kappa G}{\rho}\mathbf{K11}_g u_x + \frac{\kappa G}{\rho}\mathbf{K12}_g\varphi \quad \textbf{(1)}$$

$$\mathbf{M2}_g\ddot{\varphi} = -\frac{E}{\rho}\mathbf{K12}_g\phi - \frac{\kappa AG}{\rho I}\mathbf{K22}_g u_x - \frac{\kappa AG}{\rho I}\mathbf{K23}_g\varphi \quad \textbf{(2)}$$

$$\mathbf{M1}^e_{ij} = w^e_j\delta ijJ \qquad \mathbf{M2}^e_{ij} = w^e_j\delta_{ij}J$$

$$\mathbf{K11}^e_{ij} = \sum_k^{N+1} w^e_k\partial_\xi\ell^e_i\partial_\xi\ell^e_j JJ^{-2} \qquad \mathbf{K23}^e_{ij} = \gamma w^e_j\delta_{ij}J$$

$$\mathbf{K12}^e_{ij} = \sum_k^{N+1} w^e_k\partial_\xi\ell^e_j JJ^{-1} \qquad \mathbf{K22}^e_{ij} = \sum_k^{N+1} w^e_k\partial_\xi\ell^e_j JJ^{-1}$$

$$\mathbf{K21}^e_{ij} = \sum_k^{N+1} w^e_k\partial_\xi\ell^e_i\partial_\xi\ell^e_j JJ^{-2}$$

Inversion Results

Simulated Annealing is a global optimization method that enables a brief exploration of the model parameters space [5][6]. This method provides a substantial sampling of the misfit function.

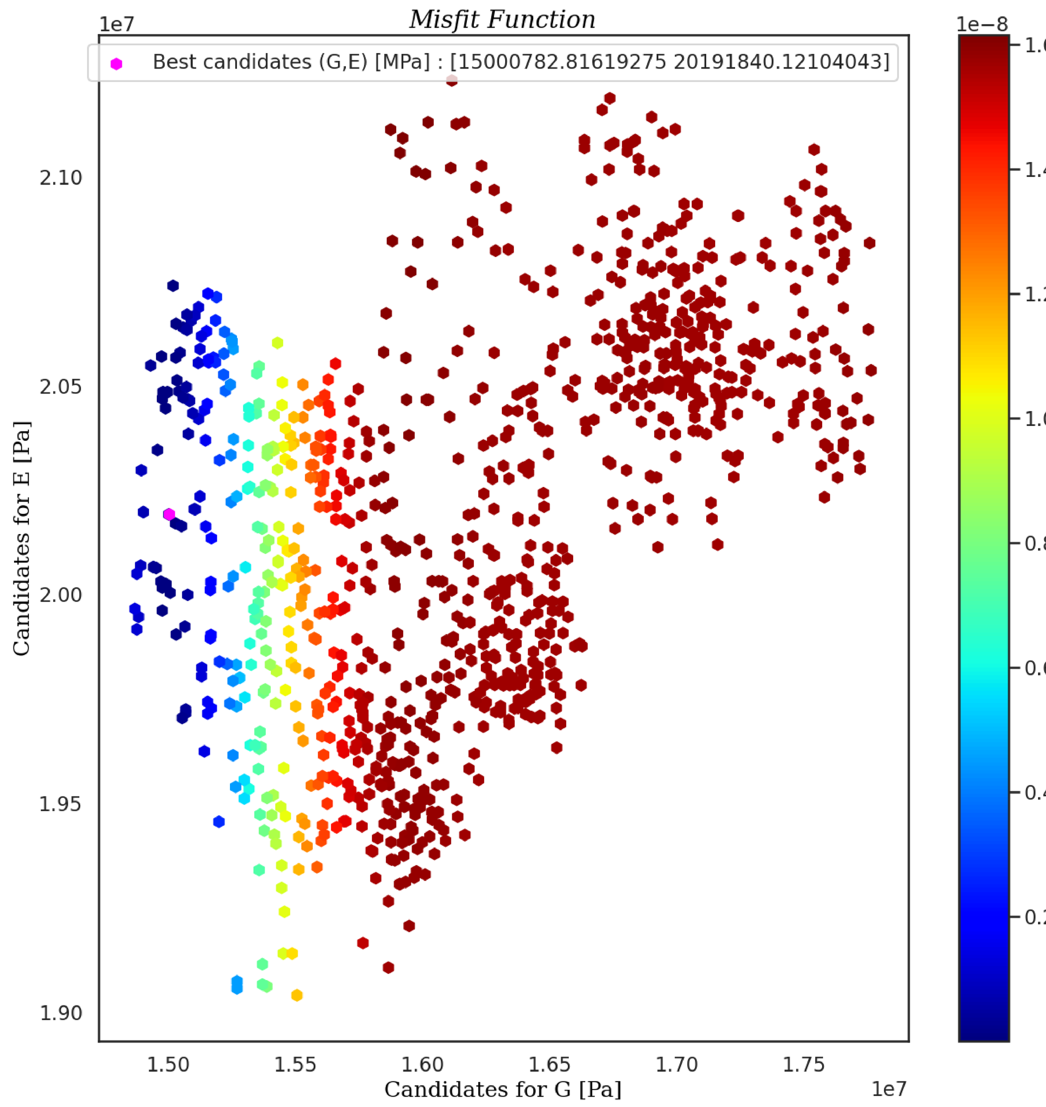


Figure3. Scattering of accepted candidates for the couple (G,E). This figure *illustrates* the shape of the misfit function behavior and its sensibility to changes in G and E. Contains rich information about the dynamics of beam-like buildings.

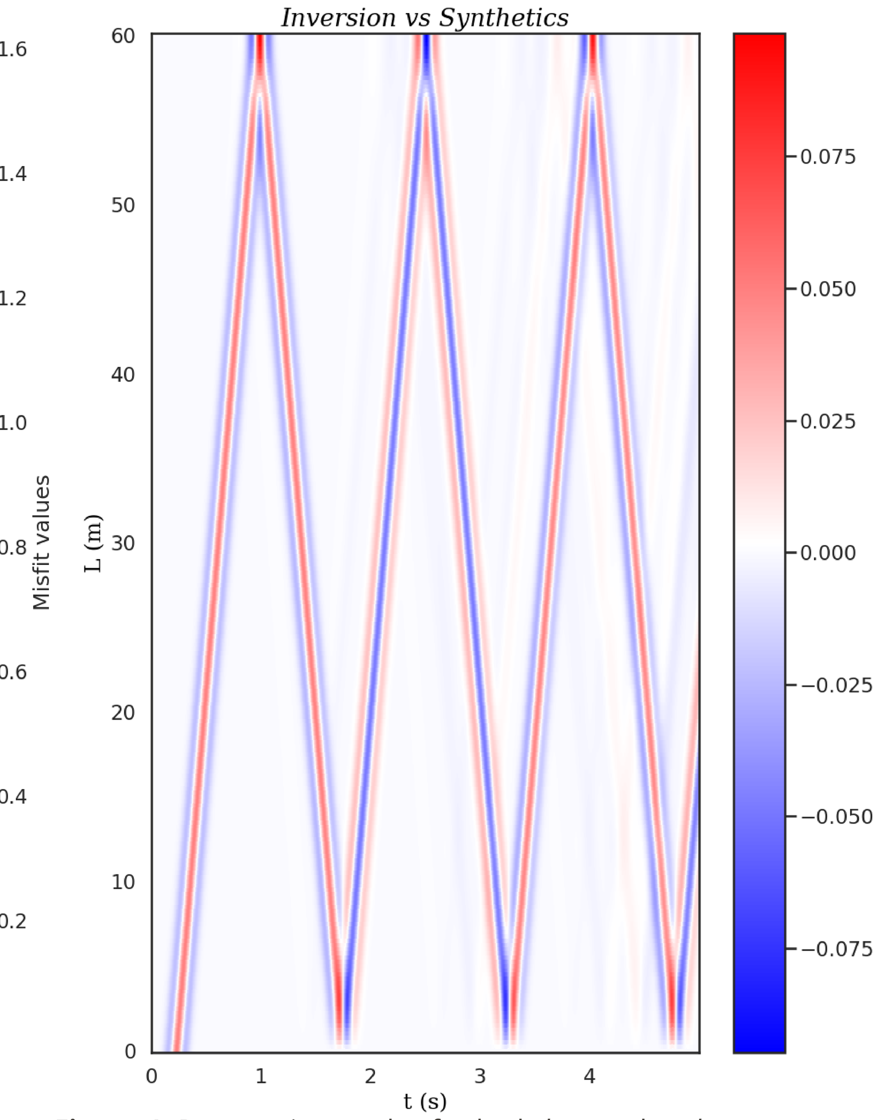


Figure 4. Propagation modes for both *best* and *real* parameters. Both modes overlap with a misfit value of 1e-9 and an error of 0.045.

Propagation Mode Results

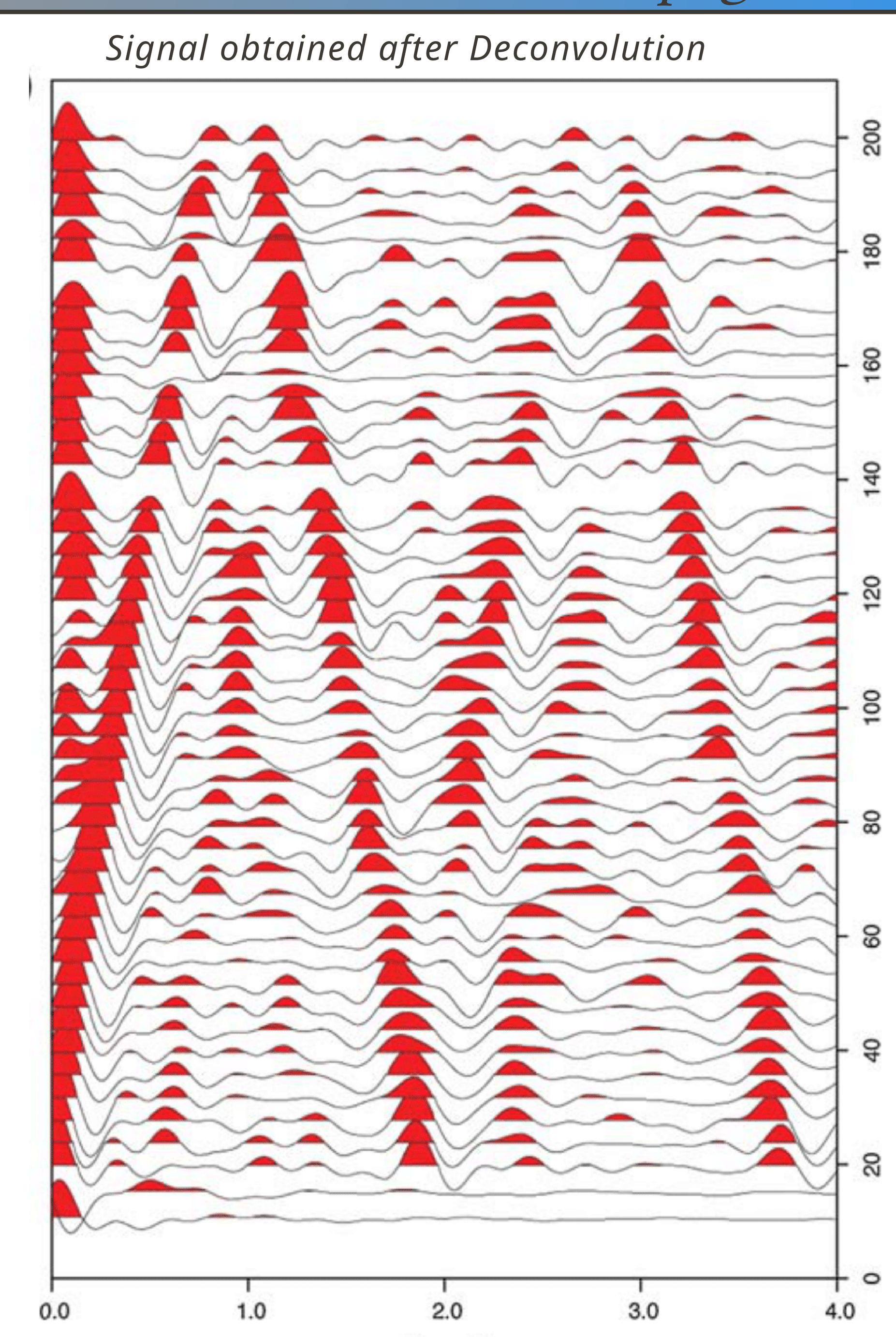


Figure1. Impulse response of seismic data from a real building of 200 m. These data was acquired after deconvolution procedure (Gueguen et. al 2019).

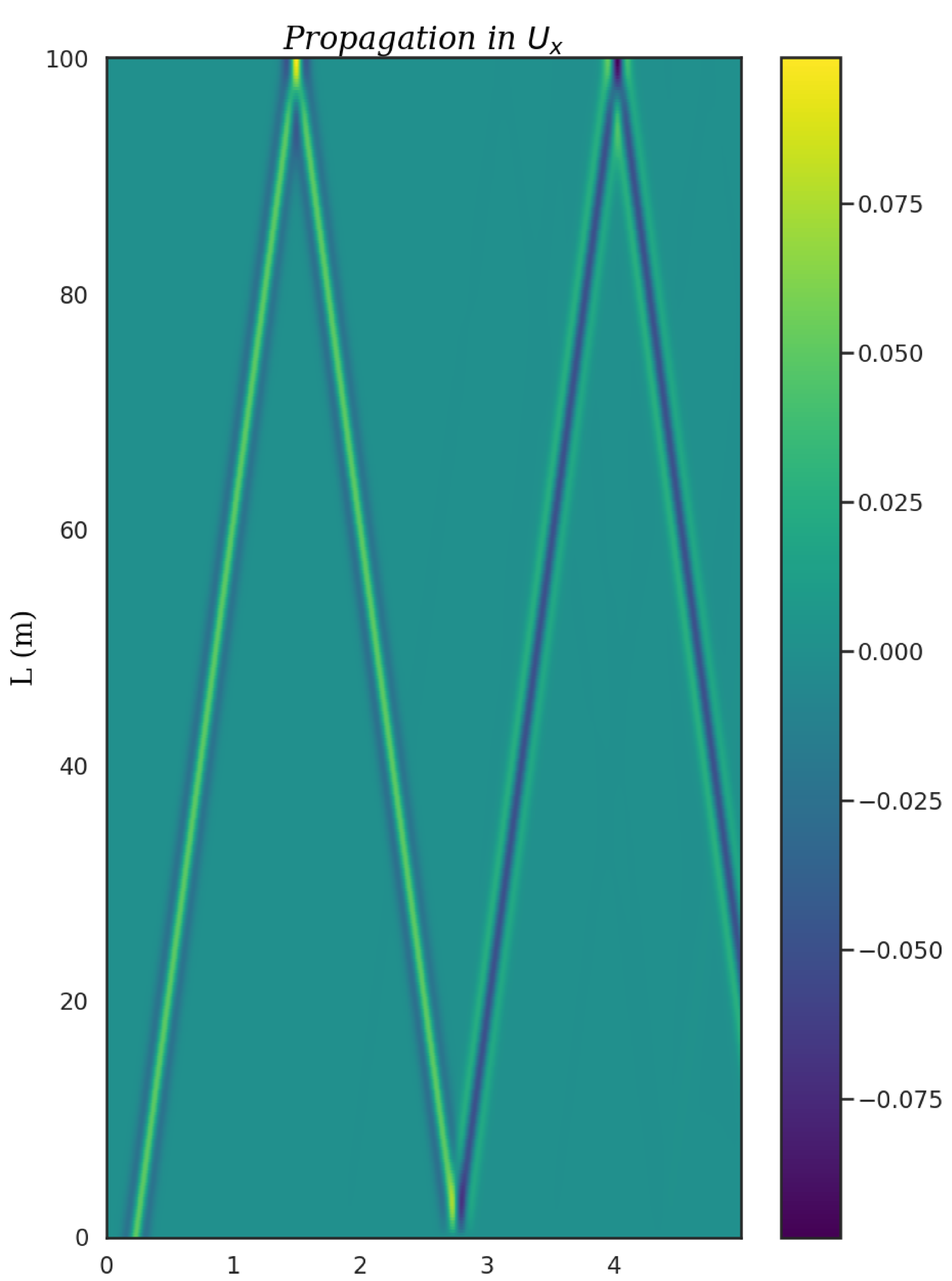


Figure2. Propagation mode of displacement field Ux. This result was obtained applying the SEM for solving equations (1) and (2).

Conclusions and Further work

- The implementation of the SEM of Timoshenko's model provides accurate results if we compare the propagation mode of field displacement with the data obtained in real buildings i.e. figures (1) and (2)
- SA results accurate fit the synthetic data and best candidate data allowing to invert (G,E) as stiffness parameters
- Being a 2 freedom degree problem, it is possible to "visualize" the misfit function shape i.e figure (3). In the shear regime, the changes in E does not interfere in the propagation mode in Ux
- The next stage of this project is the model's validation by studying the dispersion curves and propagation mode velocities and compare them with the theoretical model proposed by (Gueguen et al., 2019) [1]
- As an improvement in the project, we propose an inversion with a spatial dependence of both G and E as (G(z), E(z)), making affordable an inversion procedure per floors. This makes affordable the study of new stiffness parameters values to explore new behaviors in beam-like buildings
- Finally, we propose a local optimization process in the computing of FWI of propagation modes to try to localize damages in the structures and changes in G and E

References

1. Guéguen P, Diego Mercerat E, Alarcon F. Parametric study on the interpretation of wave velocity obtained by seismic interferometry in beam-like buildings. Bull Seismol Soc Am. 2019 [cited 2023 May 12];109(5):1829–42. Available from: https://pubs.geoscienceworld.org/ssa/bssa/article-abstract/109/5/1829/573504/Parametric-Study-on-the-Interpretation-of-Wave?redirectedFrom=fulltext
2. Ebrahimian M, Todorovska MI. Wave propagation in a timoshenko beam building model. J Eng Mech. 2014 [cited 2023 May 12];140(5):04014018. Available from: https://www.academia.edu/28489335/Wave_Propagation_in_a_Timoshenko_Beam_Building_Model
3. Menke W. Geophysical data analysis: Discrete inverse theory: Volume 45: MATLAB edition. 3rd ed. San Diego, CA: Academic Press; 2012.
4. Sen MK, Stoffa PL. Global optimization methods in geophysical inversion [Internet]. Cambridge, England: Cambridge University Press; 2013 [cited 2023 May 12]. Available from: https://www.cambridge.org/core/books/global-optimization-methods-in-geophysical-inversion/C2B23286E6BCC2177117431CB568101C
5. Igel H. Computational seismology: A practical introduction. London, England: Oxford University Press; 2016.
6. Ingber L. Simulated annealing: Practice versus theory. Math Comput Model. 1993;18(11):29–57. Available from: https://www.sciencedirect.com/science/article/pii/089571779390204C

1. ricardo.caiza@univ-grenoble-alpes.fr, Univ. Grenoble Alpes, ISTerre, 2. romain.brossier@univ-grenoble-alpes.fr, Univ. Grenoble Alpes, 3. philippe.gueguen@univ-grenoble-alpes.fr, Univ. Gustave Eiffel, ISTerre ,