Optimal Experimental Design for Full Waveform Inversion.

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I - Full Waveform Inversion

• The **Full Waveform Inversion** (Virieux et al., 2014) is a geophysical imaging technique that aims at recovering subsurface properties $m \in \mathcal{M}$ by iteratively reducing the misfit between observed data d_{obs} and calculated data $d_{cal}(m)$. It can be expressed as a PDE constrained minimization problem

$$\min_{m\in\mathcal{M}} f(m) = \frac{1}{2}\sum_{s=1}^{N_s}\sum_{r=1}^{N_r}\int_0^T |d_{cal,s}(x_r,t) - d_{obs,s}(x_r,t)|^2 dt \tag{1}$$

$$\text{subject to:}\quad A(m)u_s(x,t) = \varphi_s(x,t),\ \text{for } s = 1,\dots,N_s\ ,$$
$$d_{cal,s}(x,t) = R[u_s](x,t),\ \text{for } s = 1,\dots,N_s\ .$$

• It uses a gradient descent method to solve (1). A fundamental component of the FWI is the gradient $\nabla f(m)$ which can be computed (Plessix, 2006) as the **zero-lag time correlation** between the **incident** and **adjoint** wavefields u_s and λ_s.

• Using a **plane wave approximation** of u_s and λ_s we can express the gradient as a sum of planewaves $\nabla f(x) \approx e^{ik_0(p_s+p_r)\cdot x}$ which has a wavenumber content given by (see figure 2) :

$$k(s,r) = \frac{2\pi f_0}{c_0}(p_s + p_r) = k_0(p_s + p_r) \tag{2}$$

• As shown by figure 1 a regular acquisition on the surface does not provide a regular distribution of wavenumber points. So **what is the optimal acquisition geometry for the regularity of the wavenumebr points** and **how do we find such a geometry**?

III - Method

• In our approach we define a function $G : (S,R) \mapsto \{k(s,r), s \in S, r \in R\}$ which for sets S of sources and R of receivers computes the wavenumbers. We then apply the CVT energy function F (Liu et al., 2009) to $G(S,R)$ to get a new energy function $H : (S,R) \mapsto F \circ G(S,R)$ to express the quality of an acquisition with regards to our criterion of regularity of the wavenumbers inside the envelope.

• Finally we optimize the acquisition by minimizing the function $H(S,R)$. To that end we use a gradient-descent method.

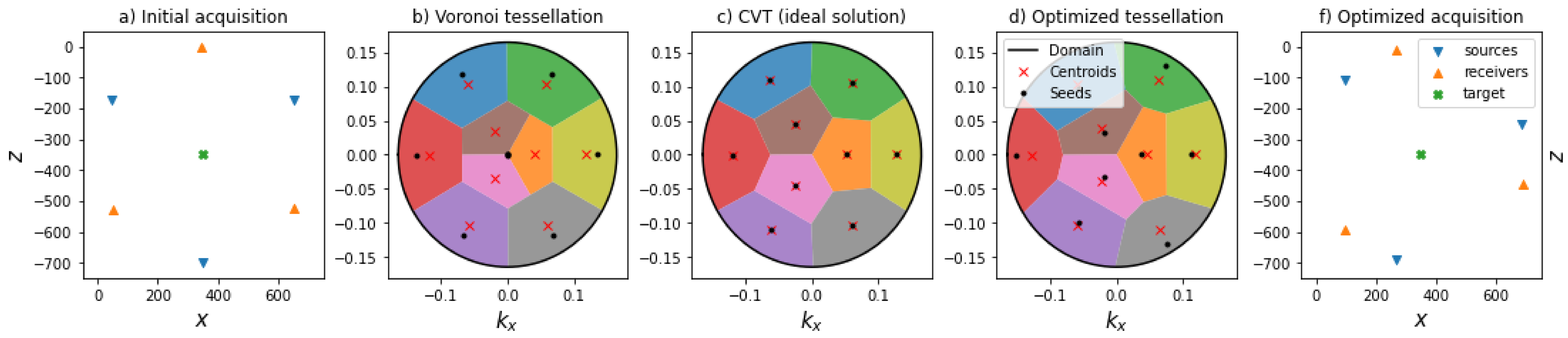

Figure 3: In this illustration of our approach we consider a simple acquisition in circle. We can compute the wavenumbers using formula (3) and generate their Voronoi tessellation inside their envelope, as in (b). Compare this tessellation with the one in (c) which corresponds to an ideal CVT. Yet, minimizing H still yields an acquisition (f) that provides a wavenumber cloud (d) the Voronoi Tessellation of which is significantly closer to a CVT.

IV - Numerical Test

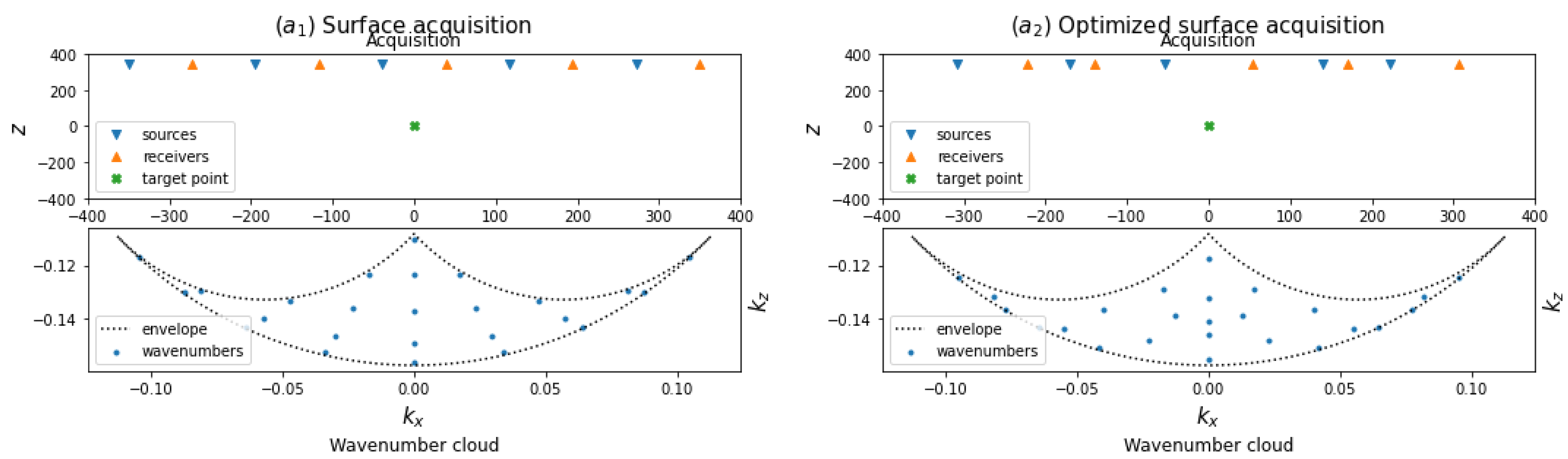

Figure 4: Two surface acquisitions alongside their corresponding wavenumber clouds. The acquisition on the left has regular spacing and the one on the right is optimized using our method.

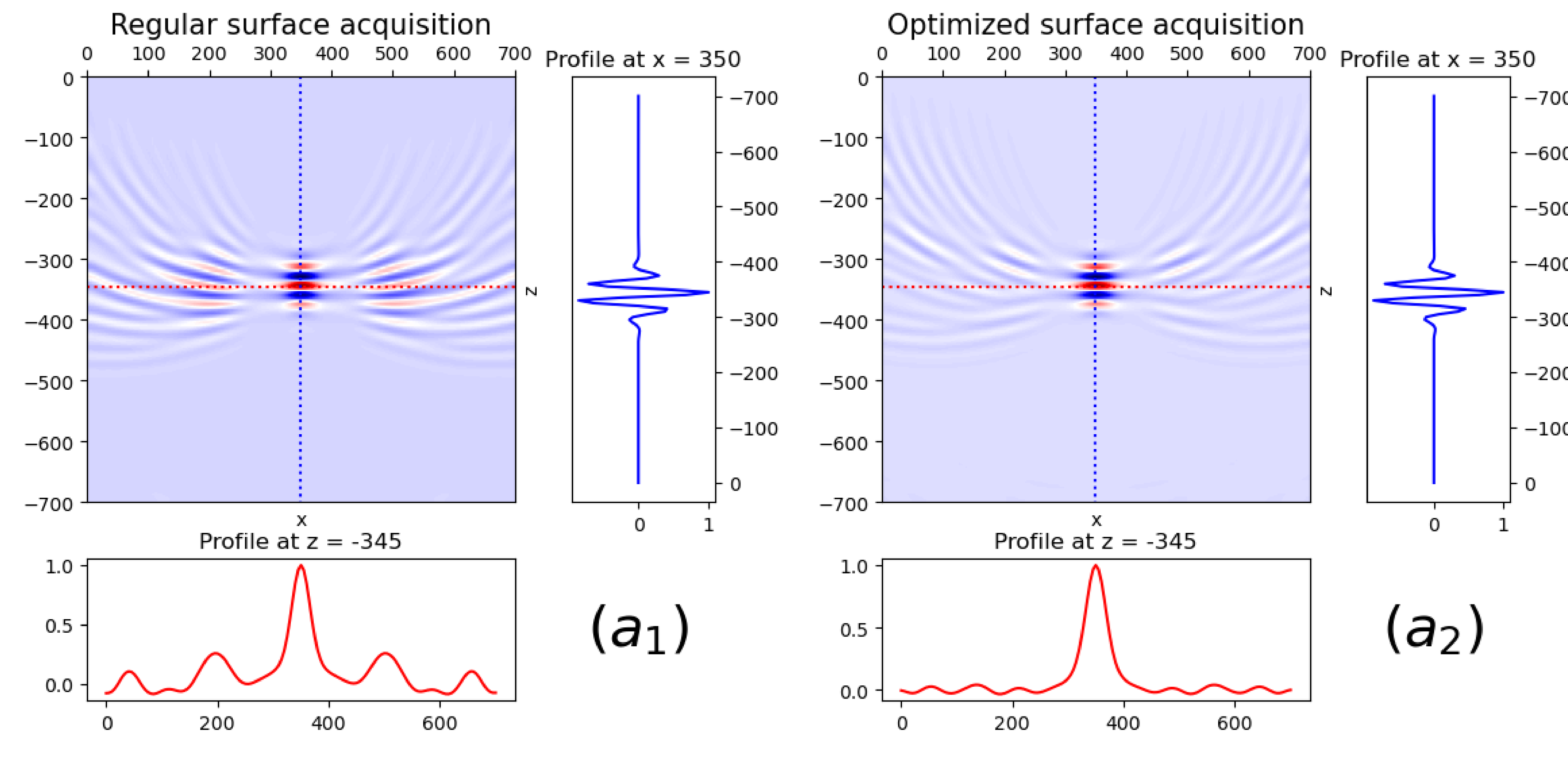

Figure 5: The first FWI gradients for each acquisition in Figure 4 with profiles on the x and z-axis. Notice that we have less noise around the target in the optimized case compared to the regular case.

II - Wavenumber cloud

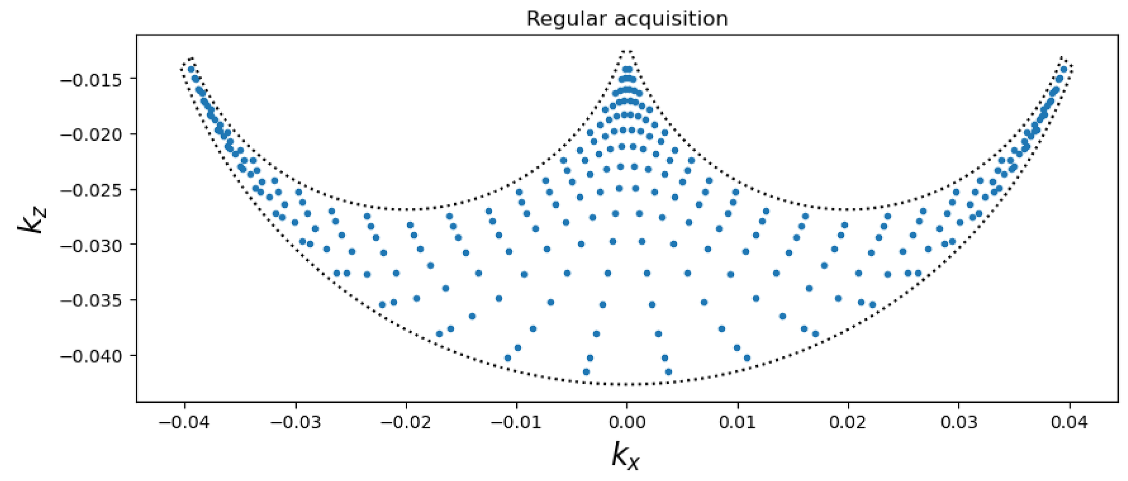

Figure 1: Exemple of a wavenumber content associated to a surface acquisition and the corresponding envelope.

• From figure 2 we can rewrite (2) using an angular formulation (Wu and Toksöz, 1987) as follows

$$k = 2k_0\cos\left(\frac{\phi_s - \phi_r}{2}\right)\begin{bmatrix}\sin(\frac{\phi_s+\phi_r}{2})\\ -\cos(\phi)\end{bmatrix} \tag{3}$$

• From this angular characterization of the wavenumbers we are able to define the envelope of the cloud of points as shown in figure 1. The size of the envelope is tied to $\phi_{\max}$ the maximum value of the angles.

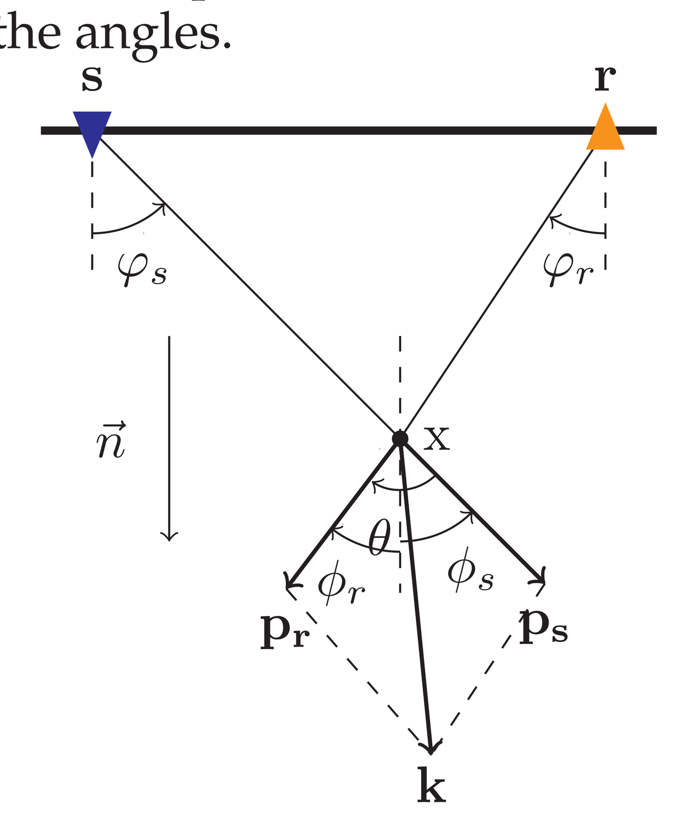

Figure 2: Illustration of a single wavenumber associated to a couple source/receiver on the surface.

• Given an acquisition range on the surface ($\phi_{\max}$ is thus fixed) and a certain number of sources and receivers, how do we **position** them so to regularly fill in the space inside the envelope? To that end we will use a tool from computational geometry : **Centroidal Voronoi Tessellation** (CVT).

• A CVT is a Voronoi tessellation where the seeds coincide with the centroids of the corresponding cells. CVT's tend to generate regular partitions of the domain and can be obtained by minimizing a certain energy function F (Liu et al., 2009).

V - Conclusion

We proposed a new approach for the optimal design in the context of the FWI, focussing on the wavenumber content of the **approximated gradient**. The problem was expressed as an instance of CVT and was solved using a **gradient descent method**. The method gives acquisition designs that fullfil the requirement of a more even distribution of the wavenumbers and the results of the experiments show FWI gradients that are more focused on the target.

VI - References

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