Receiver extension strategy for a robust full waveform inversion

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I - Context

• Full waveform inversion suffers from the cycle-skipping issue due to the non-linearity of the problem with respect to the model parameters (Virieux and Operto, 2009).
• In this work, we devise a new extension strategy of the FWI misfit function, in which we introduce additional degrees of freedom to the receiver position, generating receivers positions corrections which are time-dependent.
• Our extension strategy aims at eliminating the kinematic mismatch at earlier FWI iterations, when the model estimate is poor, and conventional FWI is prone to cycle-skipping.

• We show a simple numerical experiment to illustrate the extension strategy. A homogeneous medium is considered, the true velocity is set to 2000 $m.s^{-1}$, the misfit is computed for different velocities and it is indeed non-convex, it is shown in a dashed line in Figure 1 (top pannel), and in a black line on the bottom panel. An additional degree of freedom is introduced at the receiver's position, this allows for the fit of the data when the velocity is far from the true value. By taking the minimum corresponding to the optimal receiver position at each velocity (red lines in Figure 1). This misfit function exhibits improved convexity.

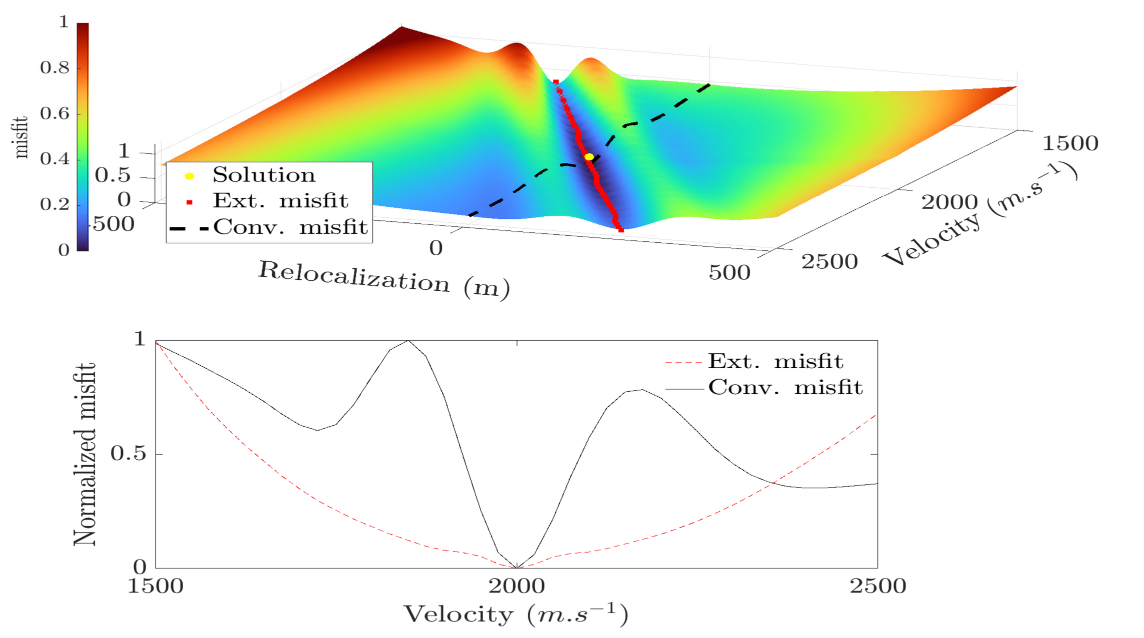

Figure 1: Misfit variation with velocity and receiver relocalization.

II - Method

• The FWI minimization is defined as PDE constrained optimization problem, where the fit between the observed (d_{obs}) and calculated ($d_{cal} = R(x_r)u_s[m]$) data is improved iteratively.

$$\min_m f(m) = \frac{1}{2}\sum_{s=1}^{N_s}\sum_{r=1}^{N_r} || R(x_r)u_s[m] - d_{obs,s,r} ||_D^2, \quad (1)$$

where s and r are the source and receiver indices, respectively. $R(x_r)$ is a linear operator that extracts wavefield values at the receivers positions x_r.

• The receiver extension problem was introduced by Métivier and Brossier (2022). We write the receiver extension problem introducing a time-dependent relocalization, that is, the receiver position shift is time-dependent:

$$\min_{m,\Delta r_s} f(m,\Delta r_s) = \frac{1}{2}\sum_{s=1}^{N_s}\sum_{r=1}^{N_r} || R(x_r+\Delta r_{s,r}(t)u_s[m]) - d_{obs,s,r} ||_D^2 + \quad (2)$$
$$\frac{\alpha}{2}\sum_{s=1}^{N_s}\sum_{r=1}^{N_r}\frac{||d_{obs,s,r}||_2^2}{L^2}||\Delta r_{s,r}(t)||^2 + \frac{\beta}{2}\sum_{s=1}^{N_s}\sum_{r=1}^{N_r}\frac{||d_{obs,s}||_2^2}{V_{max}^2}||\frac{d\Delta r_{s,r}(t)}{dt}||^2,$$

the misfit is computed using the data extracted at the new receiver position. Penalty terms are added as well, the first term on the right hand side penalizes the receiver position, the second penalizes the receiver speed. α and β are tuning parameters, Δr is the receiver position shift, L and V_{max} are the maximum allowed shift and speed, respectively.
• **How is it solved?** The problem is solved using a nested loops strategy, where the outer loop solves for the model parameters m using a quasi-Newton approach, and the inner loop solves for the optimal receiver position using a global (or semi-global) optimization scheme.
• The gradient is computed using the adjoint state strategy (Plessix, 2006), it is obtained by computing the inner-product of the scaled incident and adjoint wavefields:

$$\frac{\partial f}{\partial m} = \left\langle \frac{\partial A}{\partial m}u_s, \lambda_s \right\rangle, \quad (3)$$

where, u is the incident field, and λ is the adjoint field, the latter is computed by solving the adjoint system:

$$\begin{cases} A(m)^T\lambda = R^T(x_r+\Delta r_s)\mu_s \\ \mu_s = d_{obs,s} - R(x_r+\Delta r_s)u_s \end{cases}, \quad (4)$$

which is the same wave equation (A(m) operator), however the source term is the data residulas injected at the new receiver position.

III - Synthetic data application

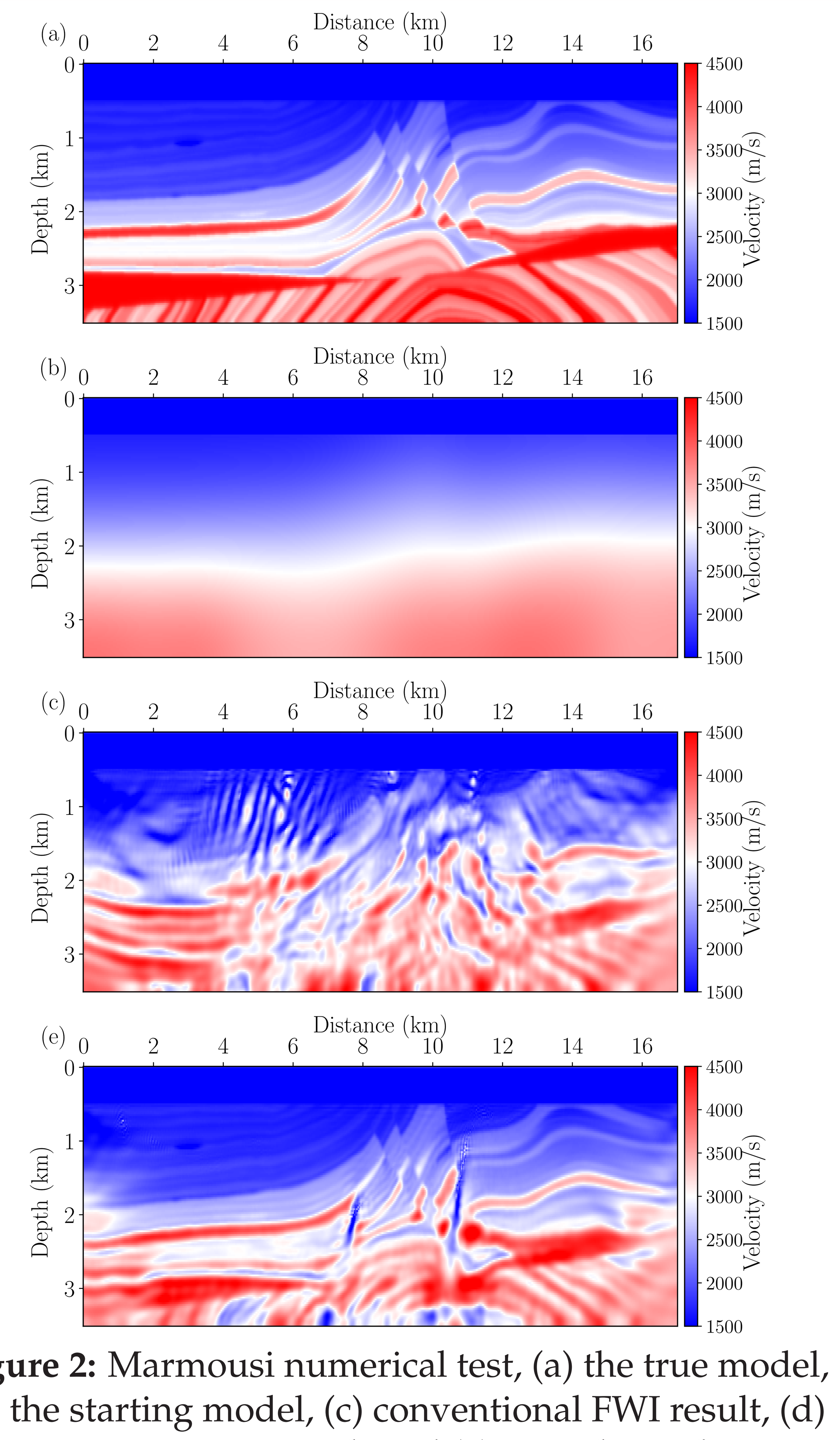

Figure 2: Marmousi numerical test, (a) the true model, (b) the starting model, (c) conventional FWI result, (d) static extension result and (e) time-dependent extension result.

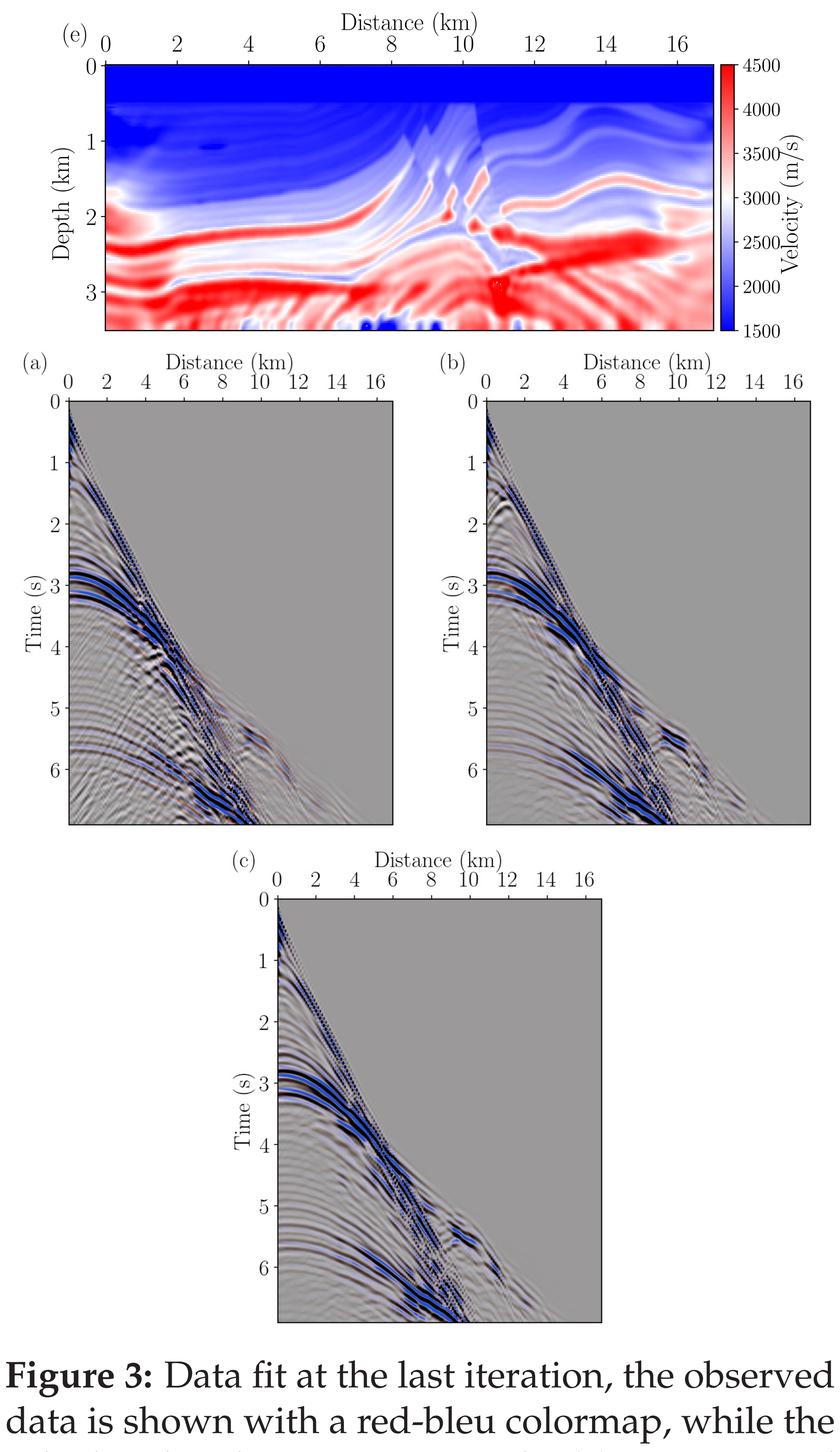

Figure 3: Data fit at the last iteration, the observed data is shown with a red-bleu colormap, while the calculated is shown in gray-scale. (a) conventional FWI , (b) static extension and (c) time-dependent extension.

We carry out a numerical experiment using the Marmousi2 model (Fig. 2a), we run conventional FWI using a heavily smooth model (Fig. 2b). The effect of cycle-skipping is clear on the final model shown in Figure 2c, the data fit a the last iteration shows the cycle-skipping impact (Fig. 3a). We start a static receiver extension waveform inversion obtaining a better model (Fig. 2d), however some artifcats are present in the center of the model at the intermediate and deeper parts of the model. The data fit is also significantly improved (Fig. 3b). Lastly, we run the time-depedent extension, yielding the best reconstruction (Fig. 2e), the data fit at the last iteratrion shows an equally improved fit (Fig.3c).

IV - Conclusion

We devise a receiver exntension strategy that uses a time-depedent relocalization, the first numerical tests show promising results which encourages us to formulate the problem for three-demensional media.

V - References

Métivier, L. and Brossier, R. [2022] Receiver-extension strategy for time-domain full-waveform inversion using a relocalization approach. *Geophysics*, **87**(1), R13–R33.
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